

Assignment

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Question : Let $\phi(\cdot)$ denote the cumulative distribution function of a standard normal random variable. If the random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \phi(x), & x < -1 \\ \phi(x+1), & x \geq -1 \end{cases} \quad (1)$$

then which one of the following statements is true?

- A) $P(X \leq -1) = \frac{1}{2}$
- B) $P(X = -1) = \frac{1}{2}$
- C) $P(X < -1) = \frac{1}{2}$
- D) $P(X \leq 0) = \frac{1}{2}$

Solution: Gaussian

Q function is defined

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du \quad (2)$$

Thus,

$$\phi(x) = \begin{cases} 1 - Q(x), & x > 0 \\ Q(-x), & x < 0 \end{cases} \quad (3)$$

From (1) and (3);

A)

$$\Pr(X \leq -1) = F(-1) = \phi(0) \quad (4)$$

$$= 1 - Q(0) \quad (5)$$

$$= 0.5 \quad (6)$$

So Option A i.e., $P(X < -1) = \frac{1}{2}$ is correct

B) The pdf of X can be defined in terms of cdf as

$$\Pr(X = b) = F(b) - \lim_{x \rightarrow b^-} F(x) \quad (7)$$

From (7);

$$\Pr(X = -1) = F(-1) - \lim_{x \rightarrow -1^-} F(x) \quad (8)$$

$$= \phi(0) - \phi(-1) \quad (9)$$

$$= 1 - Q(0) - Q(-(-1)) = 0.341 \quad (10)$$

So Option B i.e., $P(X = -1) = \frac{1}{2}$ is incorrect

C)

$$\Pr(X < -1) = \lim_{x \rightarrow -1^-} F(x) \quad (11)$$

$$= F(-1) = Q(-(-1)) \quad (12)$$

$$= 0.159 \quad (13)$$

So Option C i.e., $P(X < -1) = \frac{1}{2}$ is incorrect

D)

$$\Pr(X \leq 0) = F(0) = \phi(1) \quad (14)$$

$$= 1 - Q(1) \quad (15)$$

$$= 0.8413 \quad (16)$$

So Option D i.e., $P(X \leq 0) = \frac{1}{2}$ is incorrect

Gaussian CDF plot of X is given in fig1

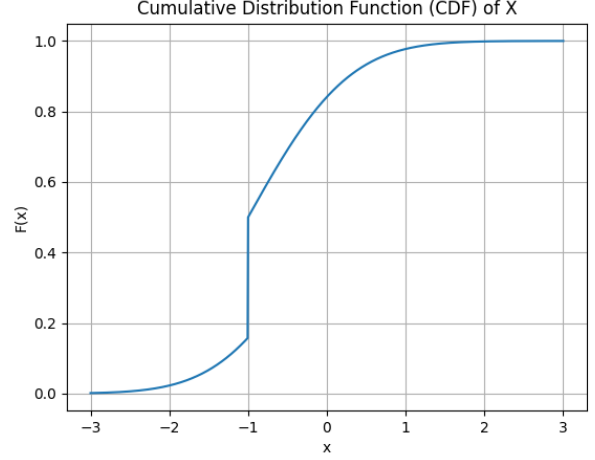


Fig. 1. Gaussian CDF