Assignment

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Question : Two natural numbers r, s are drawn one at a time, without replacement from the set S = 1,2,3, ...,n. Find P$[r \leq p | s \leq p]$

Solution: Let X and Y be random variables as defined in Table I,

RV	Value	Description
X	$\{1,2,3,\dots,n\}$	First number (s)
Y	$\{1,2,3,\dots,n\} \setminus \{s\}$	Second number (r)

TABLE I
Random variable X declaration

We need to find the value of

$$\Pr\left(Y \leq p \,|\, X \leq p\right) \tag{1}$$

The pmf's of the random variables are as follows:

$$p_X(k) = \begin{cases} \frac{1}{n} & 1 \leq k \leq n \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$p_Y(k) = \begin{cases} \frac{1}{n-1} & k \in \{1,2,3,\dots,n\} \setminus \{s\} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The cdf's of the random variables are as follows:

$$F_X(k) = \begin{cases} \frac{k}{n} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{4}$$

$$F_Y(k) = \begin{cases} \frac{k}{n-1} & 1 \leq k \leq s-1 \\ \frac{k-1}{n-1} & s \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{5}$$

From (4), (5) and $p \geq s$,

$$\Pr\left(Y \leq p,\, X \leq p\right) = F_X(p)F_Y(p) \tag{6}$$

$$= \frac{p(p-1)}{n(n-1)} \tag{7}$$

From (4),

$$\Pr\left(X \leq p\right) = F_X(p) = \frac{p}{n} \tag{8}$$

From (7) and (8),

$$\Pr\left(Y \leq p | X \leq p\right) = \frac{\Pr\left(Y \leq p,\, X \leq p\right)}{\Pr\left(X \leq p\right)} \tag{9}$$

$$= \frac{\frac{p(p-1)}{n(n-1)}}{\frac{p}{n}} = \frac{p-1}{n-1} \tag{10}$$