Q2

a $x^2+y^2=1 \qquad u = \dfrac{(ax^2+by^2)}{\sqrt{a^2x^2+b^2y^2}}$

$y^2 = 1-x^2$

$\therefore \quad u = \dfrac{ax^2+b\,(1-x^2)}{\sqrt{a^2x^2+b^2(1-x^2)}}$

$u \Rightarrow \dfrac{(a-b)x^2+b}{\sqrt{(a^2-b^2)\,x^2+b^2}}$

Differentiation of this

$$\frac{du}{dx} = \frac{\sqrt{(a^2-b^2)x^2+b^2} * \frac{d}{dx}\left[(a-b)x^2+b\right] - \left[(a-b)x^2+b\right] * \frac{d}{dx}\sqrt{(a^2-b^2)x^2+b^2}}{\left(\sqrt{(a^2-b^2)x^2+b^2}\right)^2}$$

$$= \frac{\sqrt{(a^2-b^2)x^2+b^2} * \left(((a-b)2x+0)\right) - \left[(a-b)x^2+b\right] * \left[\frac{(a^2-b^2)2x+0}{2\sqrt{(a^2-b^2)x^2+b^2}}\right]}{(a^2-b^2)x^2+b^2}$$

$$= \frac{2\left[(a^2-b^2)x^2+b^2\right] * (a-b)2x - \left[(a-b)x^2+b)\right] * (a^2-b^2)2x}{2\left((a^2-b^2)x^2+b^2\right)^{1/2} * \left[(a^2-b^2)x^2+b^2\right]}$$

$$= \frac{2x(a-b)\left\{\left[2\left[(a^2-b^2)x^2+b^2\right]\right] - \{(a-b)x^2+b\}(a+b)\right]}{2\left[(a^2-b^2)x^2+b^2\right]^{3/2}}$$

$$= \frac{2x(a-b)\left\{2\left[(a^2-b^2)x^2+b^2\right] - \left\{a^2-b^2\ x^2 + ab + b^2\right\}\right\}}{2\left[(a^2-b^2)x^2+b^2\right]^{3/2}}$$

$$= \frac{2x(a-b)\left\{2(a^2-b^2)x^2+2b^2 - (a^2-b^2)x^2 - ab - b^2\right\}}{2\left((a^2-b^2)x^2+b^2\right)^{3/2}}$$

$$= \frac{\cancel{2}x(a-b)\left\{(a^2-b^2)x^2 - ab + b^2\right\}}{\cancel{2}\left[(a^2-b^2)x^2+b^2\right]^{3/2}}$$

$$= \frac{x(a-b)\left\{[(a+b)(a-b)]x^2 - b(a-b)\right]}{\left[(a^2-b^2)x^2+b^2\right]^{3/2}}$$

$$= \frac{x(a-b)^2\left[(a+b)x^2 - b\right]}{\left[(a^2-b^2)x^2 + b^2\right]^{3/2}}$$

for $\frac{du}{dx} = 0$

$$\frac{x(a-b)^2\left[(a+b)x^2 - b\right]}{\left[(a^2-b^2)x^2 + b^2\right]^{3/2}} = 0$$

$$x(a-b)^2\left[(a+b)x^2 - b\right] = 0$$

$$(a+b)x^2 - b = 0$$

$$x^2(a+b) = b$$

$$x^2 = b/a+b$$

since

$$y^2 = 1 - x^2$$

$$y^2 = 1 - b/a+b$$

$$y^2 = a/a+b$$

Thus $x^2 = b/_{a+b}$

$$y^2 = \frac{a}{a+b}$$

substituting this in $u = \dfrac{ax^2+by^2}{\sqrt{a^2x^2+b^2y^2}}$

$$u = \frac{a \times (b/_{a+b}) + b\,(a/_{a+b})}{\sqrt{a^2 (b/_{a+b}) + b^2 (a/_{a+b})}}$$

$$= \frac{\frac{ab}{a+b} + \frac{ab}{a+b}}{\sqrt{\frac{a^2b + ab^2}{a+b}}} = \frac{\frac{2ab}{a+b}}{\sqrt{\frac{ab\cancel{(a+b)}}{\cancel{(a+b)}}}}$$

$$= \frac{2ab}{(a+b)\sqrt{ab}} = \frac{2(ab)(ab)^{-1/2}}{(a+b)}$$

$$\boxed{u = \frac{2\sqrt{ab}}{a+b}}$$

Now substitating $a = 1$ $b = 1$

$$u = \frac{2\sqrt{1 \times 1}}{1+1} = \frac{2 \times 1}{2} = 1$$

$a = 1$ $b = 2$

$$u = \frac{2\sqrt{1 \times 2}}{1+2} = \frac{2\sqrt{2}}{3}$$

$$u = 0.942809$$

$a = 2$ $b = 2$

$$u = \frac{2\sqrt{2 \times 2}}{2+2} = \frac{\cancel{2}\sqrt{4}}{\cancel{4}_2} = \frac{2}{2} = 1$$

Q2

2) Lagrange multiplier

$$\nabla f = \lambda \nabla g$$

$u = (x+a)(y+b)(z+c)$ considered as f

$xyz - k^3$ is considered g

$\nabla f_x = (y+b)(z+c)$

$\nabla g_x = yz$

$$\boxed{\begin{gathered}\nabla f_x = \lambda \nabla g_x \\ (y+b)(z+c) = \lambda yz \quad (1)\end{gathered}}$$

$\nabla f_y = (x+a)(z+c)$

$\nabla g_y = xz$

$$\boxed{\begin{gathered}\nabla f_y = \lambda \nabla g_y \\ (x+a)(z+c) = \lambda xz \quad (2)\end{gathered}}$$

$\nabla f_z = (x+a)(y+b)$

$\nabla g_z = xy$

$$\boxed{\begin{gathered}\nabla f_z = \lambda \nabla g_z \\ (x+a)(y+b) = \lambda xy \quad (3)\end{gathered}}$$

Multiplying ① with $(x+a)$

② with $(y+b)$

③ with $(z+c)$

$$\cancel{\lambda} yz(x+a) = \cancel{\lambda} xz(y+b) = \cancel{\lambda} xy(z+c)$$

$$xyz + ayz = xyz + bxz = xyz + cxy$$

$$ayz = bxz = cxy$$

$$y = \frac{b}{a} x \qquad z = \frac{c}{a} x$$

$$xyz = k^3$$

$$(x) * \left(\frac{b}{a} x\right) * \left(\frac{c}{a} x\right) = k^3$$

$$\frac{x^3 bc}{a^2} = k^3 \qquad x^3 = \frac{k^3 a^2}{bc}$$

$$x^3 = \frac{K^3 a^2 \times a}{abc}$$

$$x = \frac{ka}{\sqrt[3]{abc}}$$

$$y = \frac{kb}{\sqrt[3]{abc}}$$

$$z = \frac{kc}{\sqrt[3]{abc}}$$

$x = a/_b\, y \quad z = c/_b\, y$

$$xyz = k^3$$

$$\frac{a}{b} y * y * \frac{c}{b} y = k^3$$

$$y^3 = \frac{k^3 b^2}{ac}$$

$$y^3 = \frac{k^3 b^2 \times b}{ac \times b}$$

$$y^3 = \frac{k^3 b^3}{abc}$$

$$y = \frac{kb}{\sqrt[3]{abC}}$$

similiarly $z = \dfrac{kc}{\sqrt[3]{abc}}$

putting x y z in ~~xyz = k^3~~ u

$$u = \left[\left(\frac{ka}{\sqrt[3]{abc}} + a\right) * \left(\frac{kb}{\sqrt[3]{abc}} + b\right) * \left(\frac{kc}{\sqrt[3]{abc}} + c\right)\right]$$

$$= \left[\frac{\left(ka + a\sqrt[3]{abc}\right)}{\sqrt[3]{abc}} \frac{\left(kb + b\sqrt[3]{abc}\right)}{\sqrt[3]{abc}} \frac{\left(kc + c\sqrt[3]{abc}\right)}{\sqrt[3]{abc}}\right]$$

$$= \frac{\left[k^2ab + kab\sqrt[3]{abc} + kab\sqrt[3]{abc} + ab\left(\sqrt[3]{abc}\right)^2\right] * \left[kc + c\sqrt[3]{abc}\right]}{abc}$$

$$= \frac{\left[k^2ab + 2kab\sqrt[3]{abc} + ab\left(\sqrt[3]{abc}\right)^2\right]\left[kc + c\sqrt[3]{abc}\right]}{abc}$$

$$= \frac{k^3 abc + k^2 abc \sqrt[3]{abc} + 2k^2 abc \sqrt[3]{abc} + 2k\, abc \left(\sqrt[3]{abc}\right)^2 + k\, abc \left(\sqrt[3]{abc}\right)^2 + abc\left(\sqrt[3]{abc}\right)^3}{abc}$$

$$= abc * \left[\frac{k^3 \cancel{abc} + k^2 \sqrt[3]{abc} + 2k^2 \sqrt[3]{abc} + 3k\left(\sqrt[3]{abc}\right)^2 + \left(\sqrt[3]{abc}\right)^3}{abc} \right]$$

$$= \frac{abc \left(k + \sqrt[3]{abc}\right)^3}{abc}$$

$$= \boxed{abc \left[\frac{k}{\sqrt[3]{abc}} + 1 \right]^3}$$