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## Probability Assignment 1

## EE22BTECH11026 - KARTHIKEYA HANU PRAKASH KANITHI

Question: Suppose X is a binomial distribution  $B\left(6, \frac{1}{2}\right)$ . Show that X = 3 is the most likely outcome. (Hint: P(X = 3) is the maximum among all  $P(x_i)$ ,  $x_i = 0, 1, 2, 3, 4, 5, 6$ )

**Solution:** Given that, X is a binomial distribution with parameters

$$n = 6$$
  $p = 0.5$  (1)

the probability of getting exactly k successes in n trials is given by

$$P(X = k) = \binom{n}{k} . p^{k} . (1 - p)^{n - k}$$
 (2)

From (2), We can conclude that

For 
$$x_i = 0$$
: (3)

$$P(X=0) = \begin{pmatrix} 6 \\ 0 \end{pmatrix} .(0.5)^0 .(0.5)^6 \approx 0.016 \tag{4}$$

For 
$$x_i = 1$$
: (5)

$$P(X = 1) = {6 \choose 1}.(0.5)^1.(0.5)^5 \approx 0.094$$
 (6)

For 
$$x_i = 2$$
: (7)

$$P(X = 2) = {6 \choose 2} . (0.5)^2 . (0.5)^4 \approx 0.234$$
 (8)

For 
$$x_i = 3$$
: (9)

$$P(X = 3) = {6 \choose 3}.(0.5)^3.(0.5)^3 \approx 0.313$$
 (10)

For 
$$x_i = 4$$
: (11)

$$P(X = 4) = {6 \choose 4} \cdot (0.5)^4 \cdot (0.5)^2 \approx 0.234$$
 (12)

For 
$$x_i = 5$$
: (13)

$$P(X = 5) = {6 \choose 5} .(0.5)^5 .(0.5)^1 \approx 0.094$$
 (14)

For 
$$x_i = 6$$
: (15)

$$P(X = 6) = {6 \choose 6} \cdot (0.5)^6 \cdot (0.5)^0 \approx 0.016$$
 (16)

(17)

Here, we can conclude that P(X = 3) is the highest and hence proved that

$$X = 3 \tag{18}$$

is the most likely outcome.