

Assignment

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Question : Suppose that (X, Y) has joint probability mass function From (10),

$$P(X = 0, Y = 0) = P(X = 1, Y = 1) = \theta, \quad (1)$$

$$P(X = 1, Y = 0) = P(X = 0, Y = 1) = \frac{1}{2} - \theta. \quad (2)$$

where $0 \leq \theta \leq \frac{1}{2}$ is an unknown parameter. Consider testing $H_0 : \theta = \frac{1}{4}$ against $H_1 : \theta = \frac{1}{3}$; based on a random sample $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ from the above probability mass function. Let M be the cardinality of the set $\{i : X_i = Y_i, 1 \leq i \leq n\}$. If m is the observed value of M , then which one of the following statements is true?

- A) The likelihood ratio test rejects H_0 if $m > c$ for some c .
- B) The likelihood ratio test rejects H_0 if $m < c$ for some c .
- C) The likelihood ratio test rejects H_0 if $c_1 < m < c_2$ for some c_1 and c_2 .
- D) The likelihood ratio test rejects H_0 if $m < c_1$ or $m > c_2$ for some c_1 and c_2 .

Solution: Given that,

$$H_0 : \theta = \theta_0 = \frac{1}{4}, \quad (3)$$

$$H_1 : \theta = \theta_1 = \frac{1}{3}. \quad (4)$$

Let x represent the random sample. Then the likelihood of the data under H_0 is given by:

$$L(\theta_0 | x) = \prod_{i=1}^n P(X_i, Y_i) \quad (5)$$

$$= \left(\frac{1}{4}\right)^m \left(\frac{1}{4}\right)^{n-m} \quad (6)$$

Then the likelihood of the data under H_1 is given by:

$$L(\theta_1 | x) = \prod_{i=1}^n P(X_i, Y_i) \quad (7)$$

$$= \left(\frac{1}{3}\right)^m \left(\frac{1}{6}\right)^{n-m} \quad (8)$$

The likelihood ratio will be

$$\lambda(x) = \frac{L(\theta_1 | x)}{L(\theta_0 | x)} \quad (9)$$

$$= \frac{\left(\frac{1}{3}\right)^m \left(\frac{1}{6}\right)^{n-m}}{\left(\frac{1}{4}\right)^m \left(\frac{1}{4}\right)^{n-m}} = (2)^m \left(\frac{2}{3}\right)^n \quad (10)$$

Let the critical value be denoted by c_1 , then the likelihood ratio test rejects H_0 if

$$\Rightarrow \lambda(x) \underset{H_0}{\overset{H_1}{\geq}} c_1 \quad (11)$$

$$(12)$$

$$\Rightarrow (2)^m \left(\frac{2}{3}\right)^n \underset{H_0}{\overset{H_1}{\geq}} c_1 \quad (13)$$

$$\Rightarrow (2)^m \underset{H_0}{\overset{H_1}{\geq}} c_1 \left(\frac{2}{3}\right)^n \quad (14)$$

$$\Rightarrow m \underset{H_0}{\overset{H_1}{\geq}} \log_2 \left(c_1 \left(\frac{2}{3}\right)^n \right) \quad (15)$$

$$\Rightarrow m \underset{H_0}{\overset{H_1}{\geq}} c \quad \exists c \in \mathbb{R} \quad (16)$$

\therefore From (16), Option A is correct and Options B,C,D are incorrect