CS 461 - Computer Graphics

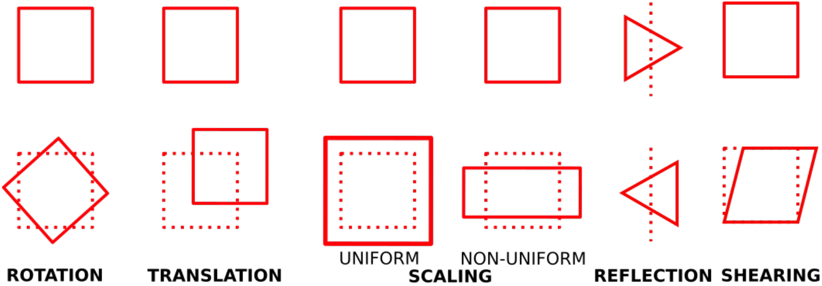
Transformations

Amal Dev Parakkat

Detailed explanation of B-Splines

- https://www.youtube.com/watch?v=_QmDr4-xUO0
- https://www.youtube.com/watch?v=wAHaJ9ASPo8&t=436s
- https://www.youtube.com/watch?v=209y6ESJIz8

2D Transformations



2D Transformation: Translation

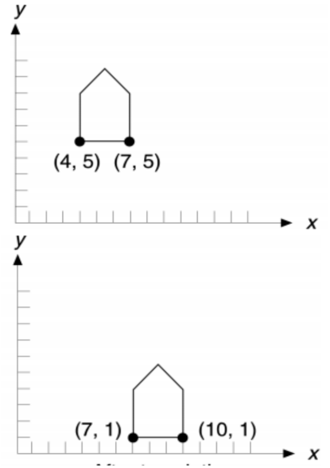
- Rigid motion of points to new locations

$$x' = x + d_x$$
$$y' = y + d_y$$

- Can be represented as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

or simply P'=T+P



2D Transformation: Scaling

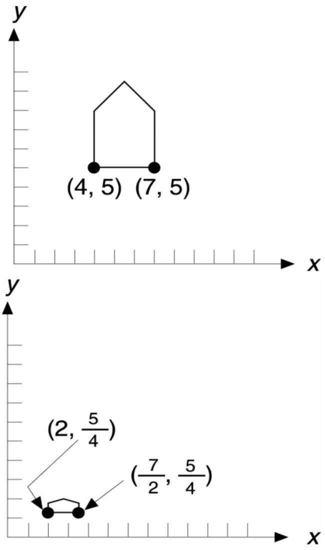
- Stretching of points

$$x' = s_x * x$$
$$y' = s_y * y$$

- Can be represented as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

or simply P'=S*P



2D Transformation: Rotation

- Rotation of points w.r.to the origin

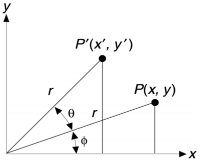
$$x' = x * cos(\theta) - y * sin(\theta)$$
$$y' = x * sin(\theta) + y * cos(\theta)$$

- Positive angle $\longrightarrow$ CCW
- Negative angle $\longrightarrow$ CW
- Can be represented as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

or simply P'=R*P

Rotation - Derivation



$$
\begin{aligned}
x &= r * cos(\phi) \\
y &= r * sin(\phi) \\
x' &= r * cos(\phi + \theta) \\
&= r * cos(\phi)cos(\theta) - r * sin(\phi)sin(\theta) \\
&= x * cos(\theta) - y * sin(\theta) \\
y' &= r * sin(\phi + \theta) \\
&= r * cos(\phi)sin(\theta) + r * sin(\phi)cos(\theta) \\
&= x * sin(\theta) + y * cos(\theta)
\end{aligned}
$$

2D Transformation: Reflection

- Mirror image of an object
- $(x,y) \longrightarrow (x,-y)$
- $(x,y) \longrightarrow (-x,y)$
- $(x,y) \longrightarrow (-x,-y)$
- Special case of scaling? rotation?

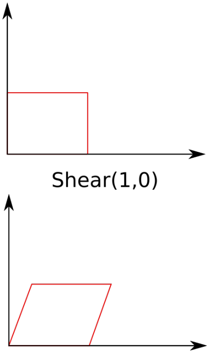
2D Transformation: Shear

- Shear (a,b):
  (x,y)⟶(x+ay,y+bx)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

  or simply P'=SH*P

- Shear(1,0) and Shear(0,1)

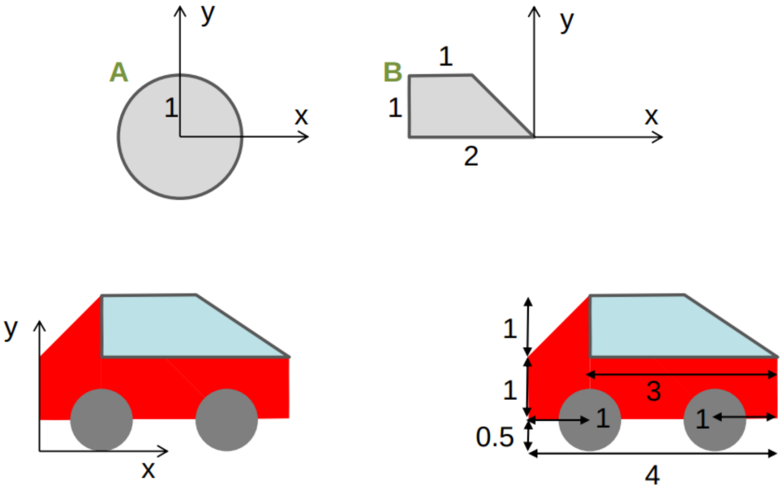


Classes of transformation

- Rigid transformation: distance and angle preserving: Translation + Rotation
- Similarity transformation: angle preserving: Translation + Rotation + Scaling
- Affine transformation: parallelism preserving (lines remain lines)

Rigid $\subset$ Similarity $\subset$ Affine

Example



- Can you animate?

Matrix Representation

- Translation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

- Rotation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

- Scaling

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

- Reflection$_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

- Shear$_x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous coordinates

- Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Rotation

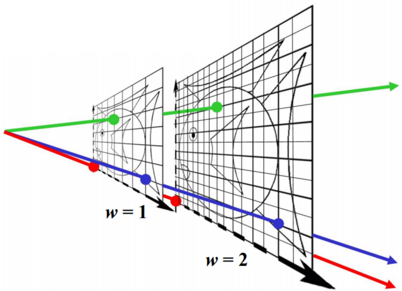
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- ...
- P'=M*P

Homogeneous coordinates

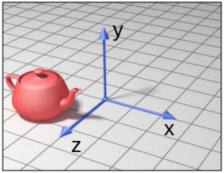
- All non-zero scalar multiples of a point are considered identical (a!=0, w!=0)

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \equiv \begin{bmatrix} ax \\ ay \\ aw \end{bmatrix} \equiv \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \\ 1 \end{bmatrix}$$

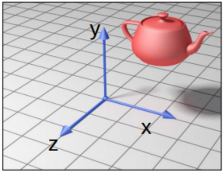


3D Transformation

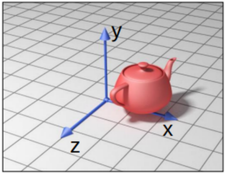
Translation:



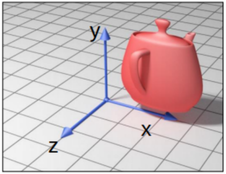
$$\mathbf{T}_{t_x,t_y,t_z} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



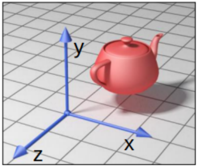
Scaling:



$$\mathbf{S}_{s_x,s_y,s_z} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

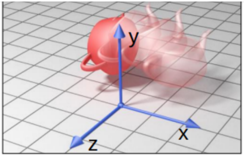


3D Transformation



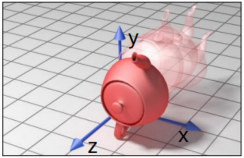
Rotation around Z:

$$\mathbf{R}_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



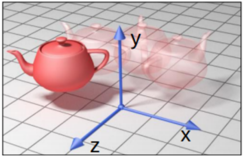
Rotation around X:

$$\mathbf{R}_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation around Y:

$$\mathbf{R}_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Next class

- Topic: 3D representations
- Date: 28^{th} Sep - Monday 9-10