# Assignment

Karthikeya hanu prakash kanithi (EE22BTECH11026)

Question : Let $\phi(.)$ denote the cumulative distribution function of a standard normal random variable. If the random variable $X$ has the cumulative distribution function

$$F(x) = \begin{cases} \phi(x), & x < -1 \\ \phi(x+1), & x \geq -1 \end{cases} \quad (1)$$

then which one of the following statements is true?

A) $P(X \leq -1) = \frac{1}{2}$
B) $P(X = -1) = \frac{1}{2}$
C) $P(X < -1) = \frac{1}{2}$
D) $P(X \leq 0) = \frac{1}{2}$

**Solution: Gaussian**
Q function is defined

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-u^2}{2}} du \quad (2)$$

From (1) and (2);

$$F_X(x) = \begin{cases} Q(-x), & x < -1 \\ 1 - Q(x+1), & x \geq -1 \end{cases} \quad (3)$$

From (7);
A)

$$\Pr(X \leq -1) = F_X(-1) = 1 - Q(0) \quad (4)$$
$$= 0.5 \quad (5)$$

So Option A i.e., $P(X < -1) = \frac{1}{2}$ is correct
B) The pdf of X can be defined in terms of cdf as

$$\Pr(X = b) = F_X(b) - \lim_{x \to b^-} F_X(x) \quad (6)$$

From (7);

$$\Pr(X = -1) = F_X(-1) - \lim_{x \to -1^-} F_X(x) \quad (7)$$
$$= 1 - Q(0) - Q(-(-1)) \quad (8)$$
$$= 0.341 \quad (9)$$

So Option B i.e., $P(X = -1) = \frac{1}{2}$ is incorrect
C)

$$\Pr(X < -1) = \lim_{x \to -1^-} F_X(x) = F_X(-1) \quad (10)$$
$$= Q(-(-1)) \quad (11)$$
$$= 0.159 \quad (12)$$

So Option C i.e., $P(X < -1) = \frac{1}{2}$ is incorrect
D)

$$\Pr(X \leq 0) = F_X(0) = 1 - Q(1) \quad (13)$$
$$= 0.8413 \quad (14)$$

So Option D i.e., $P(X \leq 0) = \frac{1}{2}$ is incorrect
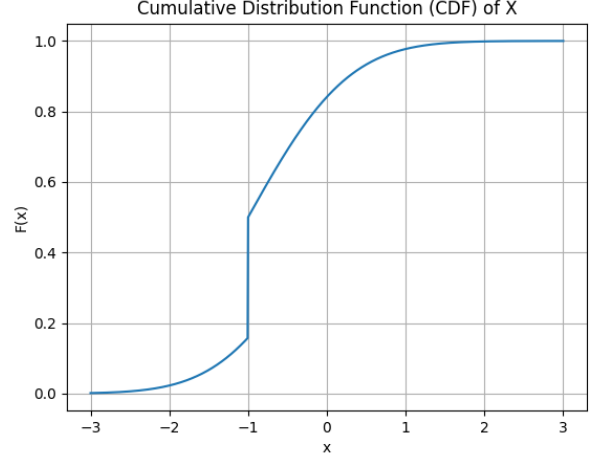
Guassian CDF plot of X is given in fig1



Fig. 1.