Probability Assignment 1

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Question : Show that $\mathbf{A}$,$\mathbf{D}$,$\mathbf{G}$ are collinear
Solution :
Given that,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

We need to show that points $\mathbf{A}$,$\mathbf{D}$,$\mathbf{G}$ are collinear.

By taking reference from the previous problems, we can conclude that

From Problem 1.2.3 We know that, The point $\mathbf{G}$ is

$$\mathbf{G} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2}$$

And from Problem 1.2.1 We know that, The point $\mathbf{D}$ is

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{3}$$

In Problem 1.1.3, There is a theorem/law mentioned i.e.,
Points $\mathbf{A}$,$\mathbf{D}$,$\mathbf{G}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix} = 2 \tag{4}$$

Using the above law/Theorem Let

$$R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-7}{2} & -2 \\ -1 & \frac{1}{2} & 0 \end{pmatrix} \tag{5}$$

The matrix R can be row reduced as follows,

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-7}{2} & -2 \\ -1 & \frac{1}{2} & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3 + R_2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{-7}{2} & -2 \\ 0 & -3 & -2 \end{pmatrix} \tag{6}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{-9}{2} & -3 \\ 0 & -3 & -2 \end{pmatrix} \tag{7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - \frac{2}{3} R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \frac{-9}{2} & -3 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

The above matrix R is in echoleon form and there are 2 independent rows. So, the rank of matrix R is 2 i.e.,

$$\rho(R) = 2 \tag{9}$$

(where $\rho(R)$ Represents the rank of matrix)

Hence, we proved that that points $\mathbf{A}$,$\mathbf{D}$,$\mathbf{G}$ are collinear.