Probability Assignment 1

EE22BTECH11026 - KARTHIKEYA HANU PRAKASH KANITHI

Question : Suppose X is a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution: Given that, X is a binomial distribution with parameters

$$n = 6 \qquad p = 0.5 \tag{1}$$

the probability of getting exactly k successes in n trials is given by

$$P(X = k) = \binom{n}{k}.p^k.(1-p)^{n-k} \tag{2}$$

From (2), We can conclude that

$$\text{For } x_i = 0 : \tag{3}$$

$$P(X = 0) = \binom{6}{0}.(0.5)^0.(0.5)^6 \approx 0.016 \tag{4}$$

$$\text{For } x_i = 1 : \tag{5}$$

$$P(X = 1) = \binom{6}{1}.(0.5)^1.(0.5)^5 \approx 0.094 \tag{6}$$

$$\text{For } x_i = 2 : \tag{7}$$

$$P(X = 2) = \binom{6}{2}.(0.5)^2.(0.5)^4 \approx 0.234 \tag{8}$$

$$\text{For } x_i = 3 : \tag{9}$$

$$P(X = 3) = \binom{6}{3}.(0.5)^3.(0.5)^3 \approx 0.313 \tag{10}$$

$$\text{For } x_i = 4 : \tag{11}$$

$$P(X = 4) = \binom{6}{4}.(0.5)^4.(0.5)^2 \approx 0.234 \tag{12}$$

$$\text{For } x_i = 5 : \tag{13}$$

$$P(X = 5) = \binom{6}{5}.(0.5)^5.(0.5)^1 \approx 0.094 \tag{14}$$

$$\text{For } x_i = 6 : \tag{15}$$

$$P(X = 6) = \binom{6}{6}.(0.5)^6.(0.5)^0 \approx 0.016 \tag{16}$$

$$\tag{17}$$

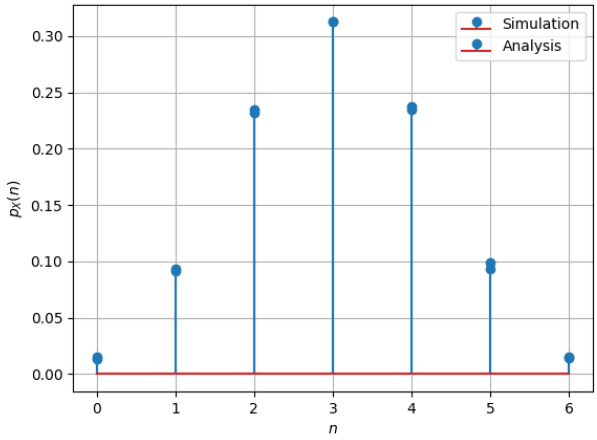

Fig. 0. Figure compares the therotical and simulation output

Here, we can conclude that $P(X = 3)$ is the highest and hence proved that

$$X = 3 \tag{18}$$

is the most likely outcome.