Control Systems

G V V Sharma*

Contents

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.1.1. The asymptotic Bode phase plot of

$$G(s) = \frac{k}{(s+0.1)(s+10)(s+p_1)} \tag{2.1.1.1}$$

with k and p_1 both positive, is shown in Fig. 2.1.1. Express it as a piecewise linear function of $\log(\omega)$.

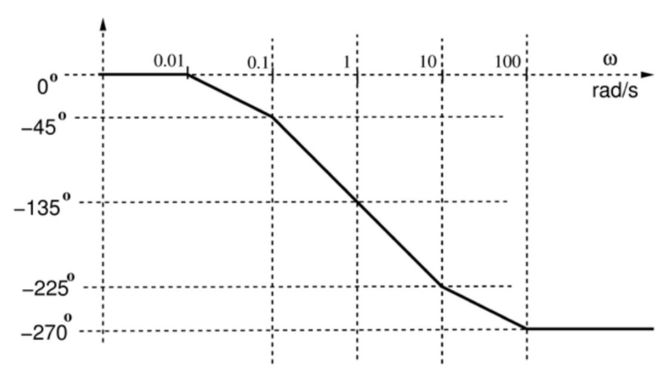

Fig. 2.1.1

Solution: The desired expression is

$$\phi(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45\log(\omega) & 0.01 < \omega < 0.1 \\ -135 - 90\log(\omega) & 0.1 < \omega < 10 \\ -180 - 45\log(\omega) & 10 < \omega < 100 \\ -90 & 100 < \omega \end{cases} \tag{2.1.1.2}$$

2.1.2. Find p_1.
Solution: Bode phase plot for a transfer function having a single pole at p_1

$$\phi_1(\omega) = \begin{cases} 0 & 0 < \omega < \frac{p_1}{10} \\ -45 \times \left(\log\left(\frac{10\omega}{p_1}\right)\right) & \frac{p_1}{10} < \omega < 10p_1 \\ -90 & 10p_1 < \omega \end{cases} \tag{2.1.2.1}$$

phase plot by considering only 0.1 and 10 poles is

$$\phi_2(\omega) = \begin{cases} 0 & 0 < \omega < 0.01 \\ -90 - 45\log(\omega) & 0.01 < \omega < 100 \\ -180 & 100 < \omega \end{cases} \tag{2.1.2.2}$$

phase plot of the given transfer function is sum of the phase plots (2.1.2.1) and (2.1.2.2)

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega) \tag{2.1.2.3}$$

and from (2.1.1.2) and (2.1.2.2)

$$\phi(\omega) = \phi_2(\omega) \text{ for } 0 < \omega < 0.1 \tag{2.1.2.4}$$

$$\implies \phi_1(\omega) = 0 \text{ for } 0 < \omega < 0.1 \tag{2.1.2.5}$$

By comparing (2.1.2.5) to (2.1.2.1),

$$\frac{p_1}{10} = 0.1 \implies p_1 = 1 \tag{2.1.2.6}$$

the bode phase plots corresponding to the poles 0.1 and 10.

2.1.3. Find the value of p_1 using phase of the transfer function.
Solution:

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right) \tag{2.1.3.1}$$

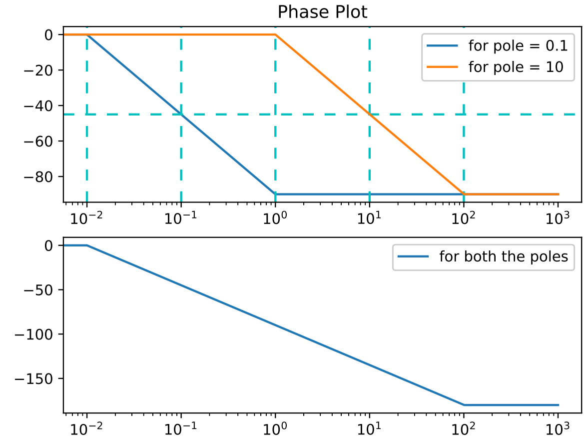

Fig. 2.1.2

From the plot 2.1.1,

$$-45^\circ = -\tan^{-1}\left(\frac{0.1}{0.1}\right) - \tan^{-1}\left(\frac{0.1}{10}\right) - \tan^{-1}\left(\frac{0.1}{p_1}\right) \tag{2.1.3.2}$$

p_1 is approximately 1, i.e, for p_1 in 0.95 to 1.05 the ϕ is approximately equals to -45°. The following code plots Fig. 2.1.2

codes/ee18btech11037.py