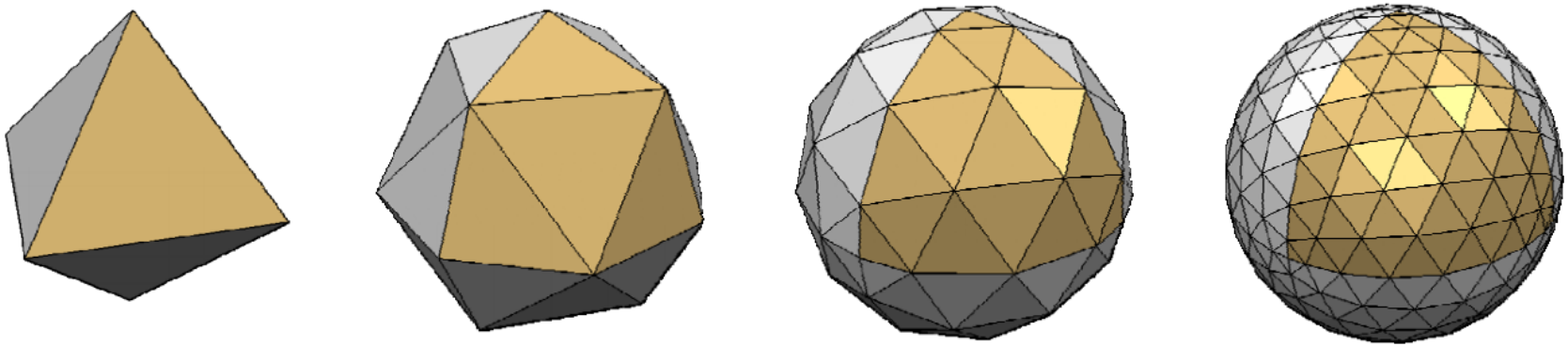
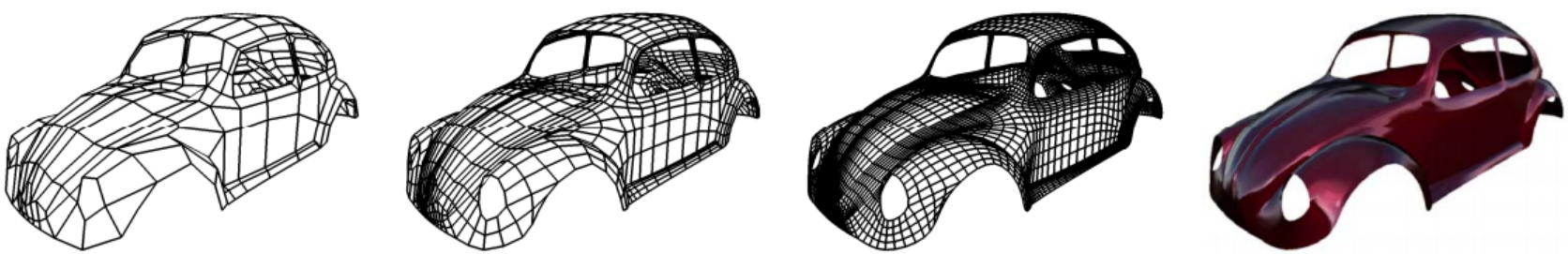
Computer Graphics

Digital Geometry Processing

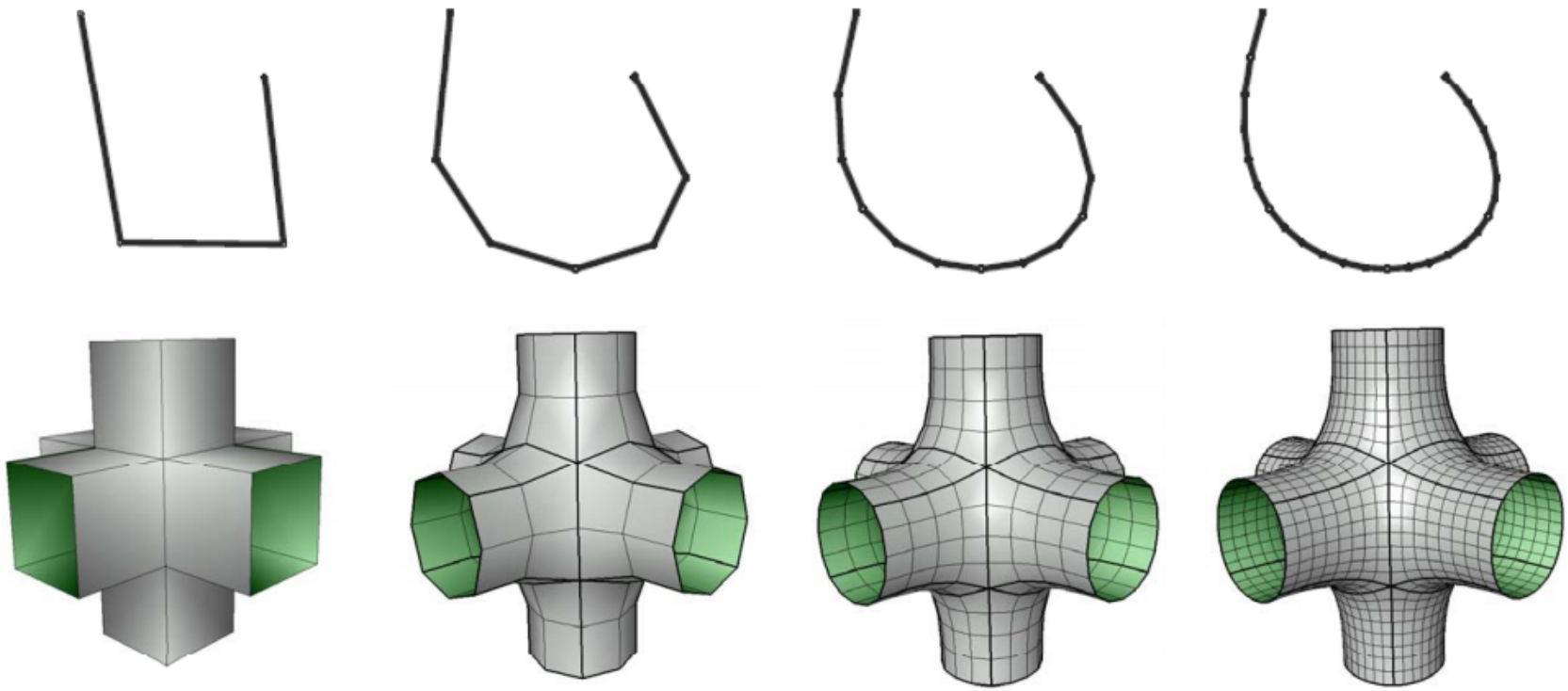
Subdivision surfaces

- Smooth surfaces as the limit of a sequence of refinements

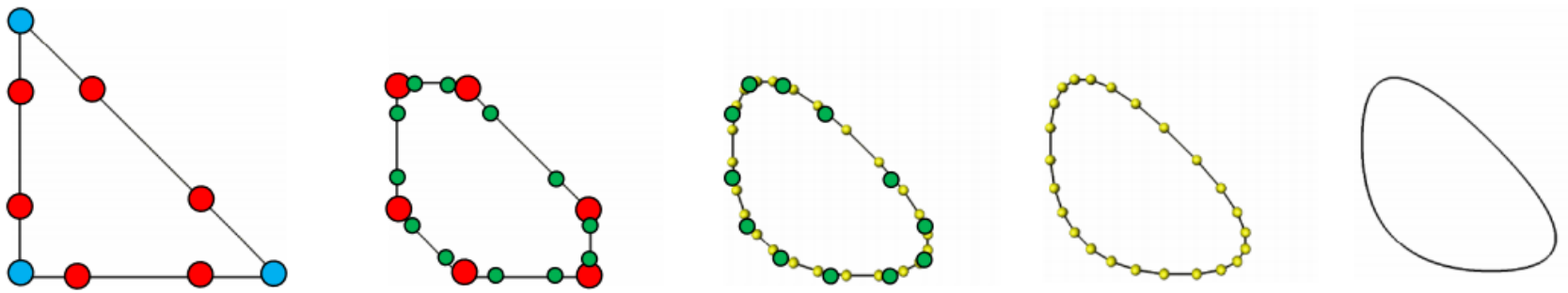




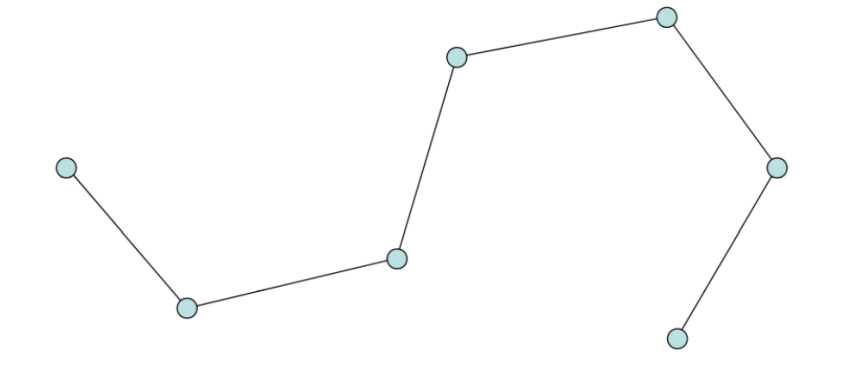
Subdivision surfaces

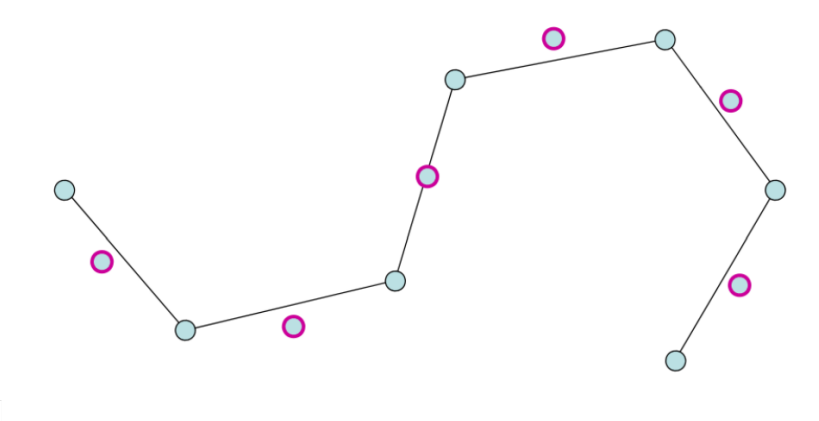


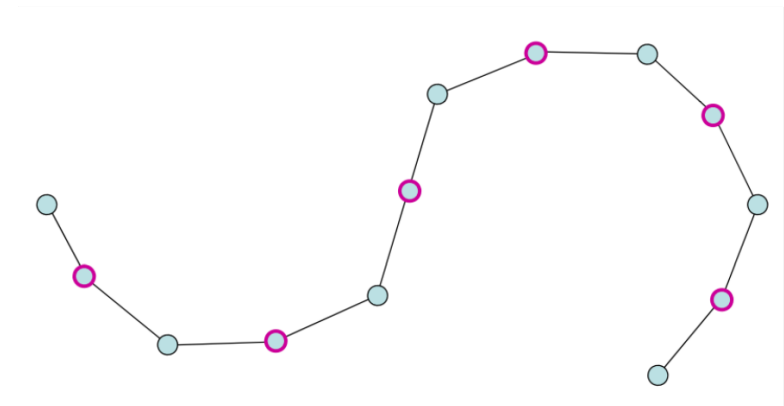
Corner cutting - Chaiken's Algorithm

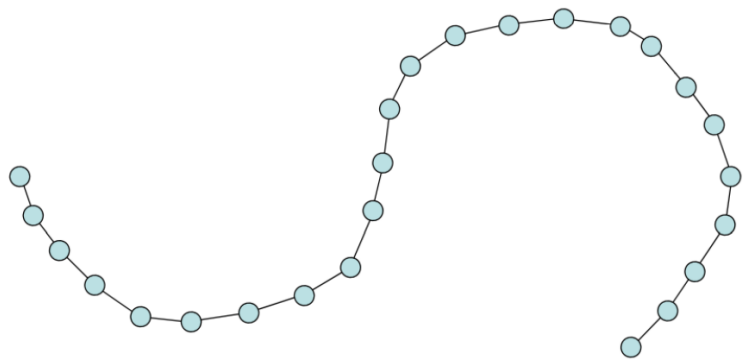


Interpolation

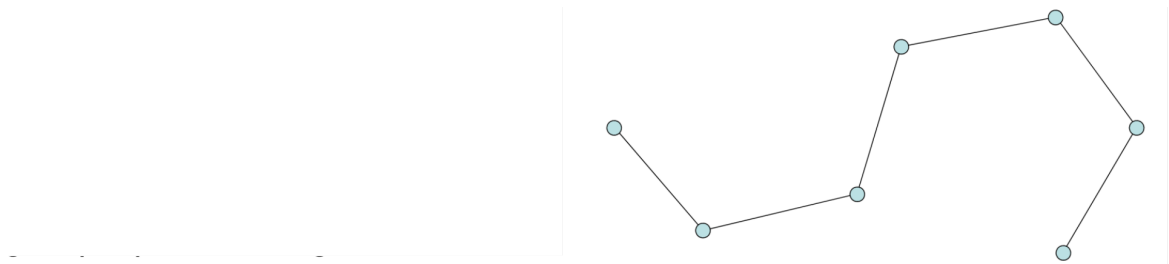


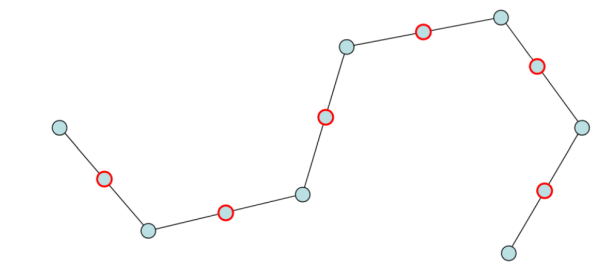


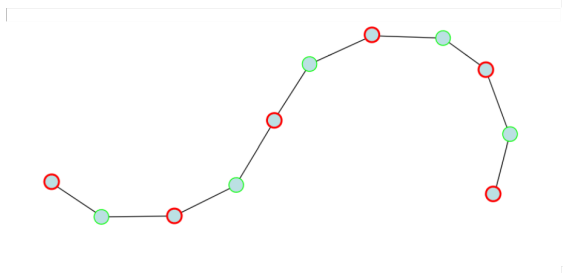


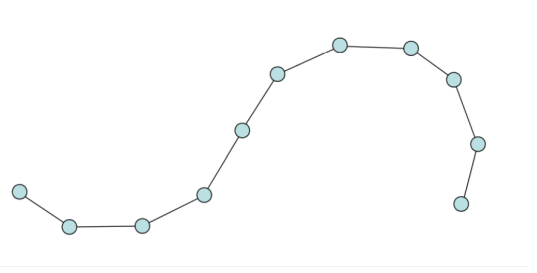


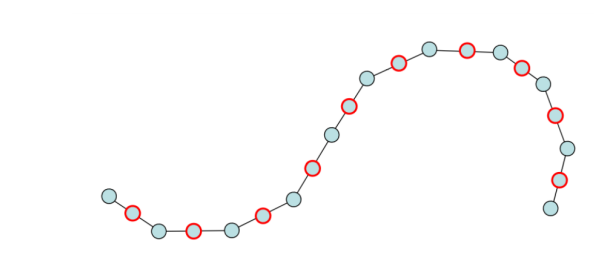
Approximation

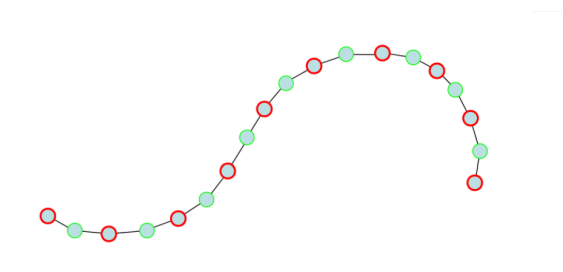


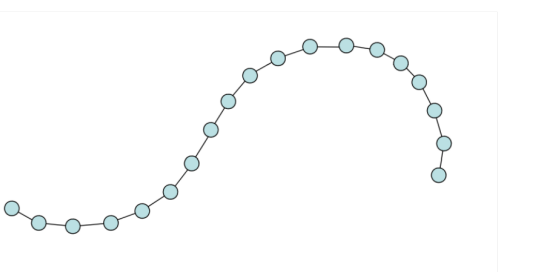




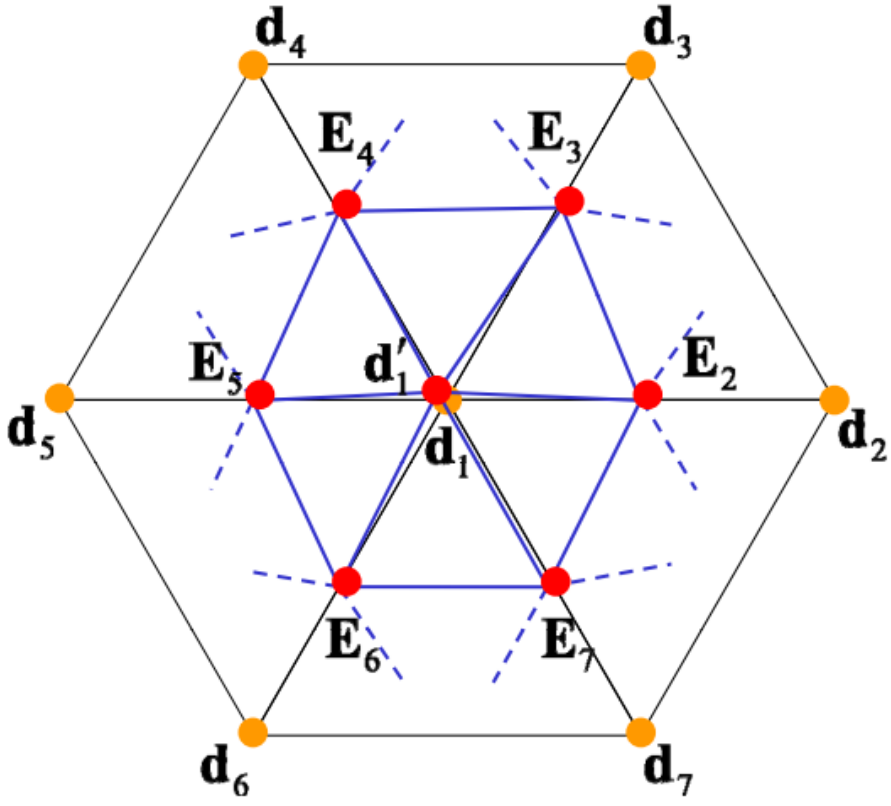








Subdivision surfaces in 3D triangular mesh

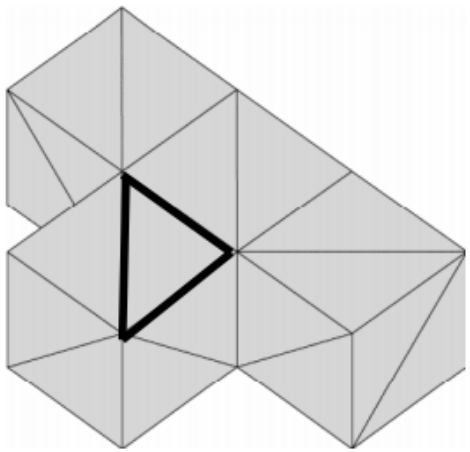


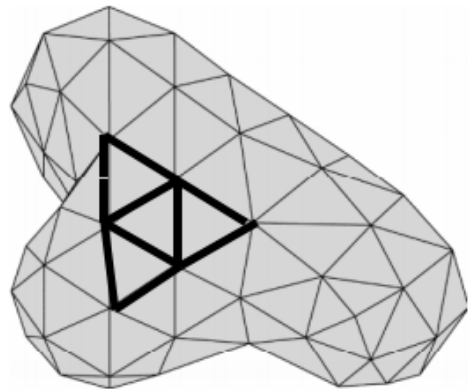
$$E_i = \frac{3}{8}(d_1 + d_i) + \frac{1}{8}(d_{i-1} + d_{i+1})$$

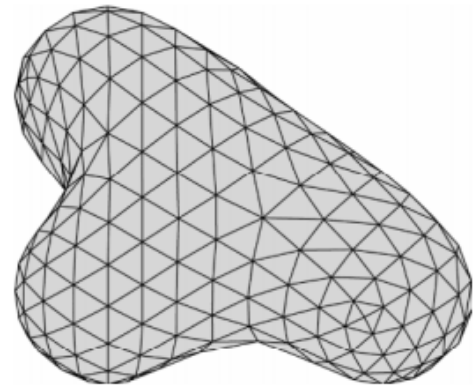
$$\mathbf{d}'_1 = \alpha_n \mathbf{d}_1 + \frac{(1-\alpha_n)}{n}\sum_{j=2}^{n+1} \mathbf{d}_j$$

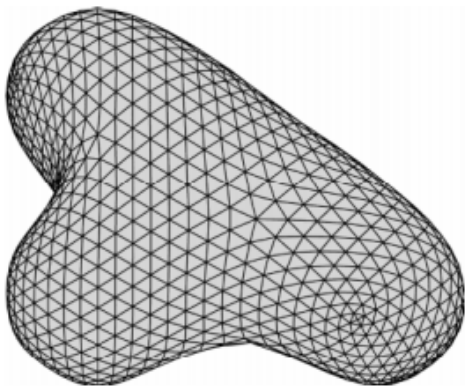
$$\alpha_n = \frac{3}{8} + \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2$$

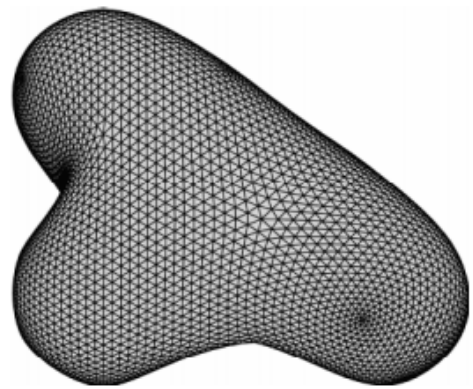
Subdivision surfaces in 3D triangular mesh



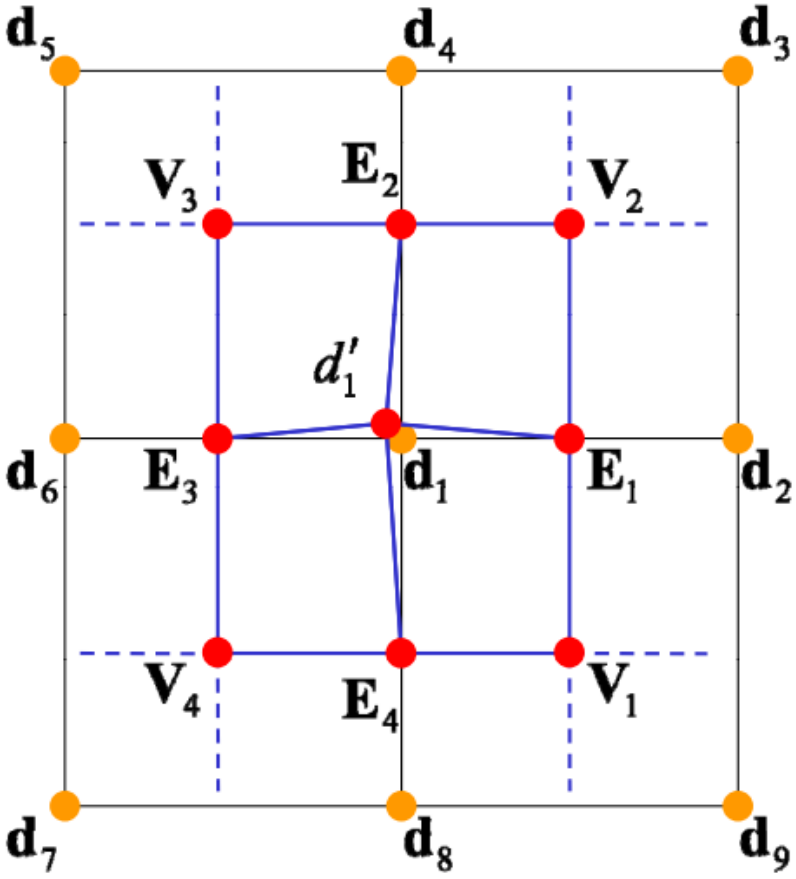








Subdivision surfaces in 3D quad mesh



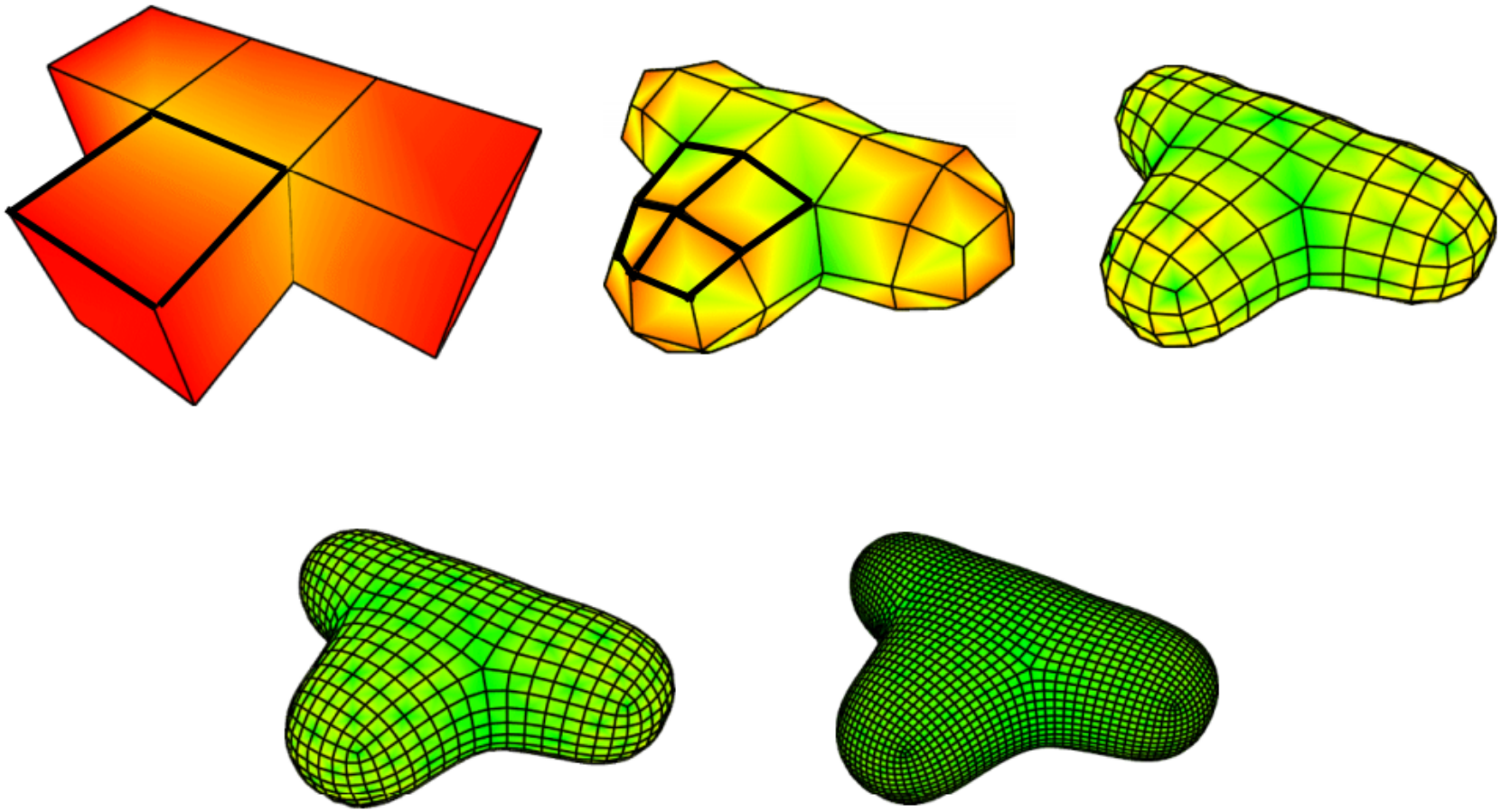
$$\mathbf{V}_2 = \frac{1}{n} \times \sum_{j=1}^{n} \mathbf{d}_j$$

$$\mathbf{E}_i = \frac{1}{4}\left(\mathbf{d}_1 + \mathbf{d}_{2i} + \mathbf{V}_i + \mathbf{V}_{i+1}\right)$$

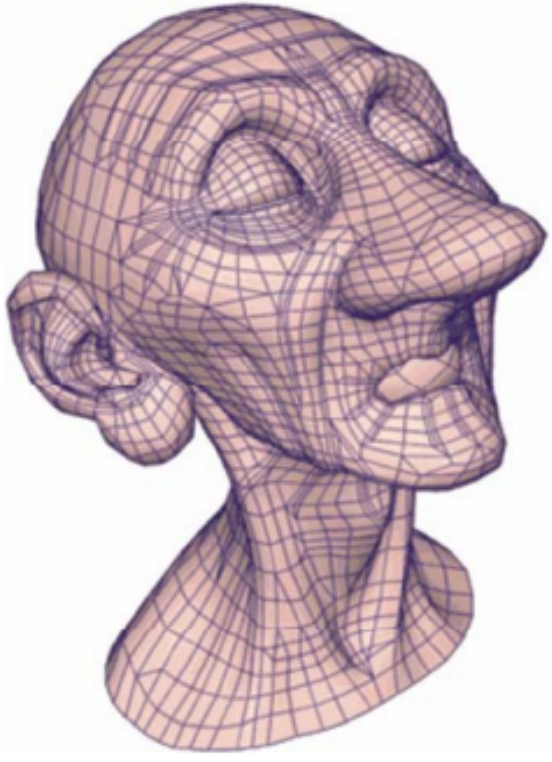
$$\mathbf{d}_1' = \frac{(n-3)}{n}\mathbf{d}_1 + \frac{2}{n}\mathbf{R} + \frac{1}{n}\mathbf{S}$$

$$\mathbf{R} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{E}_i \quad \mathbf{S} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{V}_i$$

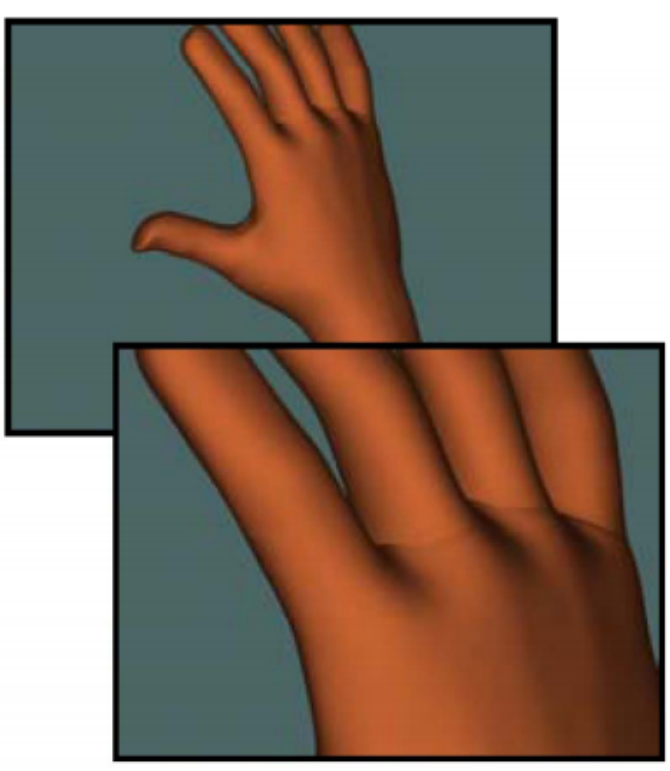
Subdivision surfaces in 3D quad mesh

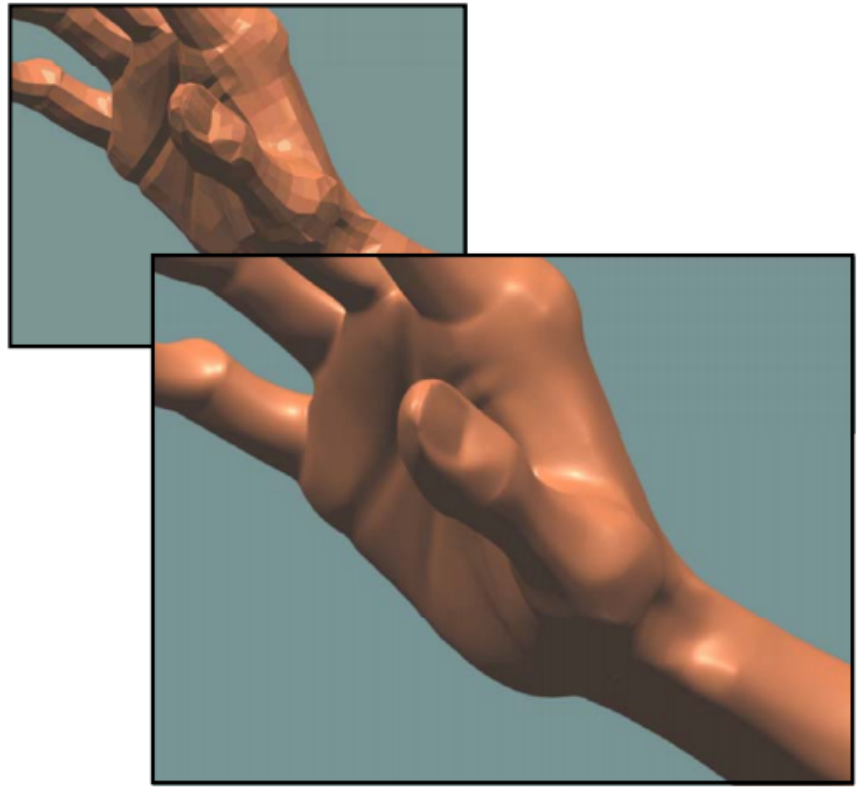


Geri's Game



Geri's Game





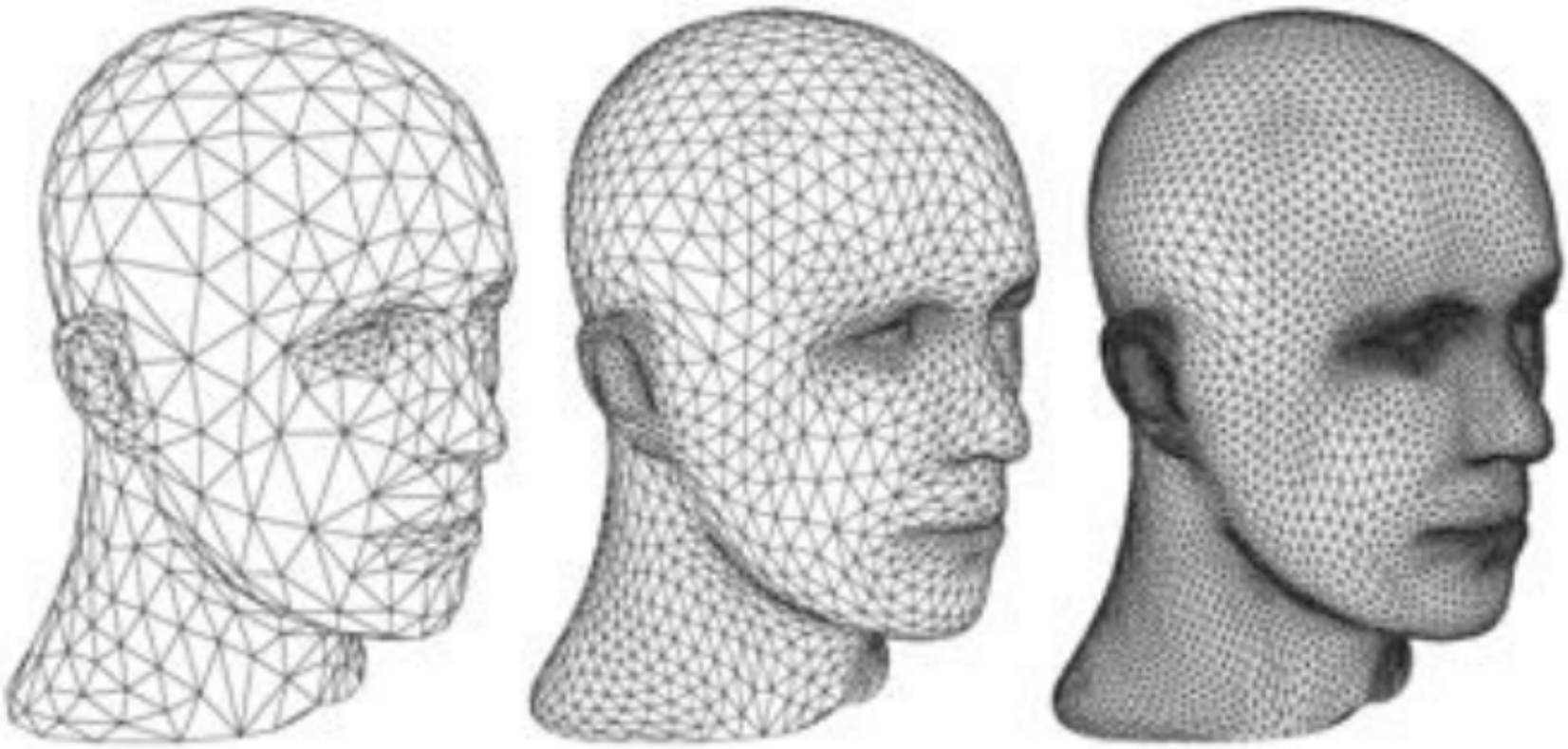
Geri's Game

Operations

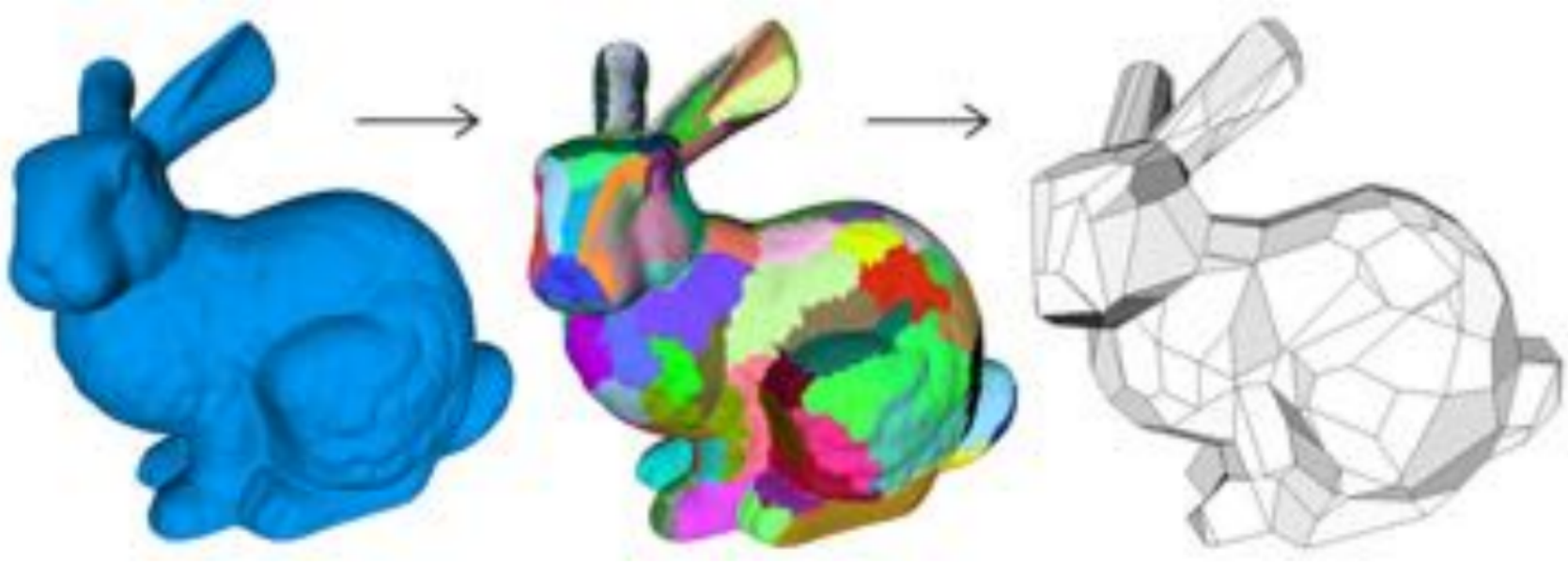
- Upsampling
- Downsampling
- Resampling
- Filtering
- Compression, Reconstruction, Parameterization, ...

Upsampling - image

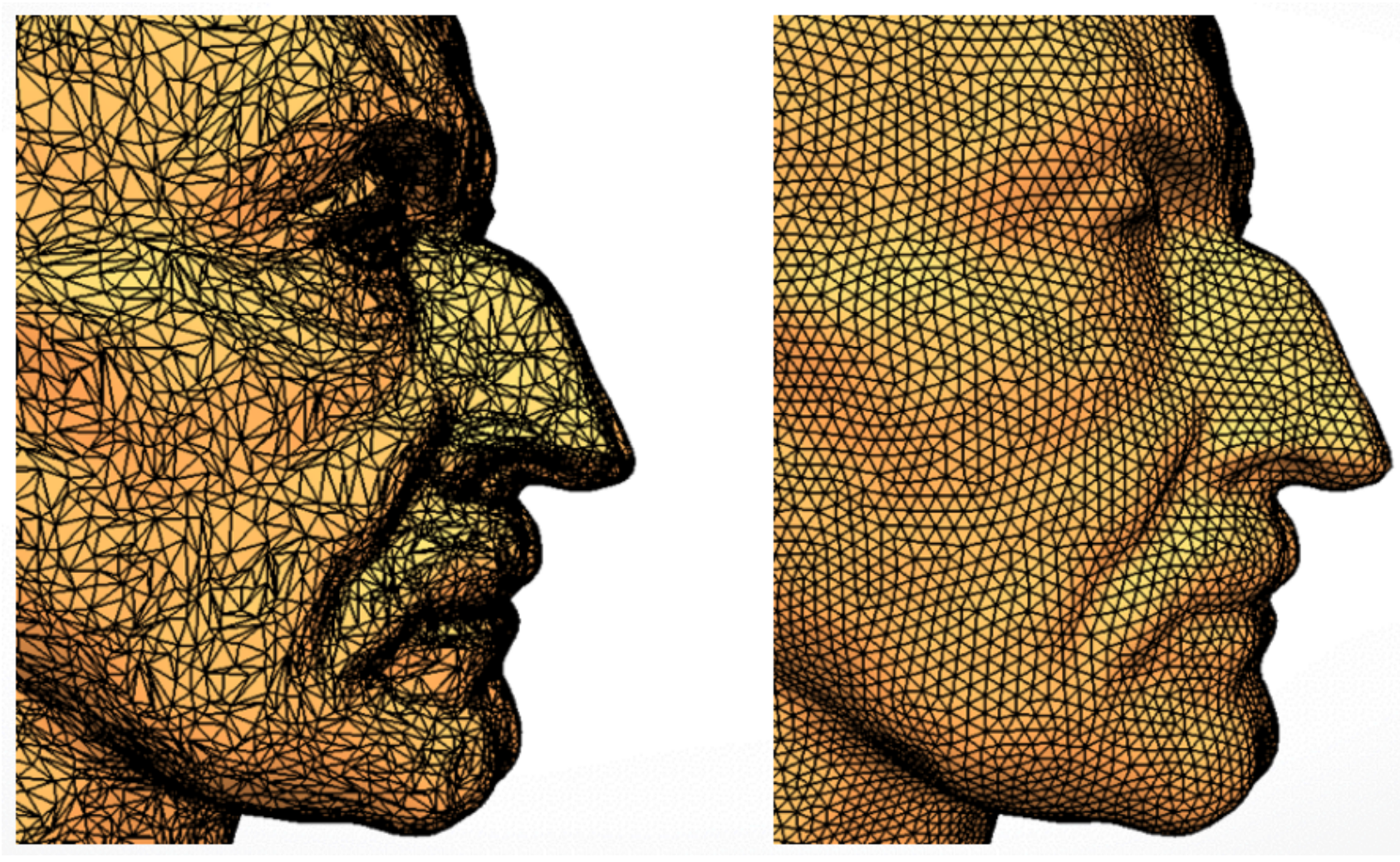
Upsampling - 3d model



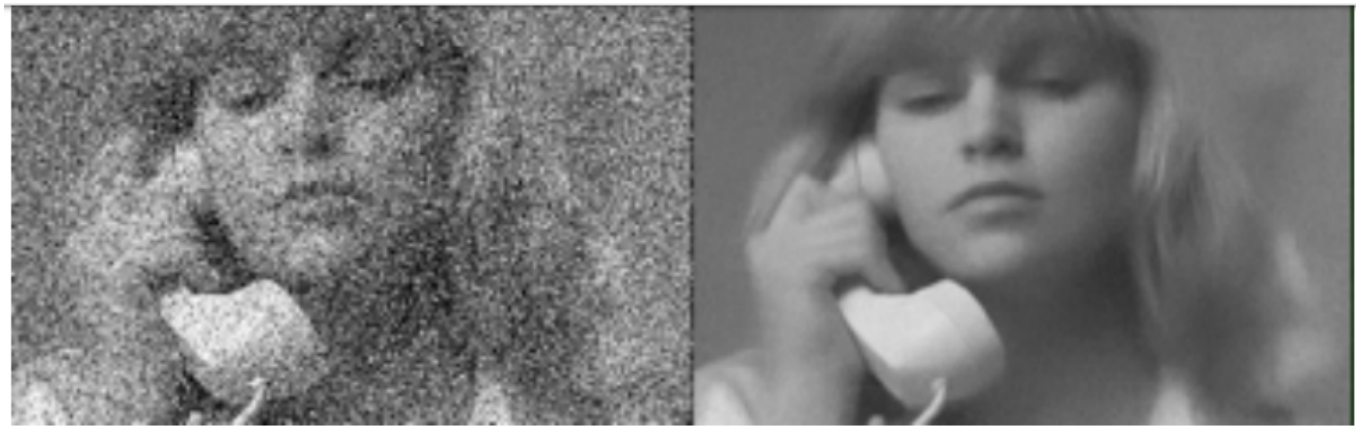
Downsampling

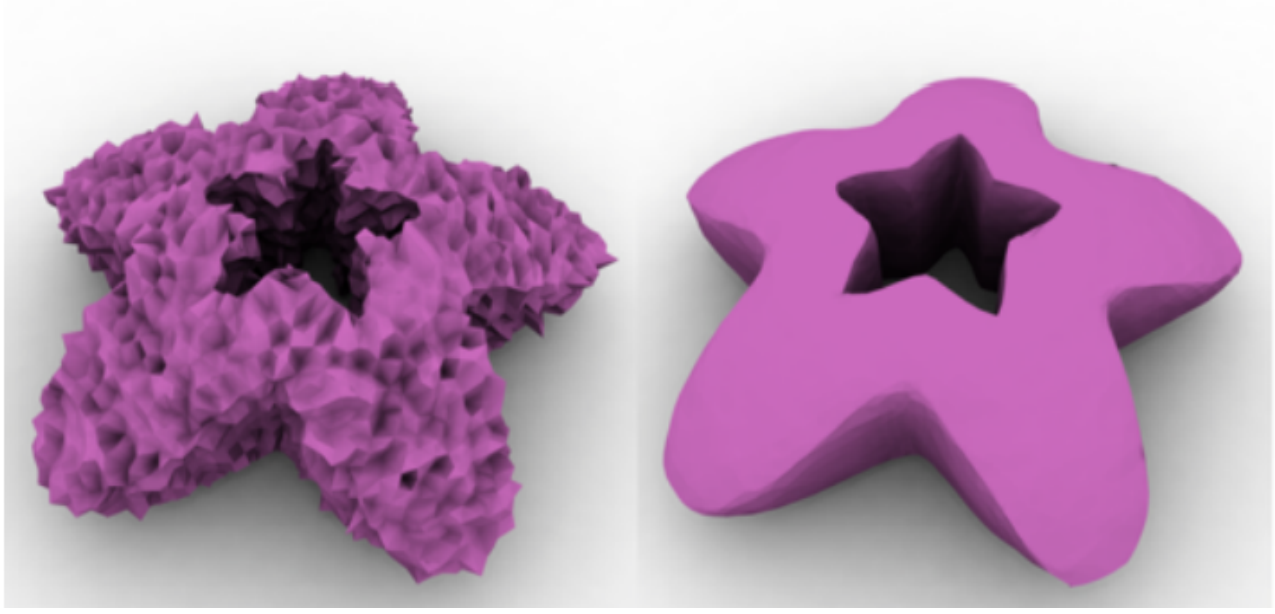


Resampling



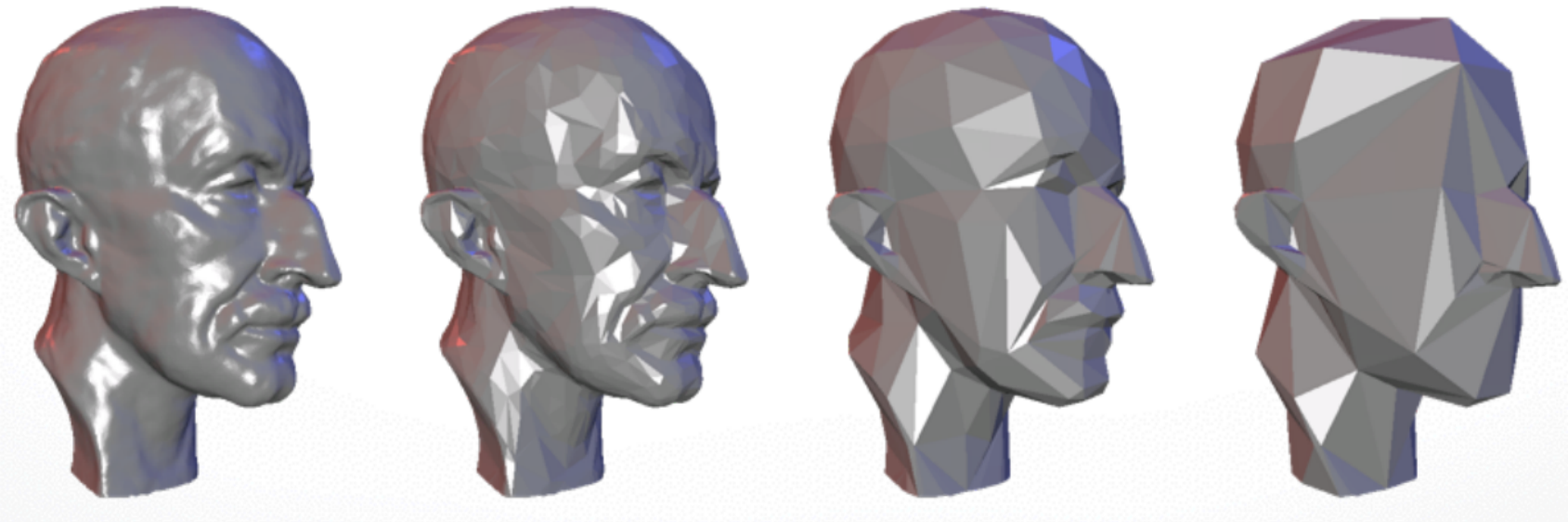
Filtering



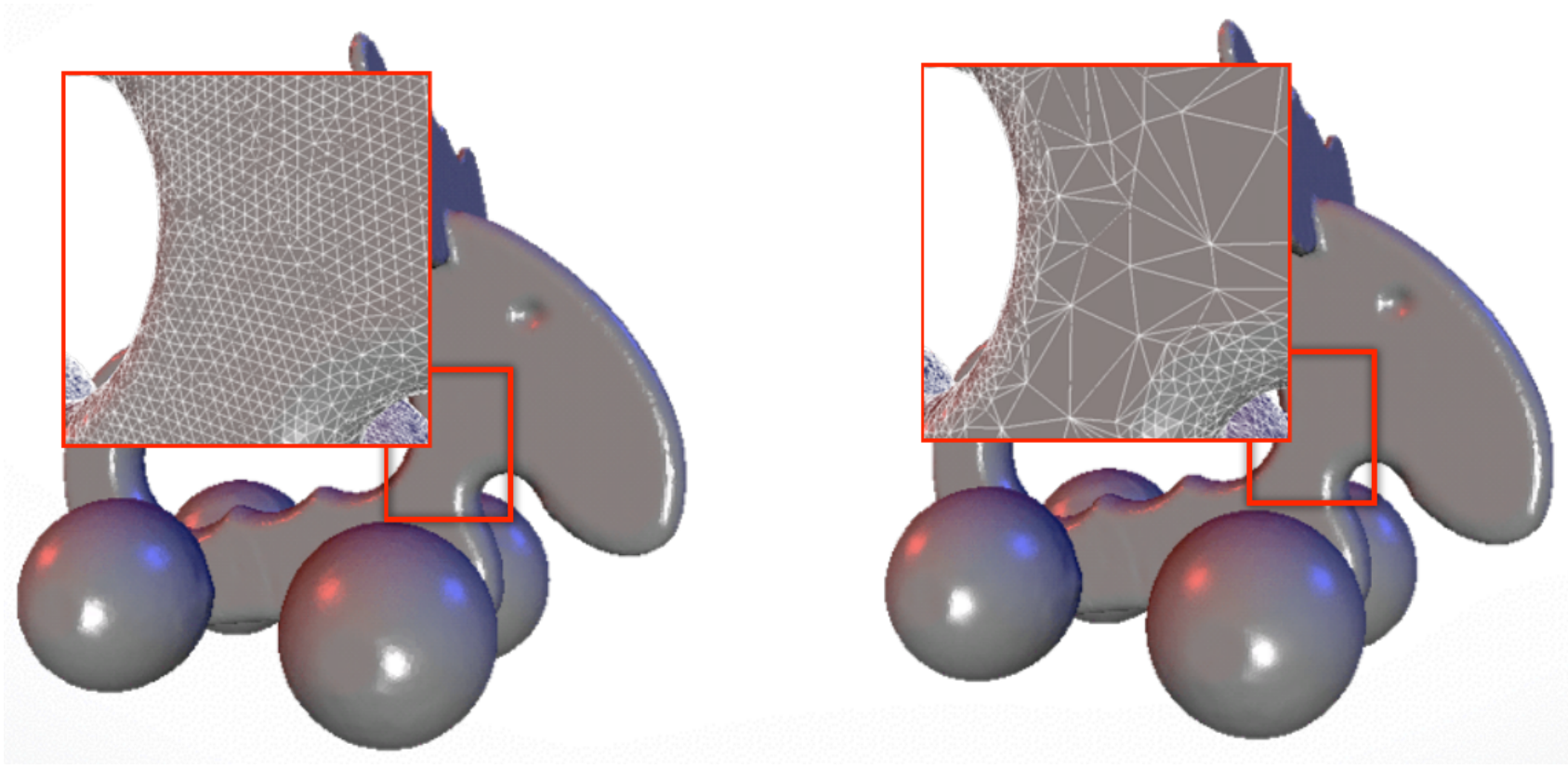


Downsampling - mesh simplification

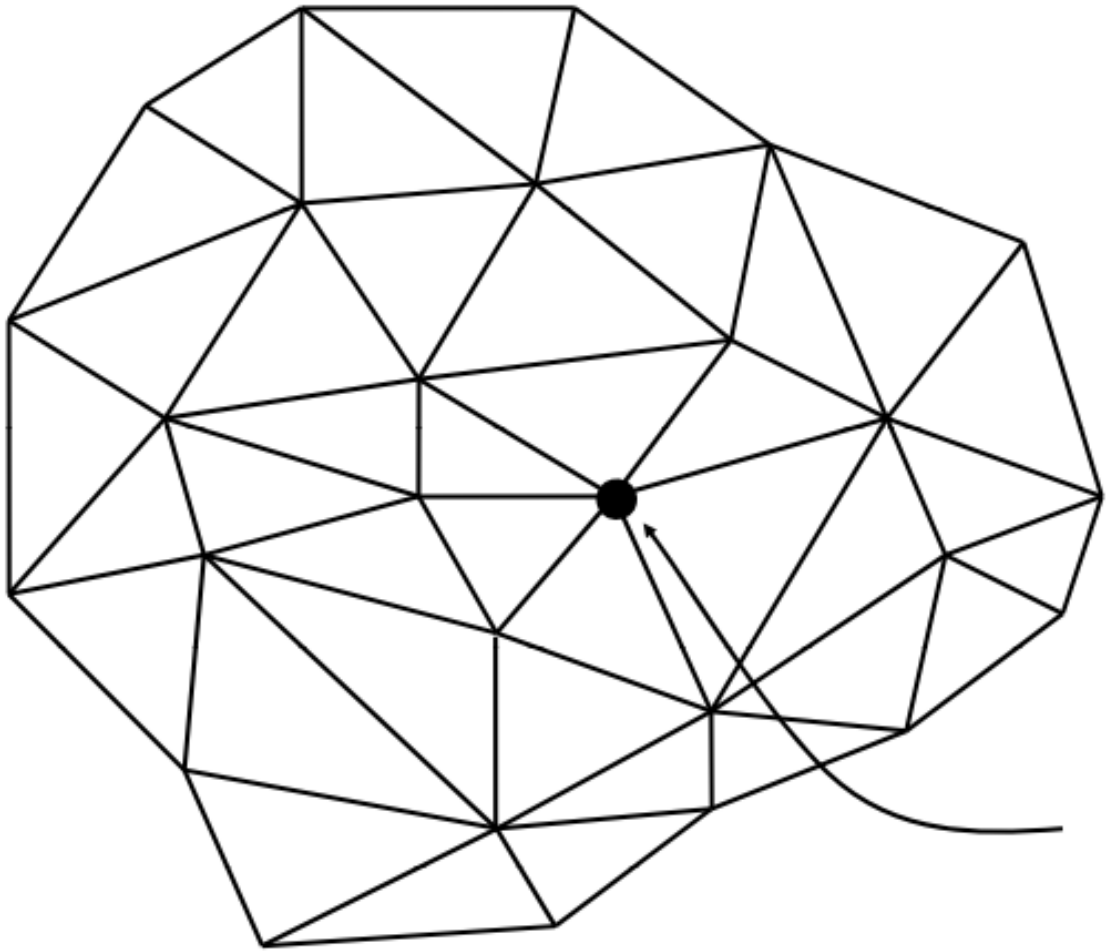
- Vertex clustering
- Incremental decimation



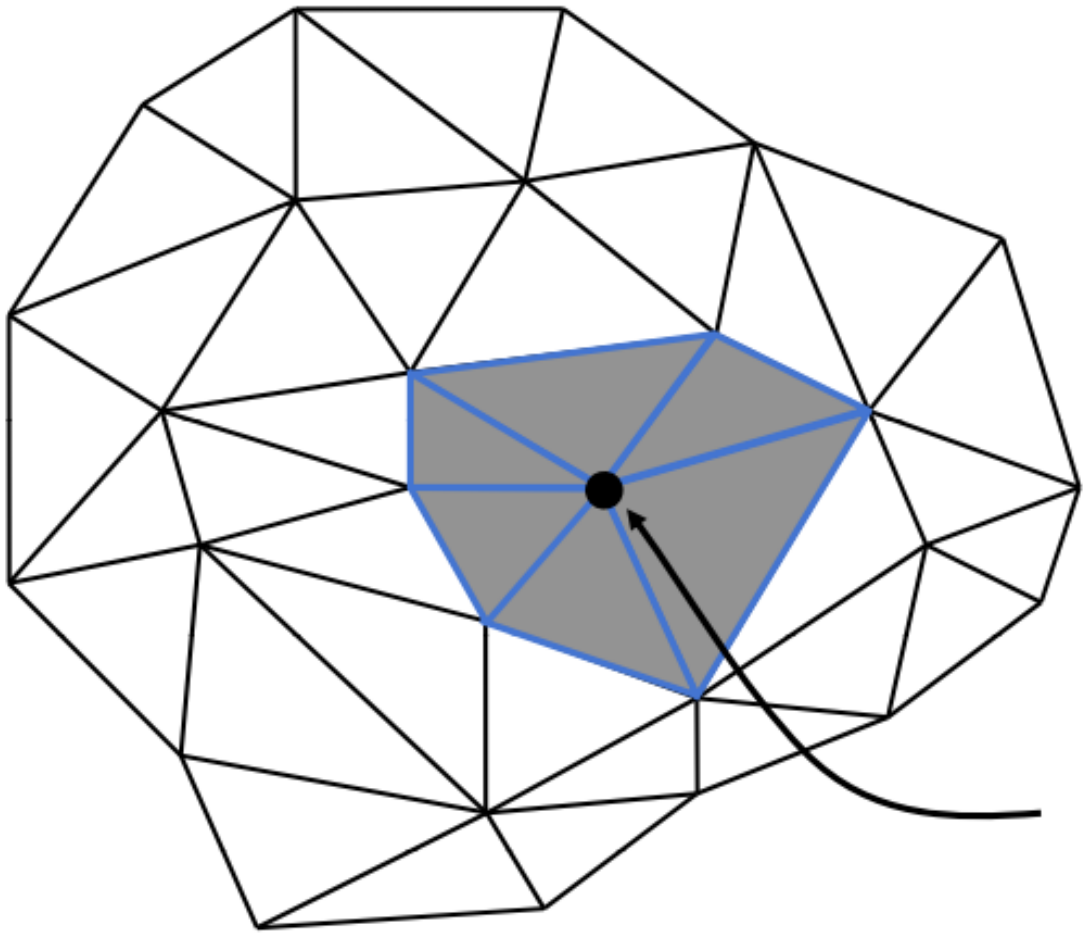
Mesh simplification



Incremental decimation

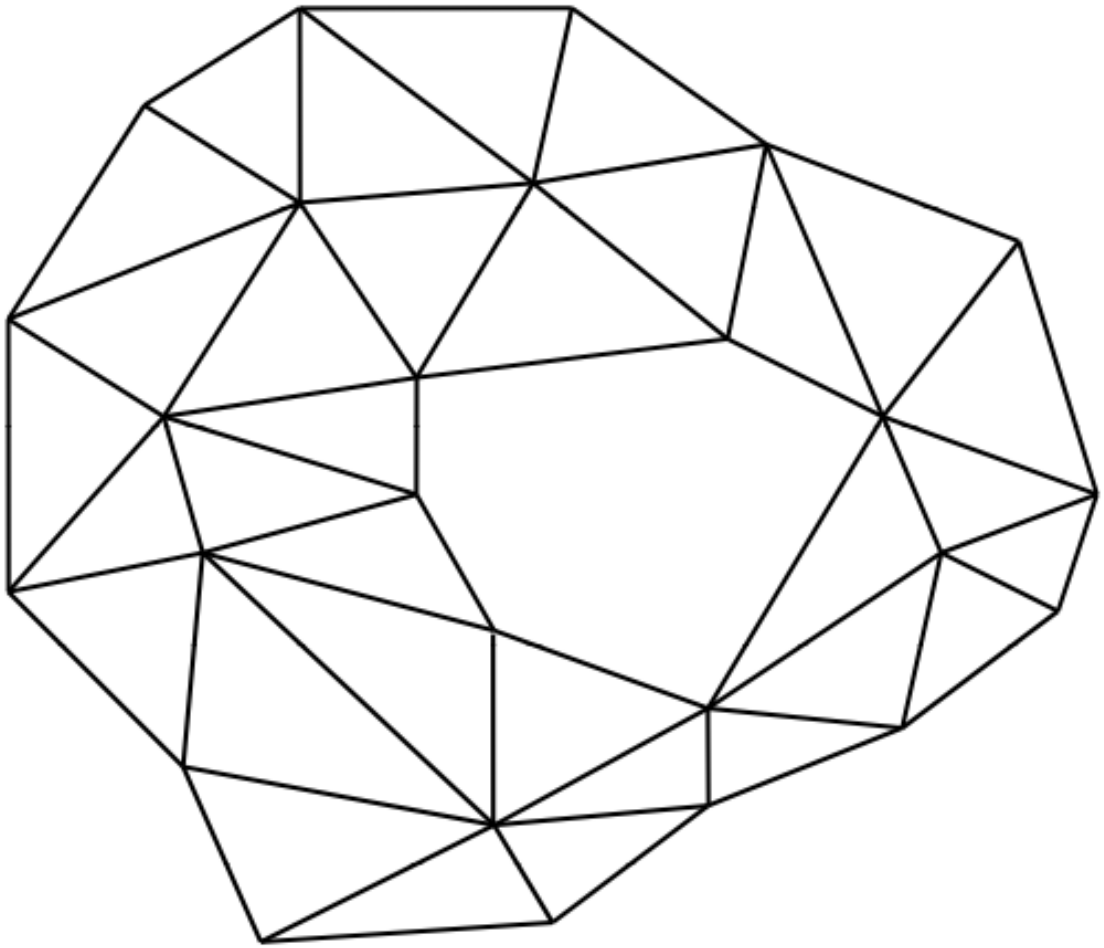


Incremental decimation



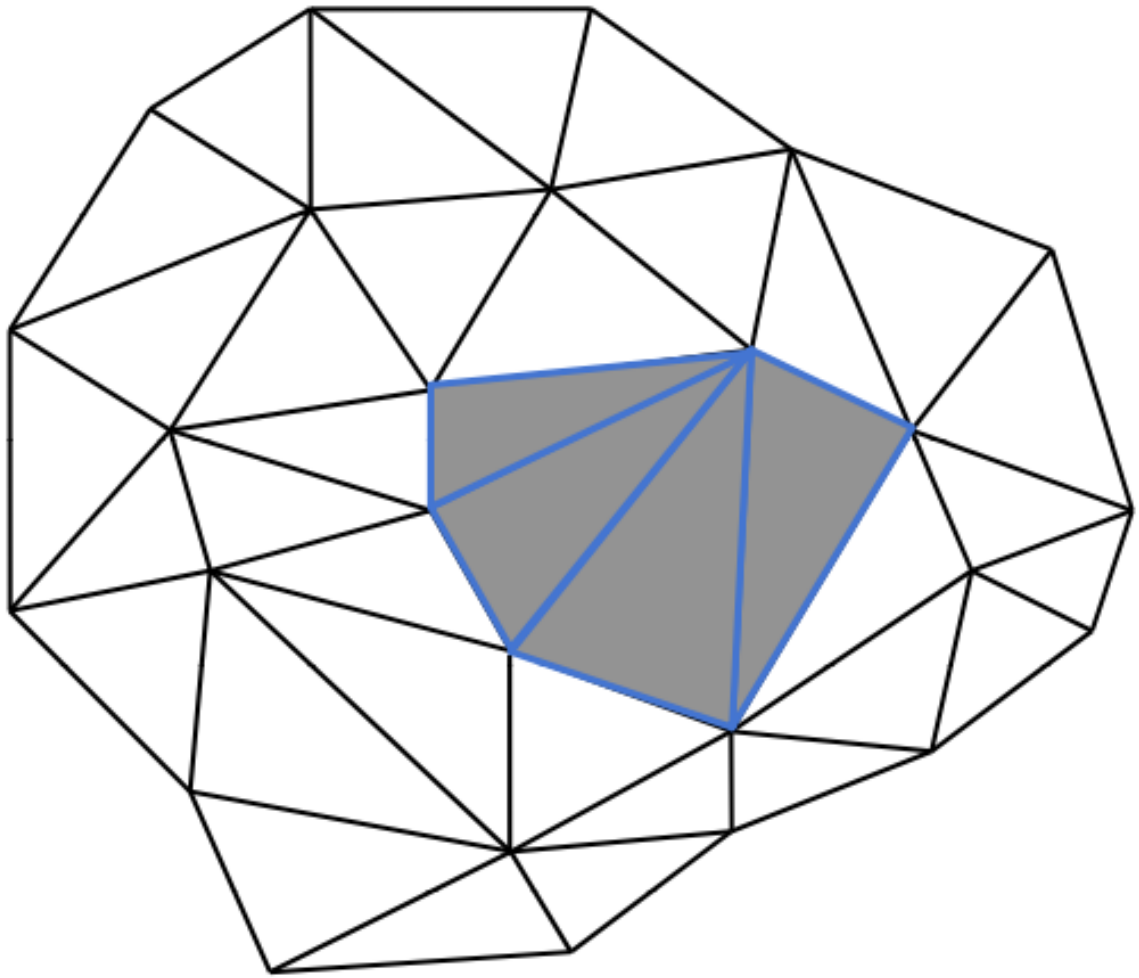
Select all triangles
sharing this vertex

Incremental decimation



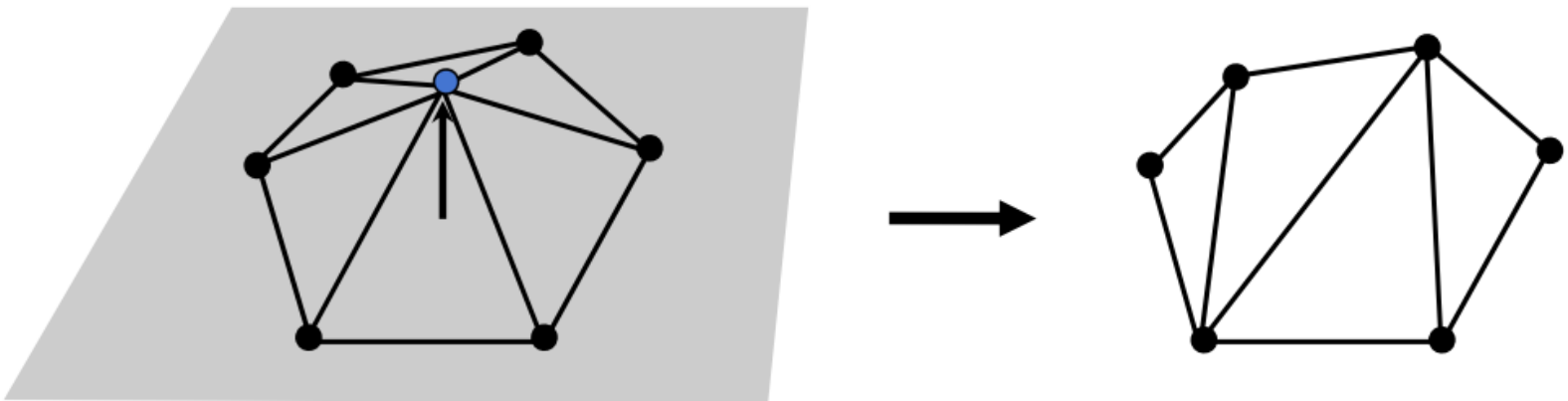
Remove the selected triangles, creating the hole

Incremental decimation



Fill the hole with new triangles

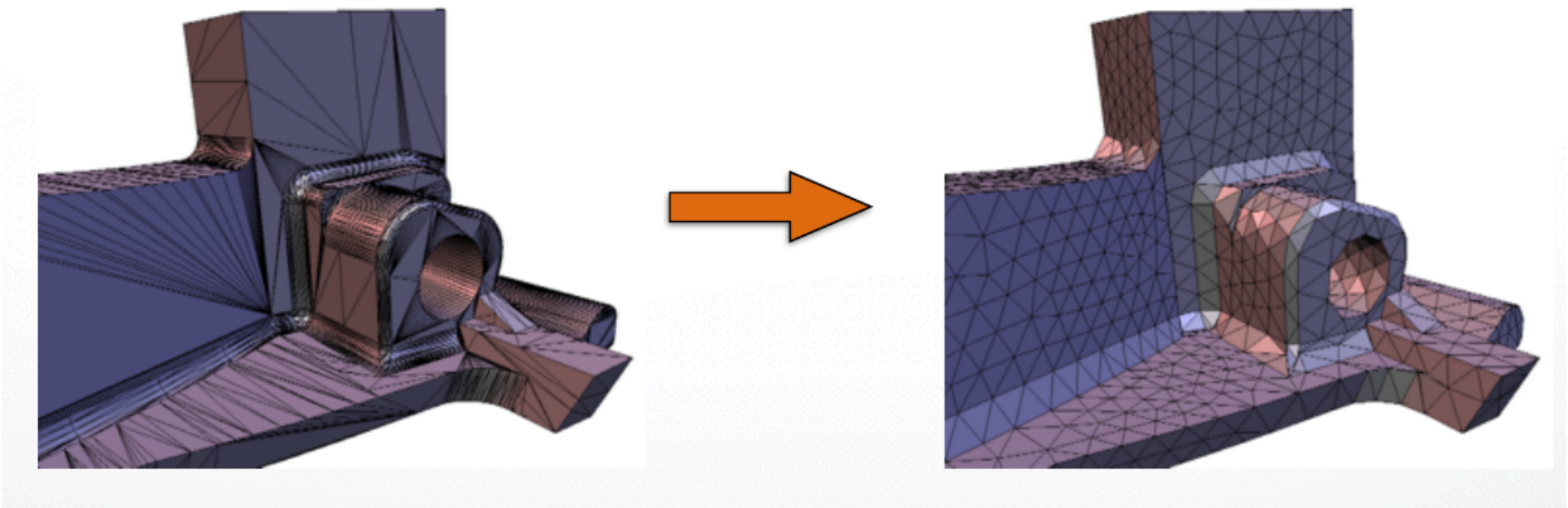
Incremental decimation



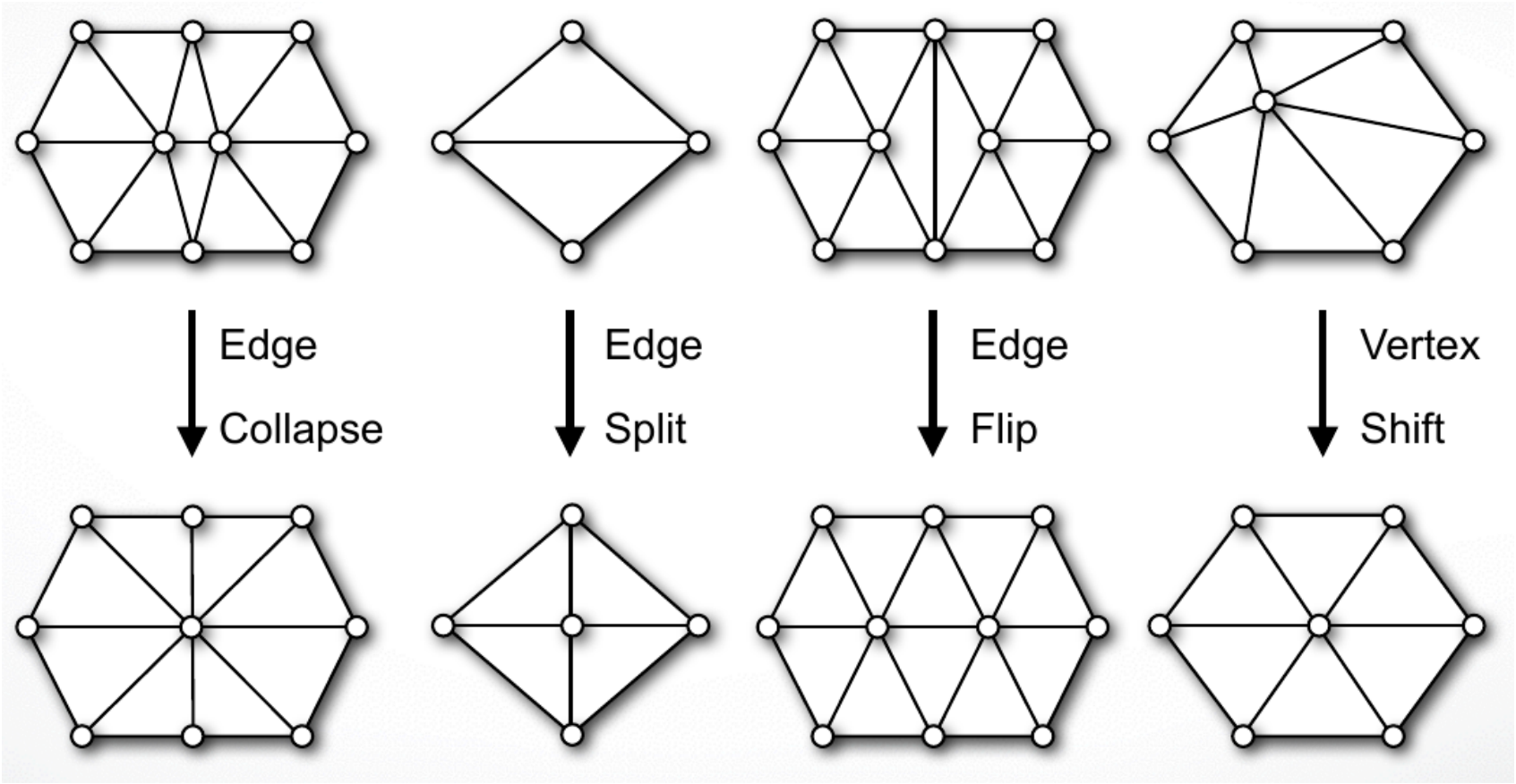
Resampling - Remeshing

- Happy with the number of triangles, but want to improve quality
- Make use of Delaunay

Remeshing



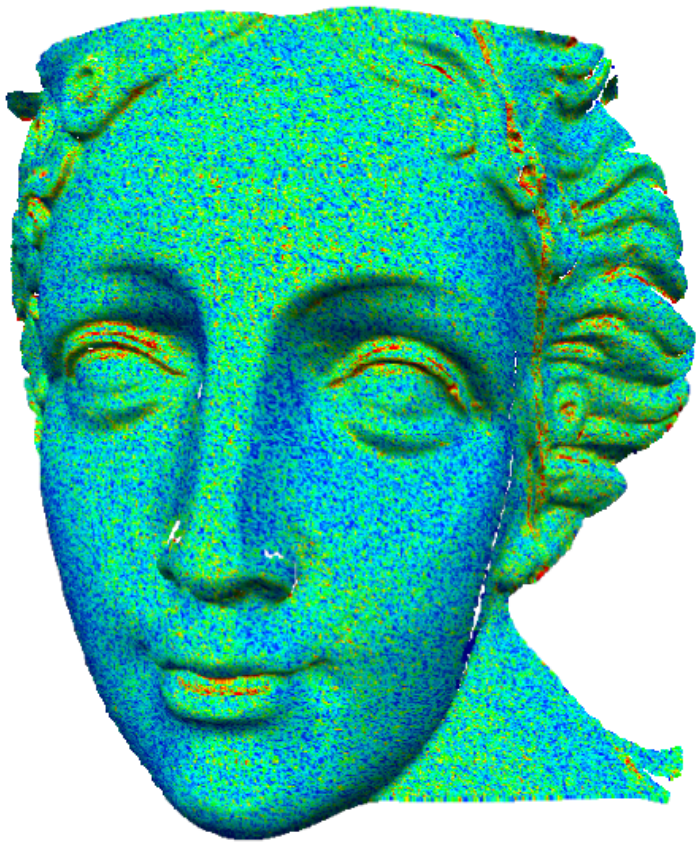
Local remeshing operators



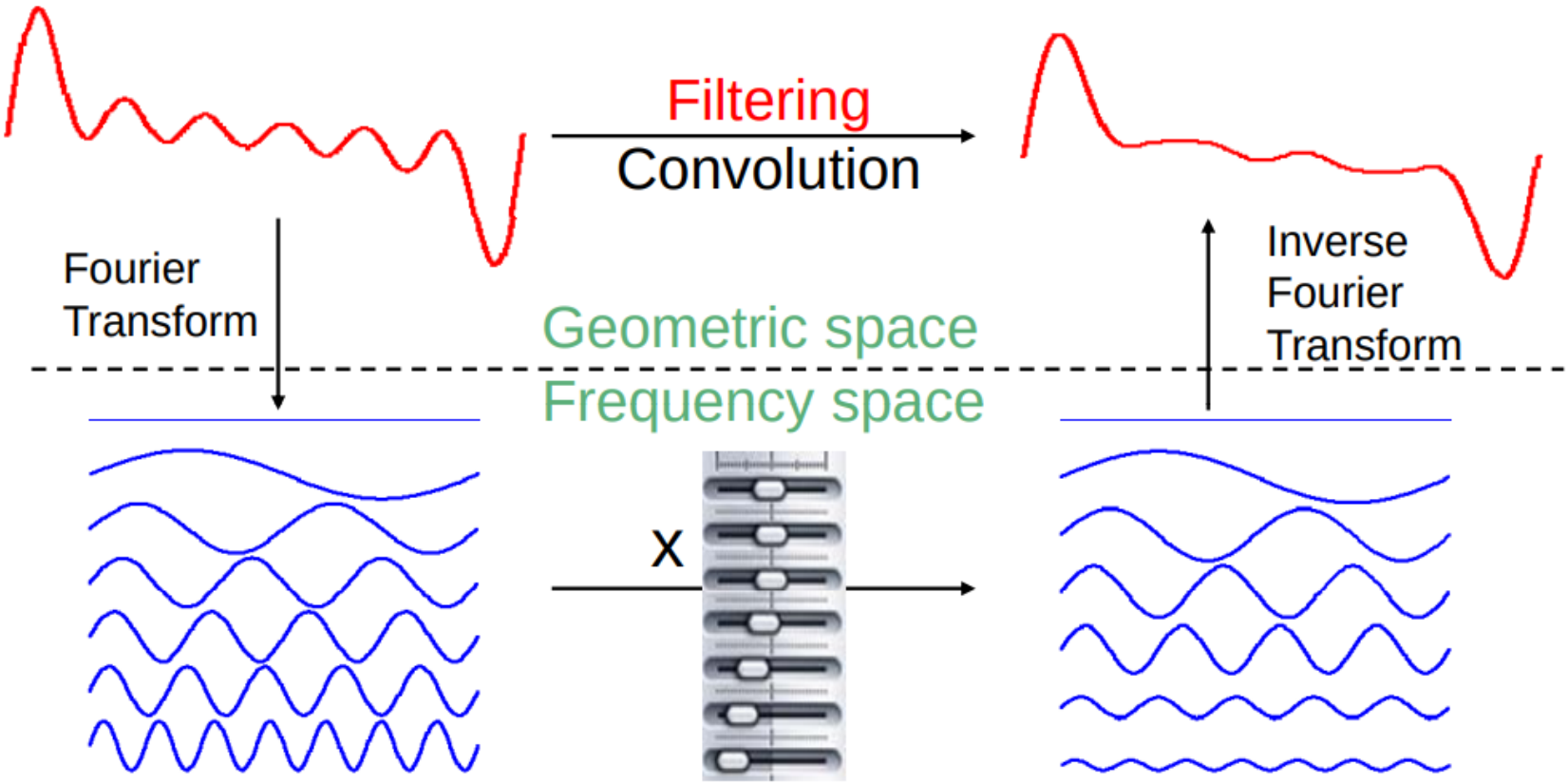
Incremental remeshing

- L_{max}=4/3*L, L_{min}=4/5*L
- Iterate:
  1. Split edges longer than L_{max}
  2. Collapse edges shorter than L_{min}
  3. Flip edges to get closer to optimal valence!
  4. Vertex shift by tangential relaxation!
  5. Project vertices onto reference mesh

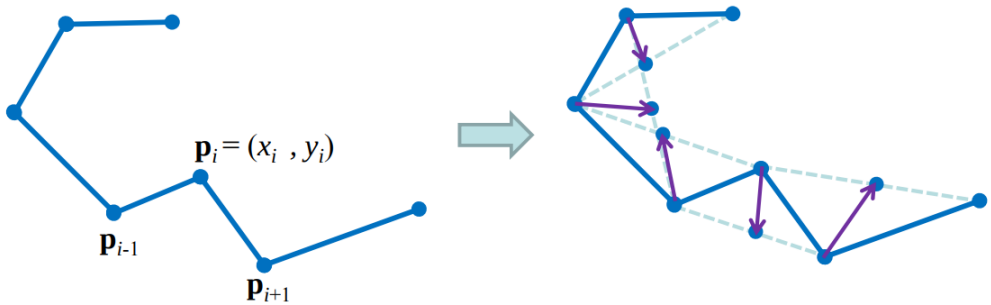
Filtering - Smoothing



Smoothing

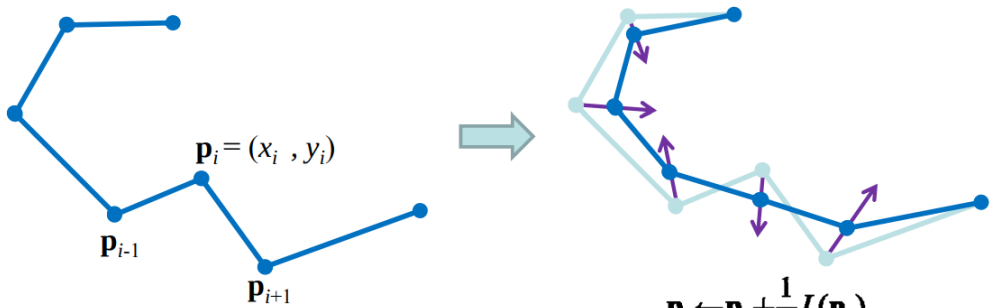

Filtering
Convolution
Fourier
Transform
Inverse
Fourier
Transform
Geometric space
Frequency space
X

Laplacian smoothing



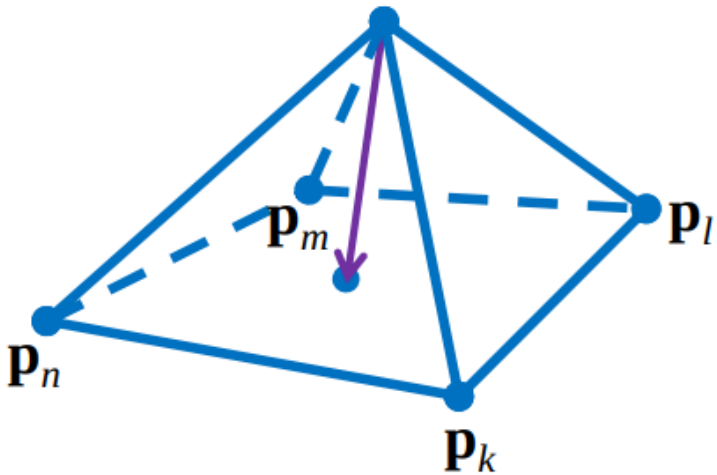
$$(\mathbf{p}_{i-1} + \mathbf{p}_{i+1})/2 - \mathbf{p}_i$$

$$L(\mathbf{p}_i)=\frac{1}{2}(\mathbf{p}_{i+1}-\mathbf{p}_i)+\frac{1}{2}(\mathbf{p}_{i-1}-\mathbf{p}_i)$$



$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \frac{1}{2}L(\mathbf{p}_i)$$

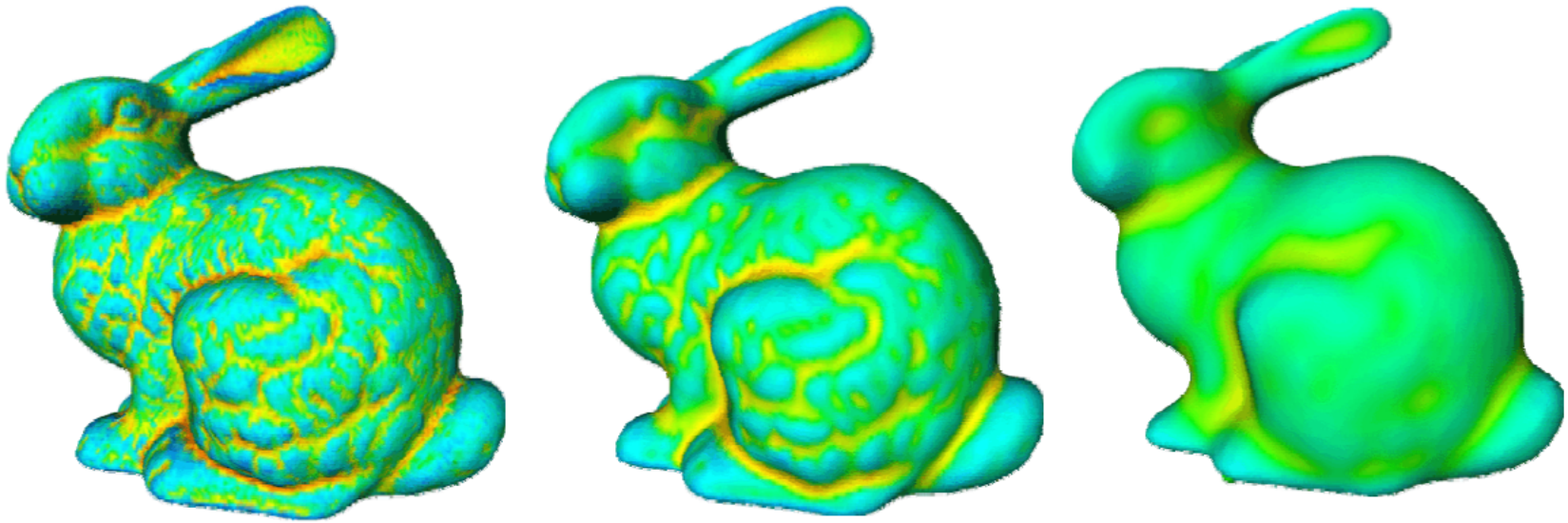
Laplacian smoothing



$$\frac{1}{2}\left(\mathbf{p}_{i+1}+\mathbf{p}_{i-1}\right)-\mathbf{p}_i$$

$$\frac{1}{|N_i|}\left(\sum_{j\in N_i}\mathbf{p}_j\right)-\mathbf{p}_i$$

Laplacian smoothing

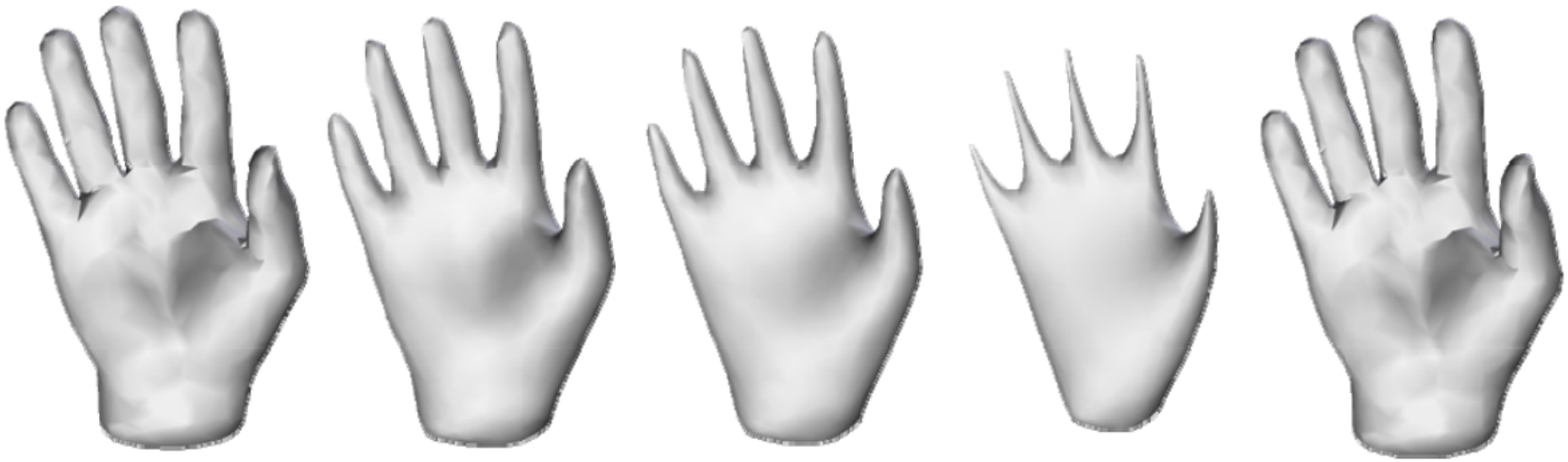


0 Iterations

5 Iterations

20 Iterations

Laplacian smoothing



Next class

- October 5^{th}, 9-10
- Topic: Culling and Hidden Surface Elimination