Probability Assignment 1

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Question : Suppose X is a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution: Given that, X is a binomial distribution with parameters

$$n = 6 \qquad p = 0.5 \tag{1}$$

the probability of getting exactly k successes in n trials is given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

From equations in (1) , The pmf simplifies as,

$$p_X(k) = \binom{6}{k} (0.5)^k (0.5)^{6-k} \tag{3}$$

$$= \binom{6}{k} (0.5)^6 \tag{4}$$

We know that $\binom{6}{k}$ is the largest when,

$$\text{If } n \text{ is even:} \quad k = \frac{n}{2} \tag{5}$$

$$\text{If } n \text{ is odd:} \quad k = \frac{n+1}{2} \text{ or } k = \frac{n-1}{2} \tag{6}$$

So $\binom{6}{k}$ is the greatest for

$$k = \frac{n}{2} \tag{7}$$

$$= 3 \tag{8}$$

Hence proved that,

$$X = 3 \tag{9}$$

is the most likely outcome and pmf for X=3 is

$$p_X(3) = \binom{6}{3} (0.5)^6 \tag{10}$$

$$= 0.3125 \tag{11}$$

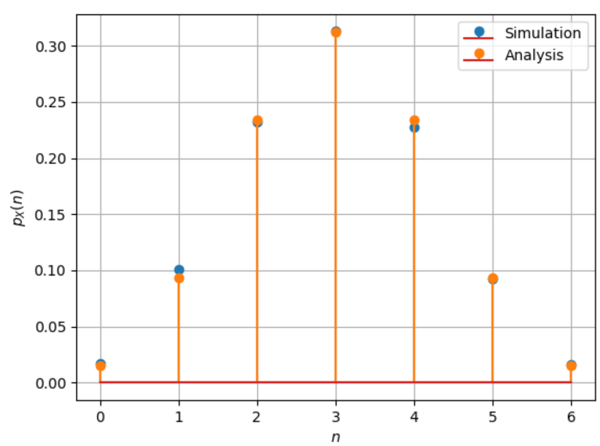

Fig. 0. Figure compares the therotical and simulation output