Control Systems

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Contents

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.1.1. The asymptotic Bode phase plot of

$$G(s) = \frac{k}{(s+0.1)(s+10)(s+p_1)} \tag{2.1.1.1}$$

with k and p_1 both positive, is shown in Fig. 2.1.1. Express it as a piecewise linear function of $\log(\omega)$.

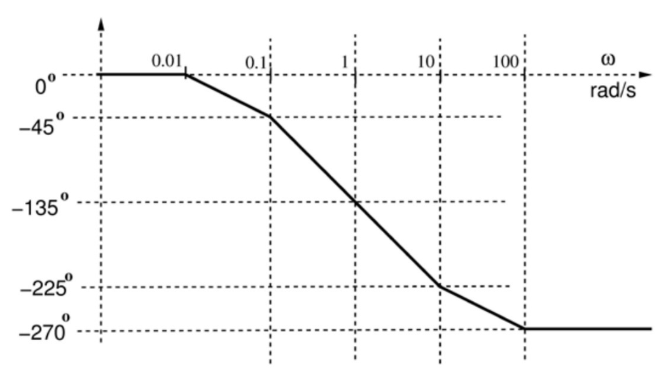

Fig. 2.1.1

Solution: The desired expression is

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45\log(\omega) & 0.01 < \omega < 0.1 \\ -135 - 90\log(\omega) & 0.1 < \omega < 10 \\ -180 - 45\log(\omega) & 10 < \omega < 100 \\ -90 & 100 < \omega \end{cases} \tag{2.1.1.2}$$

2.1.2. Find p_1.
Solution: Let

$$G_1(s) = \frac{1}{(s+p_1)} \tag{2.1.2.1}$$

The equivalent Bode phase is

$$\phi_1(\omega) = \angle G_1(\jmath\omega) = \begin{cases} 0 & 0 < \omega < \frac{p_1}{10} \\ -45 \times \left(\log\left(\frac{10\omega}{p_1}\right)\right) & \frac{p_1}{10} < \omega < 10p_1 \\ -90 & 10p_1 < \omega \end{cases} \tag{2.1.2.2}$$

Similarly, let

$$G_2(s) = \frac{k}{(s+0.1)(s+10)}. \tag{2.1.2.3}$$

The equivalent Bode phase is

$$\phi_2(\omega) = \angle G_2(\jmath\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45\log(\omega) & 0.01 < \omega < 100 \\ -180 & 100 < \omega \end{cases}. \tag{2.1.2.4}$$

Hence, from (2.1.2.2) and (2.1.2.4),

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \tag{2.1.2.5}$$

From (2.1.1.2) and (2.1.2.4)

$$\phi(\omega) = \phi_2(\omega) \quad 0 < \omega < 0.1 \tag{2.1.2.6}$$

$$\implies \phi_1(\omega) = 0 \quad 0 < \omega < 0.1 \tag{2.1.2.7}$$

Comparing (2.1.2.7) with (2.1.2.2),

$$\frac{p_1}{10} = 0.1 \implies p_1 = 1 \tag{2.1.2.8}$$

Fig. 2.1.2 generated by

```
codes/ee18btech11037.py
```

shows the bode phase plots corresponding to the poles 0.1 and 10.

Comparing this and 2.1.1, the graph remains same till 0.1 and after 0.1 slope of the line differs. So, the phase plot for (2.1.2.1) remains 0 till 0.1.

$$\implies \frac{p_1}{10} = 0.1 \implies p_1 = 1 \tag{2.1.2.9}$$

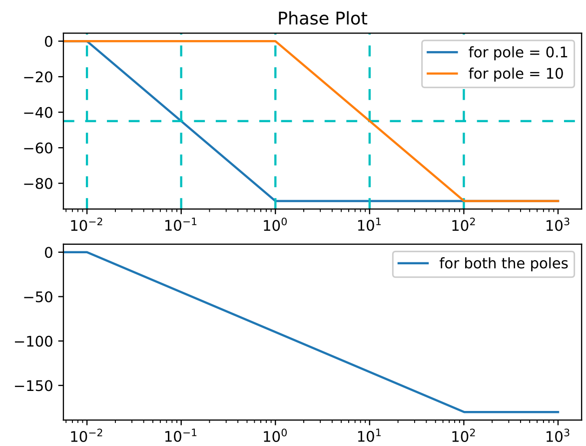

Fig. 2.1.2

2.1.3. Find the value of p_1 using phase of the transfer function.
Solution:

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right) \tag{2.1.3.1}$$

From the plot,

$$-45^\circ = -\tan^{-1}\left(\frac{0.1}{0.1}\right) - \tan^{-1}\left(\frac{0.1}{10}\right) - \tan^{-1}\left(\frac{0.1}{p_1}\right) \tag{2.1.3.2}$$

p_1 is approximately 1, i.e, for p_1 in 0.95 to 1.05 the ϕ is approximately equals to $-45°$.