

Assignment

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Question : Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability $(1 - \epsilon)$, and flipped with probability ϵ . For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error.

For $\epsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is _____ (rounded off to two decimal places). (rounded off to two decimal places).

Solution: Given that, Let X be a random variable defined in the Table I;

RV	Value	Description
X	$\{0, 1, 2, 3, 4, 5, 6, 7\}$	The number of bit errors in transmission

TABLE I
RANDOM VARIABLE X DECLARATION

Then, X is a binomial distribution with parameters

$$n = 7 \quad p = \epsilon = 0.1 \quad (1)$$

the pmf of X is given by

$$p_X(k) = {}^7C_k(\epsilon)^k(1 - \epsilon)^{7-k} \quad (2)$$

the cdf of X is given by

$$F_X(k) = \sum_{i=0}^k {}^7C_i(\epsilon)^i(1 - \epsilon)^{7-i} \quad (3)$$

From equation (3), the probability of getting one or less error is given by

$$F_X(1) = \sum_{i=0}^1 {}^7C_i(\epsilon)^i(1 - \epsilon)^{7-i} \quad (4)$$

$$= {}^7C_0(\epsilon)^0(1 - \epsilon)^7 + {}^7C_1(\epsilon)^1(1 - \epsilon)^6 \quad (5)$$

$$= (1 - \epsilon)^7 + 7(\epsilon)^1(1 - \epsilon)^6 \quad (6)$$

From (1) and (6),

$$F_X(1) = (1 - 0.1)^7 + 7(0.1)^1(1 - 0.1)^6 \quad (7)$$

$$= 0.85 \quad (8)$$

\therefore the probability that a transmitted codeword is decoded correctly is 0.85.