

Assignment

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Question : Let $\{0.13, 0.12, 0.78, 0.51\}$ be a realization of a random sample of size 4 from a population with cumulative distribution function $F(\cdot)$. Consider testing

$$H_0 : F = F_0 \quad \text{against} \quad H_1 : F \neq F_0 \quad (1)$$

where,

$$F_0(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \quad (2)$$

Let D denote the Kolmogorov-Smirnov test statistic. If $P(D > 0.669) = 0.01$ under H_0 and

$$\psi = \begin{cases} 1 & \text{if } H_0 \text{ is accepted at level } 0.01 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

then based on the given data, the observed value of $D + \psi$ (rounded off to two decimal places) equals

Solution: Its given that random sample is of size 4, So

$$n = 4 \quad (4)$$

The cdf of the random sample is given as

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \quad (5)$$

The empirical distribution function(edf) F_n for n independent and identically distributed (i.i.d.) ordered observations X_i is defined as

$$F_n(x) = \frac{\text{no of (elements in the sample } \leq x)}{n} = \frac{1}{n} \sum_{i=1}^n 1_{X_i \leq x} \quad (6)$$

where 1_A is the indicator of event A .

From (4), (5) and (6), the edf for the given data will be

$$F_n(0.13) = \frac{1}{4} \sum_{i=1}^n 1_{X_i \leq 0.13} = \frac{1}{2} \quad (7)$$

$$F_n(0.12) = \frac{1}{4} \sum_{i=1}^n 1_{X_i \leq 0.12} = \frac{1}{4} \quad (8)$$

$$F_n(0.78) = \frac{1}{4} \sum_{i=1}^n 1_{X_i \leq 0.78} = 1 \quad (9)$$

$$F_n(0.51) = \frac{1}{4} \sum_{i=1}^n 1_{X_i \leq 0.51} = \frac{3}{4} \quad (10)$$

The Kolmogorov-Smirnov statistic for a given cdf $F_X(x)$ is

$$D_n = \sup |F_n(x) - F_X(x)| \quad (11)$$

The difference between cdf and edf for the given data will be (i.e., $\forall x \in \{0.13, 0.12, 0.78, 0.51\}$)

$$F_n(0.13) - F_X(0.13) = 0.37 \quad (12)$$

$$F_n(0.12) - F_X(0.12) = 0.25 \quad (13)$$

$$F_n(0.78) - F_X(0.78) = 0.22 \quad (14)$$

$$F_n(0.51) - F_X(0.51) = 0.24 \quad (15)$$

Then

$$D_n = \sup(0.37, 0.25, 0.22, 0.24) = 0.37 \quad (16)$$

Given that,

$$P(D > 0.669) = 0.01 \quad (17)$$

Then

$$H_0 = \begin{cases} \text{accepted at level } 0.01 & \text{if } D_n \leq 0.669 \\ \text{rejected at level } 0.01 & \text{if } D_n > 0.669 \end{cases} \quad (18)$$

From (16) and (18); We can say that H_0 is accepted at level 0.01 and

$$\psi = 1 \quad (19)$$

\therefore the value will be

$$\psi + D_n = 1 + 0.37 = 1.37 \quad (20)$$