Assignment

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Question : Let $\phi(.)$ denote the cumulative distribution function of a standard normal random variable. If the random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \phi(x), & x < -1 \\ \phi(x+1), & x \geq -1 \end{cases} \tag{1}$$

then which one of the following statements is true?

A) $P(X \leq -1) = \frac{1}{2}$
B) $P(X = -1) = \frac{1}{2}$
C) $P(X < -1) = \frac{1}{2}$
D) $P(X \leq 0) = \frac{1}{2}$

Solution: Gaussian

Let Y be the standard normal random variable,

$$Y \sim \mathcal{N}(0,1) \tag{2}$$

Q function is defined

$$Q(x) = \Pr(Y > x) \ \forall x \in Y \tag{3}$$

NOTE: The range of values of X and Y are same. So,

$$x \in X \implies x \in Y \tag{4}$$

From the given values and (3),

$$\Pr(Y \leq x) = \phi(x) = \begin{cases} 1 - Q(x), & x > 0 \\ Q(-x), & x < 0 \end{cases} \tag{5}$$

From (1) and (4);

A)

$$\Pr(X \leq -1) = F(-1) = \phi(0) \tag{6}$$
$$= 1 - Q(0) \tag{7}$$
$$= 0.5 \tag{8}$$

So Option A i.e., $P(X < -1) = \frac{1}{2}$ is correct

B) The pdf of X can be defined in terms of cdf as

$$\Pr(X = b) = F(b) - \lim_{x \to b^-} F(x) \tag{9}$$

From (8);

$$\Pr(X = -1) = F(-1) - \lim_{x \to -1^-} F(x) \tag{10}$$
$$= \phi(0) - \phi(-1) \tag{11}$$
$$= 1 - Q(0) - Q(-(-1)) \quad = 0.341 \tag{12}$$

So Option B i.e., $P(X = -1) = \frac{1}{2}$ is incorrect

C)

$$\Pr(X < -1) = \lim_{x \to -1^-} F(x) \tag{13}$$
$$= F(-1) = Q(-(-1)) \tag{14}$$
$$= 0.159 \tag{15}$$

So Option C i.e., $P(X < -1) = \frac{1}{2}$ is incorrect

D)

$$\Pr(X \leq 0) = F(0) = \phi(1) \tag{16}$$
$$= 1 - Q(1) \tag{17}$$
$$= 0.8413 \tag{18}$$

So Option D i.e., $P(X \leq 0) = \frac{1}{2}$ is incorrect