

# COVID CASE INFORMATIVENESS

MINI PROJECT

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# Agenda

- Our Mission
- The Team
- New Products and Services
- Competitive Analysis
- Competitors
- Size of Market
- Our Numbers
- Path to Integration

# MOTIVATION

This document presents some computations regarding the following question:

How much information does a *negative* test result convey?

A bit more precisely: Everyone can quickly look up the official numbers for their given region. I am based in Vienna, so I check the Austrian authorities' numbers; today, there are 49113 active cases. It is said that the true number of active cases is 5-10 times larger than the measured one, so let us say 300 thousand in Austria. Austria has a population of about 9 million, so if I walk around the streets and think that everyone is COVID-free then I am wrong about 300000  
9000000  
 $=3.33\%$   
 $300000/9000000 = 3.33\%$  of cases.

Now, let us say that I meet with a friend. Naturally, I assume that the friend is COVID-free. I am interested in comparing two scenarios:

1. The friend has a fresh negative test result.
2. The friend did not get tested.

In the second case, there is a 3.33% chance that I am wrong. Put differently, every **30th** friend without a test result is expected to have COVID today.

*How much more do I know about my friends' health thanks to the negative test result?*

# 1.1 Summary

- If your friends do not have any symptoms and all have negative *antigen* test results, then you know a little bit more: instead of every 30th friend, every **50th** such friend is expected to have COVID today.
- If they all have negative PCR test results, then every **166th** friend is expected to have COVID
- today.
- Finally, you know even more if your friends have several negative test results. The number is 82 instead of 50 for two negative antigen tests and it is 1000 instead of 166 for two negative PCR results.

# 1.2 IMPORTANT NOTICE

I am a statistician doing some dirty work here. In other words:

- I am not an expert of COVID tests
- This document does not constitute any health advice
- I do not take into account estimation error; this hurts my feelings but it would be much more work to find proper information for that
- There are simplifications in these computations; however, I do not think that the conclusion is much different with 10 times as much work as I did here

# 2 Terminology

- Sensitivity: probability of correct result *if you are infected*, i.e.

$P(\text{positive antigen} \mid \text{COVID})$

- Specificity: probability of correct result *if you are not infected*,

i.e.

$P(\text{negative antigen} \mid \text{no COVID})$

# 3 INPUT

I use input from different sources. The sources were not cherry-picked; however, I did not search very thoroughly for various studies. The first study is taken that looks trustworthy.

## 3.1 ANTIGEN

This study cited by the CDC. There vastly different values for people with and without COVID symptoms. If you are symptomatic, then

- Sensitivity = 80%
- Specificity = 98.9%

If you are asymptomatic, then

- Sensitivity = 41.2%
- Specificity = 98.4%

These numbers are estimates and they have their own error. I ignore that.

As discussed in the intro, I assume that the true number of active cases in Austria is about 300000 today. That is, in the following I will use

$$P(\text{COVID})=0.0333$$

$P(\text{COVID})=0.0333$ ; that means that a random person is infected with 3.33% probability.

text

## 3.2 PCR (NAAT)

I will look into pooled saliva-based PCR tests because that is the method used by “Alles gurgelt”—an initiative of the city of Vienna.

I take the numbers from this article and the cited study therein.

- Sensitivity = 80%
- Specificity = 98.9%

The figures for the pooled nasopharyngeal swab (not saliva-based) are similar.

# 4 Computation

The interesting question is the probability if being negative given a negative result. The figures above give us the reverted probability: e.g. getting a negative result given that you are negative (that is specificity).

One uses Bayes' theorem to revert the probability:

$$P(A | B) = P(B \cap A) / P(B)$$

# 4.1 ANTIGEN

In our case, we need a more detailed variant of that formula:

$$P(\text{noCOVID} \mid \text{negativeantigen}) = P(\text{negativeantigen} \mid \text{noCOVID})P(\text{no COVID}) / P(\text{negative antigen} \mid \text{no COVID})P(\text{no COVID}) + P(\text{negative antigen} \mid \text{COVID})P(\text{COVID})$$

where

$$P(\text{negative antigen} \mid \text{COVID}) = 1 - P(\text{positive antigen} \mid \text{COVID})$$

# 4.1.1 Asymptomatic

That gives for **asymptomatic** people

$$\begin{aligned} P(\text{no COVID} \mid \text{negative antigen}) &\approx 0.984 \cdot 0.9667 / (0.984 \cdot 0.9667 + \\ &(1 - 0.412) \cdot (1 - 0.9667) \\ &= 97.98\% \end{aligned}$$

So every 50th negative COVID antigen test gives false inference for asymptomatic people.

## 4.1.2 Symptomatic

And that gives for **symptomatic** people

$P(\text{no COVID} \mid \text{negative antigen}) \approx$

$$0.989 \cdot 0.9667 / (0.989 \cdot 0.9667 + (1 - 0.8) \cdot (1 - 0.9667))$$

=99.31% Only 7 out of 1000 negative test results give false inference  
for symptomatic people.

## 4.2 PCR

Very similar computation leads to the following result:

$$P(\text{no COVID} \mid \text{negative PCR}) \approx$$

$$0.992 \cdot 0.9667 / (0.992 \cdot 0.9667 + (1 - 0.832) \cdot (1 - 0.9667))$$

$$= 99.42\%$$

Only 6 out of 1000 test results give false inference in case of a setup similar to Alles gurgelt. That is ca. every 166th negative PCR test results in wrong inference.

# 5 DISCUSSION

If it is antigen testing, then it is important to combine the test result with observing symptoms. A negative antigen test result for an asymptomatic person makes you only a bit more certain (less than twice as certain) about that person being healthy.

“Still, that 97.98% is not soooo bad.”

It is not but without the test you already know 96.67%. The error is ~2% with the antigen test which is not much better than the ~3.3% error of the base rate.

That would be the main message of the document: the interesting comparison is between the base rate (96.67%) and the rate given a negative test (97.98% for asymptomatic).

The 41.2% from above is not the final answer you are interested in because what you know is whether your friend has a negative test result; you do not know whether your friend has COVID. It is the 97.98% that tells you the probability for your friend having COVID given that you know that she/he has a negative antigen test result. Trust PCR tests more; they are more than three times as good as antigen tests and more than five times as good as the base rate guess. Do Alles gurgelt.

# 6 What about repeated tests??

Again, Bayes' theorem can be applied to answer the interesting question: how much more do I know about my friend if they have two negative tests compared to the base rate/one negative test result?

## 6.1 Asymptomatic antigen

$$\begin{aligned} P(\text{no COVID} \mid 2 \text{ negative antigens}) &\approx 0.984 \cdot 0.9667 / 0.9842 \cdot 0.9667 + \\ &(1 - 0.412)2 \cdot (1 - 0.9667) \\ &= 98.78\% \end{aligned}$$

That is every 82nd asymptomatic person is expected to have COVID when they have two negative antigen test results. Here, I assume that test results can be faulty independently; I consider it to be a realistic assumption.

## 6.2 PCR

The same idea, Bayes' rule:

$$\begin{aligned} P(\text{no COVID} \mid 2 \text{ negative PCRs}) &\approx 0.9922 \cdot 0.9667 / 0.9922 \cdot 0.9667 + (1 - 0.832) \cdot (1 - 0.9667) \\ &= 99.901\% \end{aligned}$$

Every 1000th person with two negative PCR test results is expected to have COVID.