Matrix Problem

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Contents

Geometric Question

If A is (2,5), B is (4,-11) and C lies on $9x + 7y + 4 = 0$, then the locus of the centroid of triangle ABC is

Matrix Transformation Of Geometric Question

If $\mathbf{A}\begin{bmatrix}2\\5\end{bmatrix}$ *and* $\mathbf{B}\begin{bmatrix}4\\-11\end{bmatrix}$ are two vertices of a triangle, and the third vertex lies in the line

$$\begin{bmatrix}9 & 7\end{bmatrix}\mathbf{X} = -4$$

Find the locus of the centroid of ABC.

Solution In Form Of Matrix

Consider a parameter t,

$$\mathbf{C} = \begin{bmatrix} t \\ (-9t-4)/7 \end{bmatrix}$$

Solution In Form Of Matrix

Finding the Midpoints of BC and AB
Let **D** be the midpoint of BC, then

$$\mathbf{D} = (\mathbf{B}+\mathbf{C})/2 = \begin{bmatrix} (t+4)/2 \\ (-9t-4-77)/14 \end{bmatrix} = \begin{bmatrix} (t+4)/2 \\ (-9t-81)/14 \end{bmatrix}$$

Let **F** be the midpoint of AB, then

$$\mathbf{F} = (\mathbf{A}+\mathbf{B})/2 = \begin{bmatrix} (2+4)/2 \\ (5-11)/14 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

Solution In Form Of Matrix

Now, Consider $\mathbf{N}_1$ to be normal vector to AD, and $\mathbf{N}_2$ be normal vector to CF

Let $\mathbf{G}$ be the centroid.

then,

$$\mathbf{N}_1^T(\mathbf{G}-\mathbf{A}) = 0$$

$$\Rightarrow \mathbf{N}_1^T\mathbf{G} = \mathbf{N}_1^T\mathbf{A}$$

and

$$\mathbf{N}_2^T(\mathbf{G}-\mathbf{C}) = 0$$

$$\Rightarrow \mathbf{N}_2^T\mathbf{G} = \mathbf{N}_2^T\mathbf{C}$$

Solution In Form Of Matrix

From the equations in the previous slide

$$\begin{bmatrix} \mathbf{N}_1^T \\ \mathbf{N}_2^T \end{bmatrix} \mathbf{G} = \begin{bmatrix} \mathbf{N}_1^T \mathbf{A} \\ \mathbf{N}_2^T \mathbf{C} \end{bmatrix}$$

$$\Rightarrow \mathbf{G} = \begin{bmatrix} \mathbf{N}_1^T \\ \mathbf{N}_2^T \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{N}_1^T \mathbf{A} \\ \mathbf{N}_2^T \mathbf{C} \end{bmatrix}$$

On substituting we get

$$\mathbf{G} = \begin{bmatrix} (t+6)/3 \\ (-9t-46)/21 \end{bmatrix}$$

Solution In Form Of Matrix

Simplifying,

$$\mathbf{G} = 1/21 \begin{bmatrix} 7(t+6) \\ (-9t-46) \end{bmatrix}$$

$$\begin{bmatrix} 9/7 & 1 \end{bmatrix} \mathbf{G} = 1/21 \begin{bmatrix} 9/7 & 1 \end{bmatrix} \begin{bmatrix} 7(t+6) \\ (-9t-46) \end{bmatrix}$$

$$\begin{bmatrix} 9/7 & 1 \end{bmatrix} \mathbf{G} = (1/21)(8)$$

$$\begin{bmatrix} 9 & 7 \end{bmatrix} \mathbf{G} = 8/3$$

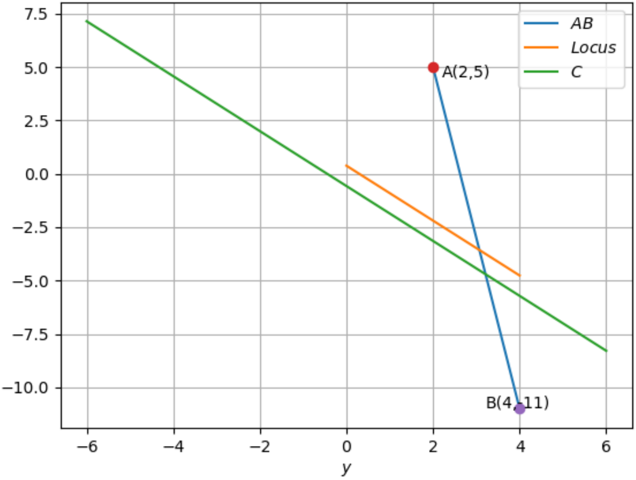
Therefore locus is a straight line

$$\begin{bmatrix} 27 & 21 \end{bmatrix} \mathbf{X} = 8$$

Locus in cartesian form is

$$27x + 21y = 8$$

Figures



Figures