ASSIGNMENT: 11

Assignment:-

T. Karthikeya.
192372125

⑥ Big omega notation: Prove that $g(n) = n3 + 2n2 + 4n$ is $\Omega(n3)$.

Solution:-

$g(n) \geq C \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants Co and no

$n^3 + 2n^2 + 4n \geq Cn^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq C$

$1 + \frac{2n}{n} + \frac{4}{n^2} \geq C$

Here: $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + \frac{2}{n} + 4/n^2 + 1$

Example:- $C = 1/2$

$1 + 2/n + \frac{4}{n^2} \geq 1/2$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1/2 \quad (n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeded $\Omega(n^3)$

⑦ Big theta notation: Determine another $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ (or) not.

solution:-

$C_1 n^2 \leq h(n) \leq C_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n \Rightarrow C_2 n^2$

$4n^2+3n \le 5n^2$

Let's $c_2=5$

Divide and sides by n^2

$4+3/n \le 5$

Let's $c_2=5$ $(c_2=5, n_0=1)$

Lower bound:- $4n^2+3n \ge c_1 n^2$

Let's $c_1=4 \Rightarrow 4n^2+3n \ge 4n^2$

Divide both sides by n^2

$4+\frac{3}{n} \ge 4$

$h(n)=4n^2+3n$ $(\Theta(n^2))$.

(8) Let $f(n)=n^3-2n^2+n$ and $g(n)-n^2$ grow whether $f(n)=\Omega g(n)$ True (or) false.

Sol:-

$f(n) \ge c.g(n)$

Substituting $f(n)$ and $g(n)$ into this inequality are yet

$n^3-2n^2+n \ge c\cdot(n^2)$

find c and n_0 holds $n \ge n_0$

$n^3 \cdot 2n^3+n \ge 1-Cn^2$

$n^3-2n^2+n+Cn^2 \ge 0$

$n^3+(c-2)n^2+n \ge 0$

$n^3+(c-2)n^2+n \ge 0$ $(n^3 \ge 0)$

$n^3+(1-2)n^2+n = n^3-n^2+n \ge 0$ $(c=1)$

$f(n)=n^3-2n^2+n$ is $\Omega g(n) = \Omega g(-n^2)$

$\therefore$ the statement $f(n)=\Omega g(n)$ is true.

(a) Determine whether $f(n) = n \log n + n$ is in $O(n \log n)$
Prove a Rigorous Proof for your Conclusion.

Sol: $c_1 n \log n \le f(n) \le c_2 n \log n$

upper bound:

$$f(n) \le c_2 n \log n$$
$$f(n) = n \log n + n$$
$$n \log n + n \le c_1 n \log n$$

Divide both sides by $n \log n$.

$$1 + \frac{n}{n \log n} \le 2$$
$$1 + \frac{1}{\log n} \le c_2 \quad \text{(simplify)}$$
$$1 + \frac{1}{\log n} \le 2 \quad (c_2 = 2)$$

Then $f(n)$ is $O(n \log n)$ $(c_2 = 2,\ n_0 = 2)$

Lower bound:

$$f(n) \ge c_1 n \log n$$
$$f(n) = n \log n + n$$
$$n \log n + n \ge c_1 n \log n$$

Divide both sides by $n \log n$

$$1 + \frac{n}{n \cdot \log n} \ge c_1$$
$$1 + \frac{1}{\log n} \ge c_1 \quad \text{(simplify)}$$
$$1 + \frac{1}{\log n} \ge 1 \quad (c_1 = 1)$$

$f(n)$ is $\Omega(n \log n)$ $\{c_1 = 1,\ n_0 = 1\}$

$h(n) = n\log n + n$ is $O(n\log n)$.

(10) Solve the following Recurrence Relations and find the order of growth for solutions.

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Sol:-

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

Applying Master theorem.

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - 1})$ $\begin{cases} \epsilon > 0 \\ T(n) = \Theta(n^{\log_b a}) \end{cases}$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + 1})$, then $T(n) = f(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$ Case = 2

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$.