Probability Assignment 1

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Question : Suppose X is a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution: Given that, X is a binomial distribution with parameters

$$n = 6 \qquad p = \frac{1}{2} \tag{1}$$

the probability of getting exactly k successes in n trials is given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

From equations in (1) , The pmf simplifies as,

$$p_X(k) = \binom{6}{k} (\frac{1}{2})^k (\frac{1}{2})^{6-k} \tag{3}$$

$$= \binom{6}{k} (\frac{1}{2})^6 \tag{4}$$

We know that $\binom{6}{k}$ can be written as,

$$\binom{6}{k} = \frac{6!}{(6-k)!k!} \tag{5}$$

If pmf is the greatest, then $\binom{6}{k}$ is the greatest for $k \in [0, 6]$, Therefore It can be said that,

$$\binom{6}{k} > \binom{6}{k-1} \quad and \tag{6}$$

$$\binom{6}{k} > \binom{6}{k+1} \tag{7}$$

From (5) and (6), we can state that

$$\frac{6!}{(6-k)!k!} > \frac{6!}{(6-k+1)!(k-1)!} \tag{8}$$

$$\implies \frac{6!}{(6-k)!k!} > \frac{6!}{(6-k)!k!} \frac{k}{6-k+1} \tag{9}$$

$$\implies 1 > \frac{k}{6-k+1} \tag{10}$$

$$\therefore k < \frac{7}{2} \tag{11}$$

From (5) and (7), we can state that

$$\frac{6!}{(6-k)!k!} > \frac{6!}{(6-k-1)!(k+1)!} \tag{12}$$

$$\implies \frac{6!}{(6-k)!k!} > \frac{6!}{(6-k)!k!} \frac{6-k}{k+1} \tag{13}$$

$$\implies 1 > \frac{6-k}{k+1} \tag{14}$$

$$\therefore k > \frac{5}{2} \tag{15}$$

We know that, $k \in R$ and $k \in [0, 6]$ and from (11) and (15),

$$k < \frac{7}{2} \quad and \quad k > \frac{5}{2} \tag{16}$$

$$\therefore k = 3 \tag{17}$$

Hence proved that,

$$X = 3 \tag{18}$$

is the most likely outcome and $p_X(3)$ is

$$p_X(3) = \binom{6}{3} (\frac{1}{2})^6 \tag{19}$$

$$= \frac{5}{16} \tag{20}$$

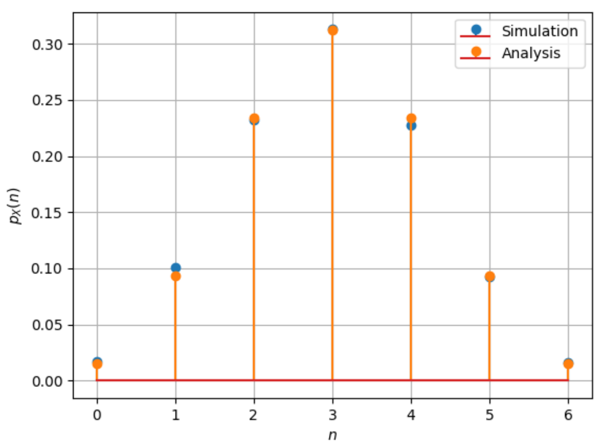

Fig. 0. Figure compares the therotical and simulation output