Assignment

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Question : Two natural numbers r, s are drawn one at a time, without replacement from the set S = 1,2,3, . . . ,n. Find P[$r \leq p | s \leq p$]

Solution: There are two conditions,

1) Case 1: s is chosen first
   Let X, Y and p be random variables as defined in Table I,

RV	Value	Description
X	$\{1, 2, 3, \ldots, n\}$	First number (s)
Y	$\{1, 2, 3, \ldots, n\} \setminus \{s\}$	Second number (r)
p	$\mathbb{Z}$	number to be compared

TABLE I
RANDOM VARIABLE X DECLARATION

We need to find the value of

$$\Pr\left(Y \leq p \,|\, X \leq p\right) = \frac{\Pr\left(Y \leq p,\, X \leq p\right)}{\Pr\left(X \leq p\right)} \tag{1}$$

There are 3 cases for the value of p;

a) if $p < 1$: This case is never possible as $X, Y \geq 1$
b) if $1 \leq p \leq n$: Then we can say that,

$$\Pr\left(Y \leq p,\, X \leq p\right) = \frac{p(p-1)}{n(n-1)}, \tag{2}$$

$$\Pr\left(X \leq p\right) = \frac{p}{n} \tag{3}$$

From (2) and (3):

$$\Pr\left(Y \leq p | X \leq p\right) = \frac{\Pr\left(Y \leq p,\, X \leq p\right)}{\Pr\left(X \leq p\right)} \tag{4}$$

$$= \frac{\frac{p(p-1)}{n(n-1)}}{\frac{p}{n}} = \frac{p-1}{n-1} \tag{5}$$

c) if $p > n$: Then we can say that,

$$\Pr\left(Y \leq p,\, X \leq p\right) = 1, \tag{6}$$

$$\Pr\left(X \leq p\right) = 1 \tag{7}$$

From (6) and (7):

$$\Pr\left(Y \leq p | X \leq p\right) = \frac{\Pr\left(Y \leq p,\, X \leq p\right)}{\Pr\left(X \leq p\right)} \tag{8}$$

$$= 1 \tag{9}$$

2) Case 2: r is chosen first
   Let X, Y and p be random variables as defined in Table II,

RV	Value	Description
X	$\{1, 2, 3, \ldots, n\}$	First number (r)
Y	$\{1, 2, 3, \ldots, n\} \setminus \{s\}$	Second number (s)
p	$\mathbb{Z}$	number to be compared

TABLE II
RANDOM VARIABLE X DECLARATION

We need to find the value of

$$\Pr\left(X \leq p | Y \leq p\right) = \frac{\Pr\left(X \leq p,\, Y \leq p\right)}{\Pr\left(Y \leq p\right)} \tag{10}$$

There are 3 cases for the value of p;

a) if $p < 1$: This case is never possible as $X, Y \geq 1$
b) if $1 \leq p \leq n$: Then we can say that,

$$\Pr\left(X \leq p,\, Y \leq p\right) = \frac{p(p-1)}{n(n-1)}, \tag{11}$$

$$\Pr\left(Y \leq p\right) = \frac{p-1}{n-1} \tag{12}$$

From (11) and (12):

$$\Pr\left(X \leq p | Y \leq p\right) = \frac{\Pr\left(X \leq p,\, Y \leq p\right)}{\Pr\left(Y \leq p\right)} \tag{13}$$

$$= \frac{\frac{p(p-1)}{n(n-1)}}{\frac{p}{n}} = \frac{p}{n} \tag{14}$$

c) if $p > n$: Then we can say that,

$$\Pr\left(X \leq p,\, Y \leq p\right) = 1, \tag{15}$$

$$\Pr\left(Y \leq p\right) = 1 \tag{16}$$

From (15) and (16):

$$\Pr\left(X \leq p | Y \leq p\right) = \frac{\Pr\left(X \leq p,\, Y \leq p\right)}{\Pr\left(Y \leq p\right)} \tag{17}$$

$$= 1 \tag{18}$$