CSA0670 – DAA

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① Solve the following recurrence relation:

ⓐ $x(n) = x(n-1)+5$ for $n>1$ with $x(1)=0$.

Sol:- ① write down the first two terms to identify the pattern

$x(1)=0$
$x(2)=x(1)+5 = 5$
$x(3) = x(2)+5 = 10$
$x(4) = x(3)+5 = 15$

② Identify the pattern (or) the general term.
→ The first term $x(1)=0$
the common difference $d=5$
The general formula for the n^{th} term of an AP is
$x(n) = x(1)+x(n-1)d$
Substituting the give values
$x(n)=5(n-1)$

ⓑ $x(n) = 3x(n-1)$ for $n \geq 1$ with $x(1)=4$.
① write down the first two terms to identify pattern.
$x(1)=4$
$x(2) = 3x(1) = 4.$
$x(3) = 3x(2) = 36$ / $x(4) = 3x(3) = 108$

② identify the general terms.
The first term $x(1)=4$
the common ratio $r=3$.
The general formula for the n^{th} form of gp is..
$x(n) = x(1) \; r^{n-1}$
$x(n) = 4 \cdot 3^{n-1}$ {The solution is $x(n) = 4 \cdot 3^{n-1}$}

(c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ solve $(n = 2^n)$

- for $n = 2^k$, we can write recurrence in terms of k.

(1) Substitute $n = 2k$ in recurrence.

$$x(2^k) = x(2^{k-1}) + 2^k$$

(2) write down the first few terms to identify pattern.

$$x(1) = 1$$

$$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$$

$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$

$$x(8) = x(2^3) = x(4) + 8 = 9 + 8 = 15.$$

(3) Identify general term by finding the pattern we observe that:- $x(2^k) = x(2^{k-1}) + 2^k$

we sum the series : $x(2^k) = 2^k + 2^{k-1} + 2^{k+2} + \ldots$

Since sum the series:

$$x(2^k) = 2^k + 2^{k-1} + 2^{k+2} + \ldots$$

Since $a(1) = 1$:

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \ldots$$

The geometric series with the term $a = 2$ & the last term 2^k except for the additional +1 term.

-> The sum of geometric series. is with ratio $r = 2$ is...

given by..., $S = \frac{ar^n - 1}{r - 1}$

Here $a = 1$, $r = 2$ & $n = k$.

$$S = 2 \cdot \frac{2^k - 1}{2 - 1} = 2(2^{k-1}) = 2^{k+1}$$

adding the +1 term: $x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

Solution is:- $x(2^k) = 2^{k+1} - 1.$

(2)

(d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve $n = 14$)

for $n = 3^k$ we can write the recurrence in terms of it.

(1) substitute $n = 3^k$ in the recurrence.

(2) write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1) + 1 = 1+1 = 2.$

$x(9) = x(3^2) = x(3) + 1 = 2+1 = 3$

$x(27) = x(3^3) = x(9) + 1 = 3+1 = 4.$

(3) Identify the general term:

we observe that:

$x(3^k) = x(3^{k-1}) + 1$

Summing up the series

$x(3^k) = 1 + \cancel{1} + 1 + \dots + 1$

$x(3^k) = k+1$

the soln is $x(3^k) = k+1$

(2) Evalute following recurrence complexity.

(i) $T(n) = T(n/2) + 1$ where $n = 2k$ for all $k \geq 0$

The recurrence relation can be solved using iteration method.

(1) Substitute $n = 2^k$ in the recurrence.

(2) iterate the recurrence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(4) = T(2)+1 = T(1)+2.$

(3) general pattern:- $T(2^k) = T(1) + k$

since $n = 2^k$, $k = \log_2 n$ $\{T(n) = T(2^k) = T(1) + \log_2 n\}$

(ii) Assume $T(1)$ is a constant c.

$T(n) = c + \log_2 n$.

The solution is $T(n) = \theta[\log n]$

(iii) $T(n) = T(n/3) + T(2n/3) + n$ (where c is constant and n is input size.

The recurrence can be solved using the masters theorem for divide and conquer recurrence at the form.

$T(n) = aT(n/b) + f(n)$

where, $a = 2$, $b = 3$ and $f(n) = cn$.

let's determine the value of $\log_b a$.

$\log_b a = \log_3 2$

$\log_3 2 = \frac{\log 2}{\log 3}$

now we compare $f(n) = n$ with $n \cdot \log_3 2 \Rightarrow f(n) = O(n)$

$n = n'$

Since $\log_3 2$ we are in the third case of master's theorem.

$f(n) = O(n^e)$ with $\omega \log_b a$

The solution is... $T(n) = O(f(n)) = O(cn) = O(n)$.

(3) Consider the following recurrence algorithm:

```
min A[0...n-2]
if n=1 return A[0]
else temp = min1[A(0...n-2)]
        if temp <= A[n-1] return temp
else:
        Return A(n-1)
```

(a) what is this algorithm compute.

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(3) Sol:-

The given algorithm min (A[0...n-1]) computes the min value in the array 'A'.. form index 0 for n-1. it does this by recursively finding the minimum value in the sub array A[0,...n-2] and then comparing it with the last element A[n-1] to determine overall max size of value.

(b) Setup a recurrence relation for the given algorithm basic operation count and solve it..

The solution is $T(n)=n$

-> This means the algorithm performs n basic operations for an input array of size n.

(4) Analyze the order of growth.

(i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation.

To analyze the order of growth and use the Ω notation, we need to compare the given function f(n) and g(n)

given functions: $f(n) = 2n^2+5$

$g(n) = 7n$

order of growth using Ω g(n) notations.

-> The notation Ω g(n) describes a lower bound on the grown rate for sufficipency large. n, f(n) grows at least as for as g(n).

$f(n) \geq c \cdot g(n)$.

Let's analyze $f(n) = 2n^2+5$ with respect to $g(n) = 7n$.

(1) Identify Dominant terms:-

The dominant terms in $f(n) = 2n^2$ since it grows faster than the constant terms as n increases.

(2) Establish the inequality:

we want to find constant c & n_0 such that:

$$2n^2 + 5 \geq c \cdot 7n \text{ for all } n \geq n_0.$$

(3) simplify the inequality:-

Ignore the lower order term 5 for larger.

$$2n^2 \geq 7cn$$

Divide both sides by n.

$$2n \geq 7c$$

Solve for n:

$$n \geq 7c/2.$$

(4) Choose constant:

let $c = 1$

$$n \geq \frac{7 \cdot 1}{2} = 3.5$$

∴ for $n \geq n$, the inequality holds:

→ we have shown that there exist constants $c=1$ and $n=n$ such that for all $n \geq n_0$:-

$$2n^2 + 5 \geq 7n$$

Thus we can conclude that:

$$f(n) = 2n^2 + 5 = \Omega\, 7(n)$$

in Ω notation the dominant term $2n^2$ in $f(n)$ clearly grows faster then $f(n)$. Hence.

$$f(n) = \Omega(n^2)$$

* showing the $f(n)$ grows at least as fast as $7n$.