

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.1.1. The asymptotic Bode phase plot of

$$G(s) = \frac{k}{(s + 0.1)(s + 10)(s + p_1)} \quad (2.1.1.1)$$

with k and p_1 both positive, is shown in Fig. 2.1.1. Express it as a piecewise linear function of $\log(\omega)$.

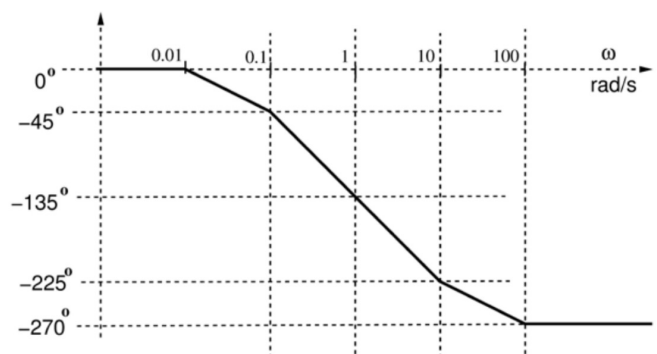


Fig. 2.1.1

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Solution: The desired expression is

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 0.1 \\ -135 - 90 \log(\omega) & 0.1 < \omega < 10 \\ -180 - 45 \log(\omega) & 10 < \omega < 100 \\ -90 & 100 < \omega \end{cases} \quad (2.1.1.2)$$

2.1.2. Find p_1 .

Solution: Bode phase plot for a transfer function having a single pole at p_1

$$\phi_1(\omega) = \begin{cases} 0 & 0 < \omega < \frac{p_1}{10} \\ -45 \times \left(\log \left(\frac{10\omega}{p_1} \right) \right) & \frac{p_1}{10} < \omega < 10p_1 \\ -90 & 10p_1 < \omega \end{cases} \quad (2.1.2.1)$$

phase plot by considering only 0.1 and 10 poles is

$$\phi_2(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45 \log(\omega) & 0.01 < \omega < 100 \\ -180 & 100 < \omega \end{cases} \quad (2.1.2.2)$$

phase plot of the given transfer function is sum of the phase plots (2.1.2.1) and (2.1.2.2)

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \quad (2.1.2.3)$$

and from (2.1.1.2) and (2.1.2.2)

$$\phi(\omega) = \phi_2(\omega) \text{ for } 0 < \omega < 0.1 \quad (2.1.2.4)$$

$$\Rightarrow \phi_1(\omega) = 0 \text{ for } 0 < \omega < 0.1 \quad (2.1.2.5)$$

By comparing (2.1.2.5) to (2.1.2.1),

$$\frac{p_1}{10} = 0.1 \Rightarrow p_1 = 1 \quad (2.1.2.6)$$

the bode phase plots corresponding to the poles 0.1 and 10.

2.1.3. Find the value of p_1 using phase of the transfer function.

Solution:

$$\phi(\omega) = -\tan^{-1} \left(\frac{\omega}{0.1} \right) - \tan^{-1} \left(\frac{\omega}{10} \right) - \tan^{-1} \left(\frac{\omega}{p_1} \right) \quad (2.1.3.1)$$

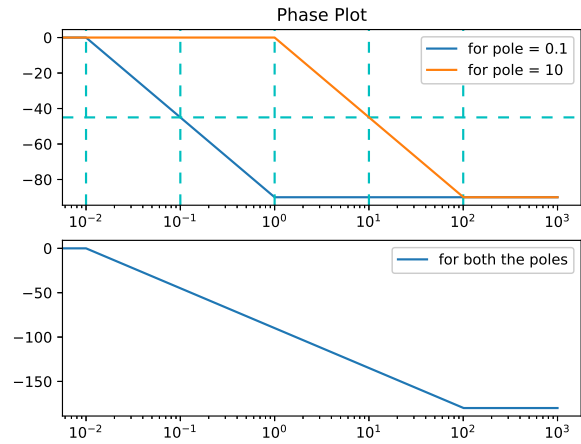


Fig. 2.1.2

From the plot 2.1.1,

$$-45^\circ = -\tan^{-1} \left(\frac{0.1}{0.1} \right) - \tan^{-1} \left(\frac{0.1}{10} \right) - \tan^{-1} \left(\frac{0.1}{p_1} \right) \quad (2.1.3.2)$$

p_1 is approximately 1, i.e., for p_1 in 0.95 to 1.05 the ϕ is approximately equals to -45° .

The following code plots Fig. 2.1.2

codes/ee18btech11037.py

2.2 *Example*

3 SECOND ORDER SYSTEM

3.1 *Damping*

3.2 *Example*

4 ROUTH HURWITZ CRITERION

4.1 *Routh Array*

4.2 *Marginal Stability*

4.3 *Stability*

4.4 *Example*

5 STATE-SPACE MODEL

5.1 *Controllability and Observability*

5.2 *Second Order System*

5.3 *Example*

5.4 *Example*

5.5 *Example*

6 NYQUIST PLOT

7 COMPENSATORS

7.1 *Phase Lead*

7.2 *Example*

8 GAIN MARGIN

8.1 *Introduction*

8.2 *Example*

9 PHASE MARGIN

10 OSCILLATOR

10.1 *Introduction*

10.2 *Example*