5.2

2. $= \int_a^b A(x)\,dx$

$= \int_0^{10} 10 e^{0.01x}\,dx$

$= 10 \cdot \frac{1}{.01} e^{.01x} \Big|_0^{10}$

$= 1000 * e^{.01(10)} - 1000 * e^{.01(0)}$

$= 1000 \cdot e^{.1} - 1000$

≈ 105.17

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5.2

6. $f(x) = mx + b$

$f(0) = 300 \quad , \quad f(160) = 6$

$$f(x) = \frac{15}{-8} x + 300$$

$$V = \int_0^{160} A(x)\, dx = \int_0^{160} \left(\frac{15}{-8} \cancel{x} + 300\right)^2 dx$$

$$u = \frac{15}{-8} x + 300 \quad , \quad du = \frac{15}{-8} dx$$

$$V = \int_{300}^{0} u^2 \frac{-8}{15} du = \frac{-8}{15} \int_{300}^{0} u^2 du$$

$$V = \frac{8}{15} \int_0^{300} u^2 du = \frac{8}{15} \frac{u^3}{3} \Big|_0^{300}$$

$$V = \frac{8}{15} \frac{(300)^3}{3} - 0$$

$$V = 48,00,000 \ ft^3$$

$\therefore$ It is $\frac{1}{8}$ of the volume of that in Eg 2.1

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5.2 (10)

$$V = \int_a^b \pi [f(x)]^2 dx$$

$$y = 120 - \frac{x^2}{120}$$

$$\frac{x^2}{120} + y = 120$$

$$x^2 = 120(120 - y)$$

$$V = \pi \int_0^{120} 120[120 - y] dy$$

$$V = 120\pi \left[120y - \frac{y^2}{2} \right]_0^{120}$$

$$V = 120\pi \left[120^2 - \frac{120^2}{2} \right]$$

$$V = 864000 \pi \ ft^3.$$

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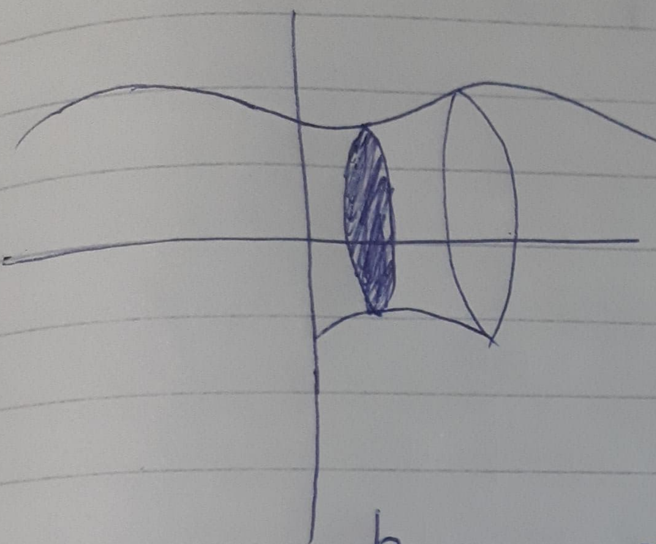
5.2 (12)



$$V = \int_a^b \pi \left[f(x)\right]^2 dx$$

$$f(x) = 4 - \sin\left(\frac{x}{2}\right) \qquad u = \frac{x}{2},\ du = \frac{1}{2}dx$$

$$V = \pi \int_0^{2\pi} \left[4 - \sin\left(\frac{x}{2}\right)\right]^2 dx$$

$$V = \pi \int_0^{2\pi} \left(16 - 8 \sin \frac{x}{2} + \sin^2 \frac{x}{2}\right) dx$$

$$V = \pi \left(16x + 16 \cos \frac{x}{2} + \frac{1}{2}x - \frac{1}{2} \sin x\right)\Big|_0^{2\pi}$$

$$V = \pi \left(16(2\pi) + 16 \cos \pi + \frac{1}{2}(2\pi) - 16\right)$$

$$V = \left(33\pi^2 - 32\pi\right) \text{ in}^3$$

PAGE

5.2 (18) ~~circle $x^2+y^2=1$ → about y-axis~~

revolved around x-axis

a) $V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi \left((4-x^2)^2 - (x^2)^2\right) dx$

$V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi (16 - 8x^2)\, dx$

$= \pi \left(16x - \frac{8x^3}{3}\right)_{-\sqrt{2}}^{\sqrt{2}} = \pi \left[\left(16(\sqrt{2}) - \frac{16\sqrt{2}}{3}\right) - \left(-16\sqrt{2} + \frac{16\sqrt{2}}{3}\right)\right]$

$= \pi \left[32\sqrt{2} - \frac{32\sqrt{2}}{3}\right]$

$= 32\sqrt{2}\pi - \frac{32\sqrt{2}\pi}{3}.$

b) revolved around $y=4$

$\int_{-\sqrt{2}}^{\sqrt{2}} \pi \left((x^2)^2 - (4-x^2)^2\right) dx$

$V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi (8x^2 - 16)\, dx = \pi \left[\frac{8x^3}{3} - 16x\right]_{-\sqrt{2}}^{\sqrt{2}}$

$= \pi \left[\frac{16\sqrt{2}}{3} - 16\sqrt{2} + \frac{16\sqrt{2}}{3} + 16\sqrt{2}\right]$

$= \frac{32\sqrt{2}\pi}{3}.$

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5.2 (20)

Volume of Solid revolving around y-axis

a) $= \int_0^1 (\sqrt{y})^2 - (y^2)^2 \, dy$

$V = \int_0^1 (y - y^4) \, dy$

$= \left(\frac{y^2}{2} - \frac{y^5}{5} \right)_0^1 = \frac{3\pi}{10}$

b) revolving around $x = 1$

$\int_0^1 \pi \left[(1-y^2)^2 - (1-\sqrt{y})^2 \right] dy$

$= \int_0^1 \pi (y^4 - 2y^2 - y + 2\sqrt{y}) \, dy$

$= \pi \left(\frac{y^5}{5} - \frac{2y^3}{3} - \frac{y^2}{2} + \frac{4y^{3/2}}{3} \right)_0^1$

$= \pi \left(\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3} \right) = \frac{11\pi}{30}$

S2 (26.a)

a) Revolve around $y=4$,

Radius $\quad r = y - f(x) = 4 - x^2$

Volume $\quad = \int_{-2}^{2} \pi r^2 \, dx$

$$= \pi \int_{-2}^{2} (4 - x^2)^2 \, dx$$

$$= \pi \int_{-2}^{2} 16 - 8x^2 + x^4 \, dx$$

$$= \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_{-2}^{2}$$

$$= \pi \left[32 \cdot 2 - \frac{16}{3} 8 + \frac{2}{5} 32 \right)$$

$$= \frac{512\pi}{15} \quad \text{units}^3$$

b) $y = x^2 \; ; \; y = 4$

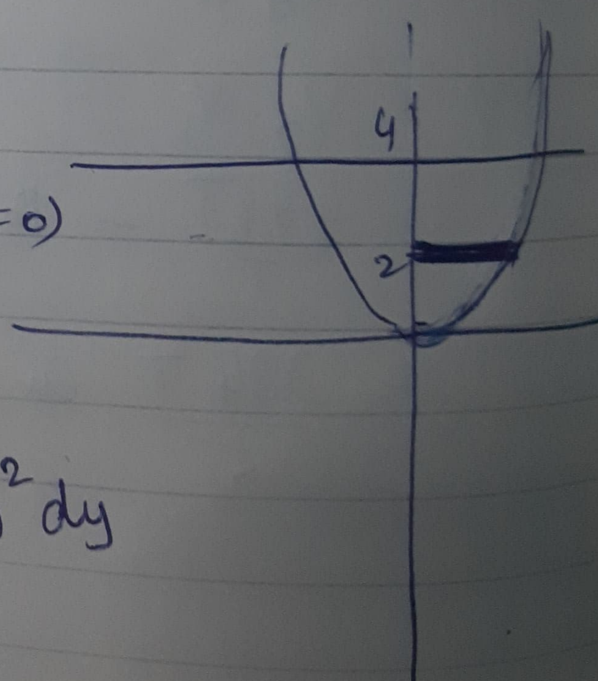
$x = \pm 2$

the radius of the disc from ($x=0$)

$r(y) = x = \sqrt{y}$

$dV = \pi r(y)^2 \, dy$

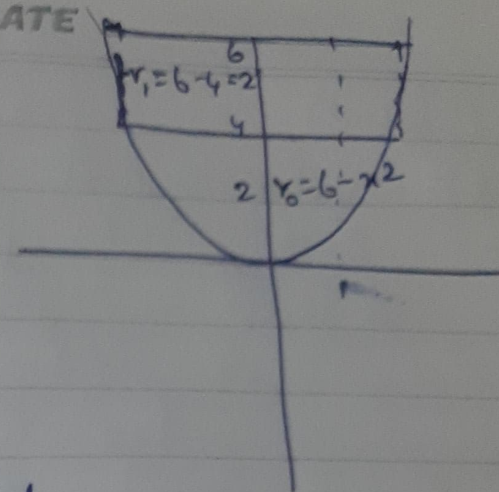
$$V = \int_{0}^{4} \pi r^2 \, dy = \pi \int_{0}^{4} (\sqrt{y})^2 \, dy$$



$$= \pi \left[\frac{y^2}{2} \right]_0^4 = 8\pi \text{ cubic units.}$$

5.2 (26.c)

Using disk method



$$V = V_0 - V_1$$

$$V_0 = \int_{-2}^{2} \pi r_0^2 \, dx = \pi \int_{-2}^{2} (6-x^2)^2 \, dx$$

$$= \pi \int_{-2}^{2} 36 - 12x^2 + x^4 \, dx$$

$$= \frac{464\pi}{5} \text{ cu. units}$$

$$V_1 = \int_{-2}^{2} \pi r_1^2 \, dx = \pi \int_{-2}^{2} (2)^2 \, dx = \pi \left[4y \right]_{-2}^{2}$$

$$= 16\pi$$

$$\begin{array}{r} 464 \\ -80 \\ \hline 384 \end{array}$$

$$V = V_0 - V_1$$

$$= \frac{464\pi}{5} - 16\pi$$

$$= \frac{384\pi}{5} \text{ Cu. Units.}$$

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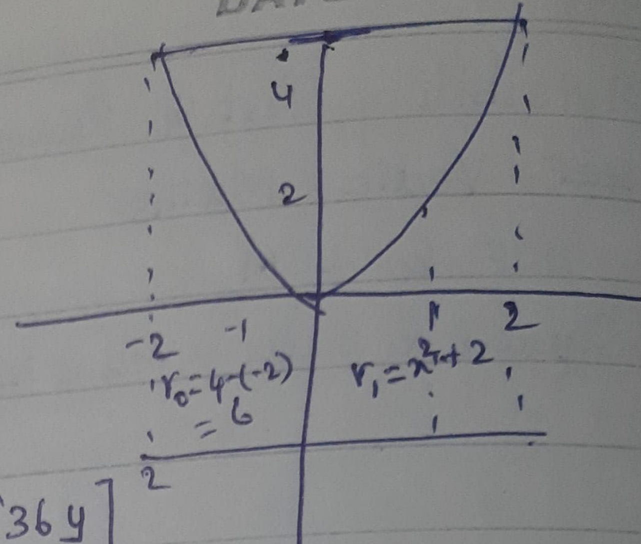
5.2.26(d)

$$V = V_0 - V_1$$

$$V_0 = \int_{-2}^{2} \pi r_0^2 \, dx =$$

$$= \pi \int_{-2}^{2} 36 \, dx = \pi \left[36y \right]_{-2}^{2}$$

$$= 144\pi$$



$$V_1 = \int_{-2}^{2} \pi r_1^2 \, dx = \pi \int_{-2}^{2} (x^2+2)^2 \, dx$$

$$= \pi \int_{-2}^{2} x^4 + 4x^2 + 4 \, dx$$

$$= \pi \left[\frac{x^5}{5} + 4\frac{x^3}{3} + 4x \right]_{-2}^{2} = \frac{752}{15}\pi$$

$$V = 144\pi - \frac{752}{15}\pi$$

$$V = \frac{1408\pi}{15} \text{ cu. units.}$$

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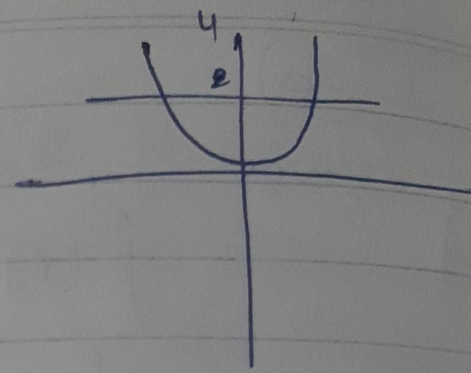
5.2(26.e) Solid revolving around $x=2$



$$V = \int_0^4 \pi\left((2+\sqrt{y})^2 - (2-\sqrt{y})^2\right) dy$$

$$V = \int_0^4 8\pi y^{1/2}\, dy$$

$$= \frac{16\pi}{3} y^{3/2}\Big|_0^4 = \frac{128\pi}{3}.$$

26. f) revolving around $x=-4$

$$V = \int_0^4 \pi(4+\sqrt{y})^2 - (4-\sqrt{y})^2\, dy$$

$$V = \int_0^4 (16\pi y^{1/2})\, dy$$

$$= \frac{32\pi}{3} y^{3/2}\Big|_0^4$$

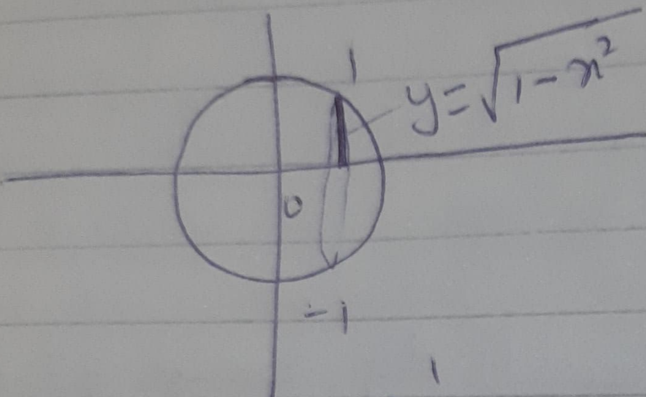
$$= \frac{256\pi}{3}.$$

5.2

32.

$$V = \int_0^b \pi [f(x)]^2 \, dx$$

Cross sectional area = πr^2



$$V = \int_{-1}^{1} \pi \left(\sqrt{1-x^2}\right)^2 \, dx$$

$$V = \pi \int_{-1}^{1} |1-x^2| \, dx$$

$$= \pi \left[x - \frac{x^3}{3} \right]_{-1}^{1}$$

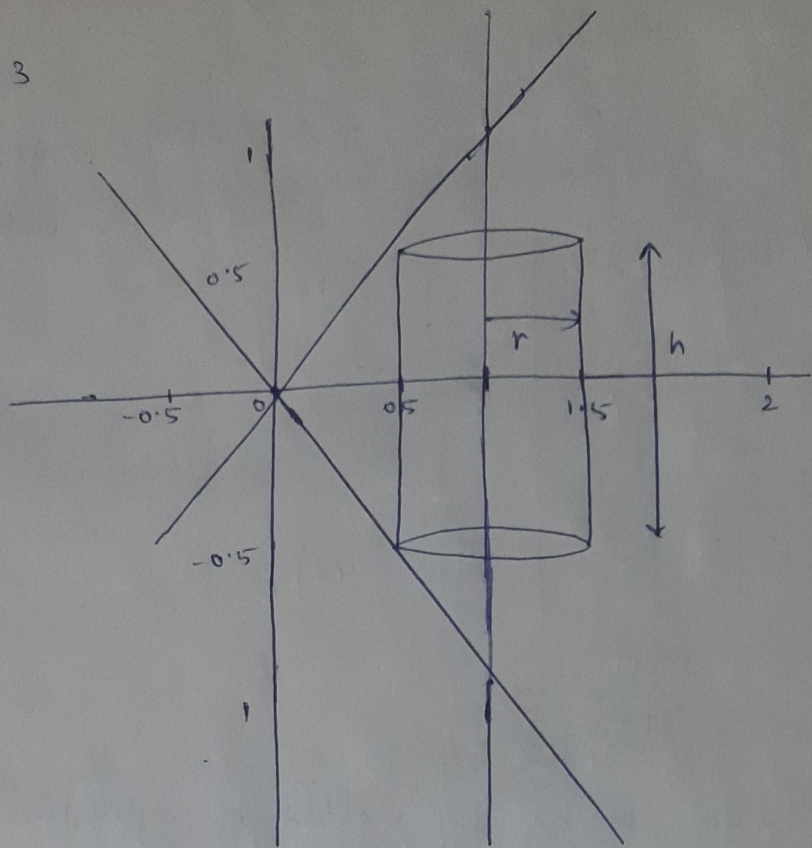
$$= \pi \left[1 - \frac{1}{3} - \left(-1 - \frac{-1}{3}\right) \right]$$

$$= \frac{4\pi}{3}$$

~~Ex 5~~

Section : 5.3

4)



The volume is given as the integral:

$\int_a^b 2\pi (\text{radius})(\text{height})\, dx$ when the region is revolved around the line $x = 1$. Putting values in the integral and computing:

$$V = \int_0^1 2\pi \, (1-x)(2x)\, dx$$

$$= \int_0^1 2\pi \, (2x - 2x^2)\, dx$$

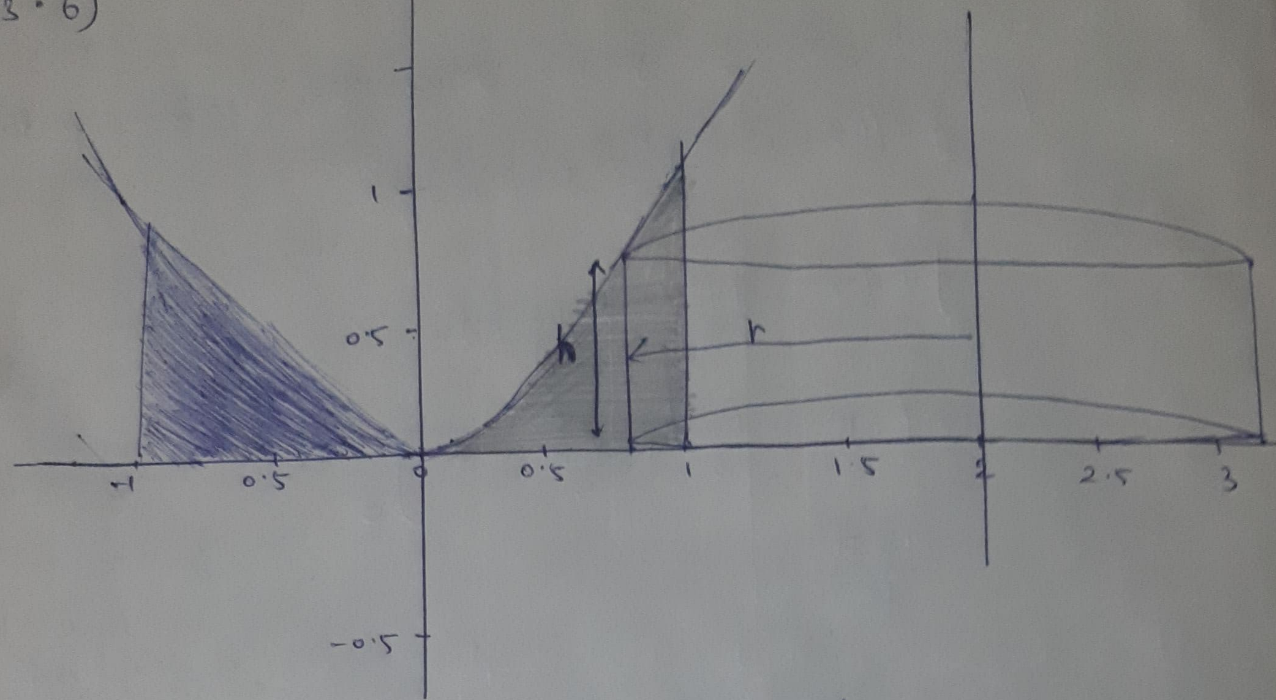
$$= 4\pi \int_0^1 (x - x^2)\, dx$$

$$= 4\pi \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = 4\pi \left(\frac{1^2}{2} - \frac{1^3}{3}\right)$$

$$= \frac{4\pi}{6} = \frac{2\pi}{3}$$

Thus, the volume of the solid so generated is $\frac{2\pi}{3}$

5.3.6)



The volume is given as the integral:

$\int_a^b 2\pi \text{(radius)(height)}\, dx$ when the region is revolved around the line $x=2$. Putting value in the integral and computing:

$$V = \int_{-1}^{1} 2\pi\,(2-x)(y)\,dx$$

$$= \int_{-1}^{1} 2\pi\,(2-x)(x^2)\,dx$$

$$= 2\pi \int_{-1}^{1} (2x^2 - x^3)\,dx$$

$$= 2\pi \left(\frac{2x^3}{3} - \frac{x^4}{4}\right)_{-1}^{1}$$

$$= 2\pi\left(\frac{2(1)^3}{3} - \frac{(1)^4}{4}\right) - 2\pi\left(\frac{2(-1)^3}{3} - \frac{(-1)^4}{4}\right)$$

$$= 8\pi/3$$

Thus, the volume of the solid so generated is $\frac{8\pi}{3}$

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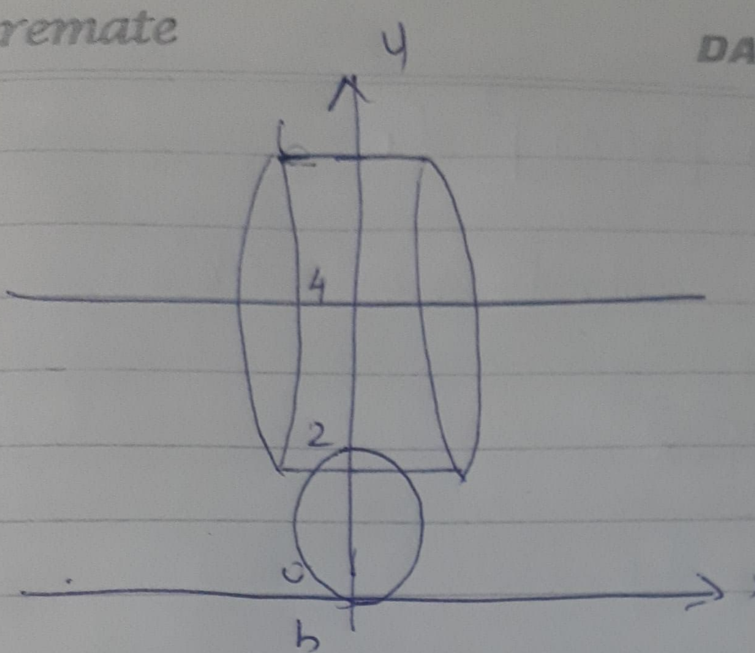
5.3 (8)



Volume $= \int_a^b 2\pi(\text{radius})(\text{height})\,dy$. around $y=4$

$$V = \int_0^2 2\pi(4-y)\left(2\sqrt{2y-y^2}\right)dy$$

$$= 4\pi \int_0^2 4\sqrt{2y-y^2} - y\sqrt{2y-y^2}\,dy$$

$$= 6\pi^2$$

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5-3

12. $x=y^2$; $x=4$ revolve about $y=2$

$$V=\int_{-2}^{2} 2\pi\,(2-y)\,(4-y^2)\,dy$$

$$V=2\pi\int_{-2}^{2}(8-2y^2-4y-y^3)\,dy$$

$2y^2$

$\frac{2y^3}{3}$

$\frac{4y^2}{2}$

$$V=2\pi\left(8y-\frac{2y^3}{3}-\frac{4y^2}{2}-\frac{y^4}{4}\right)_{-2}^{2}$$

$$V=2\pi\left(8y-\frac{2y^3}{3}-2y^2-\frac{y^4}{4}\right)_{-2}^{2}$$

$$V=2\pi\left[\left(16-\frac{16}{3}-8-\frac{32}{4}\right)-\left(-16+\frac{16}{3}-8-8\right)\right]$$

$$V=2\pi\left[16-\frac{16}{3}-8-8+16-\frac{16}{3}+8+8\right]$$

$$V=2\pi\left[32-\frac{32}{3}\right]$$

$$V=2\pi\left(\frac{64}{3}\right)$$

$$V=\frac{128\pi}{3}$$

22. a) The region bounded $y=2-x^2$ the y axis ~~ang~~ and $y=x$ for $x>0$ is revolved around the x-axis. Using, the washers method, we set up the following integral to find the volume:

$$V=\int_0^1 \pi\left((2-x^2)^2-(x^2)\right)dx.$$

$$V=\int_0^1 \pi\left(x^4-5x^2+4\right)dx$$

$$=\pi\left(\frac{x^5}{5}-\frac{5x^3}{3}+4x\right)\Big|_0^1$$

$$=\frac{38\pi}{15}$$

b) the y-axis, using the method of cylindrical shells,

$$V=\int_0^1 2\pi(x)(2-x^2-x)\,dx$$

$$=\int_0^1 2\pi(2x-x^3-x^2)\,dx$$

$$=2\pi\left(-\frac{x^4}{4}-\frac{x^3}{3}+x^2\right)\Big|_0^1$$

$$=\frac{5\pi}{12}$$

c) It revolved around $x=-1$, using the method of cylindrical shells: $V=\int_0^1 2\pi(x+1)(2-x^2-x)\,dx$

$$V=\int_0^1 2\pi(-x^3-2x^2+x+2)\,dx$$

$$=2\pi\left(-\frac{x^4}{4}-\frac{2x^3}{3}+\frac{x^2}{2}+2x\right)\Big|_0^1$$

$$=\frac{19\pi}{6}$$

~~d) is resolv~~

22.d) is revolved around $y=-1$. Using the washers method,

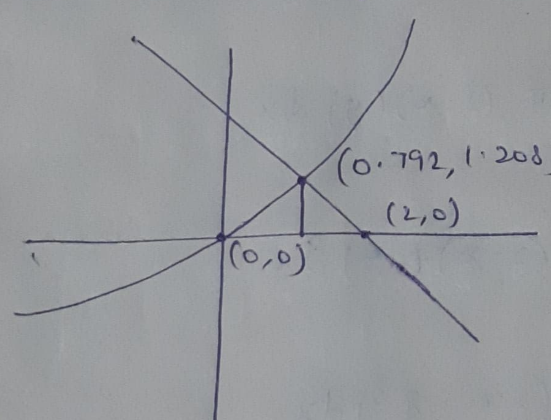
$$V = \int_0^1 \pi\left((2-x^2+1)^2 - (x+1)^2\right)dx$$

$$= \int_0^1 \pi\left(x^4 - 7x^2 - 2x + 8\right)dx$$

$$= \pi\left(8\pi - \frac{7x^3}{3} + \frac{x^5}{5} - x^2\right)\Big|_0^1$$

$$= \frac{73\pi}{15}$$

24.a) The region bounded by the given curves is shades as below. Compute the volume of the solid found by revolving this around x axis by using washers method.



The curve intersect at $x = 0.792$. Thus, we will divide this into 2 integrals:

$$\int_0^{0.792} \pi\,(e^x - 1)\,dx + \int_{0.792}^{1} (2-x)^2\,dx$$

$$V = \int_0^{0.792} \pi\left(e^{2x} + 1 - 2e^x\right)dx + \int_{0.792}^{1} (4 + x^2 - 4x)\,dx$$

$$= \pi\left(\frac{e^{2\pi}}{2} + x - 2e^x\right)\Big|_0^{0.792} + \pi\left(4x + \frac{x^3}{3} - 2x^2\right)\Big|_{0.792}^{1}$$

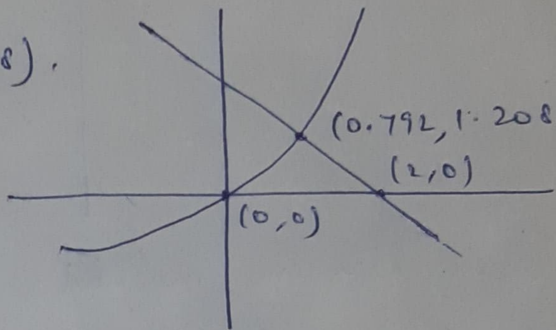
$$= \pi \left(\frac{e^{2(0.792)}}{2} + 0.792 - 2e^{0.792} \right) - \pi \left(\frac{e^{0}}{2} + 0 - 2e^{0} \right)$$

$$+ \pi \left(4(1) + \frac{1^3}{3} - 2(1)^2 \right) - \pi \left(4(0.792) + \frac{(0.792)^3}{3} - 2(0.792)^2 \right)$$

$$= 1.78\pi$$

24.
b) Revolving around y axis by using washers method:

The curve intersect at (0.792, 1.208).
The volume of the solid so obtained is given as the integral:



$$\int_0^{1.208} \pi \left((2-y)^2 - (\ln(y+1))^2 \right) dy$$

$$V = \int_0^{1.208} \pi \left(4 + y^2 - 4y - (\ln(y+1))^2 \right) dy$$

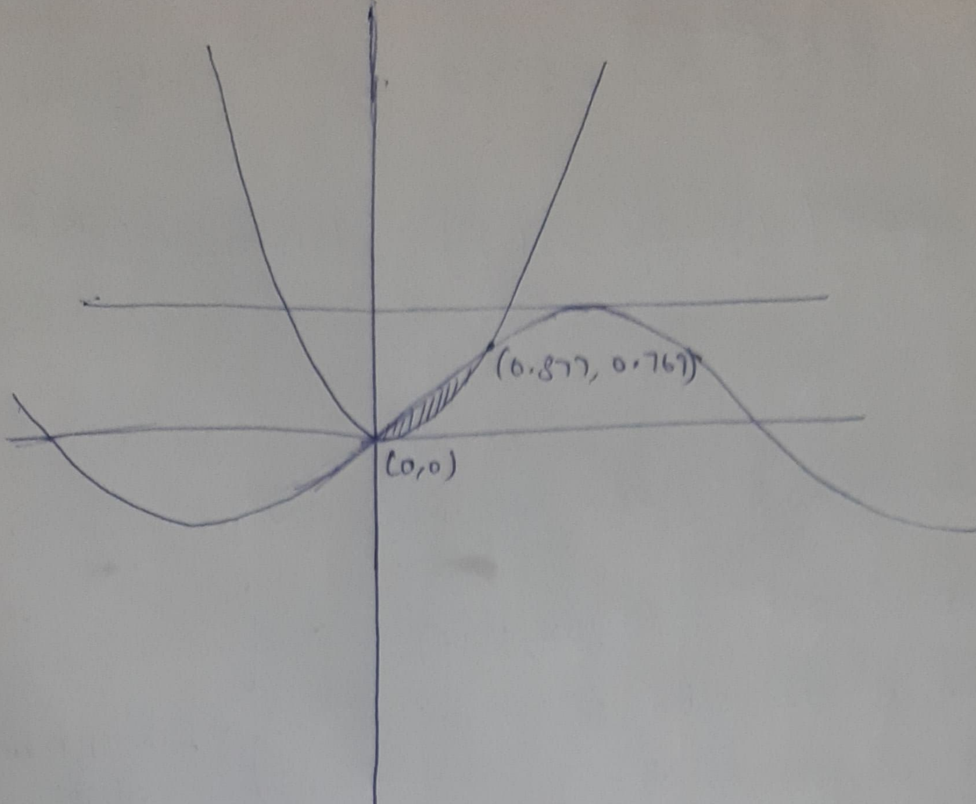
$$= \pi \left(4x + \frac{y^3}{3} - 2y^2 - (y+1)(\ln(y+1))^2 + 2(y+1)\ln(y+1) - 2y - 2 \right)_0^{1.208}$$

$$= \pi \left(4(1.208) + \frac{(1.208)^3}{3} - 2(1.208)^2 - (2.208+1)(\ln(2.208))^2 \right.$$

$$\left. + 2(1.208+1)\ln(2.208) - 2(1.208) - 2 + 2 \right)$$

$$= 2.19\pi$$

26.a)



Using washers method, the shaded region is revolved around $y=1$. The volume is given by

$$V = \int_0^{0.877} \pi \left((1-x^2)^2 - 1(1-\sin x)^2 \right) dx$$

$$= \int_0^{0.877} \pi \left((1-x^2)^2 - (1-\sin x)^2 \right) dx$$

$$= \int_0^{0.877} \pi \left(x^4 - 2x^2 - \sin^2 x + 2\sin x \right) dx$$

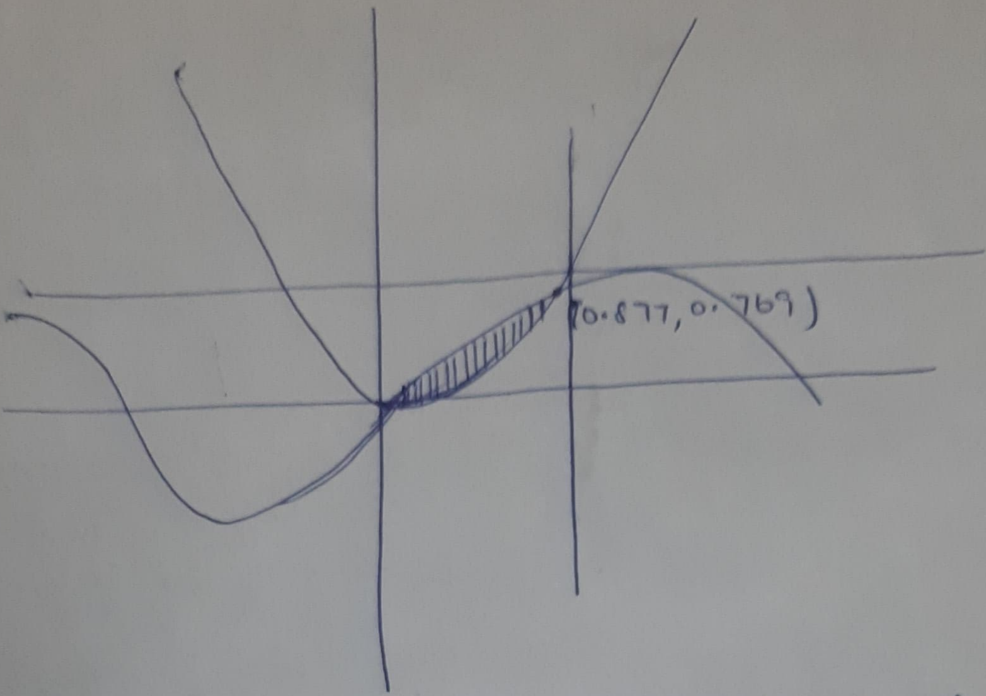
$$= \pi \left(\frac{x^5}{5} - \frac{2x^3}{3} - \frac{x}{2} + \frac{\sin 2x}{4} - 2\cos x \right)_0^{0.877}$$

$$= \pi \left(\frac{(0.877)^5}{5} - \frac{2(0.877)^3}{3} - \frac{0.877}{2} + \frac{\sin 2(0.877)}{4} - 2\cos(0.877) + 2 \right)$$

$= 0.57.$

b)

26.



Using the method of cylindrical shells to set up the integral for computing the volume. The shaded region is revolved around $x=1$.

$$V = \int_0^{0.877} 2\pi (1-x)(\sin x - x^2)\,dx$$

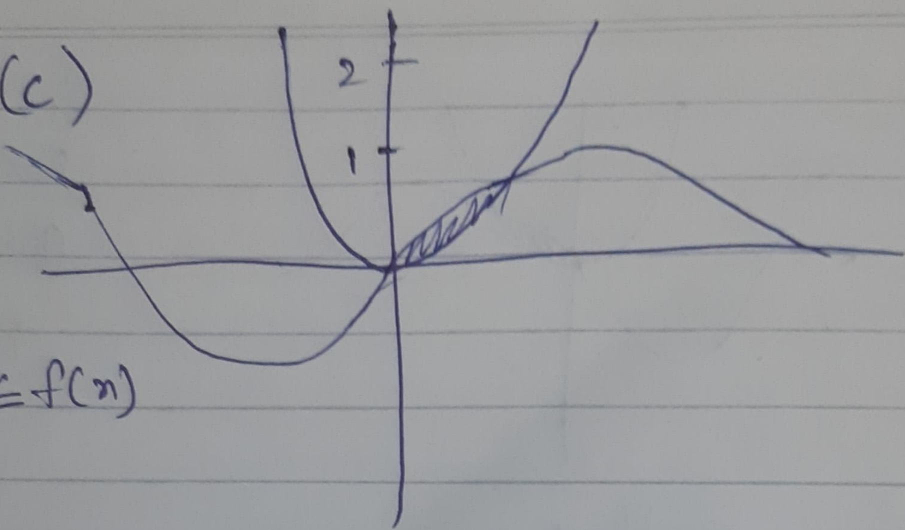
$$V = \int_0^{0.877} 2\pi (1-x)(\sin x - x^2)\,dx$$

$$= \int_0^{0.877} 2\pi (\sin x - x^2 - x\sin x + x^3)\,dx$$

$$= 2\pi \left(-\cos x - \frac{x^3}{3} + \frac{x^4}{4} + x\cos x - \sin x \right)\Bigg|_0^{0.877}$$

$= 0.4742.$

5.3 26(c)



$f(x) = \sin x$

$g(x) = x^2$

$g(x) \le y \le f(x)$

$x = 0$

Method of cylindrical shell. revolving y-axis.

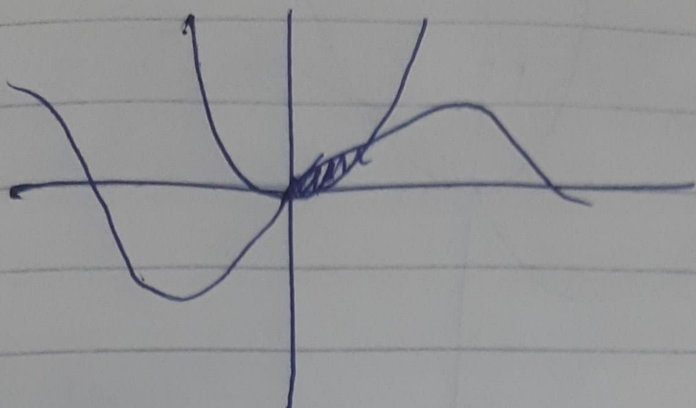
$$V = \int_0^{0.877} 2\pi(x)(\sin x - x^2)\,dx$$

$$= \int_0^{0.877} 2\pi(x \sin x - x^3)\,dx$$

$$= 2\pi\left(-x\cos x + \sin x - \frac{x^4}{4}\right)\Big|_0^{0.877}$$

$$= 0.3779$$

5.3(2b.d)



$f(x) = \sin x$

$g(x) = x^4$

$g(x) \leq y \leq f(x)$

$y = 0$

Washers method

$$V = \int_0^{0.877} \pi\left(\sin^2 x - (x^2)^2\right) dx$$

$$V = \pi \int_0^{0.877} \left(\sin^2 x - (x^2)^2\right) dx$$

$$= \pi \int_0^{0.877} \left(\sin^2 x - x^4\right) dx$$

$$= \pi \int_0^{0.877} \left(\frac{1-\cos 2x}{2} - x^4\right) dx$$

$$= \pi \left(\frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) - \frac{x^5}{5}\right)_0^{0.877}$$

$$= 0.2793.$$

5.4

4. for $1 \le x \le 3$ & $n = 2$

$x_0 = 1$, $x_1 = 2$, $x_2 = 3$.

$(1, 2), (2, 3)$

the arc length of $f(x) = \ln x$

$$S = \sqrt{(2-1)^2 + (\ln 2 - \ln 1)^2} + \sqrt{(3-2)^2 + (\ln 3 - \ln 2)^2}$$

$$\approx 2.296$$

for $n = 4$

evaluation points $1, 1.5, 2, 2.5, 3$

$$S = \sqrt{(1.5-1)^2 + (\ln 1.5 - \ln 1)^2} + \sqrt{(2-1.5)^2 + (\ln - \ln 1.5)^2} + \sqrt{(2.5-2)^2 + (\ln 2.5 - \ln 2)^2} + \sqrt{(3-2.5)^2 + (\ln 3 - \ln 2.5)^2}$$

$$\approx 2.30033$$

Futuremate

5.4 (14) $f(x) = 2\ln(4-x^2)$ $\quad 0 \le x \le 1$

arc length $L = \int_a^b \sqrt{(f'(x))^2 + 1}\, dx$

$f'(x) = (2\ln(4-x^2))' = \dfrac{4x}{x^2-4}$

$L = \int_0^1 \sqrt{\left(\dfrac{4x}{x^2-4}\right)^2 + 1}\, dx$

$= \int_0^1 \dfrac{\sqrt{(x^2+4)^2}}{(x+2)(x-2)}\, dx$

$= \int_0^1 \dfrac{x^2+4}{(x+2)(x-2)}\, dx$

$= 1 - 2\ln$

≈ -1.19722

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DATE

5.4

30) $f(x) = \sin x$ for $0 \leq x \leq \pi$

Now $f'(x) = \cos x$

Surface area of surface of revolution

$$= 2\pi \int_0^{\pi} f(x) \sqrt{1+(f'(x))^2}\, dx$$

$$= 2\pi \int_0^{\pi} \sin x \sqrt{1+(\cos x)^2}\, dx$$

$\approx$ 14.4236 square units

5.4

32) $f(x) = x^3 - 4x$ for, $-2 \le x \le 0$

Now $f'(x) = 3x^2 - 4$

Surface area of surface of revolution

$$= 2\pi \int_{-2}^{0} f(x) \sqrt{1 + [f'(x)]^2}\, dx$$

$f'($

$$= 2\pi \int_{-2}^{0} (x^3 - 4x) \sqrt{1 + (3x^2 - 4)^2}\, dx$$

≈ 67.0655 sq units

36) $f(x) = \sqrt{x}$, for $1 \le x \le 2$

Now $f'(x) = \dfrac{1}{2\sqrt{x}}$.

Surface area of surface of revolution

$$= 2\pi \int_{1}^{2} f(x) \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \cancel{2\pi \int_{1}^{2} \sqrt{x}\, \sqrt{\frac{1}{2\sqrt{x}}}}$$

$$= 2\pi \int_{1}^{2} \sqrt{x} \sqrt{\left(\frac{1}{2\sqrt{x}}\right)^2}\, dx.$$

~~≈ 8.899306~~

≈ 8.283 sq. units