

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.1.1. The asymptotic Bode phase plot of

$$G(s) = \frac{k}{(s + 0.1)(s + 10)(s + p_1)} \quad (2.1.1.1)$$

with k and p_1 both positive, is shown in Fig. 2.1.1. Express it as a piecewise linear function of $\log(\omega)$.

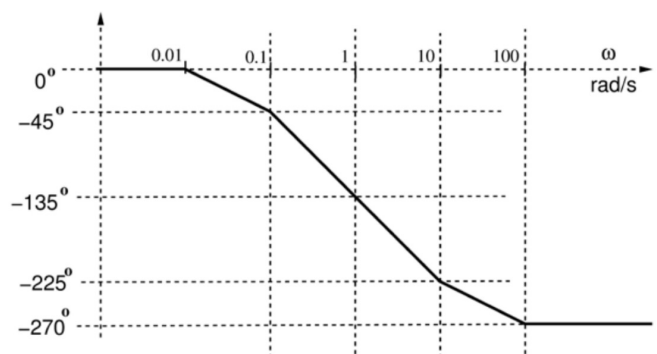


Fig. 2.1.1

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Solution: The desired expression is

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 0.1 \\ -135 - 90 \log(\omega) & 0.1 < \omega < 10 \\ -180 - 45 \log(\omega) & 10 < \omega < 100 \\ -270 & 100 < \omega \end{cases} \quad (2.1.1.2)$$

2.1.2. Find p_1 .

Solution: Let

$$G_1(s) = \frac{1}{(s + p_1)} \quad (2.1.2.1)$$

The equivalent Bode phase is

$$\begin{aligned} \phi_1(\omega) &= \angle G_1(j\omega) \\ &= \begin{cases} 0 & 0 < \omega < \frac{p_1}{10} \\ -45 \times \left(\log \left(\frac{10\omega}{p_1} \right) \right) & \frac{p_1}{10} < \omega < 10p_1 \\ -90 & 10p_1 < \omega \end{cases} \end{aligned} \quad (2.1.2.2)$$

Similarly, let

$$G_2(s) = \frac{k}{(s + 0.1)(s + 10)}. \quad (2.1.2.3)$$

The equivalent Bode phase is

$$\begin{aligned} \phi_2(\omega) &= \angle G_2(j\omega) \\ &= \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 100 \\ -180 & 100 < \omega \end{cases} \end{aligned} \quad (2.1.2.4)$$

Hence, from (2.1.2.2) and (2.1.2.4),

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \quad (2.1.2.5)$$

From (2.1.1.2) and (2.1.2.4)

$$\phi(\omega) = \phi_2(\omega) \quad 0 < \omega < 0.1 \quad (2.1.2.6)$$

$$\Rightarrow \phi_1(\omega) = 0 \quad 0 < \omega < 0.1 \quad (2.1.2.7)$$

Comparing (2.1.2.7) with (2.1.2.2),

$$\frac{p_1}{10} = 0.1 \Rightarrow p_1 = 1 \quad (2.1.2.8)$$

Fig. 2.1.2 generated by

codes/ee18btech11037.py

shows the bode phase plots corresponding to the poles 0.1 and 10 and the given phase plot.

Both the graphs remains same till 0.1 and after 0.1 slope of the line differs. So, the phase plot for (2.1.2.1) remains 0 till 0.1.

$$\Rightarrow \frac{p_1}{10} = 0.1 \Rightarrow p_1 = 1 \quad (2.1.2.9)$$

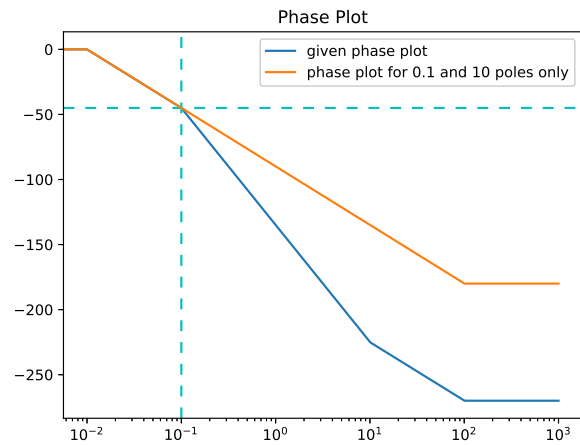


Fig. 2.1.2

2.1.3. Find the value of p_1 using phase of the transfer function.

Solution:

$$\phi(\omega) = -\tan^{-1} \left(\frac{\omega}{0.1} \right) - \tan^{-1} \left(\frac{\omega}{10} \right) - \tan^{-1} \left(\frac{\omega}{p_1} \right) \quad (2.1.3.1)$$

From the plot,

$$-45^\circ = -\tan^{-1} \left(\frac{0.1}{0.1} \right) - \tan^{-1} \left(\frac{0.1}{10} \right) - \tan^{-1} \left(\frac{0.1}{p_1} \right) \quad (2.1.3.2)$$

p_1 is approximately 1, i.e, for p_1 in 0.95 to 1.05 the ϕ is approximately equals to -45° .

2.2 *Example*

3 SECOND ORDER SYSTEM

3.1 *Damping*

3.2 *Example*

4 ROUTH HURWITZ CRITERION

4.1 *Routh Array*

4.2 *Marginal Stability*

4.3 *Stability*

4.4 *Example*

5 STATE-SPACE MODEL

5.1 *Controllability and Observability*

5.2 *Second Order System*

5.3 *Example*

5.4 *Example*

5.5 *Example*

6 NYQUIST PLOT

7 COMPENSATORS

7.1 *Phase Lead*

7.2 *Example*

8 GAIN MARGIN

8.1 *Introduction*

8.2 *Example*

9 PHASE MARGIN

10 OSCILLATOR

10.1 *Introduction*

10.2 *Example*