

Assignment

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Question : Three dice are thrown at the sametime. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

Solution: Let X_1, X_2, X_3 be Bernoulli Random Variables as defined in Table I, We need to find the value of

RV	Value	Description
X_1	0,1,2,3,4,5,6	Outcome of 1 st die
X_2	0,1,2,3,4,5,6	Outcome of 2 nd die
X_3	0,1,2,3,4,5,6	Outcome of 3 rd die

TABLE I
RANDOM VARIABLE X DECLARATION.

$$\Pr(X_1 = 2, X_2 = 2, X_3 = 2 | X_1 + X_2 + X_3 = 6) \quad (1)$$

The pmf's of the random variables are as follows:

$$p_{X_i}(k) = \begin{cases} \frac{1}{6} & x \in \{1, 2, 3, 4, 5, 6\} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Now, If

$$X = X_1 + X_2 + X_3 \quad (3)$$

Then

$$M_X(z) = \prod_{i=1}^3 M_{X_i}(z) \quad (4)$$

$$= \prod_{i=1}^3 \left(\sum_{k=1}^6 \frac{z^{-k}}{6} \right) \quad (5)$$

$$= \frac{z^{-3}}{216} + \frac{3z^{-4}}{216} + \frac{6z^{-5}}{216} + \frac{10z^{-6}}{216} + \dots \quad (6)$$

From (2), When all the three die's roll 2, then thier sum will be 6. So,

$$\Pr(X_1 = X_2 = X_3 = 2, X_1 + X_2 + X_3 = 6) \quad (7)$$

$$= p_{X_1}(2)p_{X_2}(2)p_{X_3}(2) = \left(\frac{1}{6}\right)^3 = \frac{1}{216} \quad (8)$$

From (5), the probability that the sum of all three die's is 6 is,

$$\Pr(X_1 + X_2 + X_3 = 6) = \frac{10}{216} \quad (9)$$

From (7) and (11), the probability of getting three two's, if it is known that the sum of the numbers on the dice was six will be

$$\Pr(X_1 = 2, X_2 = 2, X_3 = 2 | X_1 + X_2 + X_3 = 6) \quad (10)$$

$$= \frac{\Pr(X_1 = X_2 = X_3 = 2, X_1 + X_2 + X_3 = 6)}{\Pr(X_1 + X_2 + X_3 = 6)} \quad (11)$$

$$= \frac{\frac{1}{216}}{\frac{10}{216}} = \frac{1}{10} \quad (12)$$