

Control Systems

G V V Sharma*

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*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.1.1. The asymptotic Bode phase plot of

$$G(s) = \frac{k}{(s + 0.1)(s + 10)(s + p_1)} \quad (2.1.1.1)$$

with k and p_1 both positive, is shown below. Find the value of p_1 .

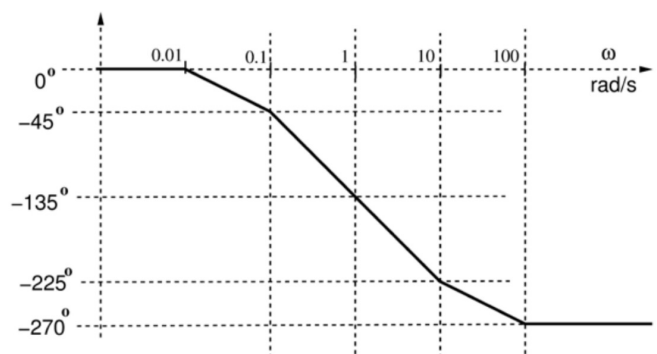


Fig. 2.1.1

Solution: Bode phase plot for a transfer function having a single pole at p

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < \frac{p}{10} \\ -45 \times \left(\log \left(\frac{10\omega}{p} \right) \right) & \frac{p}{10} < \omega < 10p \\ -90 & 10p < \omega \end{cases} \quad (2.1.1.2)$$

Phase plot of the transfer function (2.1.1.1),

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 0.1 \\ -135 - 90 \log(\omega) & 0.1 < \omega < 10 \\ -180 - 45 \log(\omega) & 10 < \omega < 100 \\ -90 & 100 < \omega \end{cases} \quad (2.1.1.3)$$

phase plot by considering only 0.1 and 10 poles is

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 100 \\ -180 & 100 < \omega \end{cases} \quad (2.1.1.4)$$

By comparing 2.1.1.3 and 2.1.1.4, $\phi(\omega)$ remains same till 0.1 so,

$$\frac{p_1}{10} = 0.1 \quad (2.1.1.5)$$

$$\Rightarrow p_1 = 1 \quad (2.1.1.6)$$

the bode phase plots corresponding to the poles 0.1 and 10.

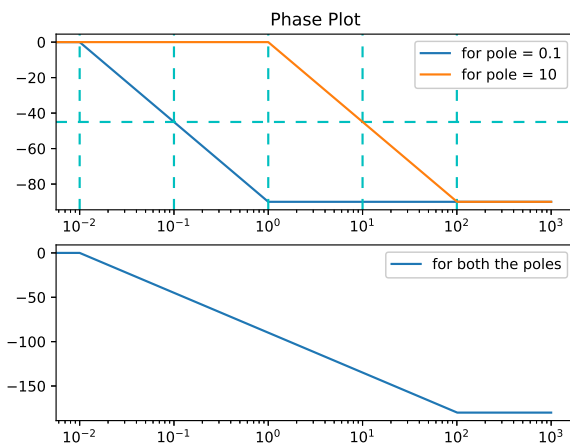


Fig. 2.1.1

2.1.2. Find the value of p_1 using phase of the transfer

function.

Solution:

$$\phi(\omega) = -\tan^{-1} \left(\frac{\omega}{0.1} \right) - \tan^{-1} \left(\frac{\omega}{10} \right) - \tan^{-1} \left(\frac{\omega}{p_1} \right) \quad (2.1.2.1)$$

From the plot,

$$-45^\circ = -\tan^{-1} \left(\frac{0.1}{0.1} \right) - \tan^{-1} \left(\frac{0.1}{10} \right) - \tan^{-1} \left(\frac{0.1}{p_1} \right) \quad (2.1.2.2)$$

p_1 is approximately 1, i.e., for p_1 in 0.95 to 1.05 the ϕ is approximately equals to -45° . The following code plots Fig. 2.1.1

codes/ee18btech11037.py

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

5.5 Example

6 NYQUIST PLOT

7 COMPENSATORS

7.1 Phase Lead

7.2 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

9 PHASE MARGIN

10 OSCILLATOR

10.1 Introduction

10.2 Example