Assignment

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Question : Let {0.13, 0.12, 0.78, 0.51} be a realization of a random sample of size 4 from a population with cumulative distribution function $F(.)$. Consider testing

$$H_0 : F = F_0 \quad \text{against} \quad H_1 : F \neq F_0 \tag{1}$$

where,

$$F_0(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases} \tag{2}$$

Let D denote the Kolmogorov-Smirnov test statistic. If $P(D > 0.669) = 0.01$ under H_0 and

$$\psi = \begin{cases} 1 & \text{if } H_0 \text{ is accepted at level } 0.01 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

then based on the given data, the observed value of $D + \psi$ (rounded off to two decimal places) equals

Solution: Its given that random sample is of size 4, So

$$n = 4 \tag{4}$$

The cdf of the random sample is given as

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases} \tag{5}$$

The empirical distribution function(edf) G_n for n independent and identically distributed (i.i.d.) ordered observations X_i is defined as

$$G_n(x) = \frac{\text{no of (elements in the sample} \le x)}{n} = \frac{1}{n}\sum_{i=1}^{n} 1(X_i \le x) \tag{6}$$

where $1(A)$ is the indicator of event A and in (6) it is defined as,

$$1(X_i \le x) = \begin{cases} 1 & X_i \le x \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

From (4), (5) and (6), the edf for the given data will be

$$G_n(0.13) = \frac{1}{4}\sum_{i=1}^{n} 1(X_i \le 0.13) = \frac{1}{2} \tag{8}$$

$$G_n(0.12) = \frac{1}{4}\sum_{i=1}^{n} 1(X_i \le 0.12) = \frac{1}{4} \tag{9}$$

$$G_n(0.78) = \frac{1}{4}\sum_{i=1}^{n} 1(X_i \le 0.78) = 1 \tag{10}$$

$$G_n(0.51) = \frac{1}{4}\sum_{i=1}^{n} 1(X_i \le 0.51) = \frac{3}{4} \tag{11}$$

The Kolmogorov–Smirnov statistic for a given cdf $F_X(x)$ is

$$D_n = \sup |G_n(x) - F_X(x)| \tag{12}$$

The difference between cdf and edf for the given data will be (i.e., $\forall x \in \{0.13, 0.12, 0.78, 0.51\}$)

$$G_n(0.13) - F_X(0.13) = 0.37 \tag{13}$$

$$G_n(0.12) - F_X(0.12) = 0.25 \tag{14}$$

$$G_n(0.78) - F_X(0.78) = 0.22 \tag{15}$$

$$G_n(0.51) - F_X(0.51) = 0.24 \tag{16}$$

Then

$$D_n = \sup(0.37, 0.25, 0.22, 0.24) = 0.37 \tag{17}$$

Given that,

$$P(D > 0.669) = 0.01 \tag{18}$$

Then

$$H_0 = \begin{cases} \text{accepted at level } 0.01 & \text{if } D_n \le 0.669 \\ \text{rejected at level } 0.01 & \text{if } D_n > 0.669 \end{cases} \tag{19}$$

From (17) and (19); We can say that H_0 is accepted at level 0.01 and

$$\psi = 1 \tag{20}$$

$\therefore$ the value will be

$$\psi + D_n = 1 + 0.37 = 1.37 \tag{21}$$