Engineering Applications of Stochastic Processes Notebook
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Summation:

In[•]:= $\sum_{i=1}^{\infty} \sum_{j=1}^{i} \frac{1}{j^2 (i+1)^2}$

Out[•]= $\frac{\pi^4}{120}$

In[•]:= $\sum_{i=1}^{\infty} \frac{1}{i^6}$

Out[•]= $\frac{\pi^6}{945}$

Integration:

In[•]:= $\int_0^{1500} y * 0.001 * \text{Exp}[-0.001 * y]\, dy + \int_{1500}^{\infty} 1500 * 0.001 * \text{Exp}[-0.001 * y]\, dy$

Out[•]= 776.87

Definite Integral:

In[•]:= `Integrate[1 / (x^3 + 1), {x, 0, 1}]`

Out[•]= $\frac{1}{18} \left(2 \sqrt{3}\, \pi + \text{Log}[64]\right)$

Indefinite Integral:

In[•]:= `Integrate[x^2 + Sin[x], x]`

Out[•]= $\frac{x^3}{3} - \text{Cos}[x]$

Binomial Random Variables:

In[•]:= $b[n\_, t\_, p\_] := \text{Binomial}[t, n] * p^n * (1-p)^{t-n}$

In[•]:= $b\left[6, 10, \frac{3.4}{5.8}\right]$

Out[•]= 0.249838

In[•]:= $B[n\_, t\_, p\_] := \sum_{i=0}^{n} \text{Binomial}[t, i] * p^i * (1-p)^{t-i}$

In[•]:= `B[1, 2, 0.2]`

Out[•]= 0.96

In[•]:= `b[0, 3, 0.2] / B[1, 3, 0.2]`

Out[•]= 0.571429

Negative Binomial Random Variables:

$b^{-1}[n, s, p]$

In[∘]:= `c[n_, s_, p_] := Binomial[s - 1, n - 1] * p^n * (1 - p)^(s-n)`

$$c[n\_, s\_, p\_] := \text{Binomial}[s-1, n-1] * p^n * (1-p)^{s-n}$$

In[∘]:= **c[4, 150, 0.05]**

Out[∘]= 0.0018886

$B^{-1}[n, s, p]$

In[∘]:=

$$d[n\_, s\_, p\_] := \sum_{i=n}^{s} \text{Binomial}[i-1, n-1] * p^n * (1-p)^{i-n}$$

In[∘]:= **d[3, 27, 0.12]**

Out[∘]= 0.644707

Single Sampling Plan:

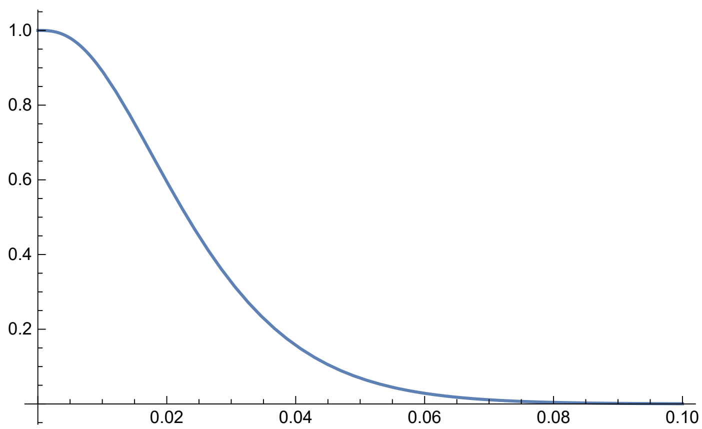
OC(p)= B(c,n,p)

In[∘]:= **B[2, 110, 0.03]**

Out[∘]= 0.355449

In[∘]:= **Plot[B[2, 115, p], {p, 0, 0.1}]**

Out[∘]=



Double Sampling Plans without Curtailment:

OC(p)= B[c1,n1,p]+ $\sum_{y=c1+1}^{c2} B[c2 - y, n2, p] * b[y, n1, p]$

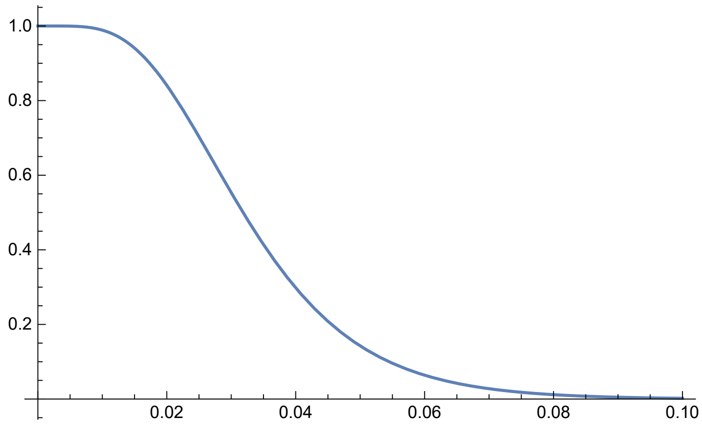
In[∘]:=

$$B[0, 40, 0.01] + \sum_{y=0+1}^{2} B[2-y, 80, 0.01] * b[y, 40, 0.01]$$

Out[∘]= 0.911506

In[•]:=
```
Plot[B[2, 100, p] + Sum[B[5 - y, 100, p] * b[y, 100, p], {y, 2+1, 5}], {p, 0, 0.1}]
```

$$\text{Plot}\left[B[2,100,p]+\sum_{y=2+1}^{5} B[5-y,100,p]*b[y,100,p],\ \{p,0,0.1\}\right]$$

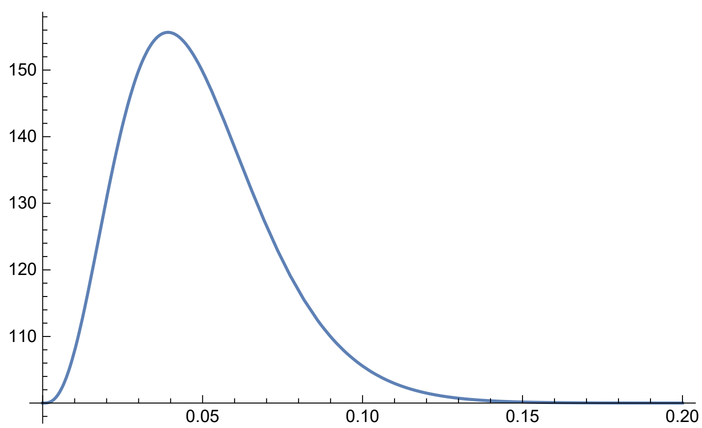
Out[•]=


ASN(p)= n1+n2*(B[c2,n1,p]-B[c1,n1,p])

In[•]:= $40 + 80 * (B[2, 40, 0.025] - B[0, 40, 0.025])$

Out[•]= 84.7055

In[•]:= $p1 = \text{Plot}\left[100 + 100\left(161\,700\,(1-p)^{97}\,p^{3} + 3\,921\,225\,(1-p)^{96}\,p^{4} + 75\,287\,520\,(1-p)^{95}\,p^{5}\right),\ \{p, 0, 0.2\}\right]$

Out[•]=


Double Sampling Plans with Curtailment:

OC(p)= B[c1,n1,p]+ $\sum_{y=c1+1}^{c2} B[c2-y,\ n2,\ p] * b[y,\ n1,\ p]$

In[•]:= $B[0, 40, 0.01] + \sum_{y=0+1}^{2} B[2-y, 80, 0.01] * b[y, 40, 0.01]$

Out[•]= 0.911506

ASN(p)=
$n1 + \sum_{y=c1+1}^{c2}\left(n2 * B[c2-y,\ n2,\ p] + \sum_{s=c2-y+1}^{n2} s * b^{-1}[c2-y+1,\ s,\ p]\right) * b[y,\ n1,\ p]$

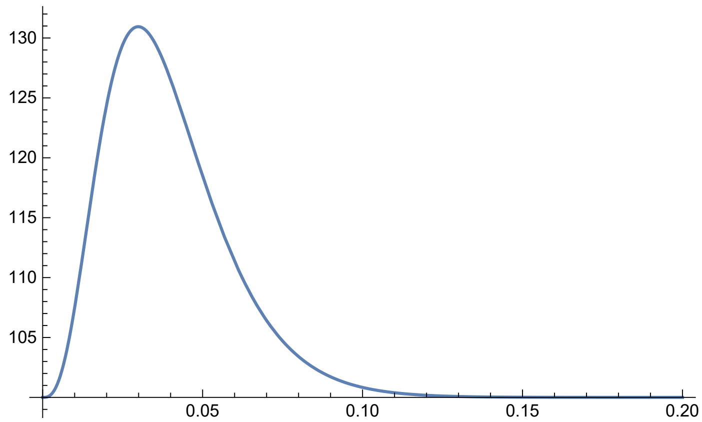
In[•]:= $40 + \sum_{y=0+1}^{2}\left(80 * B[2-y, 80, 0.025] + \sum_{s=2-y+1}^{80} s * c[2-y+1, s, 0.025]\right) * b[y, 40, 0.025]$

Out[•]= 68.3063

In[∘]:=
```
p2 =
  Plot[100 + Sum[(100 * B[5 - y, 100, p] + Sum[s * c[5 - y + 1, s, p], {s, 5-y+1, 100}]) * b[y, 100, p], {y, 2+1, 5}], {p, 0, 0.2}]
```

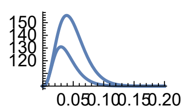
$$p2 = \text{Plot}\left[100 + \sum_{y=2+1}^{5}\left(100 * B[5-y, 100, p] + \sum_{s=5-y+1}^{100} s * c[5-y+1, s, p]\right) * b[y, 100, p], \{p, 0, 0.2\}\right]$$

Out[∘]=



In[∘]:= **Show[p1, p2]**

Out[∘]=



Single Sample Rectifying Inspection Plan

$$AOQ(p) = \frac{(m-n) \times p \times B[c, n, p]}{m}$$

In[∘]:=
$$\frac{(3000 - 100) \times 0.005 \times B[1, 100, 0.005]}{3000}$$

Out[∘]= 0.00439919

$$AFI(p) = \frac{n + (m-n) \times (1 - B[c, n, p])}{m}$$

In[∘]:=
$$\frac{110 + (2750 - 110) \times (1 - B[2, 110, 0.025])}{2750}$$

Out[∘]= 0.540006

Plotting AOQ function:

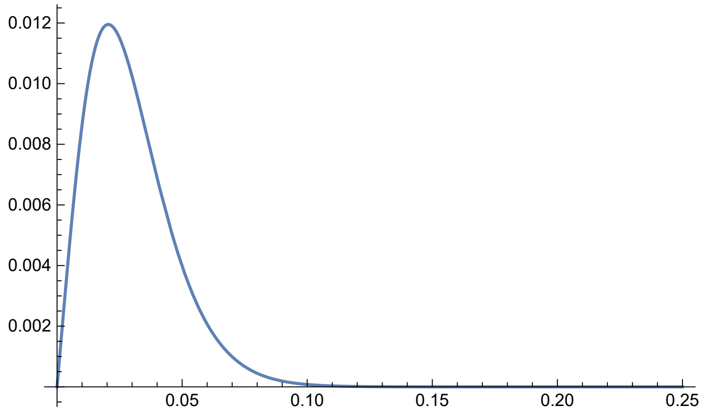
In[∘]:=
$$AOQ[m\_, n\_, p\_, c\_] := \frac{(m - n) \times p \times B[c, n, p]}{m}$$

AOQ[2750, 110, p, 2]

Out[∘]=
$$\frac{24}{25}\, p \left((1-p)^{110} + 110\,(1-p)^{109}\, p + 5995\,(1-p)^{108}\, p^2\right)$$

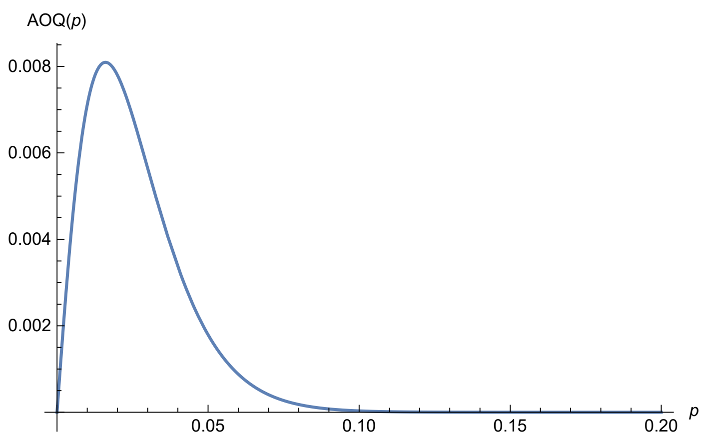
In[∘]:= Plot[$\frac{24}{25}$ p ((1 - p)^{110} + 110 (1 - p)^{109} p + 5995 (1 - p)^{108} p^2), {p, 0, 0.25}]

Out[∘]=



In[∘]:= Plot[$\frac{29}{30}$ p ((1 - p)^{100} + 100 (1 - p)^{99} p), {p, 0, 0.2}, AxesLabel → {p, AOQ[p]}]

Out[∘]=



In[∘]:= NMaximize[AOQ[2750, 110, p, 2], p]

Out[∘]= {0.0119518, {p → 0.0204855}}

Plot AFI Function:

In[∘]:= AFI[n_, m_, c_, p_] := $\frac{n + (m - n) \times (1 - B[c, n, p])}{m}$

In[∘]:= AFI[110, 2750, 2, 0.025]

Out[∘]= 0.540006

In[∘]:= AFI[110, 2750, 2, p]

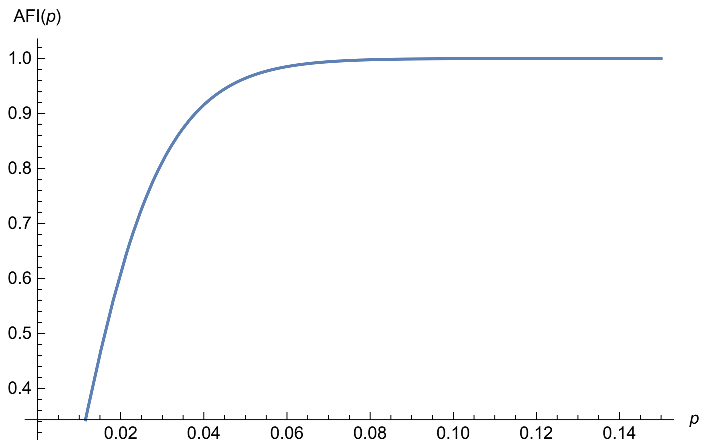
Out[∘]= $\frac{110 + 2640 \left(1 - (1 - p)^{110} - 110 (1 - p)^{109} p - 5995 (1 - p)^{108} p^2\right)}{2750}$

In[∘]:= NMaximize[AFI[110, 2750, 2, p], {p}]

Out[∘]= {1., {p → 1.15857}}

In[ ]:= $$\text{Plot}\left[\frac{100+2900\left(1-(1-p)^{100}-100\,(1-p)^{99}\,p\right)}{3000},\ \{p,0,0.15\},\ \text{AxesLabel}\to\{p,\text{AFI}[p]\}\right]$$

Out[ ]=



Continuous Sampling Plan(CSP-1)

$u(p)=\frac{\left(1-(1-p)^{i}\right)}{p\times(1-p)^{i}}$; $v(p)=\frac{1}{f\times p}$

$$\frac{\left(1-(1-p)^{i}\right)}{p\times(1-p)^{i}}$$

$$\frac{1}{f\times p}$$

$\text{AOQ}\ (p)\ =\ \frac{(1-f)\times p\times\frac{1}{f\times p}}{\frac{(1-(1-p)^{i})}{p\times(1-p)^{i}}+\frac{1}{f\times p}}$; $\text{AFI}\ (p)\ =\ \frac{\frac{(1-(1-p)^{i})}{p\times(1-p)^{i}}+f\times\frac{1}{f\times p}}{\frac{(1-(1-p)^{i})}{p\times(1-p)^{i}}+\frac{1}{f\times p}}$

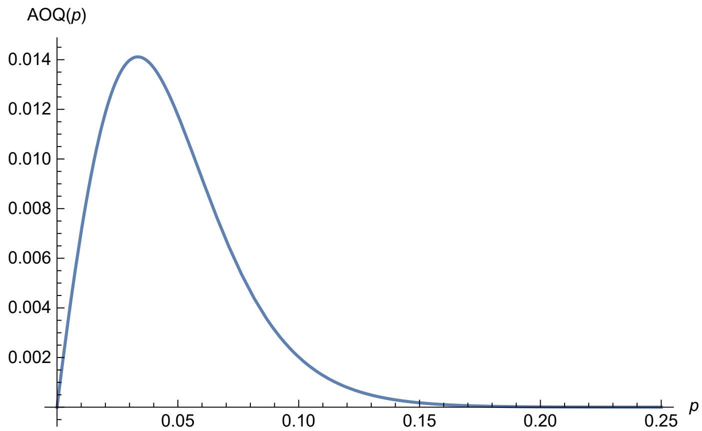
In[ ]:= $$\frac{(1-0.2)\times p\times\frac{1}{0.2\times p}}{\frac{(1-(1-p)^{50})}{p\times(1-p)^{50}}+\frac{1}{0.2\times p}}$$

Out[ ]= $$\frac{4.}{\frac{5.}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}$$

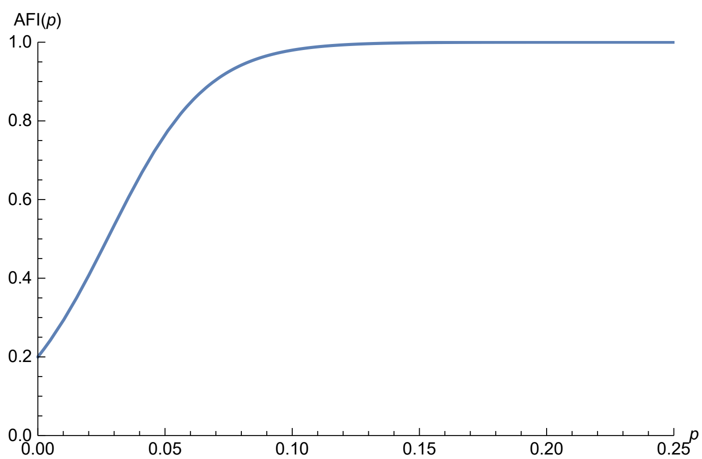
In[ ]:= $$\frac{\frac{(1-(1-p)^{50})}{p\times(1-p)^{50}}+0.2\times\frac{1}{0.2\times p}}{\frac{(1-(1-p)^{50})}{p\times(1-p)^{50}}+\frac{1}{0.2\times p}}$$

Out[ ]= $$\frac{\frac{1.}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}{\frac{5.}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}$$

In[ ]:= Plot$\left[\dfrac{4.\grave{}}{\frac{5.\grave{}}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}, \{p, 0, 0.25\}, \text{AxesLabel} \to \{p, \text{AOQ}[p]\}\right]$

Out[ ]=


In[ ]:= Plot$\left[\dfrac{\frac{1.\grave{}}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}{\frac{5.\grave{}}{p}+\frac{1-(1-p)^{50}}{(1-p)^{50}\,p}}, \{p, 0, 0.25\}, \text{PlotRange} \to \{\{0, 0.25\}, \{0, 1\}\}, \text{AxesLabel} \to \{p, \text{AFI}[p]\}\right]$

Out[ ]=


Poisson Process:

Pr(N(t)=n)= p[n_, λt_] = $\frac{e^{-\lambda t}\times(\lambda t)^n}{n!}$

In[ ]:= p[n_, λt_] := $\dfrac{e^{-\lambda t}\times(\lambda t)^n}{n!}$

In[ ]:= p[4, 0.005 * 750]

Out[ ]= 0.19378

In[ ]:= N$\left[\dfrac{9}{2\,e^3}, 2\right]$

Out[ ]= 0.22

Pr(N(t)<=n)= $P[n\_, \lambda t\_] = \sum_{i=0}^{n} \frac{e^{-\lambda t} \times (\lambda t)^{i}}{i!}$

In[ ]:= $P[n\_, \lambda t\_] := \sum_{i=0}^{n} \frac{e^{-\lambda t} \times (\lambda t)^{i}}{i!}$

In[ ]:= P[3, 0.005 * 750]

Out[ ]= 0.483767

In[ ]:= $N\left[\frac{899}{5 e^{6}}, 2\right]$

Out[ ]= 0.45

In[ ]:= $N\left[1 - \frac{2101}{7 e^{6}}, 2\right]$

Out[ ]= 0.26

Modelling Arrival Times:

Pr(S(n)<=s)= $1 - \sum_{i=0}^{n-1} \frac{e^{-\lambda s} \times (\lambda s)^{i}}{i!}$

In[ ]:= $Q[n\_, \lambda s\_] := 1 - \sum_{i=0}^{n-1} \frac{e^{-\lambda s} \times (\lambda s)^{i}}{i!}$

Q[3, 1.2 * 2]

Out[ ]= 0.430291

Pr(S(n)>s) = $\sum_{i=0}^{n-1} \frac{e^{-\lambda s} \times (\lambda s)^{i}}{i!}$

In[ ]:= $R[n\_, \lambda s\_] := \sum_{i=0}^{n-1} \frac{e^{-\lambda s} \times (\lambda s)^{i}}{i!}$

R[1, 5.8 * 0.1]

Out[ ]= 0.559898

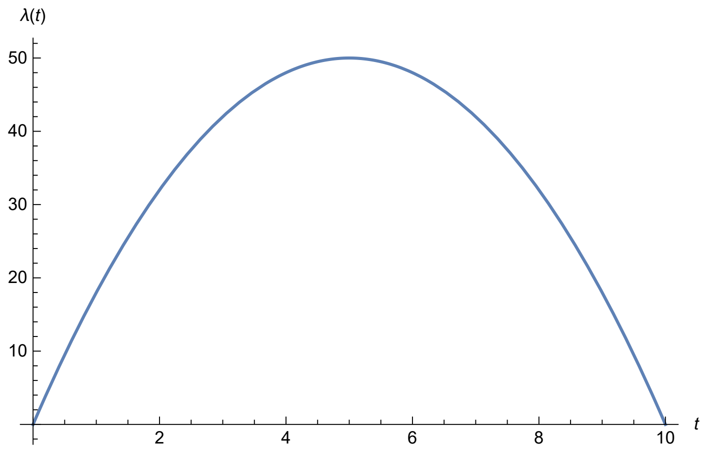
Competing Poisson Process:

$\sum_{i=n}^{n+m-1} b\left[i, n+m-1, \frac{\lambda 1}{\lambda 1+\lambda 2}\right]$

In[ ]:= $\sum_{i=8}^{8+4-1} b\left[i, 8+4-1, \frac{3.4}{3.4+1.1}\right]$

Out[ ]= 0.728482

In[]:=
```
Plot[50 - 2 (t - 5)^2, {t, 0, 10}, AxesLabel → {t, λ[t]}]
```

Out[]=


Non-Homogeneous Poisson Process

$\Lambda(t) = \int_0^t \lambda(u)\, du$

In[]:= $\int_0^4 \left(50 - 2 (u - 5)^2\right) du$

Out[]= $\frac{352}{3}$

In[]:= $N\left[\frac{352}{3}\right]$

Out[]= 117.333

In[]:= $\int_4^6 (t + 4)\, dt + \int_6^9 (22 - 2 t)\, dt$

Out[]= 39

CFR Repairable System Models:

Continuous-time Markov Chain Model:

Availability Function,

In[]:= $A[\mu\_, \lambda\_, t\_] := \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} * e^{-(\lambda+\mu)\, t}$

```
A[1/50, 1/400, 120]
```

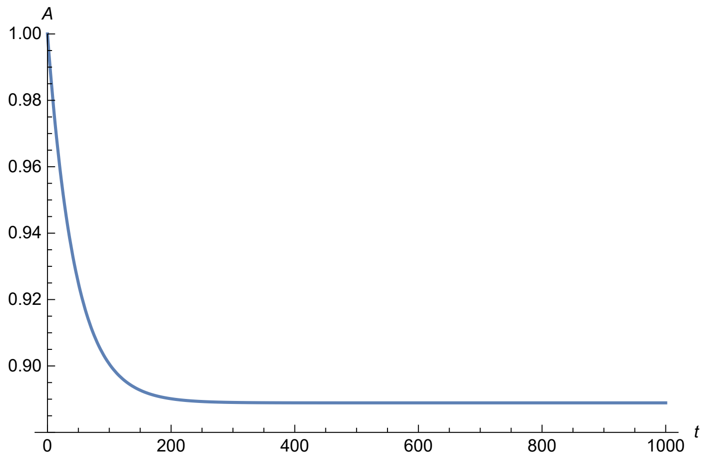
Out[]= $\frac{8}{9} + \frac{1}{9\, e^{27/10}}$

In[]:= $N\left[\frac{8}{9} + \frac{1}{9\, e^{27/10}}, 3\right]$

Out[]= 0.896

In[∘]:= Plot$\left[\frac{8}{9} + \frac{1}{9} e^{-9\,t/400}, \{t, 0, 1000\}, \text{PlotRange} \to \{0.88, 1\}, \text{AxesLabel} \to \{t, A\}\right]$

Out[∘]=



In[∘]:= N$\left[\frac{25}{26} + \frac{1}{26\, e^{39/25}}, 3\right]$

Out[∘]= 0.970

Average Availability Function,

In[∘]:= $A_{avg}[\mu\_, \lambda\_, t\_] := \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 * t} * \left(1 - e^{-(\lambda+\mu)\,t}\right)$

$A_{avg}[1/50, 1/400, 120]$

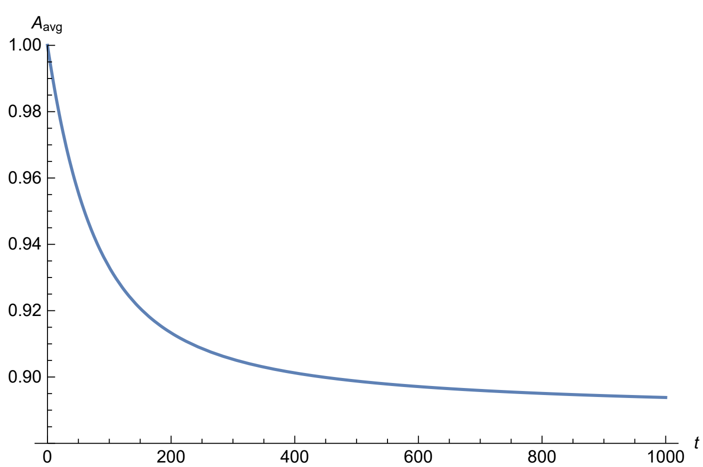
Out[∘]= $\frac{8}{9} + \frac{10}{243}\left(1 - \frac{1}{e^{27/10}}\right)$

In[∘]:= N$\left[\frac{8}{9} + \frac{10}{243}\left(1 - \frac{1}{e^{27/10}}\right), 3\right]$

Out[∘]= 0.927

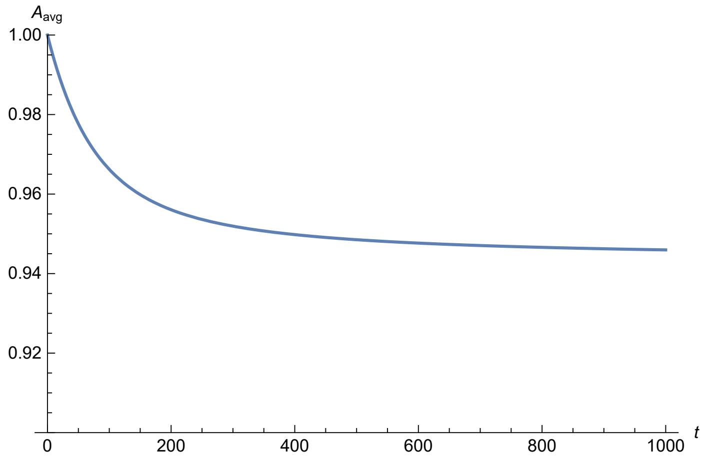
In[∘]:= Plot$\left[\frac{8}{9} + \frac{400\left(1 - e^{-9\,t/400}\right)}{81\,t}, \{t, 0, 1000\}, \text{PlotRange} \to \{0.88, 1\}, \text{AxesLabel} \to \{t, A_{avg}\}\right]$

Out[∘]=



In[ ]:=
```
Plot[50/53 + (7200 (1 - e^(-53 t/2400)))/(2809 t), {t, 0, 1000}, PlotRange → {0.90, 1}, AxesLabel → {t, A_avg}]
```

Out[ ]=



In[ ]:= $N\left[\frac{25}{26} + \frac{25\left(1 - \frac{1}{e^{39/25}}\right)}{1014}, 2\right]$

Out[ ]= 0.98

Age Replacement Model:

Availability based on optimal age-based PM policy:

$$A[\tau\_] := \frac{\int_0^\tau (t * f[t]\, dt) + (\tau * R[\tau])}{\int_0^\tau (t * f[t]\, dt) + (\tau * R[\tau]) + (D_{CM} * F[\tau] + D_{PM} * R[\tau])}$$

In[ ]:= $R[t\_] := Exp\left[-(t/\eta)^\beta\right];$

$F[t\_] := 1 - R[t];$

$f[t\_] := F'[t];$

$\beta = 2.15;\ \eta = 1600;\ D_{CM} = 180;\ D_{PM} = 12;$

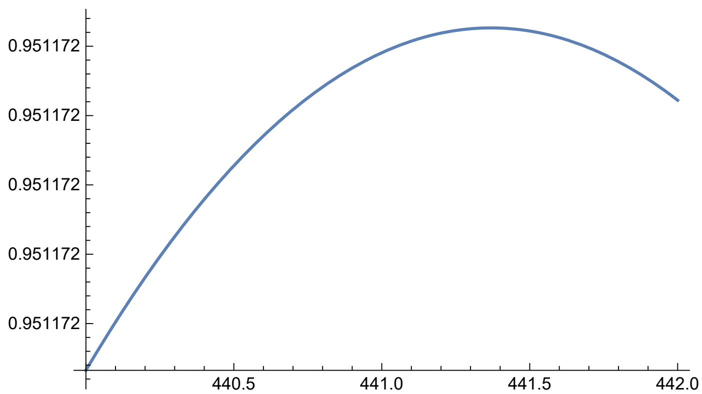
$$A[\tau\_] := \frac{\int_0^\tau (t * f[t]\, dt) + (\tau * R[\tau])}{\int_0^\tau (t * f[t]\, dt) + (\tau * R[\tau]) + (D_{CM} * F[\tau] + D_{PM} * R[\tau])}$$

In[ ]:= A[441.4]

Out[ ]= 0.951172

In[∘]:=
```
Plot[(1416.9734246232488` + E^(-1.2916416397452518`*^-7 τ^2.15`) τ -
    (1599.999999999998` τ^3.15` Gamma[1.4651162790697674`, 1.2916416397452518`*^-7 τ^2.15`]) /
    (τ^2.15`)^1.4651162790697674`) /
  (1416.9734246232488` + 12 E^(-1.2916416397452518`*^-7 τ^2.15`) +
    180 (1 - E^(-1.2916416397452518`*^-7 τ^2.15`)) + E^(-1.2916416397452518`*^-7 τ^2.15`) τ -
    (1599.999999999998` τ^3.15` Gamma[1.4651162790697674`, 1.2916416397452518`*^-7 τ^2.15`]) /
    (τ^2.15`)^1.4651162790697674`),
  {τ, 440, 442}]
```

Out[∘]=



In[∘]:=
```
Gamma[(1/2.15) + 1]
```

Out[∘]= 0.885608