Assignment

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Question : If two events are independent, then

1) they must be mutually exclusive
2) the sum of their probabilities must be equal to 1
3) (A) and (B) both are correct
4) None of the above is correct

Solution: Let X, Y be bernoulli random variables as defined in Table II, Lets us consider the pmf's as follows:

RV	Value	Description
X	$\{0,1\}$	Event A
Y	$\{0,1\}$	Event B

TABLE I
RANDOM VARIABLE X DECLARATION

$$p_X(k) = \begin{cases} x & \text{if } k = 1 \\ 1-x & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$p_Y(k) = \begin{cases} y & \text{if } k = 1 \\ 1-y & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Two events A and B are independent if and only if:

$$\Pr(X = 1\ ,\ Y = 1) = p_X(1)p_Y(1) = xy \tag{3}$$

Two events A and B are mutually-exclusive if and only if:

$$\Pr(X = 1\ ,\ Y = 1) = 0 \tag{4}$$

from (3) and (4), We can say that X and Y are not mutually exclusive as

$$\Pr(X = 1\ ,\ Y = 1) \neq 0 \tag{5}$$

The sum of the probabilities of two events A and B can be represented by:

$$\Pr(X = 1) + \Pr(Y = 1) = x + y \tag{6}$$

Here, It is not always true that $x + y$ should be 1 rather

$$0 \leq x + y \leq 2 \tag{7}$$

So option D none of these is the most appropriate answer
For example: Let event X and Y be indepedent and defined as follows: Then the pmf's as follows:

RV	Value	Description
X	$\{0,1\}$	Getting Head in a coin toss
Y	$\{0,1\}$	Getting 4 by a dice roll

TABLE II
RANDOM VARIABLE X DECLARATION

$$p_X(k) = \begin{cases} \frac{1}{2} & \text{if } k = 1 \\ \frac{1}{2} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k = 1 \\ \frac{5}{6} & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Then

$$\Pr(X = 1\ ,\ Y = 1) = p_X(1)p_Y(1) \tag{10}$$

$$= \frac{1}{12} \neq 0 \tag{11}$$

From (11), we can say that X and Y are not mutually exclusive and

$$\Pr(X = 1) + \Pr(Y = 1) = \frac{2}{3} \tag{12}$$

From (12), the sum of their probabilities may not be equal to 1