Let X_0 be the state of the floor prior to cleaning action

X_0 - Can have values either c (clean) or d (dirty)

X_1 be the state of the floor after to cleaning action

X_1 can have values either c (clean) or d (dirty)

Z be the measurement of the robot after cleaning action.

Z can have values either c (clean) or d (dirty)

u be the command for execution of cleaning action.

vc possible action.

$$P(x_1=c \mid x_0=d, u=v_c) = 0.7$$

$$P(x_1=d \mid x_0=d, u=v_c) = 0.3$$

$$P(Z=c \mid x_1=d) = 0.3$$

$$P(Z=d \mid x_1=d) = 0.7$$

$$P(Z=c \mid x_1=c) = 0.9$$

$$P(Z=d \mid x_1=c) = 0.1$$

After cleaning assumptions were made for this action.

$$P(x_1=c \mid x_0=c, u=v_c) = 1$$

$$P(x_1=d \mid x_0=c, u=v_c) = 0$$

Initial states of the floor is not given, so we need to consider with 50% probability.

$$p(x_0 = c) = 0.5 \qquad p(x_0 = d) = 0.5$$

Arbitrary prior on the initial state on the floor.

$$p(x_0 = c) = q \quad p(x_0 = d) = 1 - q$$

q - Can be chosen as any value such that $q \in [0, 1]$

Using Bayes Filter: Compute $p(x_1 = d \mid z = c, u = v_c)$

$$p(x_1 = d \mid z = c, u = v_c) = \eta\, p(z = c \mid x_1 = d) \sum_{x_0} p(x_1 = d \mid x_0, u = v_c)\, p(x_0) =$$

$$= \eta\, p(z = c \mid x_1 = d)\,\big(p(x_1 = d \mid x_0 = c, u = v_c)\, p(x_0 = c) + p(x_1 = d \mid x_0 = d, u = v_c)\, p(x_0 = d)$$

$$= \eta\, 0.3\,(0 \cdot q + 0.3 \cdot (1 - q)) = (0.09 - 0.09q)\,\eta$$

Missing the value of the normalizer η in order to obtain the probability value, we can compute it by taking into account the fact that our belief over x_1 needs to be normalized to 1 in other words. the following needs to hold,

$$p(x_1 = d \mid z = c, u = v_c) + p(x_1 = c \mid z = c, u = v_c) = 1 \quad \text{— (1)}$$

We can thus apply again Bayes Filter.

$$p(x_1 = c \mid z = c, u = v_c) = \eta\, p(z = c \mid x_1 = c) \sum_{x_0} p(x_1 = c \mid x_0, u = v_c)\, p(x_0) =$$

$$= \eta\, p(z = c \mid x_1 = c)\,\big(p(x_1 = c \mid x_0 = c, u = v_c)\, p(x_0 = c) + p(x_1 = c \mid x_0 = d, u = v_c)\, p(x_0 = d)\big) =$$

$$= \eta\, 0.9\,(1 \cdot q + 0.7 \cdot (1 - q)) = (0.63 + 0.27q)\,\eta$$

We can now plug the values we found in the ① & solve for η:

$$(0.09 - 0.09q)\eta + (0.63 + 0.27q)\eta = 1$$

$$\Rightarrow \eta = \frac{1}{0.72 + 0.18q}$$

Finally,

$$p(x_1 = d \mid z = c, u = v_c) = (0.09 - 0.09q)\eta = \frac{0.09 - 0.09q}{0.72 + 0.18q}$$

Sub q with particular values.

$q = 0.5$: $p(x_1 = d \mid z = c, u = v_c) \simeq 0.0556$

$q = 0$: $p(x_1 = d \mid z = c, u = v_c) = 0.125$

$$\eta = \frac{1}{p(z=c)} = \frac{1}{p(z=c \mid x_1 = c)\, p(x_1 = c) + p(z=c \mid x_1 = d)\, p(x_1 = d)}$$

where,

$$p(x_1 = c) = \sum_{x_0} p(x_1 = c \mid x_0, u = v_c)\, p(x_0)$$

$$p(x_1 = d) = \sum_{x_0} p(x_1 = d \mid x_0, u = v_c)\, p(x_0)$$

Lower bound probability:- $q = 1$

$p(x_0 = c) = 1$ $\quad$ $p(x_0 = d) = 0$

Minimum possible value $p(x_1 = d \mid z = c, u = v_c) = 0$