Matrix Problem

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Geometric Question

Let k be an integer such that the triangle with vertices (k,-3k),(5,k),(-k,2) has area 28. Find the orthocentre of this triangle.

Matrix Transformation Of Geometric Question

Let k be an integer such that the triangle with vertices $\begin{bmatrix} k \\ -3k \end{bmatrix}$, $\begin{bmatrix} 5 \\ k \end{bmatrix}$, $\begin{bmatrix} -k \\ 2 \end{bmatrix}$ has area 28. Find the orthocentre of this triangle.

Solution In Form Of Matrix

Area of triangle is 28

NOTE:

Area of triangle of A $\begin{bmatrix} x1 \\ y1 \end{bmatrix}$,B $\begin{bmatrix} x2 \\ y2 \end{bmatrix}$,C $\begin{bmatrix} x3 \\ y3 \end{bmatrix}$ is $1/2 \times \left\| \begin{vmatrix} x1 & y1 & 1 \\ x2 & y2 & 1 \\ x3 & y3 & 1 \end{vmatrix} \right\|$

So,

$$\left\| \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} \right\| = 56$$

.

Solution In Form Of Matrix

$$5k^2 + 13k + 10 = 56 \Rightarrow 5k^2 + 13k - 46 = 0$$

$$(OR)$$

$$5k^2 + 13k + 10 = -56 \Rightarrow 5k^2 + 13k + 66 = 0$$

NOTE:
The roots of quadratic equation $ax^2 + bx + c$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Solution In Form Of Matrix

On solving the above equations:

$$k = -4.6, 2, -1.3 + 3.39i, -1.3 - 3.39i$$

Since, k takes only integer values

$$\Rightarrow \mathbf{k = 2}$$

. The vertices of the triangle are:

$$A \begin{bmatrix} 2 \\ -6 \end{bmatrix}, B \begin{bmatrix} 5 \\ 2 \end{bmatrix}, C \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Solution In Form Of Matrix

To calculate the foot of perpendicular from A to BC
Let, $\mathrm{K}_1 = Normal - Vector - To - BC$
$\mathrm{K}_2 = Directional - Vector - Of - BC$
Then:

$$K_1^T(X - B) = 0$$

$$K_2^T(X - A) = 0$$

$$\Rightarrow \begin{bmatrix} K_1^T \\ K_2^T \end{bmatrix} X = \begin{bmatrix} K_1^T B \\ K_2^T A \end{bmatrix}$$

Solution In Form Of Matrix

Then,

$$X = \begin{bmatrix} K_1^T \\ K_2^T \end{bmatrix}^{-1} \begin{bmatrix} K_1^T B \\ K_2^T A \end{bmatrix}$$

On solving we get:

$$P = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Similarly the other coordinates are:

$$Q = \begin{bmatrix} -0.6 \\ -0.8 \end{bmatrix}$$

$$R = \begin{bmatrix} 4.13 \\ -0.30 \end{bmatrix}$$

Solution In Form Of Matrix

To solve the intersection point of the Altitudes AP CR:
Let, $N_1 = Normal - Vector - To - AP$
$N_2 = Normal - Vector - To - CR$
Then:

$$N_1^T(X - A) = 0$$

$$N_2^T(X - C) = 0$$

$$\Rightarrow \begin{bmatrix} N_1^T \\ N_2^T \end{bmatrix} X = \begin{bmatrix} N_1^T A \\ N_2^T C \end{bmatrix}$$

Solution In Form Of Matrix

Then,

$$X = \begin{bmatrix} N_1^T \\ N_2^T \end{bmatrix}^{-1} \begin{bmatrix} N_1^T A \\ N_2^T C \end{bmatrix}$$

On solving we get:

$$H = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

Figures