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UAVs Trajectory Planning by Distributed MPC under Radio Communication Path Loss Constraints

Alexandra Grancharova · Esten Ingar Grøtli · Dac-Tu Ho · Tor Arne Johansen

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Abstract In this paper, a distributed linear MPC approach to solve the trajectory planning problem for rotary-wing UAVs is proposed, where the objective of the UAV system is to form a communication network to multiple targets with given radio communication capacities. The approach explicitly incorporates constraints on radio communication path losses, computed by using SPLAT! that is able to take into account terrain models and antenna locations. In order to enhance the online optimization, at each time sample the terrain below each UAV and the communication path losses are approximated with linear functions of the spatial coordinates. This leads to linear MPC subproblems, which are solved by using convex quadratic programming. An algorithm for automatic initialization and optimal reconfiguration of the communication topology in case of failures or severe radio path loss due to e.g. channel fading, is proposed. The communication network that is provided by the UAVs is considered to be a payload communication capacity that is normally independent of the command and control datalink used to control the UAVs. The performance of the distributed linear MPC trajectory planning and the reconfiguration algorithm is studied on two simulation cases with four UAVs and two targets.

Keywords Autonomous vehicles · Radio-communication · Trajectory planning · Predictive control · Distributed control · Constraints · Multi-vehicle coordination

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A. Grancharova (✉)
Institute of System Engineering and Robotics, Bulgarian Academy of Sciences, Acad G. Bonchev str., Bl. 2, P. O. Box 79, Sofia 1113, Bulgaria
e-mail: alexandra.grancharova@abv.bg

A. Grancharova
Department of Industrial Automation, University of Chemical Technology and Metallurgy, 8 Sv. Kliment Ohridski Blvd., Sofia 1756, Bulgaria

E. I. Grøtli · D.-T. Ho · T. A. Johansen
Department of Engineering Cybernetics, Norwegian University of Science and Technology, O.S. Bragstads plass 2D, 7491 Trondheim, Norway

E. I. Grøtli
SINTEF ICT, Applied Cybernetics, PO 4760 Sluppen, 7465 Trondheim, Norway
e-mail: EstenIngar.Grotli@sintef.no

D.-T. Ho
MARINTEK, Otto Nielsens veg 10, 7052 Trondheim, Norway
e-mail: TuDac.Ho@marintek.sintef.no

T. A. Johansen
Center for Autonomous Marine Operations and Systems (AMOS), Department of Engineering Cybernetics, Norwegian University of Science and Technology, O.S. Bragstads plass 2D, 7491 Trondheim, Norway
e-mail: Tor.Arne.Johansen@itk.ntnu.no

1 Introduction

This research is motivated by emerging applications of Unmanned Aerial Vehicles (UAVs) in long range multivehicle missions in areas without a permanent communication infrastructure. The focus is on both the trajectory planning problem and the communication topology reconfiguration problem. The problem of trajectory planning for UAVs has been considered in several papers. Thus, in [20–22], a Mixed-Integer Linear Programming approach for mission and trajectory planning for UAVs under radio communication constraints is developed. In [4], relay chains are generated solving the all hops optimal path graph search problem. Location and movement of UAVs are optimized in [23], in order to improve the connectivity of a wireless network. In [7], a decentralized mobility control algorithm for an optimal end-to-end communication chain using a team of unmanned aircrafts acting solely as communication relays is presented. In [37], an optimization-based approach to the deployment of an ad-hoc wireless network between UAVs is proposed, which decomposes into two components, one for position optimization and one for communication optimization, coupled via a set of Lagrange multipliers. In [47], trajectory planning and trajectory finding algorithms for multiple UAVs are studied. In [45], a framework based on a low-dimensional abstraction is developed that enable a group of aerial robots to maintain a formation while avoiding collisions. In [14], a novel approach for multi-UAV target search using a decentralized gradient-based optimization is proposed, where the gradient is explicitly formulated. Two different trajectory planning strategies for UAVs in order to gather data in a wireless sensor network are presented and compared in [26].

Several papers have also been devoted to communication topology reconfiguration or similar problems. In [49], the problem on how to achieve reconfiguration from arbitrary initial connected formation to arbitrary final connected formation, while maintaining connectivity during reconfiguration, is considered. Two biology-inspired techniques, a Flocking Algorithm (FA) and a Particle Swarm Optimization (PSO) algorithm, are used in [56] to control a hierarchical heterogeneous wireless network to adapt to changing environments and minimize the systems energy consumption. In [44], the focus is on graph theoretic methods for the analysis and synthesis of dynamic multiagent networks. In [53], the problem of maintaining connectedness of a network while undergoing some reconfiguration maneuver is addressed. A measure is defined which quantifies the freedom individual vehicles have, to undergo arbitrary motion without disconnecting the network. A tutorial overview of information consensus in multivehicle cooperative control is given in [50]. Theoretical results regarding consensus-seeking under both time invariant and dynamically changing communication topologies are provided. An overview over research in cooperative control of multivehicle systems covering formation control, cooperative tasking, spatiotemporal planning and consensus algorithms, can be found in [48]. The paper [46] deals with directional wireless networks and use topology control to assure robust end-to-end broadband connectivity. Heuristic algorithms for topology reconfiguration both from the physical- and network layer perspective are derived and analyzed with respect to computational complexity. In [31] formation reconfiguration for fixed-wing UAVs in case of failure is considered. Two optimization problems are solved, one for the escape maneuver with the objective of reducing the possibility for collision, and one for the replacement movement for an alternative UAV. A decentralized navigation function is used in [28] to ensure that an initially connected network asymptotically converges to the desired configuration while maintaining network connectivity. In [6], a bounded distributed control law is presented, that guarantees connectivity maintenance in a network of multiple mobile agents. In [11], an architecture and algorithms for maintaining end-to-end network connectivity for autonomous robots are proposed.

Model Predictive Control (MPC) is an optimization-based method for control that can handle state and input constraints [42]. In the case of direct control, there is no trajectory planned in advance and the objective usually consists in minimizing fuel consumption, while meeting the overall mission objectives. In [8], a distributed MPC approach to multi-vehicle formation stabilization is proposed in which each subsystem is assigned its own optimal control problem, optimizes only for its own control at each update, and exchanges information only with neighboring subsystems. In [33], a decentralized MPC approach for trajectory planning for cooperative UAVs in the presence of bounded disturbances is developed. In [18], a distributed nonlinear

MPC-based trajectory planning approach for a fixed-wing UAVs communication chain is proposed (a single target is assumed), which explicitly incorporates constraints on radio communication path losses. In [19], the distributed MPC-based trajectory planning problem for a group of UAVs is solved by applying the dynamic dual decomposition method [15]. In [52], centralized and decentralized MPC methods to solve a re-planning problem are compared.

For an extensive overview over path- and motion planning techniques the interested reader is referred to [35]. A more recent survey specific for motion planning of UAVs is found in [16], where as [55] covers cooperative planning of UAVs. We will in the following summarize some of the most popular methods for vehicle motion planning. One popular technique is to represent the environment as a graph consisting of a set of possible locations and a set of edges which represent the transition between these locations. Then, motion planning can be cast as a search problem on this graph. Classical algorithms includes Dijkstras's algorithm [5] and A* [24]. These algorithms return the optimal path [13], and can be considered special cases of dynamic programming [3], see [10]. Several extensions to these algorithms have been developed. For instance were D* [54] and D* lite [32] developed to avoid complete replanning due to changes in the environment, where as different anytime algorithms such as Anytime Repairing A* [39] and Anytime Dynamic A* [38] were developed to construct an initial, possibly highly suboptimal, solution very quickly, and then improve quality if time is permitted, [10]. Graph search algorithms can be applied to terminal constraint optimization problems and set point control problems, [9]. By the way the environment is partitioned this class of methods is also often known as cell decomposition methods, [12]. Sampling based algorithms is another class of popular motion planning methods, and which aim was initially to find collision-free solutions, rather than an optimal path, [27]. One of the most popular sampling based methods is Rapidly-Exploring Random Tree (RRT) introduced in [36]. In RRT a random sequence of samples is used to incrementally construct a search tree that gradually improves the resolution. This method has later been extended in various ways, for instance to achieve minimal work paths [27] or to make use of a low-level controller to improve vehicle safety and disturbance rejection in real-time applications, [34]. In artificial potential field methods the movement of a vehicle is defined by the negative gradient of a potential function. This function provide attractive forces towards the goal, and repulsive forces away from obstacles, [12]. This way of using potential functions in robotics was proposed by [30]. Later this method has been modified in for instance [1] to include random walks to escape local minima. Particle Swarm Optimization (PSO) is another method that has successfully been applied for optimal path planning of robotic vehicles, see for instance [51] and [25]. PSO was developed in [29] and is a computational method that iteratively tries to improve a swarm of candidate solutions or particles, based on an user defined objective function.

In [16] the necessity of knowing the requirements for a particular application is pointed out. Based on our application, a receding horizon based trajectory planner is chosen over the other methods for the following reasons:

- Sensory information can be included into the online computation to account for uncertainties, [16]. In [2, p. 206] they discriminate between two different approaches to motion planning: "*deliberative* motion planning, where explicit paths and trajectories are computed based on global world knowledge, and *reactive* motion planning, which uses behavioural methods to react to local sensor information". In that respect our method falls in the second category and serves well for improving robustness towards inaccuracies in the model and uncertainties that become apparent in real time.
- A goal state or configuration need not to be known in advance for the method to work. In fact, due to the dynamically changing environment, one single goal configuration does not exist. It should be mentioned however, that a cost-to-go function is usually required to ensure completeness and optimality, [16]. An algorithm is considered *complete* if it for any input correctly reports whether there is a solution in a finite amount of time, [36].
- Sampling algorithms normally sample the output space, and it can be challenging to find the appropriate input which takes the vehicle from the current position to the calculated next sample, [9]. Receding horizon based planner finds the input sequence which minimizes the objective over the length of the horizon.

- Receding horizon based path planning provides the possibility to define hard constraints. Some of the other path- and motion planning methods does not easily handle hard inequality constraints. Instead, dissatisfied constraints are penalized in the cost function. This makes these methods hard to tune, because of the many and often competing objectives that must be weighted appropriately.
- Finally, many of the other path- and motion planning methods are not easily modified to allow for cooperation and coordination between multiple vehicles.

In this paper, a distributed linear MPC approach to solve the trajectory planning problem for rotary-wing UAVs is proposed, where the objective of the UAV system is to form a communication network to multiple targets with given radio communication capacities. The communication network is considered to be a payload communication capacity that is normally independent of the command and control datalink used to control the UAVs. The proposed distributed MPC approach explicitly incorporates constraints on radio communication path losses, computed by using the state-of-the-art simulation software SPLAT! [41]. SPLAT! is able to take into account terrain models and antenna locations and thus provides realistic prediction of radio transmission path losses. An algorithm for automatic initialization and optimal reconfiguration of the communication topology in case of failures or severe radio path loss due to e.g. channel fading, is developed. It leads to an autonomous optimization of UAV resources in order to establish the required communication network capacity. Compared to [18, 19], the communication topology optimization in combination with the distributed MPC approach represents the main extension. In comparison to other approaches (e.g. [8, 28, 31, 33, 37, 45, 49, 52]), the use of SPLAT! for radio path loss modelling in both trajectory planning and communication topology reconfiguration represents the main contribution. In comparison to the work in [20–22], this paper uses MPC to provide additional feedback that can give better performance when facing uncertainty. It means that the algorithm described in this paper can be implemented *online* where as [20–22], deal with *offline* optimization problems.

The paper is structured as follows. In Section 2, the UAVs model and constraints are presented. An approach to approximate the terrain below each UAV and the communication path losses with linear functions of the spatial coordinates is described. This leads to a linear centralized MPC problem for trajectory planning, formulated in Section 3. Further in Section 3, an approach for distributed MPC trajectory optimization is proposed. Also in this section, an algorithm for automatic initialization and optimal reconfiguration of the communication topology is developed. In Section 4, the performance of the distributed linear MPC trajectory planning approach is studied on two simulation cases with four UAVs and two targets. Section 5 concludes the paper.

2 UAVs Model and Constraints

2.1 UAVs Model in ENU Coordinate System

The East-North-Up (ENU) coordinate frame is a local geodetic coordinate system whose tangent plane is fitted to the geodetic reference ellipse at some convenient point for local measurements. The x axis points towards east, the y axis points towards true north and the z axis completes the right handed orthogonal frame by pointing away from the Earth perpendicular to the reference ellipsoid.

We consider the task where M-number of rotary-wing UAVs are assigned to communicate the data from one base station to L-number of moving targets. The base station is indexed as 0 node, the UAVs are indexed as $1, 2, \dots, M$ nodes, and the targets – as $M+1, M+2, \dots, M+L$ nodes. It is assumed that the i-th UAV is described by the following discrete time model in the ENU coordinate system:

$$\mathbf{p}_i(t+1) = \mathbf{p}_i(t) + T_{\mathrm{S}}\mathbf{v}_i(t), \tag{1}$$

$i = 1, \dots, M$, $\mathbf{p}_i(t) = [x_i(t), y_i(t), z_i(t)]$ and $\mathbf{v}_i(t) = [v_{ix}(t), v_{iy}(t), v_{iz}(t)]$, with x_i, y_i, z_i and v_{ix}, v_{iy}, v_{iz} being the positions and the velocities along the orthogonal axes in the local ENU coordinate reference frame, and T_{S} is the sampling period.

The trajectory-planning problem for the UAVs is formulated as an MPC problem, where the optimization is performed over velocity sequences $\mathbf{V}_i = [\mathbf{v}_{i,t}, \mathbf{v}_{i,t+1}, \dots, \mathbf{v}_{i,t+N-1}]$, $i = 1, \dots, M$ along a prediction horizon with length N. The predicted position $\mathbf{p}_{i,t+k|t}$ of the i-th UAV, given the initial position

$\mathbf{p}_{i,\,t|t} = \mathbf{p}_i(t)$ and the velocity sequence $\mathbf{V}_i$ is:

$$\mathbf{p}_{i,\,t+k+1|t} = \mathbf{p}_{i,\,t+k|t} + T_\mathrm{S}\mathbf{v}_{i,\,t+k}\,,\ \ k = 0,\ 1,\ \ldots\ ,\ N-1 \tag{2}$$

2.2 Constraints

The following constraints are imposed on the UAVs.

2.2.1 Constraints on the Velocity Vectors

Assume that the maximum horizontal speed for the i-th rotary-wing UAV is $\bar{S}_i$, $i = 1, \ldots, M$. This upper bound is respected by imposing the following hard constraints on the elements v_{ix} and v_{iy} of the velocity vector $\mathbf{v}_i$:

$$\begin{aligned} &-\bar{v}_i \le v_{ix,\,t+k} \le \bar{v}_i\,,\ \ -\bar{v}_i \le v_{iy,\,t+k} \le \bar{v}_i\,,\\ &\quad k = 0,\ 1,\ \ldots\ ,\ N-1 \end{aligned} \tag{3}$$

where $\bar{v}_i = \sqrt{\bar{S}_i^2/2}$. Vertical velocity is constrained as:

$$-\bar{v}_{iz} \le v_{iz,\,t+k} \le \bar{v}_{iz}\,,\ \ k = 0,\ 1,\ \ldots\ ,\ N-1 \tag{4}$$

where $\bar{v}_{iz} > 0$ is a known constant. Then, the total maximum speed of the i-th UAV is $v_{\max,i} = \sqrt{\bar{S}_i^2 + \bar{v}_{iz}^2}$.

2.2.2 Acceleration Constraints

It should be noted that the constraints (3)–(4) imply that the rate of change of the velocity vectors is bounded too. However, these bounds may be too loose and lead to unrealistically big rate of changes of the velocity vectors, i.e. rate of changes which are infeasible for the specific UAV. In order to avoid this, explicit constraints on the acceleration vectors are introduced, which are assumed to have more tight bounds than those implied by the constraints (3)–(4). Thus, the acceleration constraints for the i-th UAV are:

$$\begin{aligned} &-\boldsymbol{\Delta}_i \le (v_{i,\,t} - v_i(t-1))/T_\mathrm{S} \le \boldsymbol{\Delta}_i\\ &-\boldsymbol{\Delta}_i \le (v_{i,\,t+k+1} - v_{i,\,t+k})/T_\mathrm{S} \le \boldsymbol{\Delta}_i\,,\ \ k=0,\ 1,\ \ldots\ ,\\ &\qquad\qquad N-2 \end{aligned} \tag{5}$$

where $\boldsymbol{\Delta}_i$, $i = 1, \ldots, M$ are known vectors.

2.2.3 Anti-Grounding and Air Space Constraints

The following constraints are imposed on the position of the i-th UAV in the ENU coordinate system:

$$\begin{aligned} &\underline{x}_i - q_{x,\,i} \le x_{i,\,t+k|t} \le \bar{x}_i + q_{x,\,i}\,,\ \ \underline{y}_i - q_{y,\,i}\\ &\qquad \le y_{i,\,t+k|t} \le \bar{y}_i + q_{y,\,i}\\ &\hat{h}_i^{\mathrm{terr}}(\mathbf{p}_{i,\,t+k|t}) + \Delta h - q_{z,\,i} \le z_{i,\,t+k|t} \le \bar{z}_i + q_{z,\,i}\\ &k = 1,\ \ldots\ ,\ N \end{aligned} \tag{6}$$

where $x_{i,\,t+k|t}, y_{i,\,t+k|t}, z_{i,\,t+k|t}$ are the predicted coordinates of the i-th UAV by using the model (2) for a given velocity sequence $\mathbf{V}_i$, $\underline{x}_i$, $\underline{y}_i$ and $\bar{x}_i$, $\bar{y}_i$, $\bar{z}_i$ are lower and upper bounds, $\hat{h}_i^{\mathrm{terr}}(\mathbf{p}_{i,\,t+k|t})$ represents the approximate height of the terrain below the i-th UAV and within its maximum reach distance for the specified time horizon, and Δh is a safety distance from ground. Here, $q_{x,\,i}, q_{y,\,i}, q_{z,\,i} \ge 0$ are slack variables, which are introduced in order to avoid infeasibility. The constraints imposed on the coordinate z are such that the anti-grounding requirement is met.

The height of the terrain below the i-th UAV is upper approximated with the following linear function:

$$\hat{h}_i^{\mathrm{terr}}(\mathbf{p}_{i,\,t+k|t}) = \mathbf{a}_i^{\mathrm{T}}(x_{i,\,t|t}, y_{i,\,t|t})[x_{i,\,t+k|t}\ \ y_{i,\,t+k|t}\ \ 1]^{\mathrm{T}} \tag{7}$$

Here, the vector $\mathbf{a}_i(x_{i,\,t|t}, y_{i,\,t|t}) \in \mathbb{R}^3$ contains the coefficients $a_{i,1}$, $a_{i,2}$ and $a_{i,3}$, which depend on the initial position of the i-th UAV and they are obtained by solving the Linear Programming (LP) problem [18]:

$$\begin{aligned} &\min_{a_{i,1},\,a_{i,2},\,a_{i,3}} (a_{i,1}x_i^{\mathrm{center}} + a_{i,2}y_i^{\mathrm{center}} + a_{i,3})\\ &\text{subject to}\quad \mathbf{A}_i[a_{i,1}\ a_{i,2}\ a_{i,3}]^{\mathrm{T}} \ge \mathbf{h}_i^{\mathrm{terr}} \end{aligned} \tag{8}$$

Here, the matrix $\mathbf{A}_i$ contains the x and y coordinates of the terrain points, which are within the maximum reach distance for the i-th UAV, i.e. $\mathbf{A}_i = [x_{i,j}^{\mathrm{terr}}\ \ y_{i,j}^{\mathrm{terr}}\ \ 1]$, $j = 1, \ldots, N_i$ (N_i is the number of points), the vector $\mathbf{h}_i^{\mathrm{terr}} = [z_{i,j}^{\mathrm{terr}}]$, $j = 1, \ldots, N_i$ contains their heights, and x_i^{center}, y_i^{center} are the mean x and y coordinates of the terrain points. An example of terrain approximation with a linear function is shown in Fig. 1.

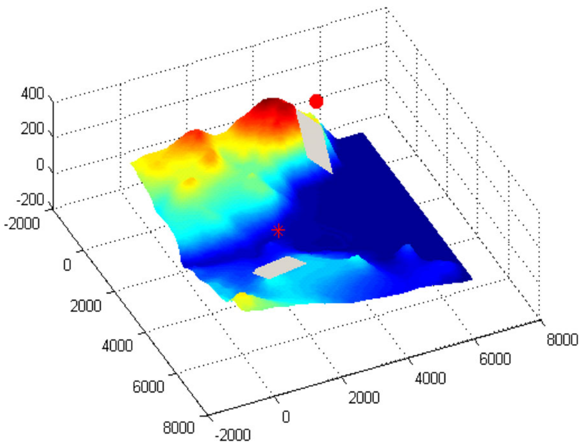

Fig. 1 Terrain approximation (*grey color*) below two UAVs (*red circle and red star*). The *dark blue color* corresponds to terrain at sea level, while the *dark red color* – to terrain with the largest height

2.2.4 Constraints on Radio Communication Path Losses

We consider the task where M-number of rotary-wing UAVs are assigned to form a communication network, which should transmit the data between one base station and L-number of targets. The communication network topology is determined automatically, as it is described in Section 3.3. The connectivity constraints represent constraints on the radio path losses in the communication between the UAVs, the UAVs and the base station and the UAVs and the targets. The radio path losses are computed by using SPLAT! [41, 43] and are approximated by linear functions of the coordinates of the UAVs, base station and target positions at each time sample. SPLAT! uses elevation data to calculate field strength and path loss based on the Longley-Rice Irregular Terrain Model [40].

Let the communication topology for transmitting the data from the base station to the l-th target, $l = 1, \ldots, L$ be specified by the adjacency matrices $\boldsymbol{\Psi}^l = \{\psi^l_{i,j}\}, i, j = 1, \ldots, M, l = 1, \ldots, L$ with $\psi^l_{i,i} = 0$, $i = 1, \ldots, M$. The elements $\psi^l_{i,j}, i, j = 1, \ldots, M$, $i \neq j$ can have the following values:

- $\psi^l_{i,j} = 0$, if the i-th UAV is not assigned to transmit data for the l-th target to the j-th UAV.
- $\psi^l_{i,j} = 1$, if the i-th UAV should communicate data for the l-th target to the j-th UAV.

In the formulation of the MPC sub-problem for the i-th UAV, only constraints on the communication path losses to those UAVs will be imposed, which correspond to $\psi^l_{i,j} = 1$, for some $l \in \{1, \ldots, L\}$, $j = 1, \ldots, M$.

In addition to the topology matrices $\boldsymbol{\Psi}^l$, $l = 1, \ldots, L$, the following vectors are supposed specified:

- $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \ldots, \varphi_M]$ with $\varphi_i = 1$ if the i-th UAV is assigned to receive data from the base station and $\varphi_i = 0$, otherwise.
- $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_M]$, where $\xi_i = 0$ if the i-th UAV is not assigned to transmit data to any of the targets, otherwise, $\xi_i \in \{M+1, M+2, \ldots, M+L\}$ is the index of the target the i-th UAV should communicate data to.

The MPC-based trajectory planning for UAVs requires the solution of an optimization problem at each sampling instant. In order to enhance the computations performed online by the optimization routine, it is necessary to simplify the communication model by approximating the radio communication path losses with an appropriate type of function. Here, we consider the case, where the sampling time, the prediction horizon in the MPC problem and the maximal velocities of the UAVs are such that a linear approximating function of the communication path losses provides the necessary level of accuracy. This results in a linear MPC problem, as it is shown in Sect. 3. By considering the above, for the i-th UAV, the following constraints on the radio communication path losses are imposed:

Constraint on Radio Communication Path Loss to the Base Station (node 0), if $\varphi_i = 1$

$$\hat{c}_{i\text{-}0,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}) \leq \bar{c}_{i\text{-}0} + q_{i\text{-}0}\,, \quad k = 1, \ldots, N \tag{9}$$

where $\hat{c}_{i\text{-}0,\,t+k|t}(\mathbf{p}_{i,\,t+k|t})$ represents the approximate path loss in the communication from the base station (assuming known fixed location) to the i-th UAV and $\bar{c}_{i\text{-}0}$ is the maximum allowed path loss. Here, $q_{i\text{-}0} \geq 0$ is a slack variable, which is introduced in order to avoid infeasibility. The path loss is upper approximated with the following linear function:

$$\hat{c}_{i\text{-}0,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}) = \mathbf{b}^{\mathrm{T}}_{i\text{-}0}(\mathbf{p}_{i,\,t|t})[\mathbf{p}^{\mathrm{T}}_{i,\,t+k|t} \;\; 1]^{\mathrm{T}} \tag{10}$$

where the vector $\mathbf{b}_{i\text{-}0}(\mathbf{p}_{i,\,t|t}) \in \mathbb{R}^4$ contains the coefficients $b_{i\text{-}0,\,j}$, $j = 1, \ldots, 4$, which depend on the position of the i-th UAV and the fixed base station position. They are obtained by solving the following LP problem:

$$\min_{b_{i\text{-}0,\,j},\, j=1,\ldots,4} \left(b_{i\text{-}0,1} x_i^{\text{center}} + b_{i\text{-}0,2} y_i^{\text{center}} + b_{i\text{-}0,3} z_i^{\text{center}} + b_{i\text{-}0,4}\right)$$
$$\text{subject to } \mathbf{B}_i [b_{i\text{-}0,1} \;\; b_{i\text{-}0,2} \;\; b_{i\text{-}0,3} \;\; b_{i\text{-}0,4}]^{\text{T}} \geq \mathbf{c}_{i\text{-}0} \quad (11)$$

Here, the matrix $\mathbf{B}_i = [x_{i,j} \; y_{i,j} \; z_{i,j} \; 1]$, $j = 1, \ldots, \tilde{N}_i$ contains the coordinates of a given number $\tilde{N}_i$ of points, which are within the maximum reach distance for the i-th UAV, the vector $\mathbf{c}_{i\text{-}0} = [c_{i\text{-}0,j}]$, $j = 1, \ldots, \tilde{N}_i$ contains the path losses between these points and the base station (computed with SPLAT!), and x_i^{center}, y_i^{center}, z_i^{center} are the mean coordinates of these points.

Constraints on Radio Communication Path Loss to the j-th UAV if $\psi_{i,j}^l = 1$ for Some $l \in \{1, \ldots, L\}$, $j = 1, \ldots, M$

$$\underline{c}_{i\text{-}j} - q_{i\text{-}j} \leq \hat{c}_{i\text{-}j,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{j,\,t|t}) \leq \bar{c}_{i\text{-}j} + q_{i\text{-}j}\,, \quad k = 1, \ldots, N \quad (12)$$

where $\hat{c}_{i\text{-}j,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{j,\,t|t})$ represents the approximate path loss in the communication from the i-th UAV to the j-th UAV, $\underline{c}_{i\text{-}j}$ and $\bar{c}_{i\text{-}j}$ are the minimum and the maximum allowed path losses, respectively, and $q_{i\text{-}j}$ is a slack variable. The lower bound $\underline{c}_{i\text{-}j}$ on the path loss is introduced as a simple method to avoid collision between the UAVs. The rationale behind this is that the communication path loss between two UAVs will depend on the terrain and the distance between them. Therefore, it is possible to avoid collision by keeping the path loss above the specified minimum value. The advantages of this approach in comparison to the approach of directly constraining the Euclidean distance between the UAVs are that it does not lead to additional computations (the communication path loss is already computed) and the resulting constraint is linear (while it is nonlinear if a constraint on the distance is considered). The slack variable in the communication constraints (12) has to be bounded such that even the maximal possible violation of the lower limit of communication path loss between the UAVs will be safe in sense of collision avoidance. Also, additional and independent systems for detect-and-avoid may be required for certain operations by authorities, and this will provide an additional layer of safety in particular regarding anti-collision with other air traffic than the cooperative UAVs. The path loss in Eq. 12 is upper approximated with the following linear function:

$$\hat{c}_{i\text{-}j,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{j,\,t|t}) = \mathbf{b}_{i\text{-}j}^{\text{T}}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t}) \times [\mathbf{p}_{i,\,t+k|t}^{\text{T}} \;\; 1]^{\text{T}} \quad (13)$$

The coefficients vector $\mathbf{b}_{i\text{-}j}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t}) \in \mathbb{R}^4$ depends on the positions of the i-th UAV and the j-th UAV. It is obtained by solving an LP problem that is similar to the LP problem (11).

Constraint on Radio Communication Path Loss to the l-th Target (node $M + l$), if $\xi_i = M + l$

$$\hat{c}_{i\text{-}(M+l),\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{M+l,\,t|t}) \leq \bar{c}_{i\text{-}(M+l)} + q_{i\text{-}(M+l)}\,, \quad k = 1, \ldots, N \quad (14)$$

where $\hat{c}_{i\text{-}(M+l),\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{M+l,\,t|t})$ is the approximate path loss in the communication from the i-th UAV to the l-th target, $\bar{c}_{i\text{-}(M+l)}$ is the maximum allowed path loss, and $q_{i\text{-}(M+l)} \geq 0$ is a slack variable. The path loss is upper approximated with the following linear function:

$$\hat{c}_{i\text{-}(M+l),\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{M+l,\,t|t}) = \mathbf{b}_{i\text{-}(M+l)}^{\text{T}}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{M+l,\,t|t})[\mathbf{p}_{i,\,t+k|t}^{\text{T}} \;\; 1]^{\text{T}} \quad (15)$$

The coefficients vector $\mathbf{b}_{i\text{-}(M+l)}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{M+l,\,t|t}) \in \mathbb{R}^4$ depends on the positions of the i-th UAV and the l-th target. It is obtained by solving an LP problem, similar to above.

For simplicity, the approximated path loss in Eq. 12 is computed as dependent on the predicted coordinates of the considered UAV only, i.e. it is assumed that the positions of the other UAVs do not change during predictions when solving the corresponding MPC sub-problem. The prediction horizon, sampling interval and safety margins should be tuned such that prediction errors made due to non-zero sampling and finite prediction horizon are much smaller than the safety margins, taking into account the vehicle velocity, turning rate constraints, disturbances such as wind, as well as computational resource limitations. Simulation provides an effective tool to support the MPC tuning process. It is also assumed that each UAV communicates its current measured position to the other UAVs.

3 Trajectory Optimization by Distributed Linear MPC

3.1 Centralized MPC Problem

For the current known positions $\mathbf{p}_i(t)$, $i = 1, \ldots, M+L$ of the UAVs and targets, and a given communication network topology, the centralized MPC trajectory planning solves the following optimization problem:

Problem $\mathbf{P}_0$:
(Centralized MPC)

$$J^* = \min_{\substack{\mathbf{V}_i,\, \mathbf{q}_i \\ i=1,\, \ldots\, ,\, M}} J(\mathbf{V}_1, \ldots, \mathbf{V}_M, \mathbf{q}_1, \ldots, \mathbf{q}_M) \tag{16}$$

subject to $\mathbf{p}_{i,\,t|t} = \mathbf{p}_i(t)$, $i = 1, \ldots, M+L$, constraints (2)–(6) for $i = 1, \ldots, M$, and: $\hat{c}_{i\text{-}0,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}) \le \bar{c}_{i\text{-}0} + q_{i\text{-}0}\,,\ k = 1, \ldots, N$

$$\text{for } \varphi_i = 1\,,\ i = 1, \ldots, M \tag{17}$$

$$\begin{aligned} &\underline{c}_{i\text{-}j} - q_{i\text{-}j} \le \hat{c}_{i\text{-}j,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{j,\,t|t}) \le \bar{c}_{i\text{-}j} + q_{i\text{-}j}\,,\\ &\qquad k = 1, \ldots, N\\ &\text{for } \psi^l_{i,j} = 1 \text{ for some } l \in \{1, \ldots, L\}\,,\\ &\qquad i, j = 1, \ldots, M \end{aligned} \tag{18}$$

$$\begin{aligned} &\hat{c}_{i\text{-}(M+l),\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{M+l,\,t|t})\\ &\quad \le \bar{c}_{i\text{-}(M+l)} + q_{i\text{-}(M+l)}\,,\ k = 1, \ldots, N\\ &\text{for } \xi_i \ne 0\,,\ M + l = \xi_i\,,\ i = 1, \ldots, M \end{aligned} \tag{19}$$

$$\mathbf{q}_i \ge 0\,,\ i = 1, \ldots, M \tag{20}$$

with the cost function:

$$\begin{aligned} &J(\mathbf{V}_1, \ldots, \mathbf{V}_M, \mathbf{q}_1, \ldots, \mathbf{q}_M)\\ &\quad = \sum_{i=1}^{M} \left[\mathbf{q}_i^{\mathrm{T}} \boldsymbol{\Lambda}_i \mathbf{q}_i + \sum_{k=0}^{N-1} \alpha_i ||\mathbf{v}_{i,\,t+k}||^2 \right] \end{aligned} \tag{21}$$

Here, $\mathbf{q}_i$ is the vector of all slack variables, associated to the constraints for the i-th UAV, α_i is a weighting coefficient and $\boldsymbol{\Lambda}_i$ is a weighting matrix. The term $||\mathbf{v}_{i,\,t+k}||^2$ represents an approximation to the i-th UAV's energy consumption. The terms $\mathbf{q}_i^{\mathrm{T}} \boldsymbol{\Lambda}_i \mathbf{q}_i$, $i = 1, \ldots, M$ are introduced in order to minimize the slack variables.

A coordination procedure is performed at each time instant, which allows the automatic reconfiguration of the communication topology in case of failure (for example, a lost communication with some UAV). This situation can be recognized if a UAV did not communicate its current position to the other UAVs. Then, this UAV will be considered as disconnected and its task will be taken over by another UAV. The reconfiguration of the communication topology will reflect on the constraints in the MPC problem $\mathbf{P}_0$.

3.2 Distributed MPC Problem

For the current known positions $\mathbf{p}_r(t)$, $r = 1, \ldots, M+L$ of the UAVs and targets, and a given communication network topology, the distributed MPC trajectory planning solves a distributed optimization problem consisting of the following sub-problems:

Problem $\mathbf{P}_i$, $i = 1, 2, \ldots, M$:
(MPC sub-problem for the i-th UAV)

$$J_i^* = \min_{\mathbf{V}_i,\, \mathbf{q}_i} J_i(\mathbf{V}_i, \mathbf{q}_i) \tag{22}$$

subject to $\mathbf{p}_{r,\,t|t} = \mathbf{p}_r(t)$, $r = 1, \ldots, M+L$, constraints (2)–(6), and:

$$\begin{aligned} &\hat{c}_{i\text{-}0,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}) \le \bar{c}_{i\text{-}0} + q_{i\text{-}0}\,,\ k = 1, \ldots, N\\ &\text{for } \varphi_i = 1 \end{aligned} \tag{23}$$

$$\begin{aligned} &\underline{c}_{i\text{-}j} - q_{i\text{-}j} \le \hat{c}_{i\text{-}j,\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{j,\,t|t}) \le \bar{c}_{i\text{-}j}\\ &\qquad + q_{i\text{-}j}\,,\ k = 1, \ldots, N\\ &\text{for } \psi^l_{i,j} = 1 \text{ for some } l \in \{1, \ldots, L\}\,,\\ &\qquad j = 1, \ldots, M \end{aligned} \tag{24}$$

$$\begin{aligned} &\hat{c}_{i\text{-}(M+l),\,t+k|t}(\mathbf{p}_{i,\,t+k|t}, \mathbf{p}_{M+l,\,t|t}) \le \bar{c}_{i\text{-}(M+l)} + q_{i\text{-}(M+l)}\,,\\ &\qquad k = 1, \ldots, N\\ &\text{for } \xi_i \ne 0\,,\ M + l = \xi_i \end{aligned} \tag{25}$$

$$\mathbf{q}_i \ge 0 \tag{26}$$

with the cost function:

$$J_i(\mathbf{V}_i, \mathbf{q}_i) = \mathbf{q}_i^{\mathrm{T}} \boldsymbol{\Lambda}_i \mathbf{q}_i + \sum_{k=0}^{N-1} \alpha_i ||\mathbf{v}_{i,\,t+k}||^2 \tag{27}$$

The linear MPC sub-problem, formulated above, can be solved by convex quadratic programming.

3.3 Automatic Reconfiguration of Communication Topology

The reconfiguration is performed by using the maximum flow algorithm [17], which solves the maximum flow problem in a directed graph (in optimization theory, maximum flow problems involve finding a feasible flow through a single-source, single-sink flow network that is maximum). A directed graph with K-number of nodes is represented as a sparse matrix $\mathbf{H}$ with elements $h_{i,j}$, $i,\ j = 1, 2, \ldots, K$. Any nonzero entries in $\mathbf{H}$ represent the edges of the graph, and the values of these entries are the associated capacity of the edge. In $\mathbf{H}$, the row (column) index indicates the source (target) of the edge. In the case we consider, the l-th directed graph will have as nodes the UAVs, the base station and the l-th target. The capacity $h_{i,j}$ of a given edge in the graph will represent the difference between the approximated communication path loss between the source node (i-th node) andthe target node (j-th node) of the edge and the maximum allowed loss $\bar{c}_{i\text{-}j}$ needed in order to achieve the required datarate, i.e.:

$$h_{i,j} = \bar{c}_{i\text{-}j} - \hat{c}_{i\text{-}j,\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t}) \tag{28}$$

This simple model does not account for data coding and other communication properties, but can be readily extended with models to capture more accurately the effect of radio path loss on the actual data rate of a specific communication system. Assume that $h_{i,j} \geq 0$, $i,\ j = 1, 2, \ldots, K$. By solving the maximum flow problem, a feasible reconfiguration of the communication topology will be found for which the minimum difference (28) along the edges of the path will be maximum. The rationale behind that is to find a feasible path in the graph, where the approximated communication path losses between the nodes are as far as possible from the maximum allowed values, i.e. this communication topology can for relatively long time fulfill the task of transmitting the signal from the base station to the target.

Let the set of UAVs indices be $\mathcal{I}_{\text{UAV}} = \{1, 2, \ldots, M\}$ and the set of all nodes indices be $\mathcal{I}_{\text{Nodes}} = \{0, 1, 2, \ldots, M, M+1, \ldots, M+L\}$. Let the fixed base station position is denoted $\mathbf{p}_{0,\,t|t}$. Since the maximum flow algorithm can accept only non-negative values of the capacities of edges, the expression (28) is modified as follows:

$$h_{i,j} = \bar{c} - \hat{c}_{i\text{-}j,\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t}) \tag{29}$$

where $\bar{c}$ is the solution of the following Linear Programming (LP) problem:

$$\begin{aligned} &\min_{\bar{c}>0} \bar{c} \\ &\text{subject to} \quad \bar{c} - \hat{c}_{i\text{-}j,\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t}) > \varepsilon_c\,,\ \ i \in \mathcal{I}_{\text{UAV}}\,, \\ &\quad j \in \mathcal{I}_{\text{Nodes}} \end{aligned} \tag{30}$$

Here, ε_c is a small positive constant which guarantees that $\bar{c} - \hat{c}_{i\text{-}j,\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t})$ is strictly positive.

Let $\mathbf{H}^l_r$, be the r-th possible graph, where the source node is the base station, the target node is the l-th target, and the other nodes are all UAVs. It is assumed that all UAVs communicate with the base station and the l-th target. It is supposed that it is not allowed to have reciprocal edges in the graph, i.e. i-th UAV can communicate to the j-th UAV, but not vice versa (the communication of the j-th UAV to the i-th UAV will belong to another graph). The matrix $\mathbf{H}^l_r$ has $M+1$ rows (the 1-st row corresponds to the base station and the other rows – to all UAVs) and $M+1$ columns (the columns from 1 to M correspond to all UAVs and the $M+1$ column – to the l-th target), and its elements are defined as follows:

$$\begin{aligned} &h^{r,l}_{1,j} = \bar{c} - \hat{c}_{j\text{-}0,\,t|t}(\mathbf{p}_{j,\,t|t}) \\ &j \in \mathcal{I}_{\text{UAV}} \end{aligned} \tag{31}$$

$$\begin{aligned} &h^{r,l}_{i+1,j} = \begin{cases} \bar{c} - \hat{c}_{i\text{-}j,\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{j,\,t|t})\,, & \text{if } \psi^{r,l}_{i,j} = 1 \\ 0\,, \ \text{if } \psi^{r,l}_{i,j} = 0 \end{cases} \\ &i,\ j \in \mathcal{I}_{\text{UAV}} \end{aligned} \tag{32}$$

$$\begin{aligned} &h^{r,l}_{i+1,M+l} = \bar{c} - \hat{c}_{i\text{-}(M+l),\,t|t}(\mathbf{p}_{i,\,t|t}, \mathbf{p}_{M+l,\,t|t}) \\ &i \in \mathcal{I}_{\text{UAV}} \end{aligned} \tag{33}$$

For a given graph $\mathbf{H}^l_r$, there is an associated communication topology matrix $\boldsymbol{\Psi}^l_r$ with dimension $M \times M$ with elements $\psi^{r,l}_{i,j} = 1$ if the i-th UAV is assigned to transmit the data to the j-th UAV, and $\psi^{r,l}_{i,j} = 0$, otherwise.

Let $\mathbf{H}^l_r$, $r = 1, \ldots, R_l$ be all possible graphs, which can be used to transmit the signal from the base station to the l-th target. By running the maximum flow algorithm [17], the optimal graph with the corresponding graph matrix $\mathbf{H}^l_{\text{opt}}$ and topology matrix $\boldsymbol{\Psi}^l_{\text{opt}}$ can be determined. In the general case, the optimal graph $\mathbf{H}^l_{\text{opt}}$ will contain several paths $p^l_{\text{opt},i}$, $i = 1, 2, \ldots, K_l$ from the base station to the l-th target with the associated optimal values of the flows $f^l_{\mathrm{opt},i}$, $i = 1, 2, \ldots, K_l$ (these values represent the capacity of the path to transmit the signal form the base station to the l-th target). The presence of parallel paths is related to the task being solved, namely to obtain a maximum total flow $\sum_{i=1}^{K_l} f^l_{\mathrm{opt},i}$ (maximum total capacity to transmit the signal) from the base station to the l-th target. The best combination of paths to transmit the signal from the base station to all targets can be determined by solving the following optimization problem:

$$\max_{\substack{p^l_{\mathrm{opt},i},\, i\in\{1,2,\ldots,K_l\} \\ l=1,\ldots,L}} \Bigg(\sum_{i\in\{1,\ldots,K_1\}} f^1_{\mathrm{opt},i} + \sum_{i\in\{1,\ldots,K_2\}} f^2_{\mathrm{opt},i} + \ldots + \sum_{i\in\{1,\ldots,K_L\}} f^L_{\mathrm{opt},i} - \Big| \sum_{i\in\{1,\ldots,K_1\}} f^1_{\mathrm{opt},i} - \sum_{i\in\{1,\ldots,K_2\}} f^2_{\mathrm{opt},i} \Big| - \Big| \sum_{i\in\{1,\ldots,K_1\}} f^1_{\mathrm{opt},i} - \sum_{i\in\{1,\ldots,K_3\}} f^3_{\mathrm{opt},i} \Big| - \ldots - \Big| \sum_{i\in\{1,\ldots,K_{L-1}\}} f^{L-1}_{\mathrm{opt},i} - \sum_{i\in\{1,\ldots,K_L\}} f^L_{\mathrm{opt},i} \Big| \Bigg) \tag{34}$$

$$\text{subject to } \mathcal{I}^{l_1}_{\mathrm{UAV},i} \cap \mathcal{I}^{l_2}_{\mathrm{UAV},j} = \emptyset,\ \forall l_1, l_2 = 1, \ldots, L,\ l_1 \neq l_2, \forall i \in \{1, \ldots, K_{l_1}\}, \forall j \in \{1, \ldots, K_{l_2}\} \tag{35}$$

Here, $\mathcal{I}^{l_1}_{\mathrm{UAV},i}$ is the set of the indices of the UAV nodes, which participate in the optimal path $p^{l_1}_{\mathrm{opt},i}$ for transmitting the signal to the l_1-th target. The constraint (35) imposes the requirement for the communication resources used in the optimal paths to the different targets not to be shared. Thus, in a given sum in Eq. 34, only those optimal paths are considered, which do not share nodes with the optimal paths in the other sums. There may be cases where the problem (34)–(35) may be infeasible. Then, the constraints (35) can be relaxed according to some priorities in order to ensure feasibility.

Remark 1 The meaning of the expression in Eq. 34 is that the best paths to transmit the signal from the base station to all targets are obtained by maximizing the flows associated to these paths, while minimizing the difference between the flows of paths used to transmit to different targets (i.e. to ensure similar flows of transmission to the different targets). For example, if there are two targets (the case considered in Section 4), then the problem (34)–(35) will have the form:

$$\max_{\substack{p^1_{\mathrm{opt},i},\, i\in\{1,2,\ldots,K_1\} \\ p^2_{\mathrm{opt},i},\, i\in\{1,2,\ldots,K_2\}}} \Bigg(\sum_{i\in\{1,\ldots,K_1\}} f^1_{\mathrm{opt},i} + \sum_{i\in\{1,\ldots,K_2\}} f^2_{\mathrm{opt},i} - \Big| \sum_{i\in\{1,\ldots,K_1\}} f^1_{\mathrm{opt},i} - \sum_{i\in\{1,\ldots,K_2\}} f^2_{\mathrm{opt},i} \Big| \Bigg) \tag{36}$$

$$\text{subject to } \mathcal{I}^1_{\mathrm{UAV},i} \cap \mathcal{I}^2_{\mathrm{UAV},j} = \emptyset,\ \forall i \in \{1, \ldots, K_1\},\ \forall j \in \{1, \ldots, K_2\} \tag{37}$$

The constraint (37) means that the communication resources are not shared.

The algorithm for automatic initialization and optimal reconfiguration of communication topology is described as follows:

Algorithm for automatic initialization and optimal reconfiguration of communication topology:

1. Input data: the base station position, a threshold $\varepsilon > 0$ for switching to the optimal communication topology, the positive constant ε_c, the terrain data. Let $\mathcal{I}_{\mathrm{UAV}} := \emptyset$, $M := 0$, and $f^l_{\mathrm{current}} := 0$, $l = 1, \ldots, L$, where f^l_{current} denotes the maximum flow of the current topology for transmitting the signal from the base station to the l-th target.
2. Update the list $\mathcal{I}_{\mathrm{UAV}}$ of available UAV nodes. Let M be the number of these nodes. Get the current positions of the M-number of UAVs $\mathbf{p}_{i,t|t}$, $i \in \mathcal{I}_{\mathrm{UAV}}$ and the L-number of targets $\mathbf{p}_{M+l,t|t}$, $l = 1, \ldots, L$. Determine $\bar{c}$ by solving the LP problem (30).
3. For the l-th target, define all possible graphs $\mathbf{H}^l_r$, $r = 1, \ldots, R_l$ (R_l is the number of combinations), where the source node is the base station, the target node is the l-th target, and the other nodes are all UAVs. It is assumed that all UAVs communicate with the base station and the l-th target. The elements of each graph matrix $\mathbf{H}^l_r$ are determined according to Eqs. 31–33. The associated communication topology matrices $\boldsymbol{\Psi}^l_r$,

Table 1 Parameter values

Parameter	Value
$\underline{x}_i,\ \bar{x}_i,\ \underline{y}_i,\ \bar{y}_i,\ \bar{z}_i,\ i = 1,\ 2,\ 3,\ 4$	500, 5800, 1250, 5500, 600
$\bar{S}_1,\ \bar{S}_2,\ \bar{S}_3,\ \bar{S}_4$	20.4 [m/s], 20.4 [m/s], 26.9 [m/s], 26.9 [m/s]
$\bar{v}_{1z},\ \bar{v}_{2z},\ \bar{v}_{3z},\ \bar{v}_{4z}$	14.4 [m/s], 14.4 [m/s], 19.1 [m/s], 19.1 [m/s]
$v_{\max,1},\ v_{\max,2},\ v_{\max,3},\ v_{\max,4}$	25 [m/s], 25 [m/s], 33 [m/s], 33 [m/s]
$\boldsymbol{\Delta}_i,\ i = 1,\ 2,\ 3,\ 4$	$[1\ 1\ 1]^{\mathrm{T}}$ [m/s^2]
Δh	100 [m]
$\bar{c}_{i\text{-}0},\ \bar{c}_{i\text{-}5},\ \bar{c}_{i\text{-}6},\ i = 1,\ 2,\ 3,\ 4$	110 [dB], 112 [dB], 108 [dB]
$\underline{c}_{i\text{-}j},\ \bar{c}_{i\text{-}j},\ i,\ j = 1,\ 2,\ 3,\ 4,\ i \neq j$	85 [dB], 105 [dB]
$T_{\mathrm{S}},\ N$	1 [s], 10
$\alpha_i,\ i = 1,\ 2,\ 3,\ 4$	$0.1/v^2_{\max,i}$

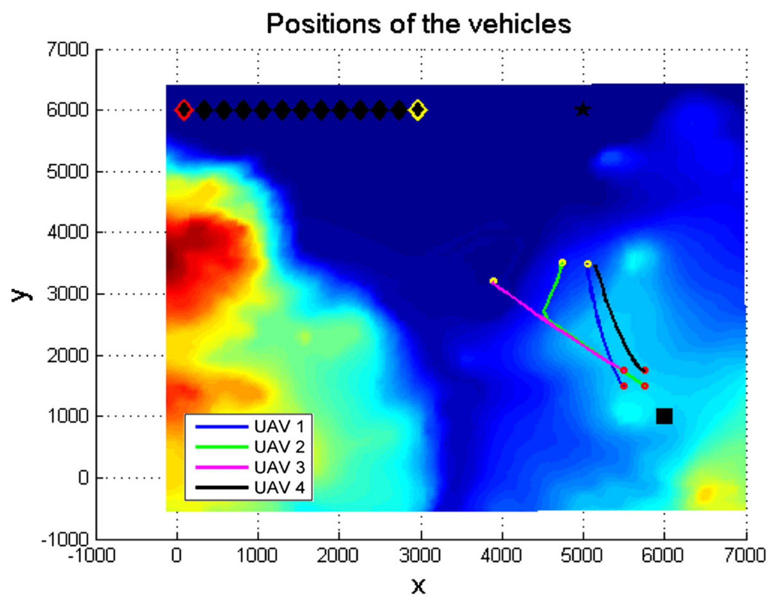

Fig. 2 Trajectories of the four UAVs and the moving target (*diamond*) – the initial positions are marked with *red* and the final positions are with *yellow*, and the positions of the base station (*square*) and the stationary target (*pentagram*). The *dark blue color* corresponds to terrain at sea level, while the *dark red color* – to terrain with the largest height (see Fig. 1)

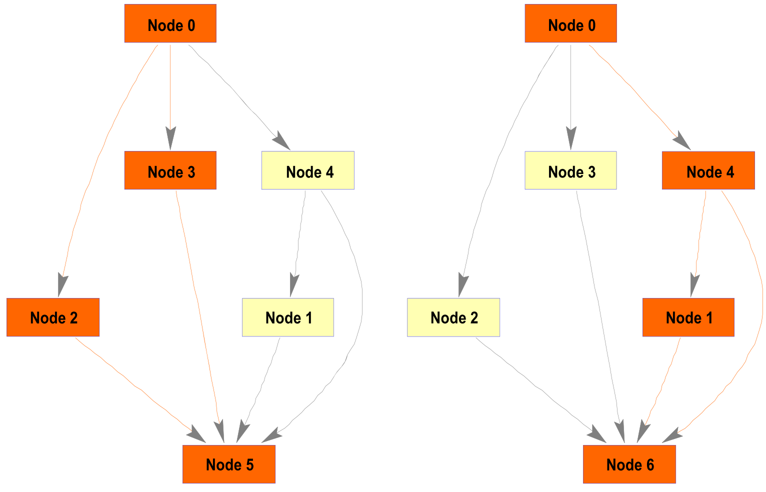

Fig. 3 The optimal paths (depicted in *orange*) at times $t = 0$ [s] and $t = 60$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

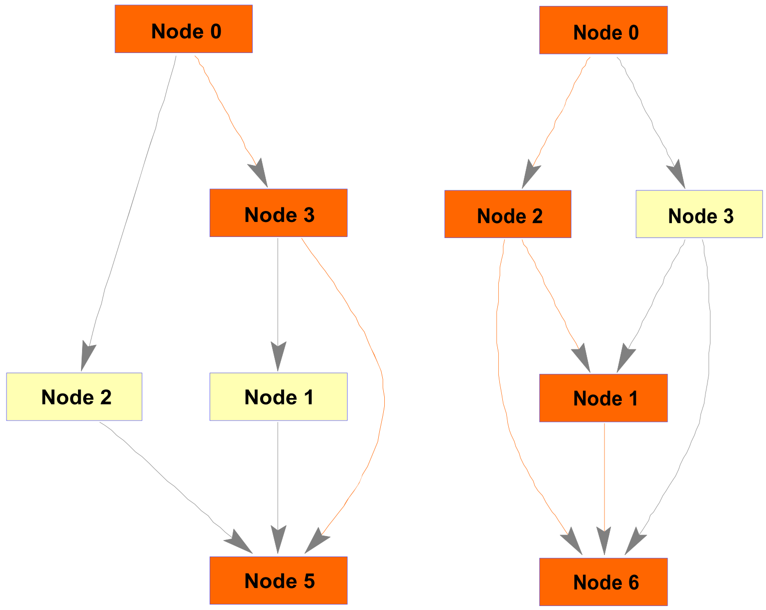

Fig. 4 The optimal paths (depicted in *orange*) at time $t = 120$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

$r = 1, \ldots, R_l$ are also deteremined. All possible graphs are found by solving a combinatorial task.

4. For the graphs $\mathbf{H}^l_r$, $r = 1, \ldots, R_l$, run the maximum flow algorithm [17] and determine the optimal graph with the corresponding graph matrix $\mathbf{H}^l_{\text{opt}}$ and topology matrix $\boldsymbol{\Psi}^l_{\text{opt}}$. Let $p^l_{\text{opt},i}$, $i = 1, 2, \ldots, K_l$ be the paths from the base station to the l-th target in the optimal graph $\mathbf{H}^l_{\text{opt}}$. Let $\mathcal{I}^l_{\text{UAV},i}$ includes the indices of the UAV nodes in the path $p^l_{\text{opt},i}$.
5. Repeat steps 3 and 4 for $l = 1, \ldots, L$.
6. Determine the best combination of paths to transmit the signal from the base station to all targets by solving the optimization problem (34)–(35). Let the best combination of paths be $p^l_{\text{best},i}$ and the associated flow values be $f^l_{\text{best},i}$, $i \in \{1, 2, \ldots, K_l\}$, $l = 1, \ldots, L$.

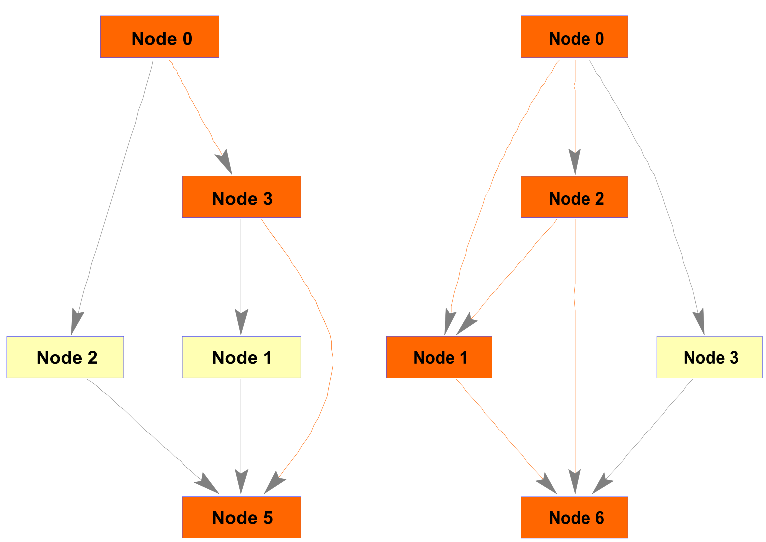

Fig. 5 The optimal paths (depicted in *orange*) at time $t = 180$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

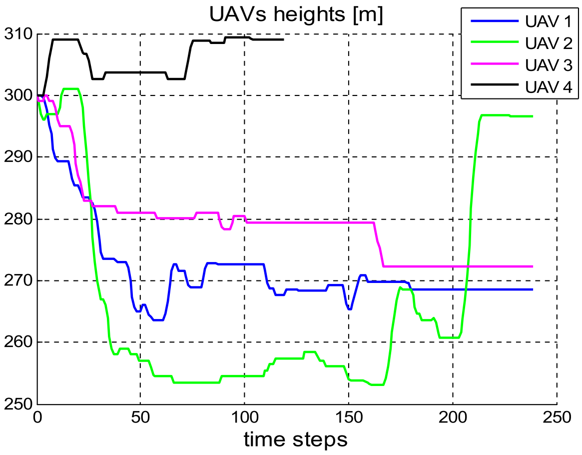

Fig. 6 Heights of the UAVs

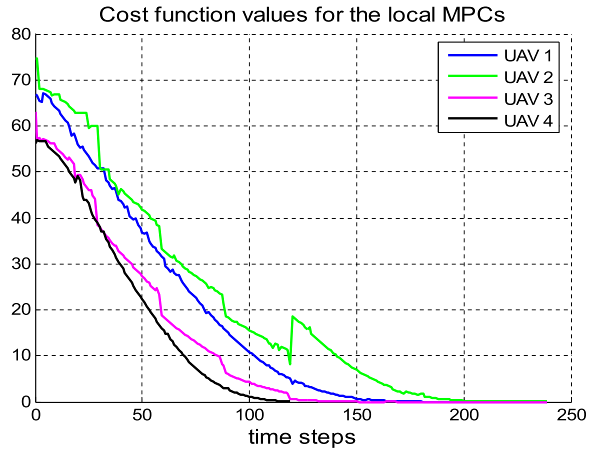

Fig. 8 Cost function values for the local MPCs

7. Compute $f_{\text{best}}^{l} = \sum_{i \in \{1, 2, \ldots, K_l\}} f_{\text{best},i}^{l}$. If $f_{\text{best}}^{l} > f_{\text{current}}^{l} + \varepsilon$, then the communication topology for transmitting the data from the base station to the l-th target will be changed to the best determined topology (the paths $p_{\text{best},i}^{l}$, $i \in \{1, 2, \ldots, K_l\}$) and $f_{\text{current}}^{l} := f_{\text{best}}^{l}$. This is repeated for $l = 1, \ldots, L$. Go to step 2.

The result of the automatic reconfiguration will be the update of the communication topology matrices $\boldsymbol{\Psi}^{l}$, $l = 1, \ldots, L$ and the vectors $\boldsymbol{\varphi}$ and $\boldsymbol{\xi}$. This will lead to an update of the respective MPC sub-problems $\mathbf{P}_i$.

It is supposed that the above algorithm is executed by a central computing unit at a specified sampling time, for example at every 60 seconds. The central computing unit can typically be the ground control station (or base station), and the communication topology configuration can be transmitted from the ground control station to the UAVs over the separate and independent command and control datalink.

Remark 2 The number R_l of all possible graphs $\mathbf{H}_r^l$, $r = 1, \ldots, R_l$, generated in step 3 of the algorithm, might be large if the number of nodes is large. In such cases, some heuristics can be used in order to consider only a small number of promising candidate graphs rather than all possible graphs. For example, for the purpose of the automatic initialization of the communication network topology, only the nodes (UAVs)

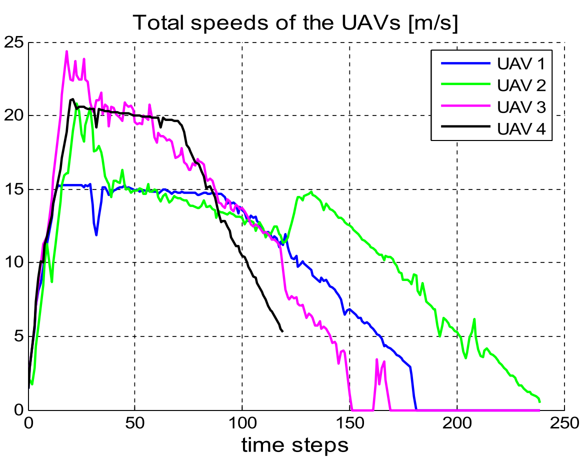

Fig. 7 Speeds of the UAVs

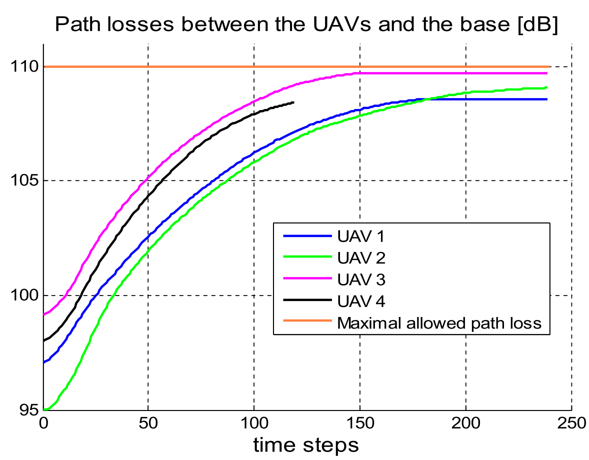

Fig. 9 Radio communication path losses between the UAVs and the base station

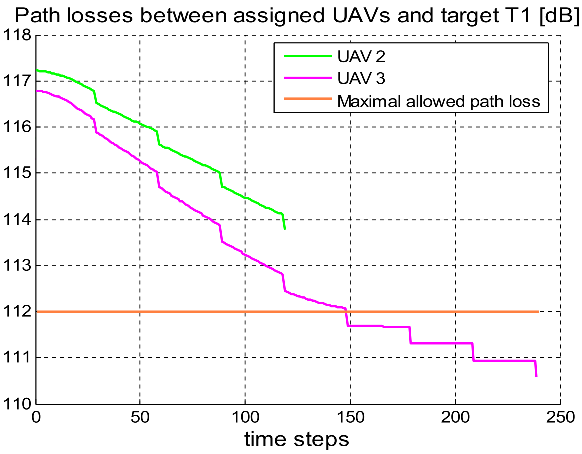

Fig. 10 Radio communication path losses between the moving target T1 and the UAVs, assigned to communicate with it

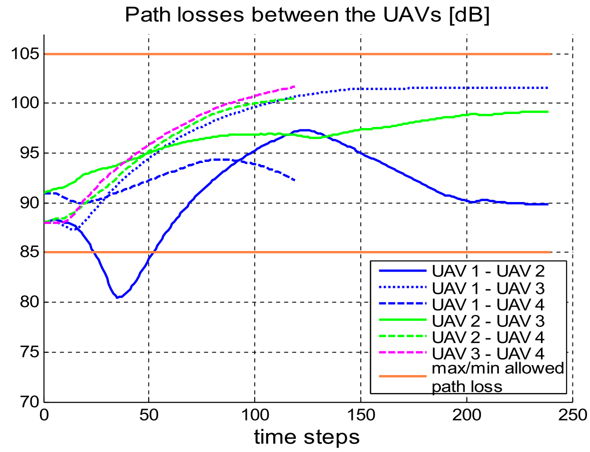

Fig. 12 Radio communication path losses between the UAVs

which belong to a sub-space including the base station and only the specific target (the l-th target) may be considered. When a reconfiguration is to be made, then only the nodes from the previous optimal configuration can be used in constructing the graphs $\mathbf{H}^l_r, r = 1, \ldots, R_l$.

4 Simulation Cases

In this section we consider two simulation case studies, where we use several UAVs to form a radio communication network, which should transmit the signal from a base station to a moving target (T1) and to a stationary target (T2). The base station and the target T2 are located at (6000, 1000, 150) and (5000, 6000, 10), respectively. The initial position of target T1 is (100, 6000, 10) and its speed is 8 [m/s] towards east. The parameter values for the UAVs, as well as for the optimization problem are given in Table 1. In the two case studies, the base station is denoted as node 0, UAV1 to UAV4 – as node 1 to node 4, the target T1 – as node 5, and the target T2 – as node 6. The UAVs have omni-directional antennas and the communication is characterized with frequency bandwidth of 2400 [MHz]. The two simulation cases use the full nonlinear SPLAT! simulations in order to validate the proposed distributed approach and its simplifying assumptions used in the MPC.

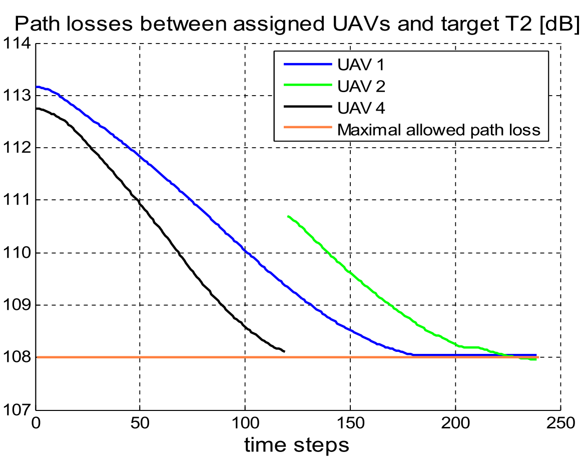

Fig. 11 Radio communication path losses between the stationary target T2 and the UAVs, assigned to communicate with it

4.1 Scenario 1

In this scenario, initially 4 UAVs are used and it is supposed that at time $t = 120$ [s] the communication with the UAV4 is lost. The initial positions of the UAVs are (see Fig. 2, where the initial positions of the UAVs are marked with red):

$$
\begin{aligned}
p_1(0) &= (5500,\ 1500,\ 300)\\
p_2(0) &= (5750,\ 1500,\ 300)\\
p_3(0) &= (5500,\ 1750,\ 300)\\
p_4(0) &= (5750,\ 1750,\ 300)
\end{aligned} \tag{38}
$$

The distributed linear MPC approach, described in the previous section, is applied for trajectory planning of the four UAVs. The automatic reconfiguration algorithm runs at every 60 seconds and the optimal paths

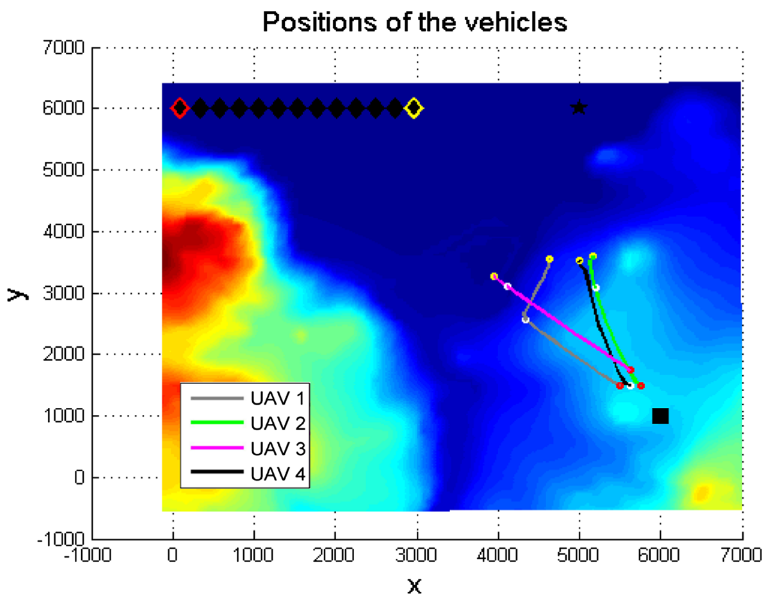

Fig. 13 Trajectories of the four UAVs, the moving target (*diamond*), and the positions of the base station (*square*) and the stationary target (*pentagram*). The positions of UAV1, UAV2, UAV3, and the moving target at time $t = 0$ [s] are marked with *red*, the positions of all UAVs at time $t = 120$ [s] – with white, and the final positions – with *yellow*

for transmitting the signal from the base station to the two targets are given in Figs. 3 to 5. The nodes participating in the optimal paths, as well as the directions of transmitting the signal are depicted in orange. It can be seen from Figs. 4 and 5 that the optimal paths for transmitting the signal from the base station to the two targets change both at times $t = 120$ [s] and $t = 180$ [s].

The corresponding trajectories of the UAVs, their heights and speeds, the cost function values (27) of the local MPC problems, as well as the communication path losses are presented in Figs. 2 and 6, 7, 8, 9, 10, 11 to 12. Since at time $t = 120$ [s] the communication with UAV4 (node 4) is lost, all graphics associated to UAV4 are depicted only until this time. From Fig. 10, it can be seen that until time $t = 120$ [s] the UAV2 is tasked to communicate with the target T1 (node 5) according to the optimal paths in Fig. 3. It is noticed in Fig. 11 that after this time the UAV2 has to communicate with the target T2 (node

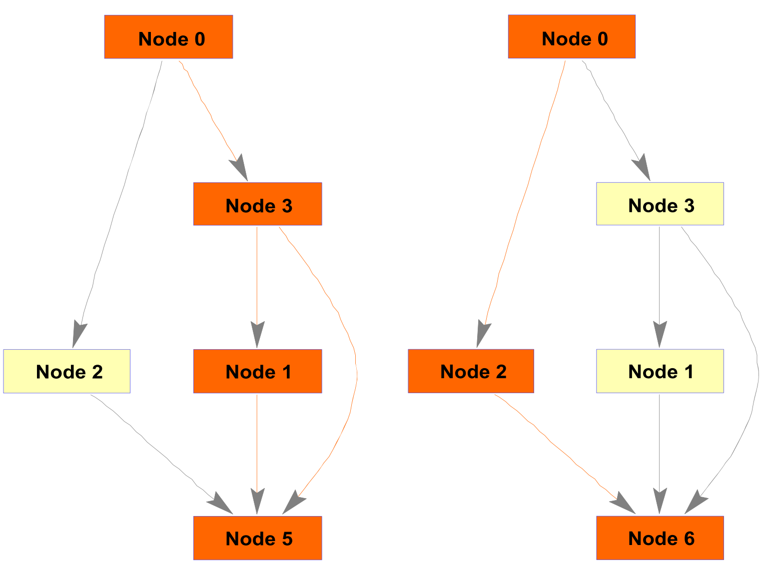

Fig. 14 The optimal paths (depicted in *orange*) at times $t = 0$ [s] and $t = 60$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

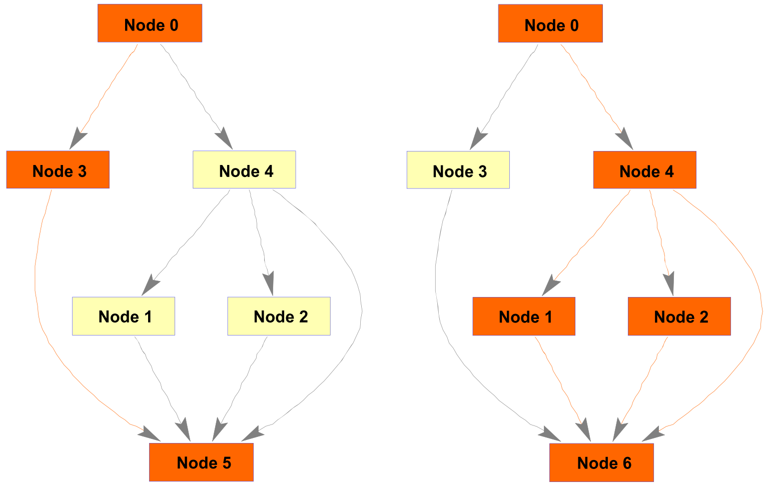

Fig. 15 The optimal paths (depicted in *orange*) at time $t = 120$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

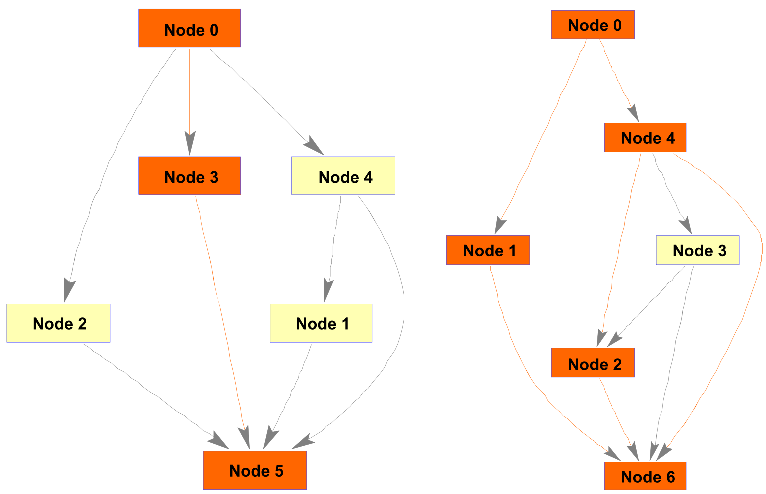

Fig. 16 The optimal paths (depicted in *orange*) at time $t = 180$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

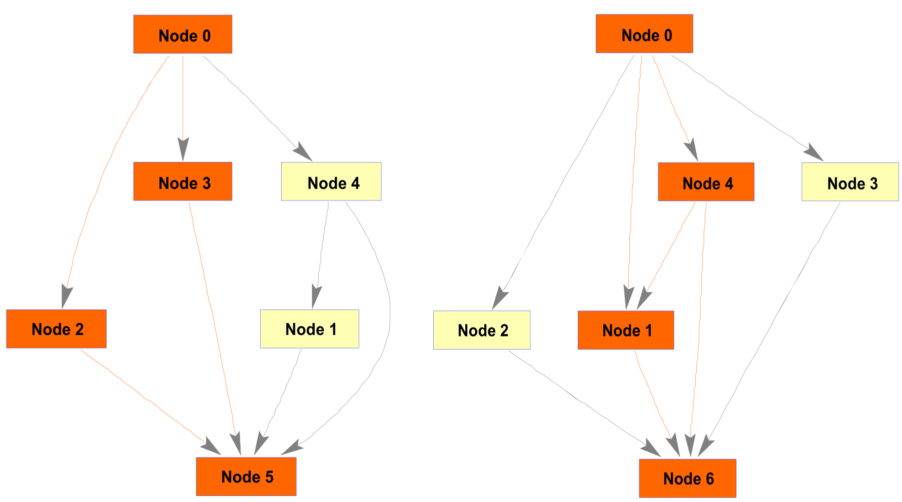

Fig. 17 The optimal paths (depicted in *orange*) at time $t = 240$ [s]. *Left* The optimal paths to transmit the signal from the base station to the moving target T1 (node 5). *Right* The optimal paths from the base station to the stationary target T2 (node 6)

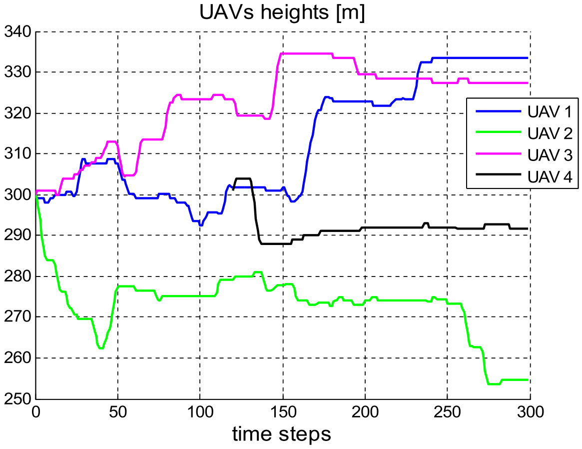

Fig. 18 Heights of the UAVs

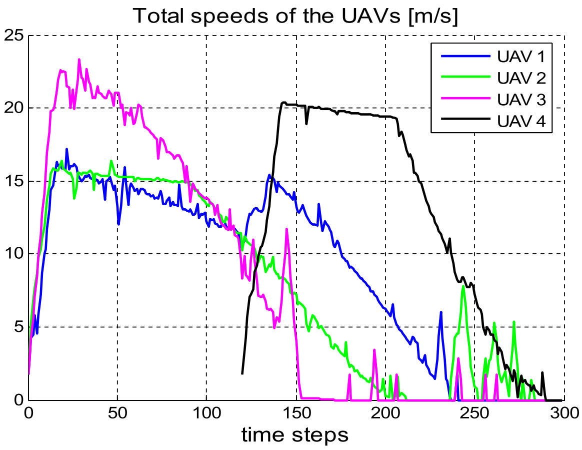

Fig. 19 Speeds of the UAVs

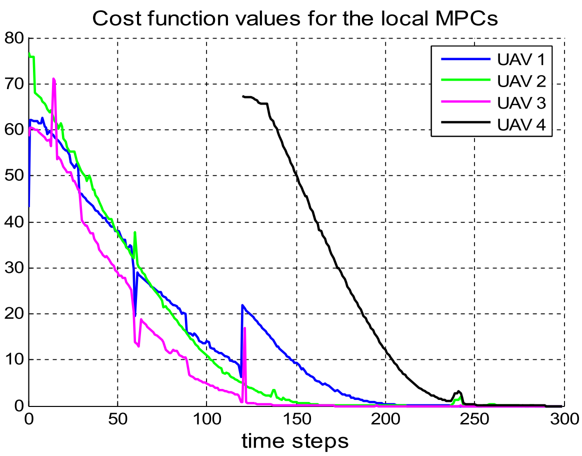

Fig. 20 Cost function values for the local MPCs

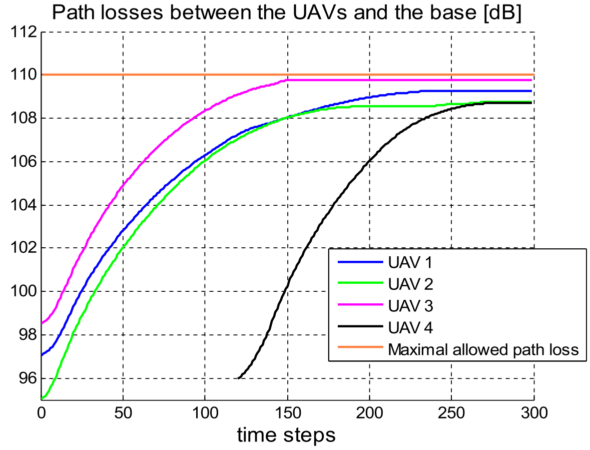

Fig. 21 Radio communication path losses between the UAVs and the base station

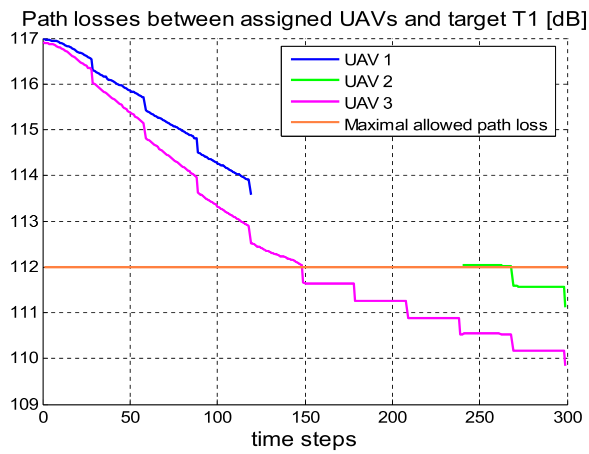

Fig. 22 Radio communication path losses between the moving target T1 and the UAVs, assigned to communicate with it

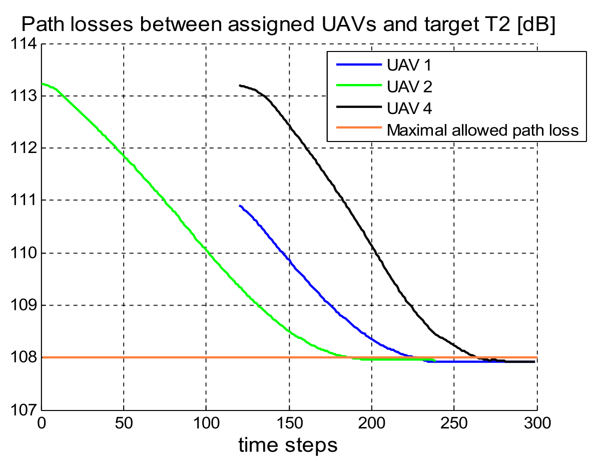

Fig. 23 Radio communication path losses between the stationary target T2 and the UAVs, assigned to communicate with it

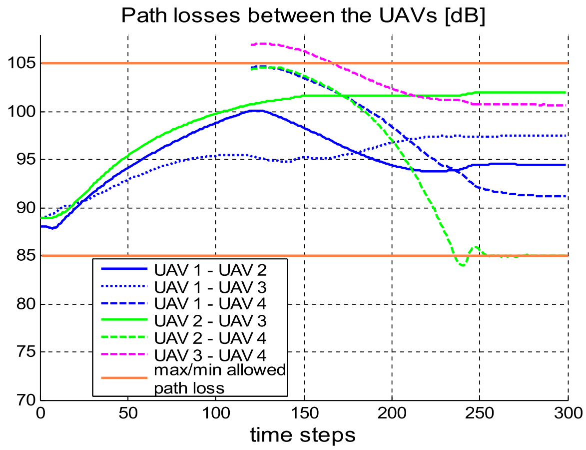

Fig. 24 Radio communication path losses between the UAVs

6) in accordance with the optimal paths in Figs. 4 and 5, and it quickly establishes communication with this target. From Fig. 10 it can be observed that the UAV3 establishes communication with the target T1 in accordance with the optimal paths in Figs. 3 to 5. The non-smoothness of the path loss curves in Fig. 10 is due to the fact that the position of the moving target T1 is updated every 30 [s]. In Fig. 8, the jumps in the local cost function values result from the non-smooth change of the values of the slack variables, associated to the communication constraints, the reconfiguration of the communication network topology, where some other slack variables may become non-zero, and the update of the position of the moving target every 30 [s]. From Fig. 12 it can be noticed that the communication path losses between the UAVs never exceed the maximum allowed values. However, the radio communication path loss between the UAV1 and the UAV2 falls below the minimal allowed value. This can be explained with the use of slack variables in the constraints on communication path losses (see Section 2.2.4). Therefore, it may happen that some slack variable is non-zero. The slack variables in the communication constraints are bounded such that even the maximal possible violation of the lower limits of communication path losses between the UAVs is not critical with respect to the safety distance between the UAVs. In the considered simulation case, these slack variables are bounded by 20 [dB]. In Fig. 12, the minimal distance between the UAV1 and the UAV2, corresponding to the minimal value of the communication path loss, is 108 [m] and it is far from the critical value.

4.2 Scenario 2

In this scenario, initially 3 UAVs are used to form a communication network and it is supposed that at time $t = 120$ [s] another UAV (UAV4) joins the network. The starting positions of all UAVs are (see Fig. 13):

$$
\begin{aligned}
p_1(0) &= (5500,\ 1500,\ 300)\\
p_2(0) &= (5750,\ 1500,\ 300)\\
p_3(0) &= (5500,\ 1750,\ 300)\\
p_4(120) &= (5750,\ 1750,\ 300)
\end{aligned}
\tag{39}
$$

The optimal paths for transmitting the signal from the base station to the two targets are given in Figs. 14, 15, 16 to 17 (as in the previous case study, the automatic reconfiguration algorithm runs at every 60 seconds). It can be seen from these figures that during the whole period of simulation the UAV3 is tasked to communicate with the target T1 (node 5), the UAV2 is tasked to communicate with the target T2 (node 6) and later – with the target T1, while the UAV1 has to initially communicate with the target T1 and later – with the target T2. The UAV4 (which joins the communication network at time $t = 120$ [s]) is assigned to establish connection with the target T2.

The trajectories of the UAVs, their heights and speeds, the cost function values (27), and the communication path losses are presented in Figs. 13 and 18, 19, 20, 21, 22, 23 to 24. All graphics associated to the UAV4 are depicted from time $t = 120$ [s] (when it joins the communication network) onward. It can be seen from Figs. 21 to 24 that the UAVs quickly establish communication with the respective targets and the communication constraints are kept, except some minor violations of the bounds on the communication path losses between some UAVs, which are not critical (as discussed in the simulation scenario 1).

5 Conclusions

A distributed linear MPC approach to solve the trajectory planning problem for rotary-wing UAVs is proposed, where the objective of the UAV system is to form a communication network to multiple targets with given radio communication capacities. An algorithm for automatic initialization and optimal reconfiguration of the communication topology in case of failures or severe radio path loss is proposed. The simulation results show that the distributed MPC trajectory planning approach achieves the goal of forming a communication network, which can in case of failures be successfully reconfigured by applying the developed optimal topology reconfiguration algorithm.

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