Probability Assignment 1

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Question : Show that **A**,**D**,**G** are collinear
Solution :
Given that,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

We need to show that points **A**,**D**,**G** are collinear

By taking reference from the previous problems, we can conclude that

From Problem 1.2.3 We know that, The point G is

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2}$$

And from Problem 1.2.1 We know that, The point D is

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{3}$$

In Problem 1.1.3, There is a theorem/law mentioned i.e.,

Theorem 1. Points **A**,**B**,**C** are defined to be collinear if

$$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} = 2 \tag{4}$$

Using Theorom 1: Let

$$R = \begin{pmatrix} 1 & 1 & 1 \\ a_{11} & d_{11} & g_{11} \\ a_{21} & d_{21} & g_{21} \end{pmatrix} \tag{5}$$

Then **A**,**D**,**G** are collinear points if and only if

$$\rho(R) = 2 \tag{6}$$

(Where $\rho(R)$ represents rank of matrix R)
So, lets convert the matrix R into echoloen form by using Elementary operations

The matrix R can be row reduced as follows,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-7}{2} & -2 \\ -1 & \frac{1}{2} & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + R_2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-7}{2} & -2 \\ 0 & -3 & -2 \end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{-9}{2} & -3 \\ 0 & -3 & -2 \end{pmatrix} \tag{8}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - \frac{2}{3} R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{-9}{2} & -3 \\ 0 & 0 & 0 \end{pmatrix} \tag{9}$$

The above matrix R is in echoleon form and we can see that every element of row 3 is zero.
So, we can say that there are only 2 independent rows and the rank of matrix R is 2 i.e.,

$$\rho(R) = 2 \tag{10}$$

As, we have proved that $\rho(R)$ is 2, The theorom states that points **A**,**D**,**G** are collinear.