Probability Assignment 1

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Question : FInd the equations of the altitudes BE_1 and CF_1 to the sides AC and AB respectively.

Solution: Given that,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{1}$$

The direction vector of AC and line AC in vector form is given by

$$\mathbf{m_{AC}} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 \end{pmatrix} \tag{3}$$

The direction vector of AB and line AB in vector form is given by

$$\mathbf{m_{AB}} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \end{pmatrix} \tag{5}$$

The line passing through $\mathbf{B}$ and is perpendicular to AC (line BE_1)is given by

$$\mathbf{n_{BE_1}}^\top (\mathbf{x} - \mathbf{B}) = 0 \tag{6}$$

$$\implies \mathbf{n_{BE_1}}^\top \mathbf{x} = \mathbf{n_{BE_1}}^\top \mathbf{B} \tag{7}$$

Here, $\mathbf{n_{BE_1}}$ is the normal vector of line BE_1 and so it is parallel to AC and from (2)

$$\mathbf{n_{BE_1}} = \mathbf{m_{AC}} \tag{8}$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{9}$$

Hence the equation of BE_1 is

$$\begin{pmatrix} -4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -4 & -4 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{10}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2 \tag{11}$$

Similarly, The line passing through $\mathbf{C}$ and is perpendicular to AB (line CF_1)is given by

$$\mathbf{n_{CF_1}}^\top (\mathbf{x} - \mathbf{C}) = 0 \tag{12}$$

$$\implies \mathbf{n_{CF_1}}^\top \mathbf{x} = \mathbf{n_{CF_1}}^\top \mathbf{C} \tag{13}$$

Here, $\mathbf{n_{CF_1}}$ is the normal vector of line CF_1 and so it is parallel to AB and from (4)

$$\mathbf{n_{CF_1}} = \mathbf{m_{AB}} \tag{14}$$

$$= \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{15}$$

Hence the equation of CF_1 is

$$\begin{pmatrix} -5 & 7 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{16}$$

$$\implies \begin{pmatrix} -5 & 7 \end{pmatrix} \mathbf{x} = -20 \tag{17}$$