-: ELECTROSTATICS :-

> CHARGES :-

∵ charges are everywhere

∵ charge is always associated with mass.

∵ Intrinsic property like mass (always exist along with Particles)

> ELECTRIC CHARGE :-

Property associated with matter due to which it produces and experiences electrical & magnetic effects.

charges at rest produces only electric effects.

charges in motion produces Both electric & mag. effects

Accelerated charge produces Both E and B and also radiates energy.

#

1) $V=0$	2) $V = \text{constant}$	3) $V \neq \text{constant}$
Q (+)	Q (+) $\longrightarrow$	Q (+) $\Longrightarrow$
> Produce only $\vec{E}$	> Produce Both $\vec{E}$ and $\vec{B}$	> Produces $\vec{E}, \vec{B}$ and radiates energy

> FUNDAMENTAL CHARGES :-

> SI unit :- Coulomb (C)

> dimensional formula :- $Q = AT$

It is a scaler Quantity and can be added algebraically electronic charge(e) = 1.6×10^{-19} C

1> Charge on electron ($-e$)

2> charge on Proton ($+e$)

3> Charge on Neutron 0C

TYPES OF CHARGES :-

(i) Positive charge

deficiency of e^- as compared to proton

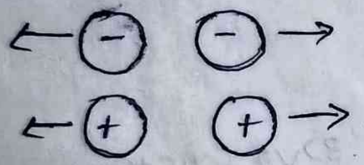
(ii) Negative charge

excess of e^- as compared to proton

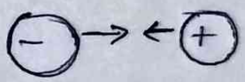
> Properties of CHARGES :-

#

1> like charges Repel



2> Unlike charges Attract



charge is Conserved :-

(Neither Created Nor destroyed)

charge is Relativistic Invarient :-

(Mass of Particle varies with speed v)

Unlike Mass, charge on a Body never vary with speed.

charge is Quantized :-

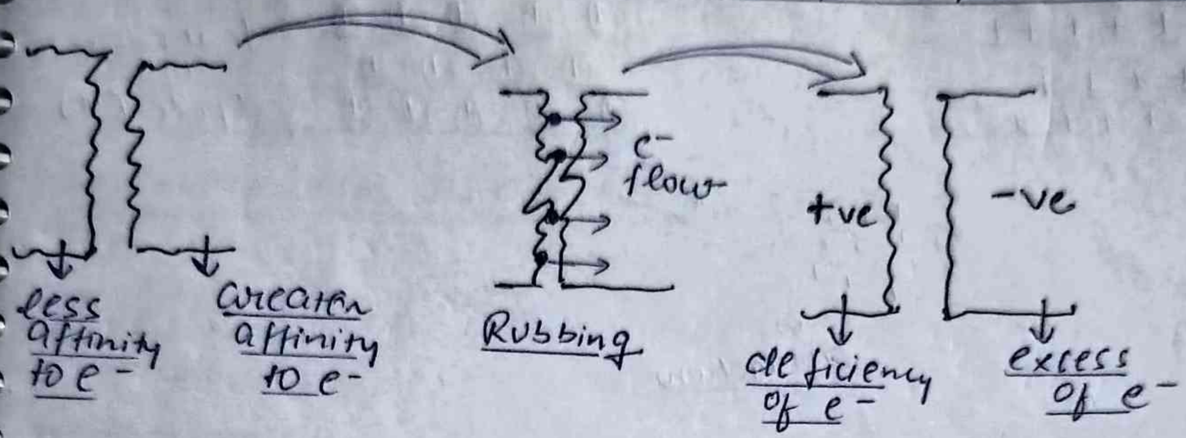
charge on a Body must always exists as integral multiple of fundamental unit of charge (e)

$\boxed{Q = ne}$

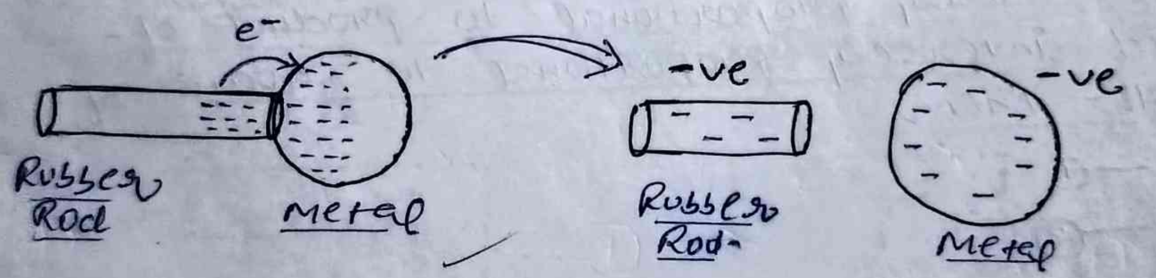
n → integer

>Different Method of Charging :-

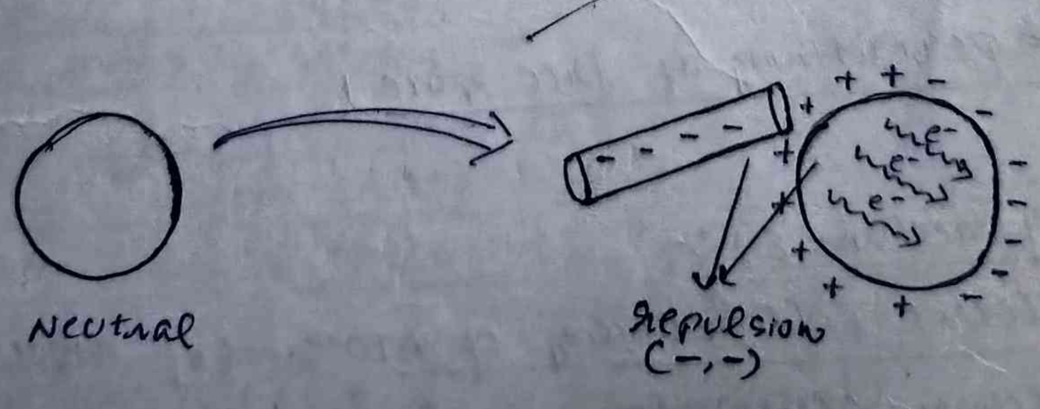
(i) **Charging by Friction :-** (rubbing of two object)
&
e^- are Transfered from one Body to anot



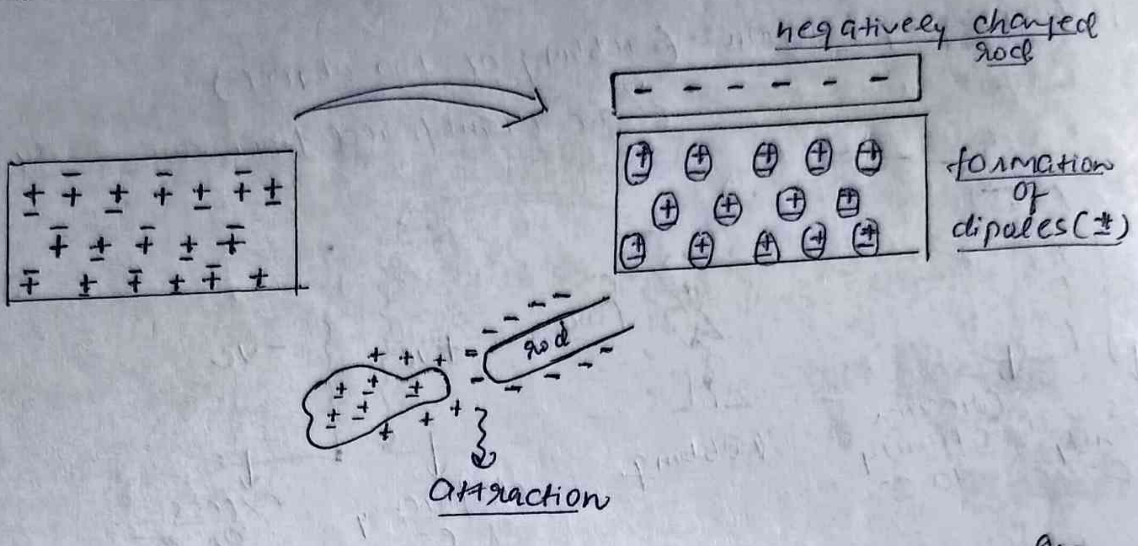
(ii) **Charging by Conduction :-** (Contact b/w two objects)
The Conductors will be charged with same sign.



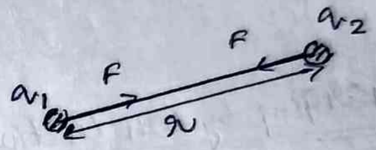
(iii) **Charging by Induction :-** (e^- inside conductor move to create surface charge)



lifting of paper Bits by charged rod :-



> COULOMB's LAW :-



According to this law;
Magnitude of electrostatic force acting b/w two Point charges is directly proportional to product of charge and inversely proportional to square of distance b/w them

$\because F \propto q_1 q_2$ —— ①

$\because F \propto 1/r^2$ —— ②

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{k q_1 q_2}{r^2} \; ; \; K = 9 \times 10^9 \ Nm^2/C^2$$

$K = \frac{1}{4\pi\varepsilon_0}$; ε_0 = permittivity of free space !

> Permittivity of Medium (ε) :-

measure of electric polarizability of Material of Medium
(how well Medium allow/permit electric field to pass through it!)

> RELATIVE PERMITTIVITY OF MEDIUM (K / ε_r)

$$\varepsilon = K\varepsilon_0$$

ε_0 ← Permittivity of free space.

K ↑ relative permittivity (dielectric constant)

> COULOMB'S LAW IN DIFFERENT MEDIUM :-

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi(K\varepsilon_0)} \times \frac{q_1 q_2}{r^2} = \frac{1}{K}\left[\frac{1}{(4\pi\varepsilon_0)}\frac{q_1 q_2}{r^2}\right] = \frac{1}{K}F_0$$

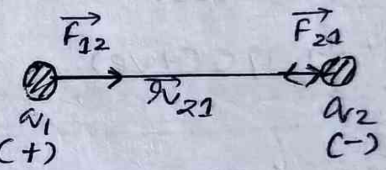
$$\boxed{F = \frac{1}{K}F_0}$$

> COULOMB'S LAW IN VECTOR FORM :-

$$\vec{F}_{21} = \frac{k q_1 q_2}{r^2}(+\hat{r}_{21})$$

$$\Rightarrow \frac{k q_1 q_2}{r^3}(+\vec{r}_{21})$$

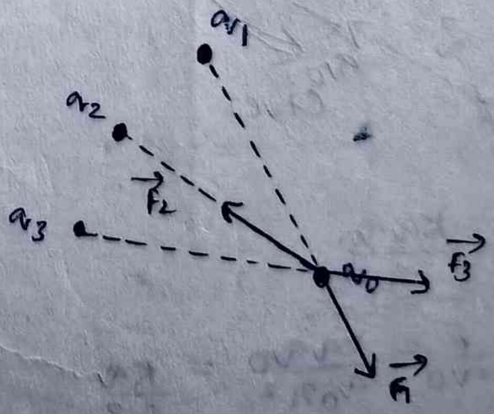
$$\boxed{\vec{F}_{21} = +\frac{k q_1 q_2}{r^3}(\vec{r}_{21})}$$



$\boxed{q_1 q_2 < 0}$

> SUPERPOSITION IN COLUMB'S LAW :-

Net force on a charge can be found by vector addition of forces due to each of these charges as if they were acting alone



$$\because \vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

> ELECTRIC FIELD :-

Region around charged particle into which when another charge is brought in, exp. e-static force

electrostatic Interactⁿ b/w charged particle is two step process

CHARGE $\xrightarrow{(I)}$ FIELD $\xrightarrow{(II)}$ FORCE

> ELECTRIC FIELD INTENSITY :- ($\vec{E}$)

electric field Int. at a point is equal to e-static force experienced by unit +ve charge.

$\because \vec{E} = \dfrac{\vec{F}}{q_0}$ ← vector! / ← scaler!

; q_0 is test charge (+ve)

> Methods of determing $\vec{E}$:-

1.

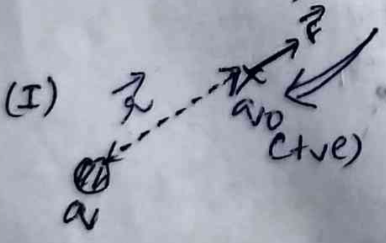
$\because$ Bring +ve Test charge q_0 (q_0)

$\because$ determine F $\left(F = \dfrac{kq_1q_2}{r^2}\right)$

2.

$\because$ determ $\vec{E}$ $\left(\vec{E} = \dfrac{\vec{F}}{q_0}\right)$

(I)



(II) $F = \dfrac{kqq_0}{r^2}$

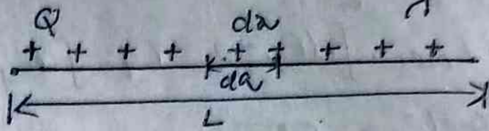
(III) $E = \dfrac{F}{q_0} = \dfrac{kqq_0}{q_0r^2} = \dfrac{kq}{r^2}$

$\because \boxed{E = \dfrac{kq}{r^2}}$ or $\vec{E} = \dfrac{kq}{r^2}(\hat{r}) = \dfrac{kq}{r^3}(\vec{r})$

> TYPE OF CHARGE DISTRIBUTION :-

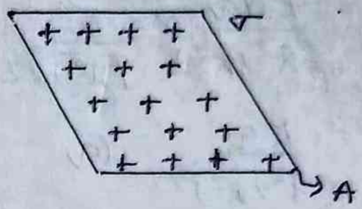
1> LINEAR CHARGE DENSITY (λ) :- Q/L

$$\lambda = \frac{Q}{L} = \frac{dQ}{dx} \quad \text{——} \; (1)$$



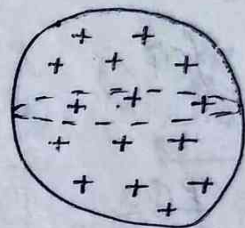
2> SURFACE CHARGE DENSITY (σ) :-

$$\sigma = \frac{dq}{dA} \quad \text{——} \; (2)$$



3) VOLUME CHARGE DENSITY :- (ρ)

$$\rho = \frac{dq}{dV} \quad \text{——} \; (3)$$



> electric field due to Continuous charge distribution :-

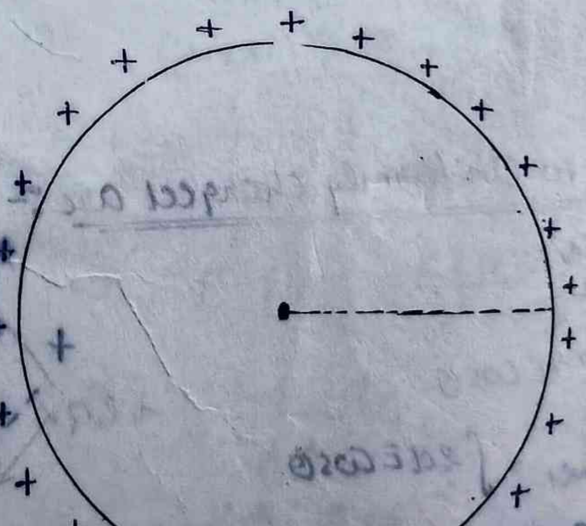
(i) $\vec{E}$ due to Circular ring at its centre :-

$$\lambda = \frac{Q}{2\pi R} \quad \text{——} \; (1)$$

$$dE = \frac{K(dq)}{R^2}$$

$$\int dE = \int \frac{K\,dq}{R^2}$$

$$E = +K \int \frac{dq}{R^2}$$



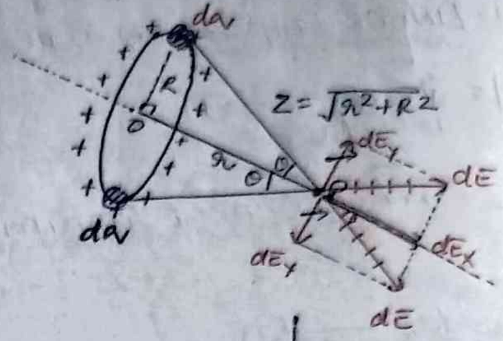
(ii) $\vec{E}$ due to uniform circular ring at a point on axis :-

$$dE = \frac{k\,dq}{z^2} \quad -(1)$$

$E_{Net} = \int dE_x = ?$ $\quad E_y = \int dE_y = 0$

and; $z = \sqrt{r^2 + R^2}$

$dE_x = dE\cos\theta$



∵ dE_y cancels out

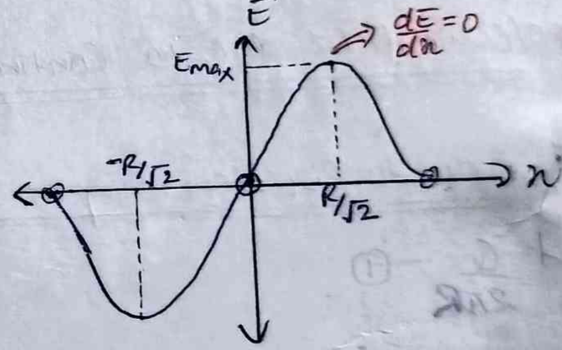
$$dE_x = \frac{k\,dq}{(r^2+R^2)} \times \frac{r}{\sqrt{r^2+R^2}}$$

$$dE_x = \frac{kr}{(r^2+R^2)^{3/2}}\,dq$$

$$E_{Net} = \int dE_x = \frac{kr}{(r^2+R^2)^{3/2}} \int_0^Q dq = \frac{krQ}{(r^2+R^2)^{3/2}}$$

$$\therefore \boxed{E_{Net} = \frac{KQr}{(r^2+R^2)^{3/2}}}$$

$$|E_{max}| = |E_{min}| = \frac{2KQ}{3\sqrt{3}R^2}$$



> $\vec{E}$ due to Uniformly charged arc :-

$$dE = \frac{K\,dq}{R^2}$$

$$dE_{Net} = 2\,dE\cos\theta$$

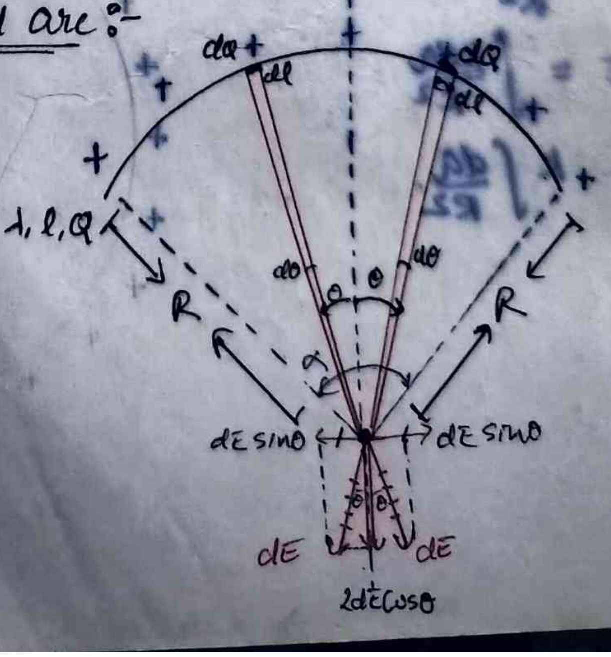
$$E_{Net} = \int dE_{Net} = \int 2\,dE\cos\theta$$

$$\Rightarrow \int \frac{2K\,dq\cos\theta}{R^2}$$

and;

$dQ = \lambda\,dl \quad -(1)$

$dl = R\,d\theta \quad -(2)$



from eq 1 and 2

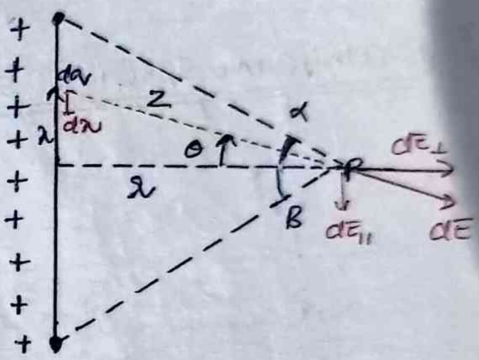
$$E_{Net} = \int_0^{\alpha/2} \frac{2K(\lambda R d\theta)\cos\theta}{R^2}$$

$$= \frac{2K\lambda}{R}\int_0^{\alpha/2} \cos\theta \, d\theta$$

$$\frac{2K\lambda}{R}\sin\frac{\alpha}{2}$$

$$\therefore \boxed{E_{Net} = \frac{2K\lambda}{R}\left(\sin\frac{\alpha}{2}\right)}$$

(iv) $\vec{E}$ due to line of charge :-



$dq = \lambda dx$ — ①

$z = \sqrt{x^2+r^2}$ — ②

$$dE = \frac{Kdq}{z^2} = \frac{Kdq}{(x^2+r^2)}$$

$$dE = \frac{K\lambda dx}{(x^2+r^2)}$$

(I)

$$dE_\perp = dE\cos\theta = \frac{K\lambda dx}{(x^2+r^2)} \times \frac{r}{\sqrt{x^2+r^2}} = \frac{(K\lambda r)dx}{(x^2+r^2)^{3/2}}$$

$$\int dE_\perp = K\lambda r\int \frac{1}{(r^2+x^2)^{3/2}}dx$$

$$E_\perp = \frac{K\lambda}{r}[\sin\theta]_{-\beta}^{+\alpha}$$

$$\Rightarrow \frac{K\lambda}{r}(\sin\alpha + \sin\beta)$$

$$\therefore \boxed{E_\perp = \frac{K\lambda}{r}(\sin\alpha + \sin\beta)}$$

(II)

$$dE_{\parallel} = dE\sin\theta = \frac{K\lambda dx}{(x^2+r^2)} \times \frac{x}{\sqrt{x^2+r^2}}$$

$$dE_{\parallel} = \frac{K\lambda x dx}{(x^2+r^2)^{3/2}}$$

$$E_{\parallel} = \frac{K\lambda}{r}[-\cos\theta]_{-\beta}^{\alpha}$$

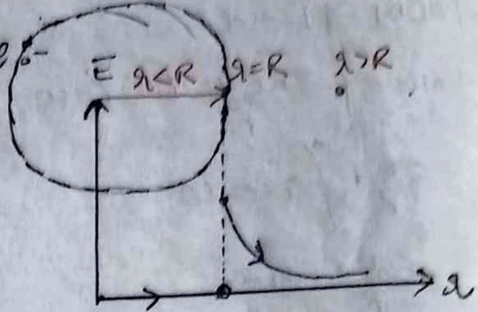
$$= \frac{K\lambda}{r}(\cos\beta - \cos\alpha)$$

$$\therefore \boxed{E_{\parallel} = \frac{K\lambda}{r}(\cos\beta - \cos\alpha)}$$

> $\vec{E}$ due to uniform spherical shell :-

∵ Outside shell ($r>R$) $\quad E = \frac{KQ}{r^2}$

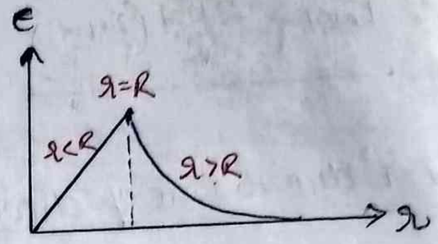
∵ inside shell ($r<R$) $\quad E = 0$



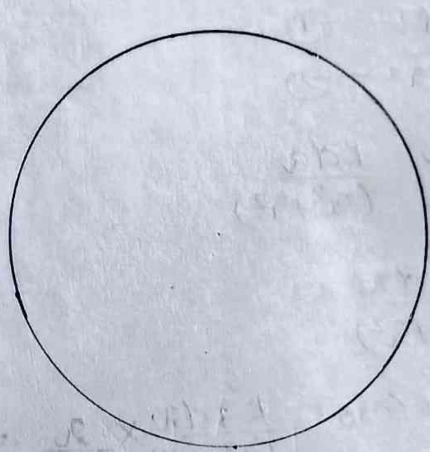
> $\vec{E}$ due to unif. solid sphere :-

∵ outside sphere ($r>R$) : $\quad E = \frac{KQ}{r^2}$

∵ inside shell ($r<R$) : $\quad E = \frac{KQr}{R^3}$

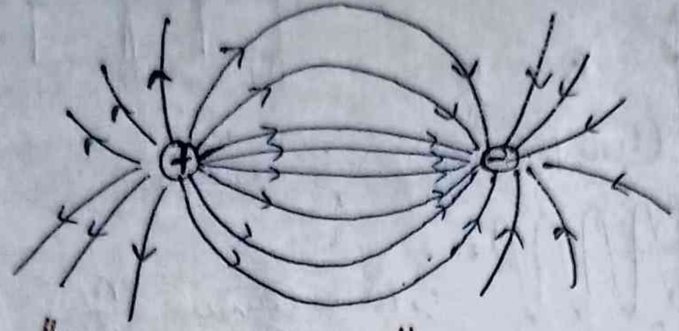


$\vec{E}$ for uniform spherical shell :-



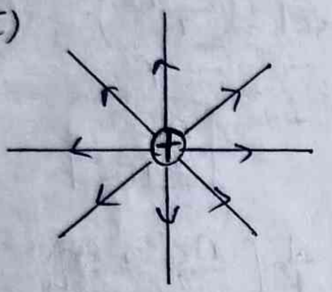
> Electric field lines :- (Given by faraday!)

∵ lines of forces [starts from +ve charge & end on -ve charge]



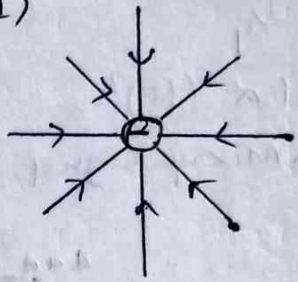
They are "imaginary lines" pictorially maping elc field!

(I)



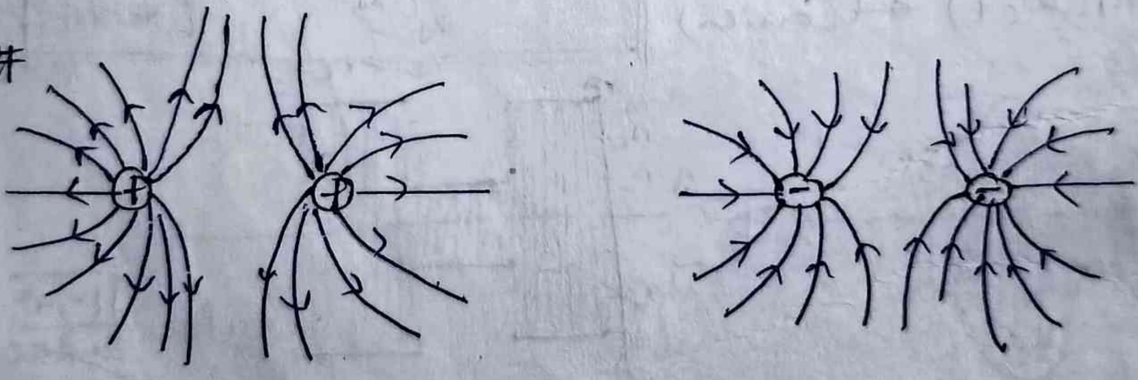
∵ starts out of +ve charge and terminate at ∞

(II)



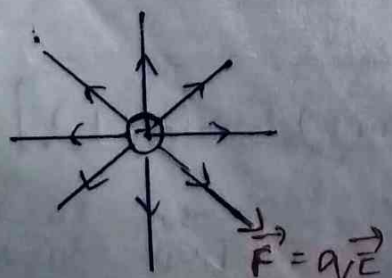
∵ starts out from ∞ & terminate at -ve charge

#



Tangent to line of force at Point in electric field Gives direction of Intensity

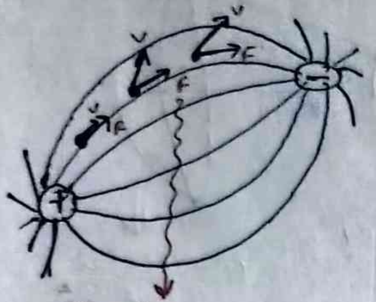
(I)



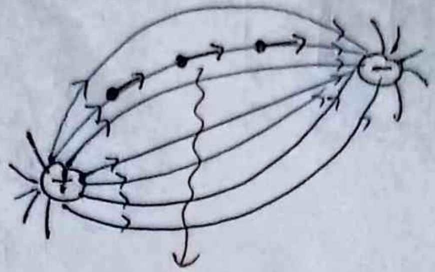
∵ $[\vec{F} \parallel \vec{E}]$

(II)

It is not necessary that a charge will follow one particular EFL If released from rest.



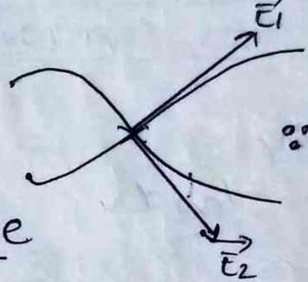
∵ deviates from EFL since $\vec{v}(\uparrow)$ and $\vec{F}$ remains Tangent to EFL!



∵ Not possible!

never cross eachother

we cannot have two direction of $\vec{E}_{Net}$ at same place

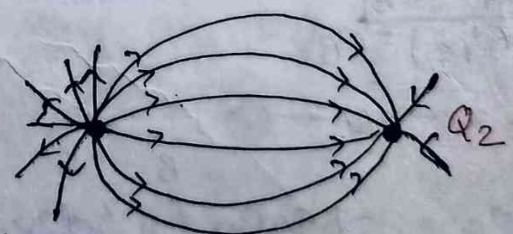


∵ Absurd

never be in closed loops :-
(except in Induced electric field)

Number of line originating or Term. on a charge ∝ to magnitude of charge

∵ $\boxed{\text{lines} \propto |\text{charge}|}$

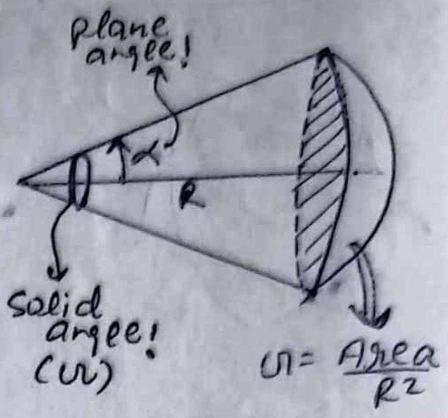


(12 lines) (8 lines)

$|Q_1| > |Q_2|$

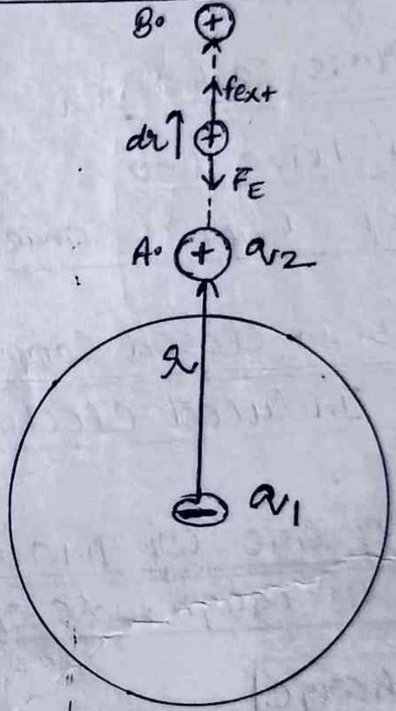
> RLT^n B/W PLANE ANGLE & SOLID ANGLE :-

$\because$ $\boxed{\Omega = 2\pi(1-\cos\alpha)}$



> ELECTROSTATIC POTENTIAL.E :-

> Work done ag. electrostatic force b/w two charged particles in Bri q. them together from infinity to a particular seperation!



$dW_{ext} = F_{ext} \cdot dr$

$(dW)_{Fe} = -dW_{ext} = -F_{ext}\, dr$

and since;

$|F_{ext}| = |F_e| = \frac{Kq_1q_2}{r^2}$

$dW_{Fe} = -\frac{Kq_1q_2}{r^2}\, dr$ —— (1)

and;

$\boxed{U_A + W_{ext} = U_B}$

$U_A + (-W_{Fe}) = U_B$

$W_{Fe} = -\Delta U$ —— (2)

$W_{Fe} = \int dW_{Fe} = -Kq_1q_2 \int_A^B r^{-2}\, dr$

$\Rightarrow -Kq_1q_2\left[\frac{-1}{r}\right]_A^B$

$\Rightarrow -Kq_1q_2\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

$-\Delta U = -Kq_1q_2\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

$U_A - U_B = -Kq_1q_2\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$

If: $B \to \infty$; $r_B \to \infty$; $U_B = 0$

So;

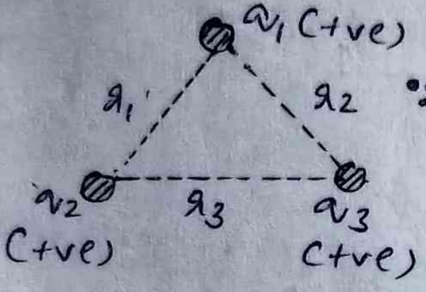
$\boxed{U_A = -\frac{Kq_1q_2}{r_A}}$

; $q_1\, q_2$ are opp. charges!

> Similarly;

$$\boxed{U = +\frac{Kq_1 q_2}{r}}$$ when; q_1 and q_2 are of same sign!

> **Electrostatic U for system of Particles :-**

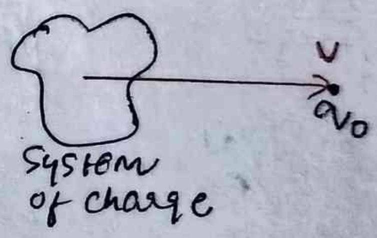


$$\therefore U_{Net} = \frac{Kq_1q_2}{r_1} + \frac{Kq_2q_3}{r_2} + \frac{Kq_1q_3}{r_3}$$

> **Electrostatic Potential :- (V)**

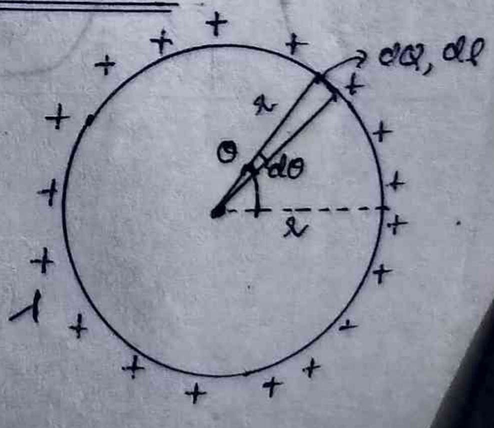
If U is e-static U of int^ren of system of charge with q_0 at particular Point P then; Electric potential (V) is Given by;

$$\therefore \boxed{V = \frac{U}{q_0}}$$; V is scaler Quantity.



∴ **Unit**: Joule / Coulomb [volts]

> **V due to Uniform Circular ring at its centre :-**



$$\lambda = \frac{Q}{2\pi r} \; ; \; dQ = \lambda dl$$

$$dV = \frac{KdQ}{r}$$

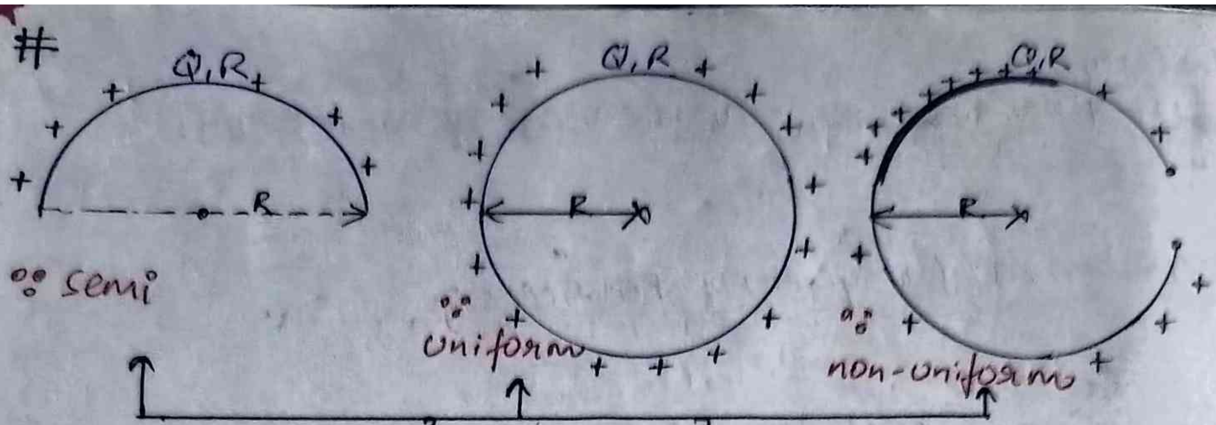
$$= \frac{K}{r}(\lambda dl)$$

$$= \frac{K}{r}\left(\frac{Q}{2\pi r}\right)(r d\theta)$$

$$= \frac{KQ}{2\pi r} d\theta$$

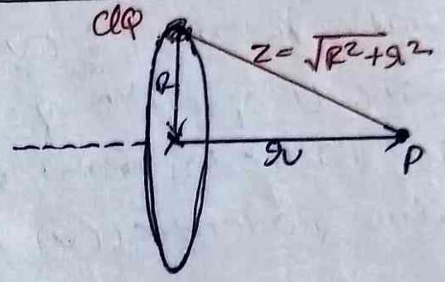
$$V = \int_0^{2\pi} \frac{KQ}{2\pi r} d\theta \Rightarrow \therefore \boxed{V = \frac{KQ}{r}}$$

#



$$\therefore V = \frac{KQ}{R}$$

(ii) V due to Unif Cir ring at a Point on its axis :-



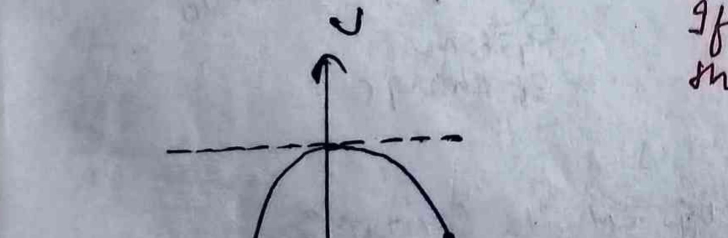
$$dV = \frac{K\,dQ}{\sqrt{R^2+r^2}}$$

$$\int dV = \int_0^Q \frac{K\,dQ}{\sqrt{R^2+r^2}}$$

$$V = \frac{KQ}{\sqrt{R^2+r^2}} \Rightarrow \therefore \boxed{V = \frac{KQ}{\sqrt{R^2+r^2}}}$$

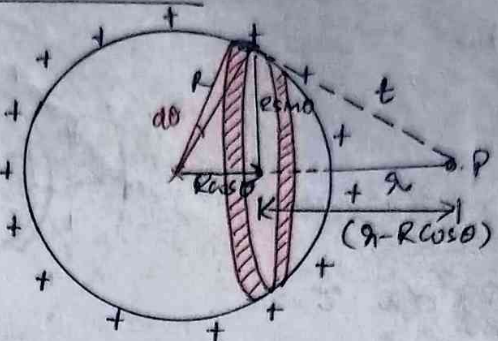
If $r \gg R$

then; $V = \frac{KQ}{r}$



> V due to uniform Spherical sheet :-

$$dV = \frac{K\,dQ}{\sqrt{(R\sin\theta)^2 + (r - R\cos\theta)^2}}$$



$$dV = \frac{K\,dQ}{\sqrt{R^2 + r^2 - 2Rr\cos\theta}} \quad \text{—(1)}$$

and :

$\underline{dQ = \sigma\,dA}$; $\sigma = \frac{Q}{4\pi R^2}$ $\underline{dA = 2\pi R^2 \sin\theta\, d\theta}$

$$dQ = \frac{Q}{2}\sin\theta\, d\theta \quad \text{—(2)}$$

from eq 1 and 2 :-

$$dV = \frac{KQ \cdot \sin\theta\, d\theta}{2\sqrt{R^2 + r^2 - 2Rr\cos\theta}}$$

from figure :-

$$t^2 = R^2 + r^2 - 2Rr\cos\theta$$

$$2t\,dt = 2Rr\sin\theta\, d\theta$$

$$\therefore \sin\theta\, d\theta = \frac{t\,dt}{Rr}$$

$$dV = \frac{KQ}{2}\,\frac{t\,dt}{tRr}$$

$$dV = \frac{KQ}{2Rr}\,dt$$

$$V = \frac{KQ}{2Rr}[t]_{r-R}^{r+R} \Rightarrow \therefore \boxed{V = \frac{KQ}{r}}$$

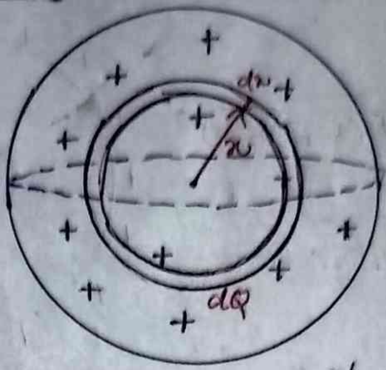
Inside the sheet :-

$$V = \frac{KQ}{2rR}[t]_{R-r}^{R+r} = \frac{KQ}{2\pi R}(2r) = \frac{KQ}{R}$$

$$\therefore \boxed{V = \frac{KQ}{R} = \text{Constant}}$$

> V due to uniform solid sphere :-

$$\rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)} \quad \text{—(1)}$$



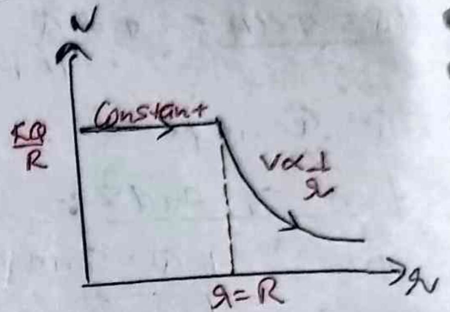
$$dQ = \rho\, dV$$

$$dQ = \frac{Q}{\frac{4}{3}\pi R^3} \times 4\pi x^2 \times dx$$

$$dQ \Rightarrow \frac{3Qx^2\,dx}{R^3}$$

Now;

$$dV = \frac{K\,dQ}{r}$$



$$V_{shell} = \frac{K}{r}\int dQ$$

$$= \frac{K}{r}\int \frac{3Qx^2}{R^3}\,dx$$

$$\Rightarrow \frac{3KQ}{rR^3}\int_0^R x^2\,dx = \frac{3KQ}{rR^3} \times \frac{R^3}{3} = \frac{KQ}{r}$$

V_{shell} ∴ $\boxed{V_{sphere} = \frac{KQ}{r}}$

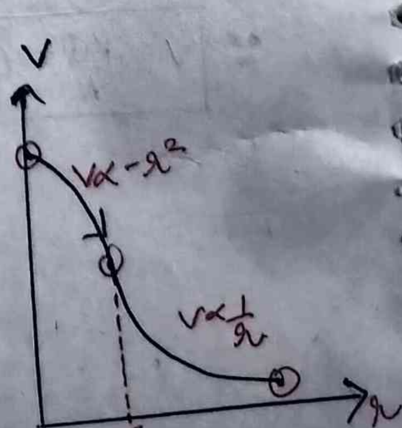
inside sphere :-

$$\int dV_{inside} = \int_0^r dV_{shell\ out} + \int_r^R dV_{shell\ in}$$

$$\Rightarrow \int_0^r \frac{K\,dQ}{r} + \int_r^R \frac{K\,dQ}{x}$$

$$\Rightarrow \frac{3KQ}{R^3 r}\int_0^r x^2\,dx + \frac{3KQ}{R^3}\int_r^R x\,dx$$

$$\Rightarrow \frac{3KQ}{R^3 r} \times \frac{r^3}{3} + \frac{3KQ}{R^3}\left[\frac{R^2}{2} - \frac{r^2}{2}\right]$$



$$\therefore \boxed{V_{inside} = \left[3R^2 - r^2\right]\frac{KQ}{2R^3}}$$

> Relᵗʰ B/w E and V :-

By def :- $dV = \frac{dU}{q_0} = \frac{dW_{ext}}{q_0} = -\frac{dW_E}{q_0} = -\frac{\vec{F_e}\cdot\vec{dr}}{q_0} = -\vec{E}\cdot\vec{dr}$

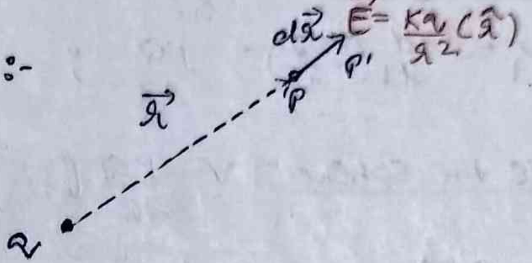
$$\therefore \boxed{dV = -\vec{E}\cdot\vec{dr}}$$

or

$$\int_{V_i}^{V_f} dV = -\int \vec{E}\cdot\vec{dr}$$

$$\Rightarrow \boxed{\Delta V = -\int \vec{E}\cdot\vec{dr}}$$

> determing V from $\vec{E}$? :-



$\because dV = -\vec{E}\cdot\vec{dr}$

$\Rightarrow dV = -E\,dr\cos 0$

$\Rightarrow dV = -E\,dr$

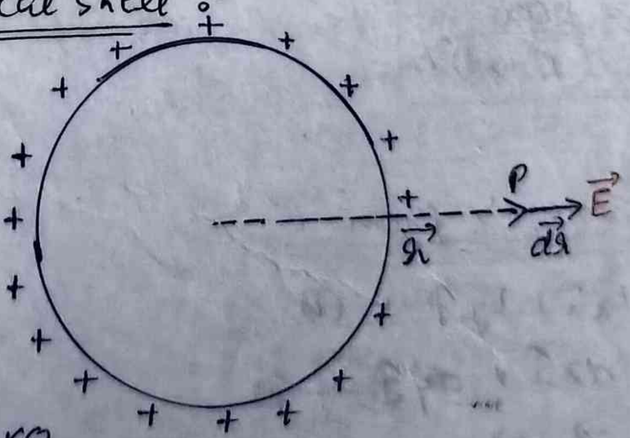
$\Rightarrow dV = \frac{-Kq}{r^2}dr$

$\Rightarrow \int dV = -Kq\int r^{-2}dr$

$\Rightarrow [V]_0^V\ -Kq\left(\frac{-1}{r}\right]_\infty^r$

$$\therefore \boxed{V = \frac{Kq}{r}}$$

> $\vec{E}$ from V due to Spherical shell :-



$dV = -\vec{E}\cdot\vec{dr}$

$\Rightarrow dV = -E\,dr\cos 0$

$\Rightarrow dV = -E\,dr$

$\Rightarrow E = -\frac{dV}{dr}$

$\Rightarrow E - \frac{d}{dr}\left(\frac{KQ}{r}\right)$; $V_{out} = \frac{KQ}{r}$

$\Rightarrow E = \frac{KQ}{r^2}$ ☺

Inside the Shell :-

$V = \frac{KQ}{R} = \text{Constant}$

$E = -\frac{dv}{dr} = -\frac{d}{dr}\left(\frac{KQ}{R}\right) = 0$

$\because \boxed{E=0}$

> $\vec{E}$ from V due to sphere :-

$\because$ outside the sphere ; $V = \frac{KQ}{r}$

$E = -\frac{dv}{dr} = -\frac{d}{dr}\left(\frac{KQ}{r}\right) = \frac{KQ}{r^2}$; $\boxed{E = \frac{KQ}{r^2}}$

$\because$ inside the sphere ; $V = \frac{KQ}{2R^3}[3R^2 - r^2]$

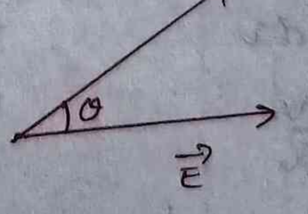
$E = -\frac{dv}{dr} = +\frac{KQ}{R^3}r$; $\boxed{E = \frac{KQ\,r}{R^3}}$

> $\vec{E}$ from V due to ring along its axis :-

$V = \frac{KQ}{\sqrt{R^2+x^2}}$

$E = -\frac{dv}{dx} = -\frac{d}{dx}\left[\frac{KQ}{\sqrt{R^2+x^2}}\right] = \frac{KQx}{(R^2+x^2)^{3/2}}$

$\boxed{\because E = \frac{KQx}{(R^2+x^2)^{3/2}}}$



> determining V from $\vec{E}$:-

$\vec{E} = E_x\hat{i} + E_y\hat{j}$ —— (1)

$\vec{dr} = dx\hat{i} + dy\hat{j}$ —— (2)

$dv = -\vec{E}\cdot\vec{dr}$

$\Rightarrow -[E_x dx + E_y dy]$

$\because \boxed{dv = -E_x dx - E_y dy}$

In 3-dimension

$\because \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ —— (1)

$\because d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ —— (2)

$dV = -\vec{E}\cdot d\vec{r} = -E_x dx - E_y dy - E_z dz$

$\therefore \boxed{dV = -E_x dx - E_y dy - E_z dz}$

Suppose particle move in x-axis ; $dy, dz = 0$

then; $dV = -E_x dx$

only in y-axis ! only in z-axis !

$\therefore \boxed{E_x = -\frac{\partial V}{\partial x}}$; $\boxed{E_y = -\frac{\partial V}{\partial y}}$; $\boxed{E_z = -\frac{\partial V}{\partial z}}$

$\boxed{\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]}$

> Relⁿ b/w E and V :-

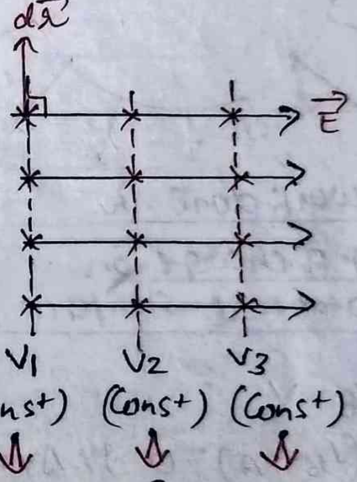
$\because \perp$ to electric field lines potential does not change (remains constant)

$dV = -\vec{E}\cdot d\vec{r}$

$\Rightarrow dV = -E\,dr\cos 90$

$\Rightarrow dV = 0$

$\Rightarrow \boxed{V = \text{Constant}}$

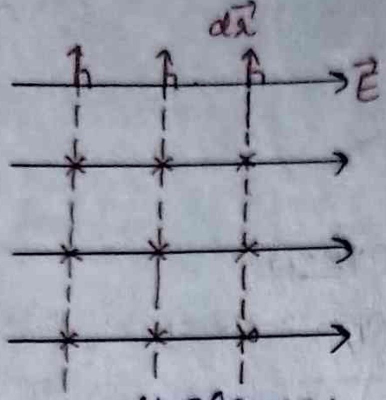


$\therefore$ locus of all point on which potential is const is equipotential surface

> (They are the planes ∥ to each other and ⊥ to the field)

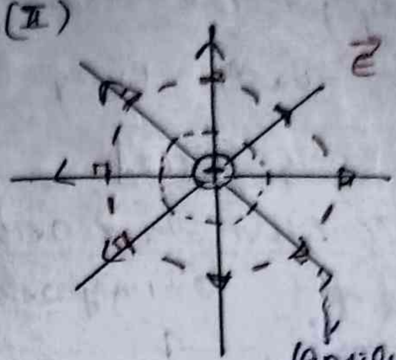
Equipotential surfaces :-

(I)



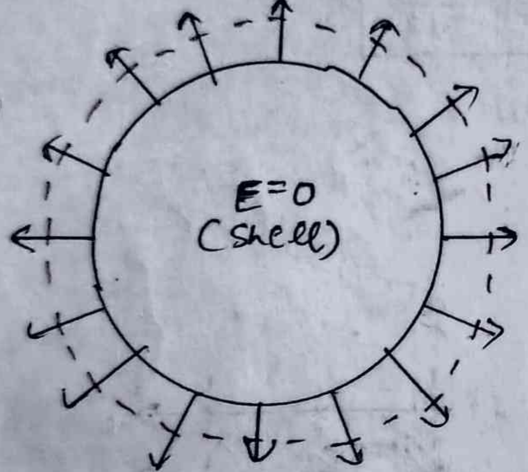
|| planes!

(II)

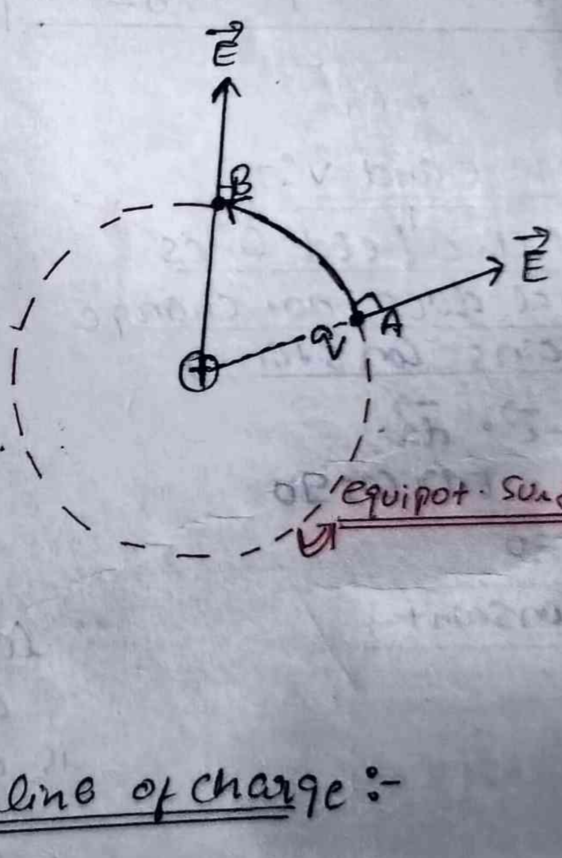


family of Concentric sphere!

(III)



(IV)



Net work done in moving a charge on equipotential surface is 0.

$V_A = V_B = V$

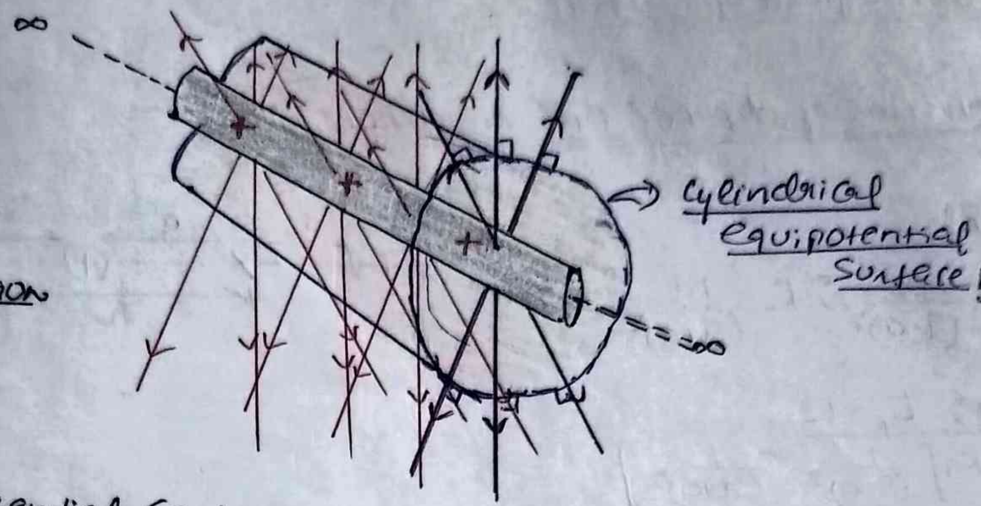
$\Rightarrow \Delta V = (V_B - V_A) = 0 \quad \because \Delta V = 0$

$\therefore W_{ef} = 0$

> equipotential surf. of Inf line of charge :-

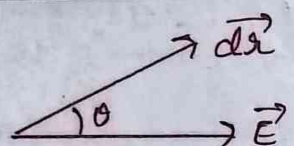
$\because$ They are Co-axial Cylinders having common axes at line charge

Killer visualization



equipotential surface never intersect each other

potential decreases along field!



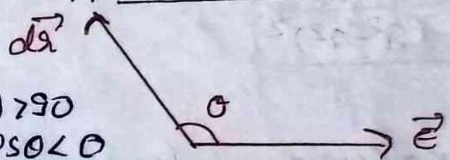
$\theta < 90$
$\cos\theta > 0$

$dV = -\vec{E}\cdot\vec{dr}$

$dv = -E\,dr\cos\theta$

$\because$ $\boxed{dv = -ve}$

potential increases opposite to field!



$\theta > 90$
$\cos\theta < 0$

$dv = -E\,dr\cos\theta$

$\because$ $\boxed{dv = +ve}$

♡:- $\boxed{\Delta U = q\Delta V = -W_E = +W_{ext}}$

> Electric dipole :-

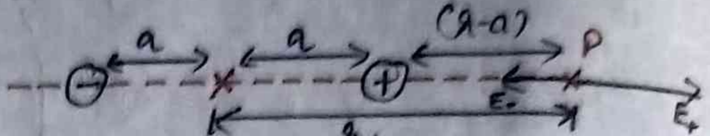
Arrangement of two equal & opp. charges seperated by distance

$(-q) \xrightarrow[\vec{p}]{2\vec{a}} (+q)$

$\because$ $\boxed{\text{dipole moment}(\vec{p}) = q \times 2\vec{a}}$

> Electric field of a dipole :-

(I) on Axis of dipole (end-on dipole) :-



at P :-

$E_+ = \frac{Kq}{(r-a)^2}$; $E_- = \frac{Kq}{(r+a)^2}$

E_{net} ; $E_+ - E_-$

$\Rightarrow Kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$

$\Rightarrow \frac{Kq(4ra)}{(r^2-a^2)^2}$

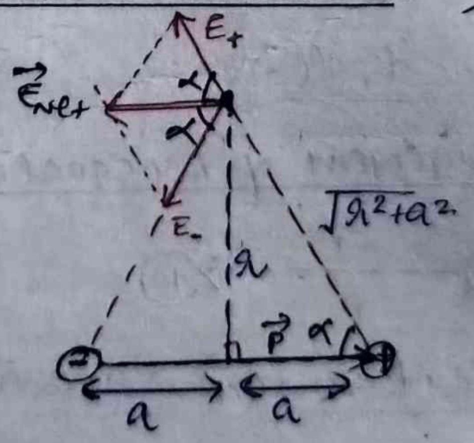
$\therefore \boxed{E_{net} = \frac{Kq(4ra)}{(r^2-a^2)^2}}$

If $r >>> a$

$E_{net} = \frac{Kq(4ra)}{r^4} = \frac{Kq(4a)}{r^3} = \frac{2Kq(2a)}{r^3} = \frac{2KP}{r^3}$

$\therefore \boxed{\vec{E}_{net} = \left(\frac{2K}{r^3}\right)\vec{P}}$

(II) On a Equitorial line :- (⊥ bisector | broadside-on post^n)



$E_+ = E_- = \frac{Kq}{(\sqrt{r^2+a^2})^2} = \frac{Kq}{(r^2+a^2)}$

$E_{net} = 2E\cos\alpha$

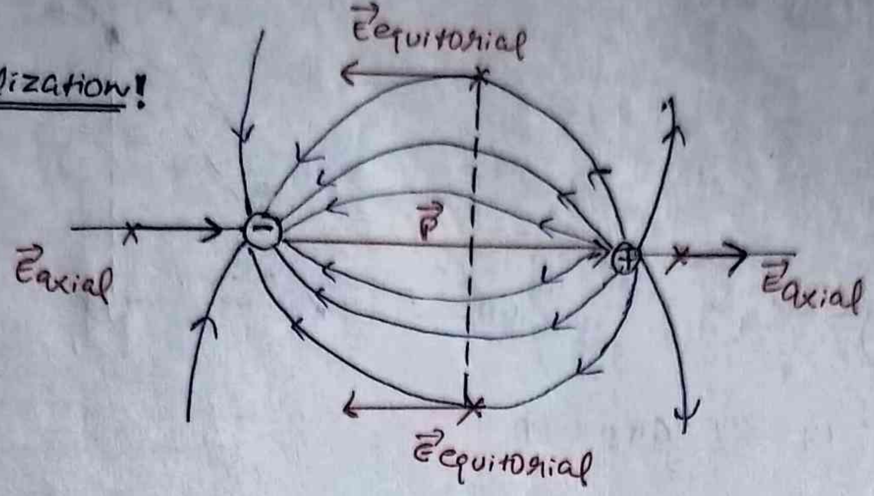
$= 2 \times \frac{Kq}{(r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}}$

$= \frac{Kq(2a)}{(r^2+a^2)^{3/2}}$

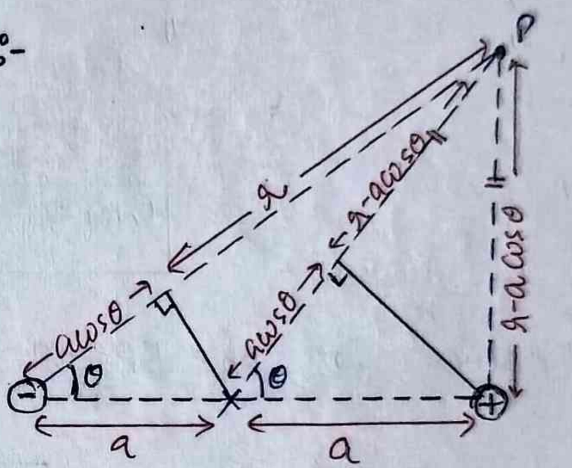
If $r >>> a$

$E_{net} = \frac{KP}{r^3} \Rightarrow \boxed{E_{net} = \frac{KP}{r^3}}$ and; $\boxed{\vec{E}_{net} = \left(\frac{-K}{r^3}\right)\vec{P}}$

visualization!



Potential due to Dipole :-



$$V_{at\,P} = (V_+) + (V_-)$$

$$= \frac{Kq}{(r-a\cos\theta)} + \frac{K(-q)}{(r+a\cos\theta)}$$

$$Kq\left[\frac{1}{(r-a\cos\theta)} - \frac{1}{(r+a\cos\theta)}\right]$$

$$\frac{Kq(2a)\cos\theta}{(r^2-a^2\cos^2\theta)}$$

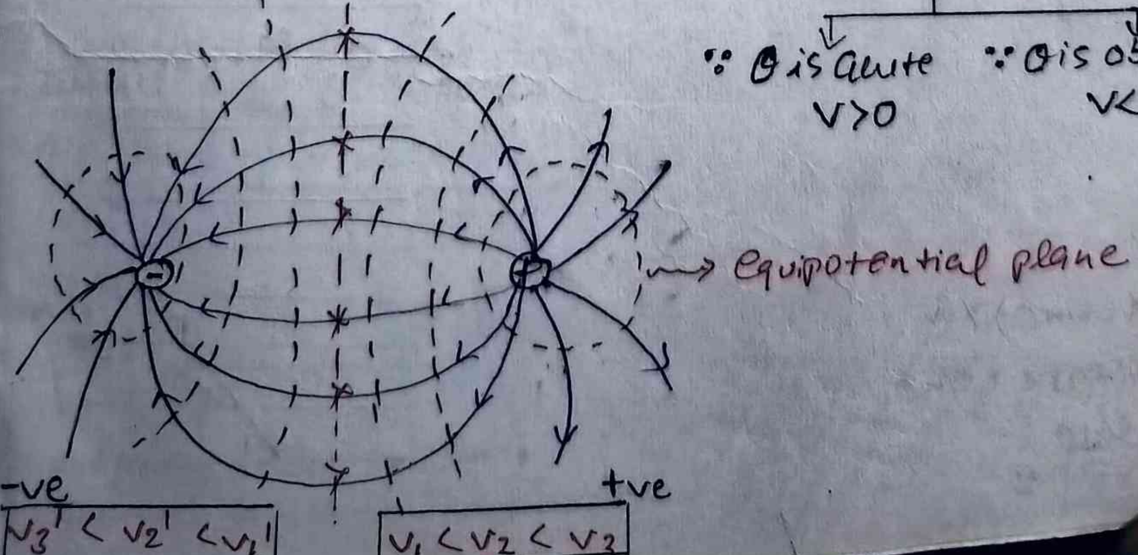
and $r \gg a$

$$V_P = \frac{Kq(2a)\cos^2\theta}{r^2}$$

$$\boxed{V_P = \frac{Kp\cos\theta}{r^2}} \quad V = \frac{K(pr\cos\theta)}{r^3} \Rightarrow \boxed{V = \frac{K\,\vec{p}\cdot\vec{r}}{r^3}} \because$$

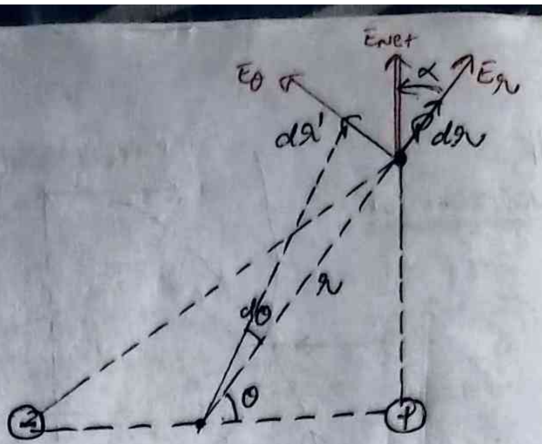
$\because$ θ is acute $V>0$

$\because$ θ is obtuse $V<0$



$\boxed{V_3' < V_2' < V_1'}$ $\boxed{V_1 < V_2 < V_3}$

> **field due to dipole :-**



$V = \dfrac{Kp\cos\theta}{r^2}$

$E_r = -\dfrac{dV}{dr}$

$\Rightarrow -\dfrac{d}{dr}\left(\dfrac{Kp\cos\theta}{r^2}\right)$

$\Rightarrow \dfrac{2Kp\cos\theta}{r^3}$ So; $E_r = \dfrac{2Kp\cos\theta}{r^3}$ — (1)

$E_\theta = \dfrac{-dV}{dr'}$

$\Rightarrow \dfrac{-dV}{r\,d\theta}$

$\Rightarrow \dfrac{-1}{r}\dfrac{dV}{d\theta}$

$\Rightarrow \dfrac{-1}{r}\dfrac{Kp}{r^2}\dfrac{d(\cos\theta)}{d\theta}$

$\Rightarrow \dfrac{+Kp\sin\theta}{r^3}$ So; $E_\theta = \dfrac{Kp\sin\theta}{r^3}$ — (2)

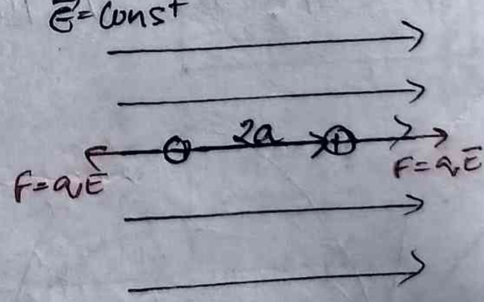
$E_{net} = \sqrt{E_r^2 + E_\theta^2}$

$\Rightarrow \dfrac{Kp}{r^3}\sqrt{(2\cos\theta)^2 + \sin^2\theta}$

$$\boxed{E_{net} \Rightarrow \dfrac{Kp}{r^3}\sqrt{1+3\cos^2\theta}}$$

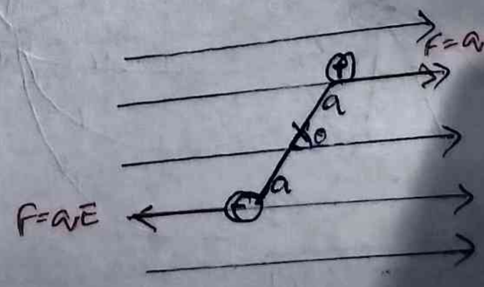
$\tan\alpha = \dfrac{E_\theta}{E_r} = \dfrac{\sin\theta}{2\cos\theta} = \dfrac{\tan\theta}{2}$; $\because$ $\boxed{\tan\alpha = \dfrac{\tan\theta}{2}}$

> **Torque on dipole in Unif. $\vec{E}$:-**



(F_{net}) on dipole $= qE - qE = 0$

$\therefore \boxed{\vec{F}_{net} = \vec{0}}$



$\tau = F_\perp \times r$

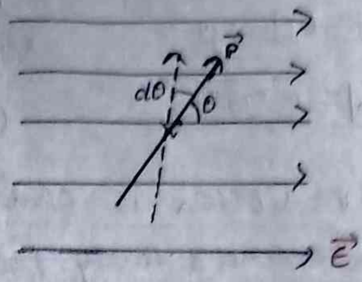
$\Rightarrow \tau_{net} = 2(qE\sin\theta) \times a$

$\tau_{net} = q(2a) \times E\sin\theta$

$\tau = pE\sin\theta$

$\therefore \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

∵ U of dipole in $\vec{E}$:-



$\tau_E = \vec{P} \times \vec{E} = PE\sin\theta$ — ②

$dw = \vec{\tau_e} \cdot \vec{d\theta}$

$dw = \tau\, d\theta \cos\pi$

$dw = -\tau d\theta$ — ①

and

$dU = -dw$

$dU = \tau d\theta$ — ③

from eq 2 and 3

$dU = PE \sin\theta\, d\theta$

$\int dU = \int PE \sin\theta\, d\theta$

$\int_{U_1}^{U_2} dU = PE \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$

$U_2 - U_1 = PE[-\cos\theta]_{\theta_1}^{\theta_2}$

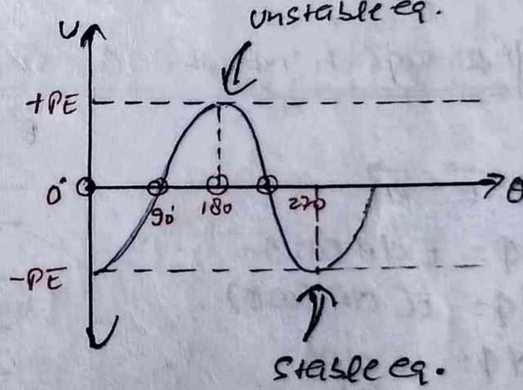
$U_2 - U_1 = PE[\cos\theta_1 - \cos\theta_2]$

If $U_1 = 0$ then $\theta_1 = 90$

$U_2 = U$ then $\theta_2 = \theta$

$U - 0 = PE[0 - \cos\theta]$

$$\boxed{U = -PE\cos\theta}$$



> Electric flux :-

Measure of Number of electric field lines Crossing a Given area

∵ flux depend on

(i) $|\vec{E}|$ Magnitude of electric field

(ii) Area

(iii) θ (Angle)

$\Rightarrow \phi \propto EA\theta$

$\Rightarrow \phi = EA\cos\theta$

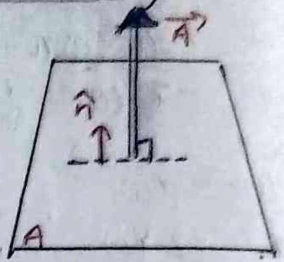
$\boxed{\phi = \vec{E}\cdot\vec{A}}$

Area vector :-

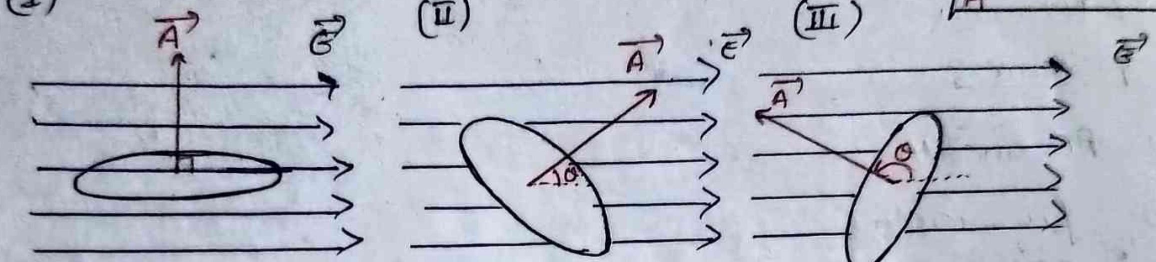
Area vector of surface is

$\because \boxed{\vec{A} = |A|\hat{A}}$

($\hat{n}$ is unit vector $\perp$ to surface)



$\because$ Area vector is always outward & away from surface!



$\because$ If $\theta = 90$

$\phi = \vec{E}\cdot\vec{A}$

$\phi = EA\cos 90$

$\boxed{\phi = 0}$

$\because$ If θ is acute

$\phi = \vec{E}\cdot\vec{A}$

$\phi = EA\cos\theta$

$\boxed{\phi > 0}$

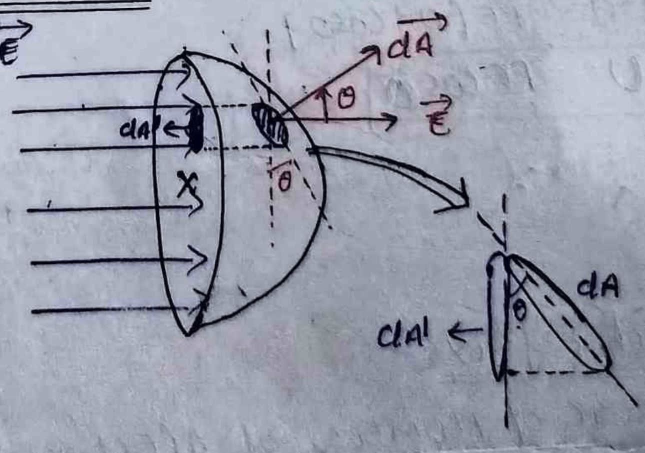
$\because$ If θ is obtuse

$\phi = \vec{E}\cdot\vec{A}$

$\phi = EA\cos\theta$

$\boxed{\phi < 0}$

> ϕ through hemispherical surface :-



$d\phi = \vec{E}\cdot\vec{dA}$

$d\phi = E\,dA\cos\theta$

$d\phi = E(dA\cos\theta)$

$d\phi = E\,dA'$

$\phi = E\int_0^{\pi r^2} dA'$

$\because \boxed{\phi = E\pi r^2}$

ϕ through Half-Cylindrical surface :-

$d\phi = \vec{E}\cdot\vec{dA}$ —— (1)

$\Rightarrow d\phi = E\,dA\cos\theta$

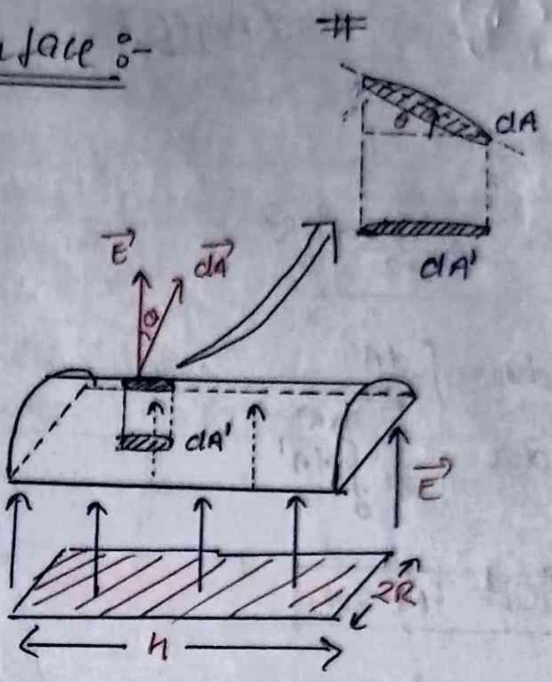
$\Rightarrow d\phi = E(dA\cos\theta)$

$\Rightarrow d\phi = E\,dA'$

$\Rightarrow \phi = E\int dA'$

$\Rightarrow \phi = E\int_0^{2Rh} dA'$

$$\boxed{\phi = E(2Rh)}$$



ϕ through closed surface :-

If;

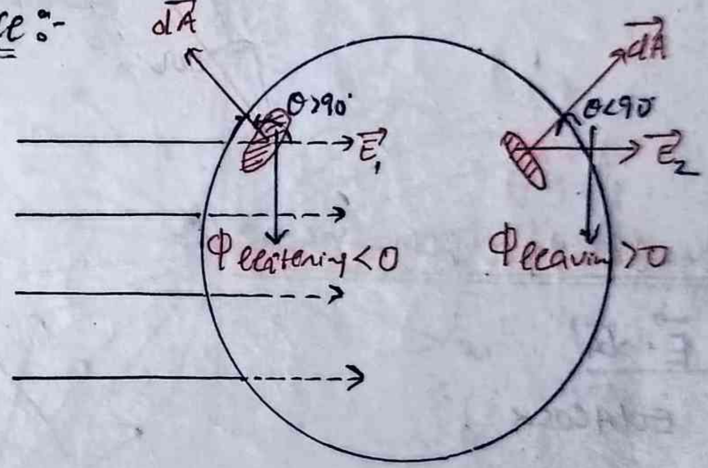
$E_2 > E_1$

$\therefore \boxed{|\phi_{leaving}| > |\phi_{entering}|}$

and if;

$\because E_2 = E_1 (\vec{E} = \text{constant})$

$$\boxed{|\phi_{leaving}| = |\phi_{entering}|}$$



Relⁿ b/w Plane Angle & Solid Angle :-

$\Omega = A/R^2$

$dA = 2\pi(R\sin\theta)\times R\,d\theta$

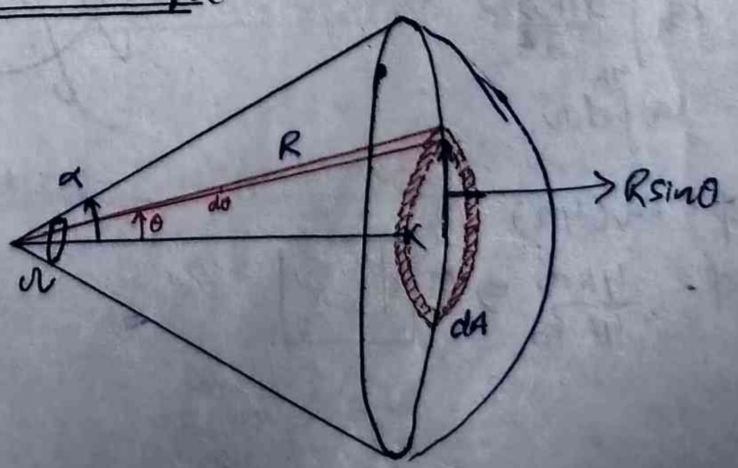
and

$d\Omega = \frac{dA}{R^2}$

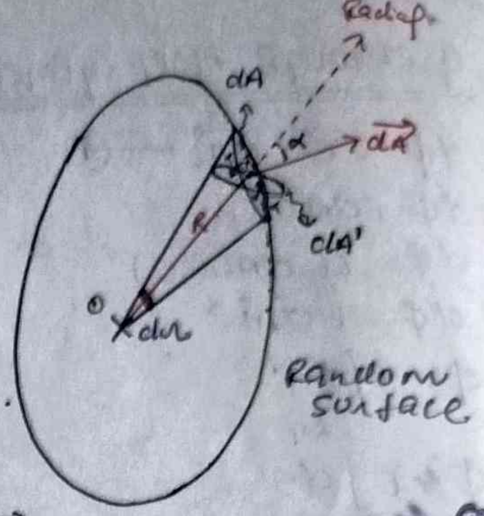
$d\Omega = \frac{2\pi R^2\sin\theta\,d\theta}{R^2}$

$d\Omega = 2\pi\sin\theta\,d\theta$

$\Omega = 2\pi[-\cos\theta]_0^{\alpha}$ $\therefore$ $\boxed{\Omega = 2\pi(1-\cos\alpha)}$



> Concept of Solid Angle :-



$d\Omega = \frac{dA'}{R^2}$

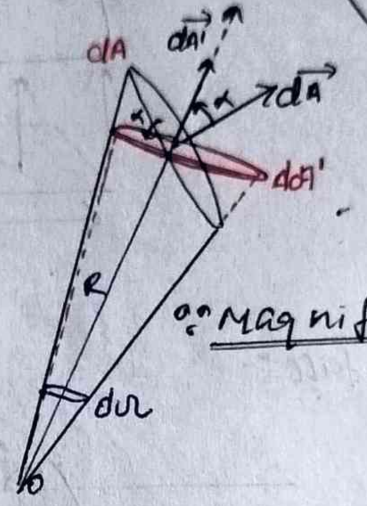
$$\boxed{d\Omega = \frac{dA\cos\alpha}{R^2}}$$

$dA' = dA\cos\alpha$ —— (1)

$\int_{\Omega} d\Omega = \int \frac{dA'}{R^2}$

$\int_0 d\Omega = \frac{1}{R^2}\int_0^{4\pi R^2} dA'$

$\therefore \boxed{\Omega = 4\pi\ Sr}$



∴ Magnified!

> Total ϕ By a point charge :-

$d\phi = \vec{E}\cdot\vec{dA}$

$d\phi = E\,dA\cos\alpha$

$d\phi = \frac{Kq}{r^2}\,dA\cos\alpha$

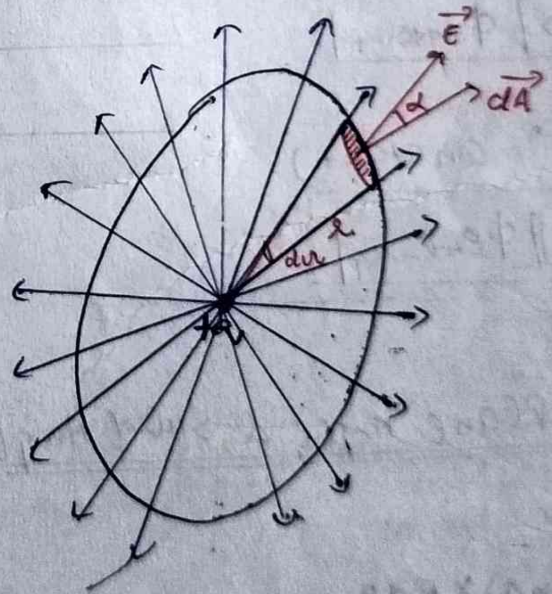
$d\phi = Kq\left(\frac{dA\cos\alpha}{r^2}\right)$

$\phi = Kq\int \frac{dA\cos\alpha}{r^2}$

$\phi = Kq\int_0^{4\pi} d\Omega$

$\phi = Kq(4\pi)$

$\phi = \frac{4\pi q}{4\pi\varepsilon_0} \Rightarrow \therefore \boxed{\phi = \frac{q}{\varepsilon_0}}$



Gauss's law!

$$\boxed{\phi_{total} = \oint \vec{E}\cdot d\vec{a} = \frac{q}{\varepsilon_0}}$$

∵ Surface integral of $\vec{E}$ due to all charges over any closed Guassian surface is $\left(\frac{1}{\varepsilon_0}\right)$ times net charge enclosed by surface :-

or

> Gauss law :-

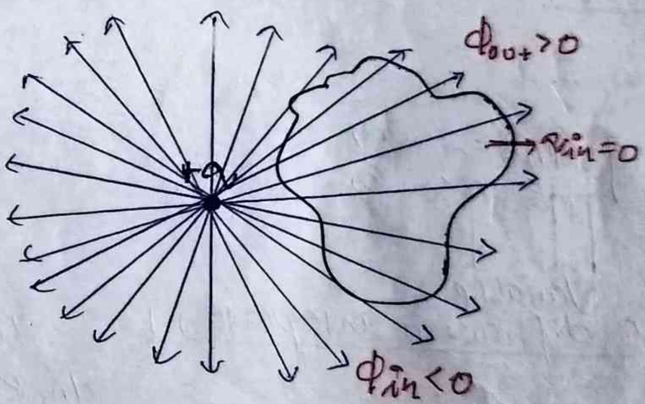
total flux emerging out of a closed surface is equal to $\left(\frac{1}{\varepsilon_0}\right)$ times total charge enclosed by surface.

$$\boxed{\phi_{total} = \frac{q_{in}}{\varepsilon_0}}$$

#

If charge is present outside the closed surface!

$$\phi_{total} = \phi_{in} + \phi_{out} \simeq 0$$



#

ϕ depend only on enclosed charge and not on shape of enclosing surface

#

If ϕ is zero then $\vec{E}$ is not necessarily zero.

#

ϕ_{total} only on enclosed charge & $\vec{E}$ depend on q_{in} & q_{out}.

> $\vec{E}$ due to Point charge :- :: to Prove "Couloms law" from "Gauss's law!"

(I) By Gauss's law :-

$\Rightarrow \phi_{total} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

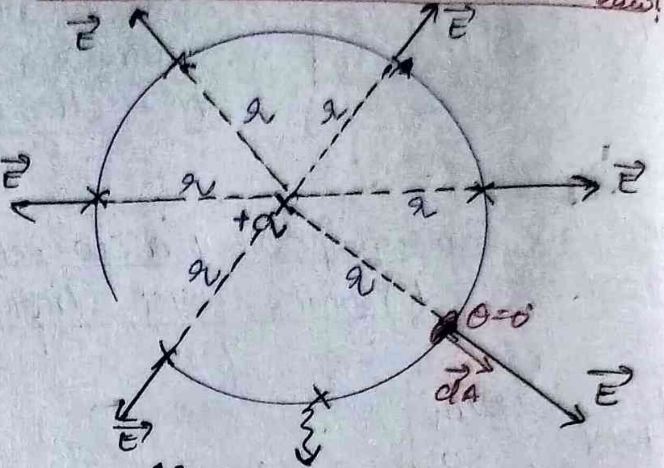
$\Rightarrow \int E dA = \frac{q}{\varepsilon_0}$

$\Rightarrow E \oint dA = \frac{q}{\varepsilon_0}$

$E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$

$\therefore \boxed{E = \frac{Kq}{r^2}}$ :: (Coulomb's law)

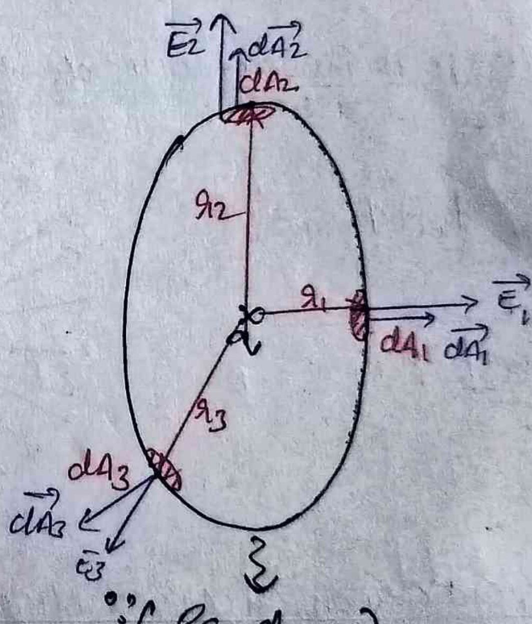


:: (Symmetrical)

(II)

$\phi_{total} = \int E dA \cos\theta = \frac{q}{\varepsilon_0}$

Variable
(difficult to integrate) !



:: (Random)

:: Gauss's law is valid for all type of surface
However using Gauss law in Symm. surface is easier

> field due to ∞ line of charge :-

$$\phi_{total} = \oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

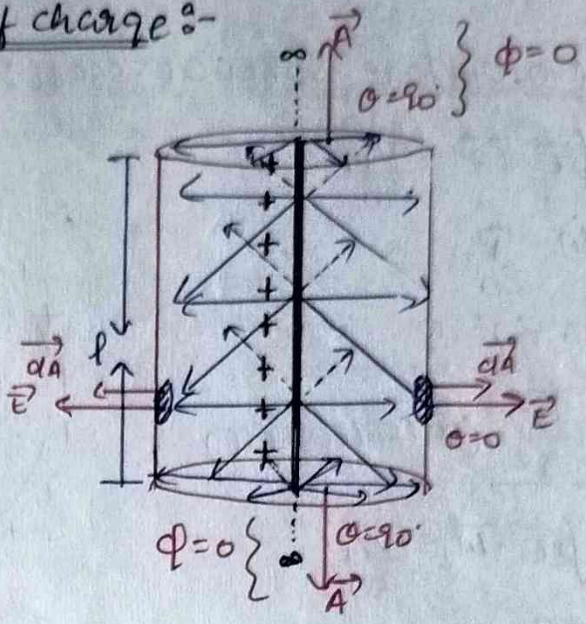
$$\Rightarrow \oint_{2\pi r l} E\,dA\cos 0 = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E \oint_0 dA = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{1}{2\pi\varepsilon_0} \times \frac{\lambda}{r}$$

$$\therefore \boxed{E = \frac{2K\lambda}{r}}$$



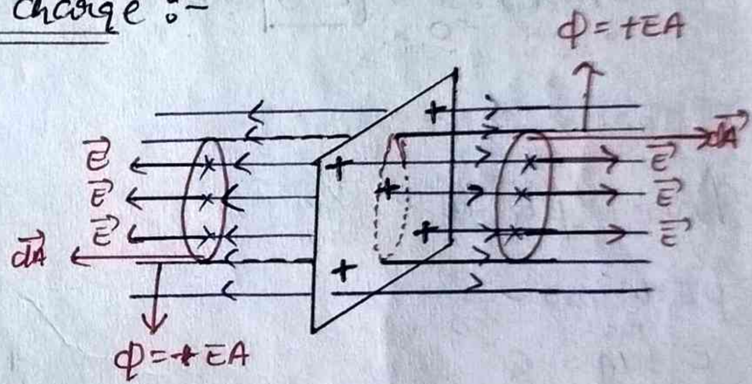
> field due ∞ sheet of charge :-

$$\phi_{total} = \oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \int E\,dA\cos 0 = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow E \int dA = \frac{\sigma A}{\varepsilon_0}$$

$$\Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$



> Electric field of a disc :-

$$\therefore E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

> $\vec{E}$ due to uniform spherical shell :-

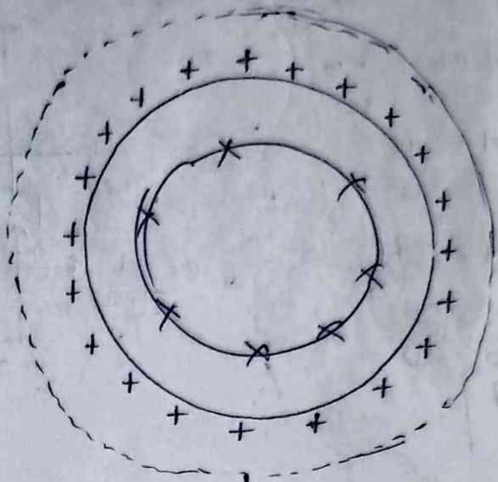
#

$$\phi_{total} = \oint \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\oint dA = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{KQ}{r^2}}$$ (outside shell)



#

$$\phi_{total} = \oint \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\oint d\vec{A} = \frac{0}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{0}{\varepsilon_0} \Rightarrow \boxed{E = 0}$$ (Inside shell)

> $\vec{E}$ due to solid sphere :-

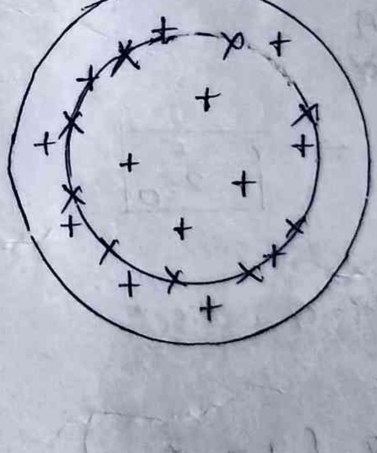
#

$$\phi_{total} = \oint \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

$$\Rightarrow \oint E\, dA\cos 0 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E\oint_0^{4\pi r^2} dA = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E = \frac{KQ}{r^2}}$$



#

$$\phi_{total} = \oint \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

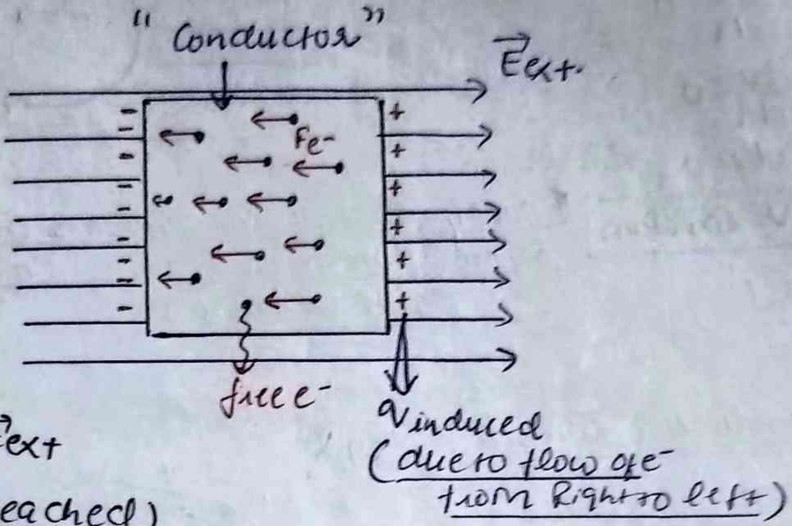
$$\Rightarrow E\oint_0^{4\pi r^2} dA = \frac{\rho \times dv}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{\frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_0} \Rightarrow \frac{Qr^3}{R^3\varepsilon_0}$$

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3} \Rightarrow \boxed{\frac{KQr}{R^3}}$$

> Conductors :-

∴ (fixed metal ions with freely flowing e^-)



$\vec{E}_{induced}$:-
from ⊕ to ⊖ ✓
after some time
∴ F_{net} on a $e^- = 0$

$\vec{E}_{in} \longleftarrow \underset{e^-}{\times} \longrightarrow \vec{E}_{ext}$

∴ (steady state is reached)

> $\vec{E}_{inside\ conductor} = 0$! ⟶ ∴ Conductor shield it interior point from $\vec{E}_{ext}$
(shielding effect)

> charge inside a conductor ?

$$\phi_{total} = \oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

and since ;
$\vec{E}_{inside\ Conductor} = 0$
then ;

$$\oint \vec{0} \cdot \vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

$\Rightarrow \boxed{q_{in} = 0}$ (Net charge inside Gaussian surface is zero)

> Example :- (Metallic Sphere)

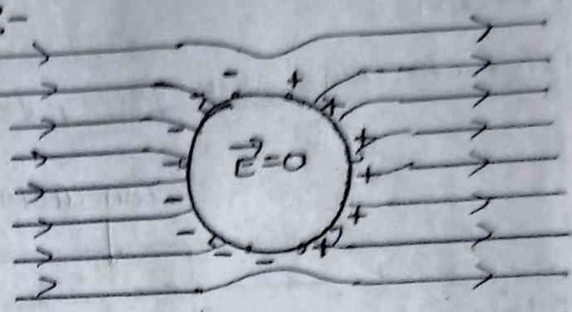
$E = 0$ (inside sphere)

$\Rightarrow \phi_{total} = \oint \vec{E} \cdot \vec{dA}$

$\Rightarrow \frac{q_{in}}{\varepsilon_0} = 0$

$\Rightarrow \boxed{q_{in} = 0}$

Potential of a Conductor :-



$\vec{E}_{inside\ Cond} = 0$

$\Rightarrow -\frac{dv}{dr} = 0$

$\Rightarrow \frac{dv}{dr} = 0$

$\Rightarrow dv = 0$

$\Rightarrow \boxed{V = \text{Constant}}$ (every point of conductor is at same potential & thus entire volume is equipotential)

Conductor with a Cavity :-

$\Rightarrow E = 0$

$\Rightarrow \oint \vec{E} \cdot \vec{dA} = 0$

$\Rightarrow \frac{q_{in}}{\varepsilon_0} = 0$

$\boxed{\therefore q_{inside} = 0}$