Probability Assignment 1

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Question : Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability$(1 - \epsilon)$, and flipped with probability ϵ. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error.
For $\epsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is ______(rounded off to two decimal places). (rounded off to two decimal places).
Solution: Given that, Let X be a random variable representing the number of bit errors introduced during transmission over the binary symmetric channel.Then, X is a binomial distribution with parameters

$$n = 7 \quad p = \epsilon = 0.1 \tag{1}$$

the pmf of X is given by

$$p_X(k) = {}^7C_k(\epsilon)^k(1-\epsilon)^{7-k} \tag{2}$$

the cdf of X is given by

$$F_X(k) = \sum_{i=0}^{k} {}^7C_i(\epsilon)^i(1-\epsilon)^{7-i} \tag{3}$$

From equation (3), the probability of getting one or less error is given by

$$\begin{aligned} F_X(1) &= \sum_{i=0}^{1} {}^7C_i(\epsilon)^i(1-\epsilon)^{7-i} && (4)\\ &= {}^7C_0(\epsilon)^0(1-\epsilon)^7 + {}^7C_1(\epsilon)^1(1-\epsilon)^6 && (5)\\ &= (1-\epsilon)^7 + 7(\epsilon)^1(1-\epsilon)^6 && (6)\end{aligned}$$

From (1) and (6),

$$\begin{aligned} F_X(1) &= (1-0.1)^7 + 7(0.1)^1(1-0.1)^6 && (7)\\ &= 0.85 && (8)\end{aligned}$$

$\therefore$ the probability that a transmitted codeword is decoded correctly is 0.85.