-: OSCILLATIONS & SHM :-

> **Periodic Motion :-**

Motion which repeats itself after a specific Time

Time after which Motion is repeated is called Time Period of Motion.

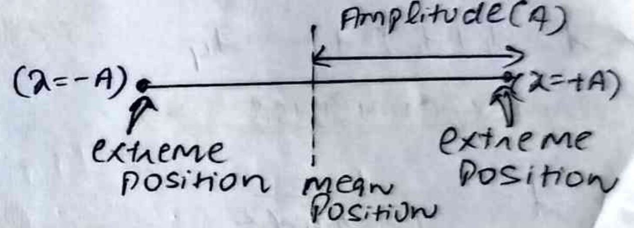
> **FREQUENCY (ν) :-**

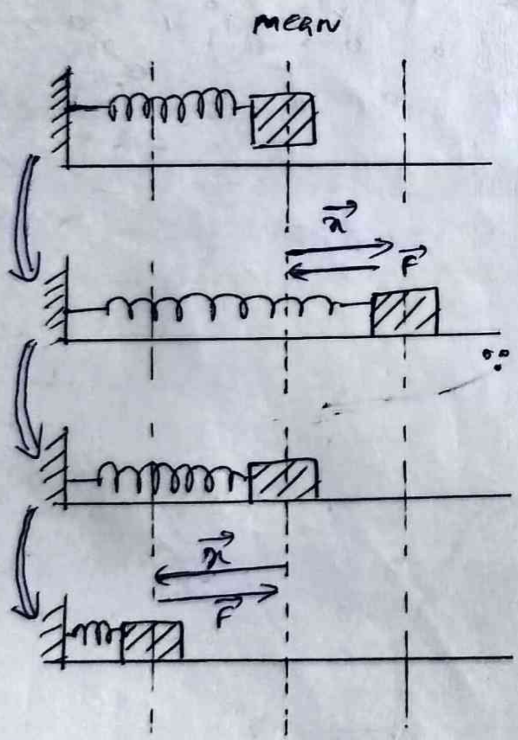
No. of Cycle per unit time

$$\boxed{\nu(Hz) = \frac{1}{T(sec)}}$$

> **OSCILLATIONS :-**

Periodic Motion in which Particle Move to and fro about fixed Point.





∵ The Body Oscillate About the Mean Position!

> **TYPES OF OSCILLATIONS :-**

∵ Free Oscillations (No energy loss)

∵ Damped Oscillations (Presence of opposition)

∵ Forced oscillation. (Compensated by ext forces)

> SIMPLE HARMONIC MOTION :-

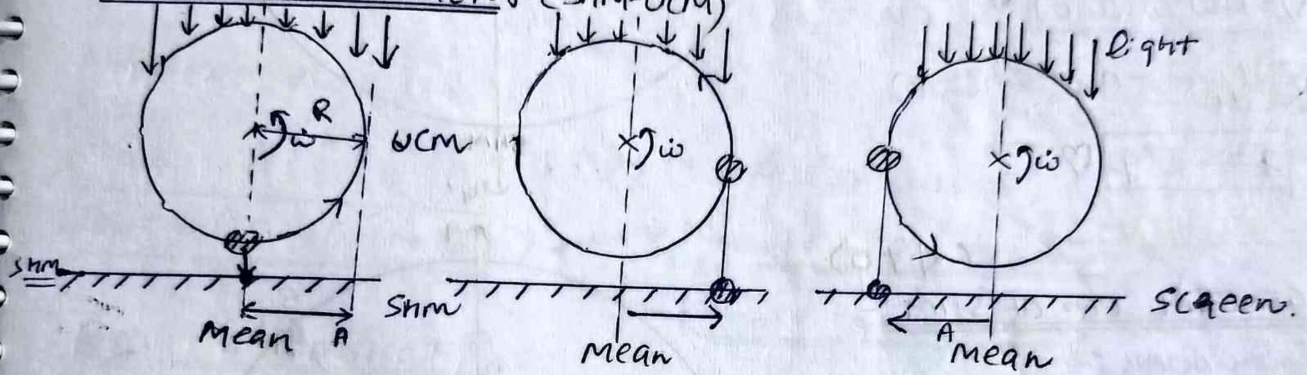
The equation of Harmonic / Oscillatory Motion ! is

$\boxed{F = -Kx^n \; ; \; n = 1, 3, 5, 7 \ldots}$

On $n = 1$, Harmonic Motion Become simple Harmonic Motion.

> Graphical representation of SHM :-

KILLER VISUALIZATION :- (SHM-UCM)



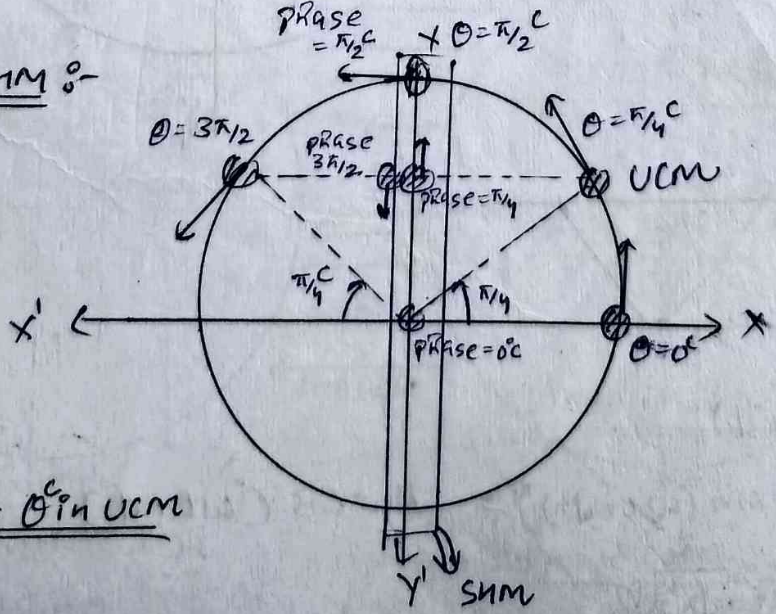
SHADOW IS DOING SHM!

∵

> Time Period of SHM = Time Period of UCM
> Amplitude of SHM = Radious of UCM.
> Ang. frequancy of SHM = Angular velocity of UCM

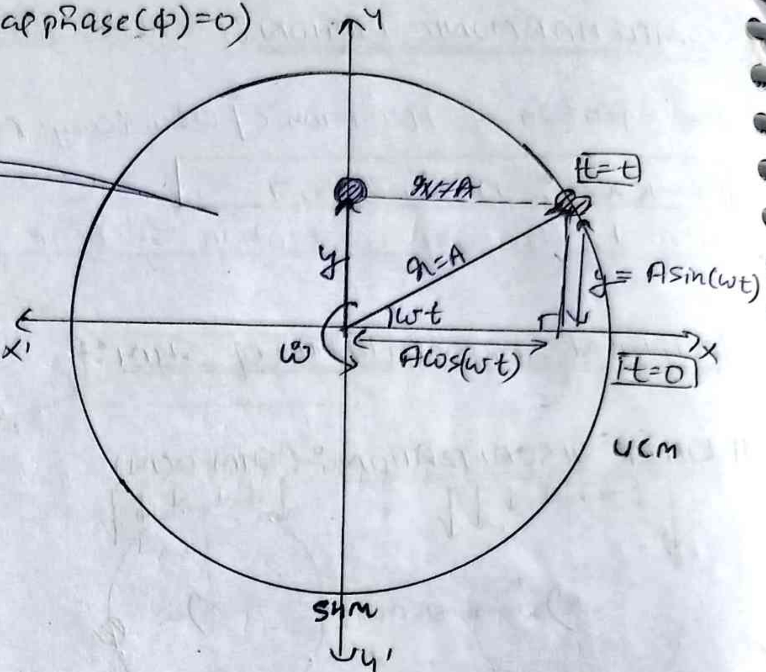
} PURE GOLD!

> PHASE OF SHM :-



PHASE IN SHM = θ^c in UCM

> EQUATION OF SHM :- (Initial phase (ϕ) = 0)



$$\boxed{y = A\sin(\omega t)}$$ ♡

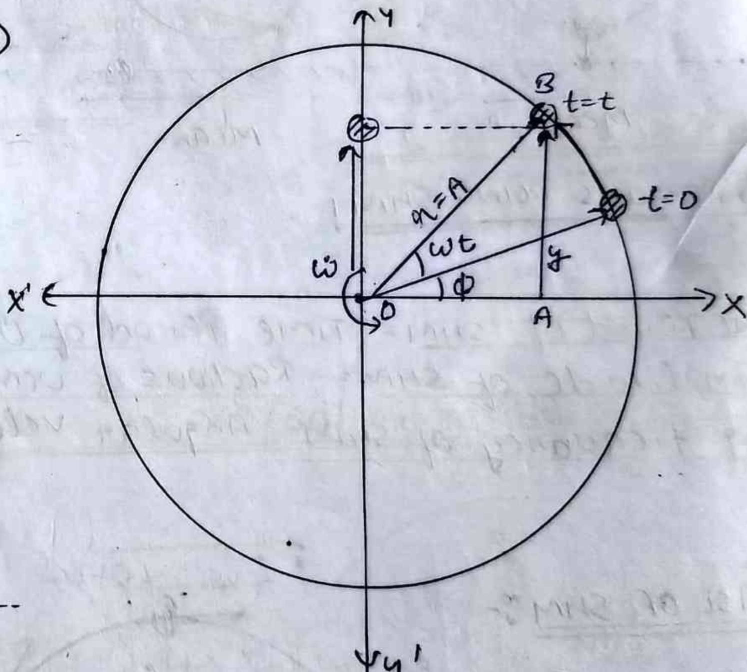
$$V = \frac{dy}{dt} = \frac{d}{dt}[A\sin(\omega t)]$$

$$\boxed{V = A\omega\cos(\omega t)}$$ ♡

$$a = \frac{dv}{dt} = -A\omega^2\sin(\omega t)$$

$$\boxed{a = -\omega^2 y}$$ ♡

> GENERAL EQ. OF SHM :- ($\phi \neq 0$)

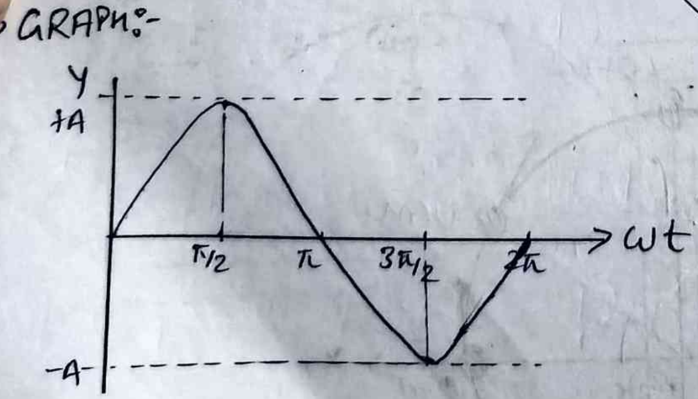


From ΔOAB :-

$$\boxed{y = A\sin(\omega t + \phi)}$$ ♡

A :- Amplitude
ϕ :- Initial phase
y :- Position in SHM

> GRAPH :-



$$y = A\sin(\omega t + \phi)$$

$$V = \frac{dy}{dt} = \frac{d}{dt}(A\sin(\omega t + \phi)) = A\omega\cos(\omega t + \phi)$$

$$\boxed{V = A\omega\cos(\omega t + \phi)}$$ ♡

Now;

$y = A\sin(\omega t + \phi) \Rightarrow y/A = \sin(\omega t + \phi)$ — ①

$v = A\omega\cos(\omega t + \phi)$ — ②

from eq 1 and 2 :-

$v = A\omega\sqrt{1 - y^2/A^2}$

$v = \frac{A\omega}{A}\sqrt{A^2 - y^2} \Rightarrow \boxed{v = \omega\sqrt{A^2 - y^2}}$ ♡

$\Rightarrow$ $V_{Max} = \omega A$; $y = 0$ ♡

$\Rightarrow$ $V_{Min} = 0$; $y = +A, -A$ ♡

$v = A\omega\cos(\omega t + \phi)$

$a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos(\omega t + \phi)) = -A\omega^2\sin(\omega t + \phi)$

$\boxed{a = -A\omega^2\sin(\omega t + \phi)}$ ♡

$\Rightarrow$ $|a_{max}| = \omega^2 A$; extremes ♡

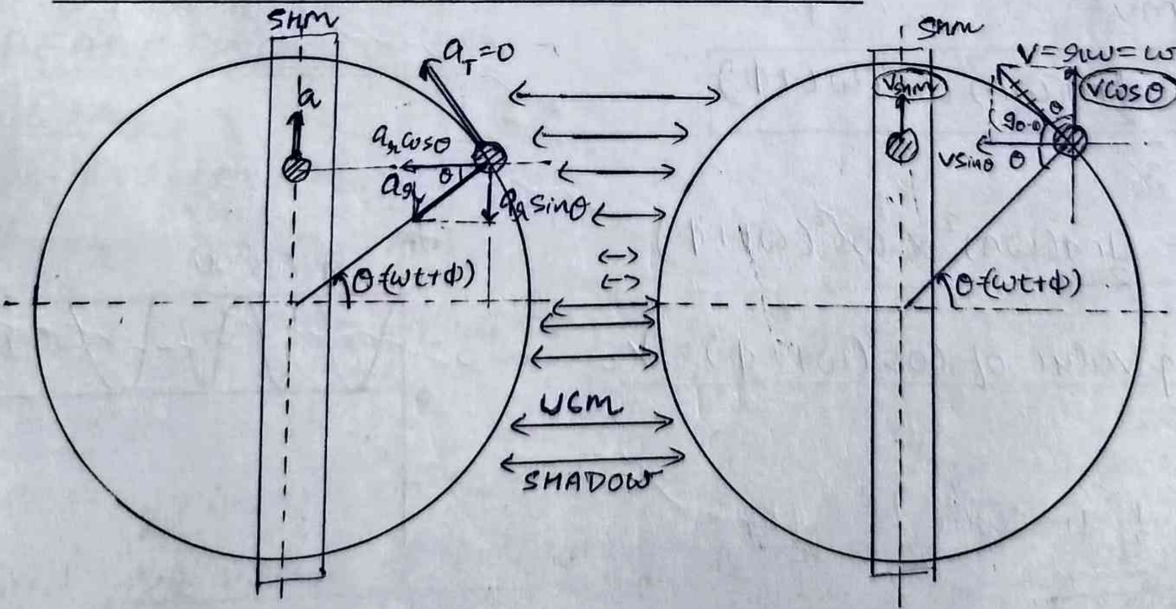
$\Rightarrow$ $|a_{Min}| = 0$; means ♡

Basic differential eq :-

$a = -\omega^2 y$

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

KILLER VISUALIZATION FOR V & a :-(IN SHM):-



$a = -a_r\sin\theta$

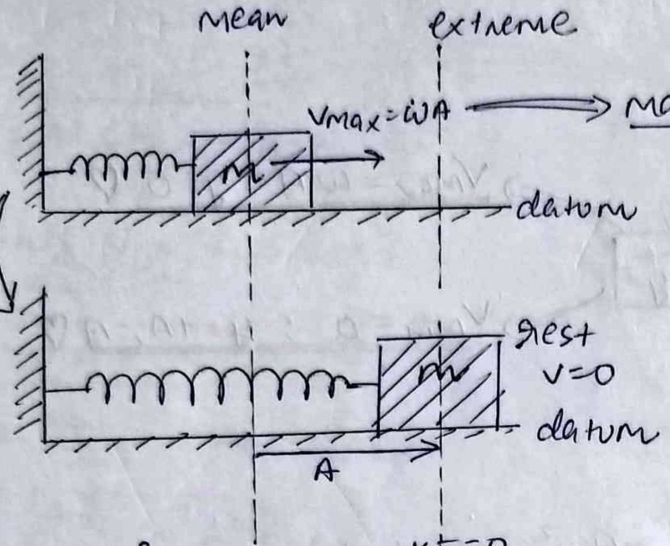
$a = -(\omega^2 r)\sin\theta$

$\boxed{a = -\omega^2 A\sin(\omega t + \phi)}$

$V_{SHM} = V\cos\theta$

$\boxed{V_{SHM} = \omega A\cos(\omega t + \phi)}$

> ENERGY OF PARTICLE IN SHM :-



Maximum velocity in SHM = ωA

Also;
By Energy Conservation

$E_{mean} = E_{extreme}$

$\frac{1}{2}m(\omega A)^2 = \frac{1}{2}KA^2$

$m\omega^2 = K$

$\boxed{\omega = \sqrt{K/m}}$

$\underbrace{\begin{array}{l} \frac{1}{2}m(\omega A)^2 = KE_{max} \\ PE = 0 \end{array}}$

> At Mean
$E_T = \frac{1}{2}m(\omega A)^2$

$\underbrace{\begin{array}{l} KE = 0 \\ PE_{max} = \frac{1}{2}KA^2 \end{array}}$

> At extreme
$ET = \frac{1}{2}KA^2$

> KINETIC.E IN SHM AS A FUNCTION OF TIME :-

We know that in SHM:-

$V = A\omega \cos(\omega t + \phi)$

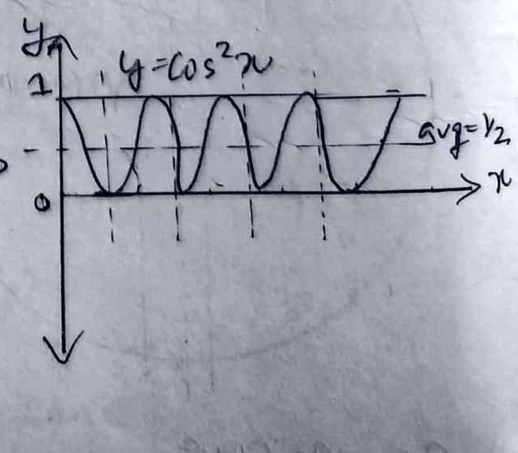
$KE = \frac{1}{2}mv^2$

$$\boxed{KE(t) = \frac{1}{2}m(\omega A)^2 \cos^2(\omega t + \phi)}$$

Avg. KE :-

$\underbrace{\langle KE \rangle}_{Avg.} = \frac{1}{2}m(\omega A)^2 \times \cos^2(\omega t + \phi)$

and Avg. value of $\cos^2(\omega t + \phi) = 1/2$



so,

$\langle KE \rangle = \frac{1}{2}\left[\frac{1}{2}m(\omega A)^2\right] = \frac{1}{2}E_T$

$$\boxed{\langle KE \rangle = \frac{1}{2}K_T}$$

KE AS A FUNCTION OF x :-

We know;

$v = \omega\sqrt{A^2 - x^2}$ (for SHM in x-axis)

$KE = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}m\omega^2(A^2 - x^2)$

$$\boxed{KE(x) = \frac{1}{2}m\omega^2(A^2 - x^2)}$$

POTENTIAL E AS A FUNCTION OF x :-

We know :-

$KE(x) = \frac{1}{2}m\omega^2(A^2 - x^2)$

$KE(x) = \frac{1}{2}m\omega^2A^2 - \frac{1}{2}m\omega^2x^2$

also;

$PE(x) + KE(x) = T.E(x)$

$PE(x) + \left(\cancel{\frac{1}{2}m\omega^2A^2} - \frac{1}{2}m\omega^2x^2\right) = \cancel{\frac{1}{2}m\omega^2A^2}$

$$\boxed{PE(x) = \frac{1}{2}m\omega^2x^2}$$

P.E AS A FUNCTION OF t :-

We know;

$PE(x) = \frac{1}{2}m\omega^2x^2$

and $x = A\sin(\omega t + \phi)$

$PE(x) = \frac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$

Avg. PE :-

$\langle PE \rangle = \frac{1}{2}m(\omega A)^2 \sin^2(\omega t + \phi)$

and Avg. value of $\sin^2(\omega t + \phi) = 1/2$

$\langle PE \rangle = \frac{1}{2}\left[\frac{1}{2}m\omega^2A^2\right]$

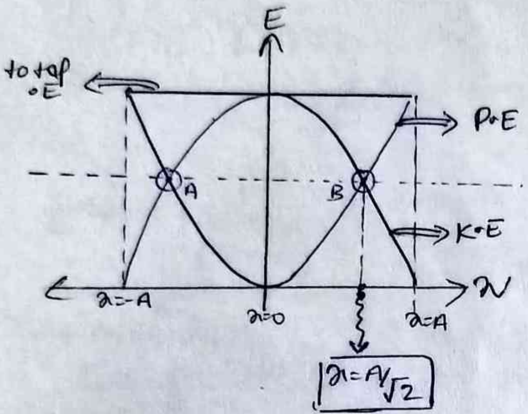
$$\boxed{\langle PE \rangle = \frac{1}{2}E_T}$$

> GRAPH FOR K·E AND P·E IN SHM :-

> $P.E = \frac{1}{2} m\omega^2 x^2 \Rightarrow$ upward Parabola

> $K.E = \underbrace{\frac{1}{2} m\omega^2 A^2}_{\text{constant}} - \frac{1}{2} m\omega^2 x^2 \Rightarrow$ downward Parabola

> total E $= \cancel{\frac{m}{2}\omega^2 x^2} + \frac{m}{2}\omega^2 A^2 - \cancel{\frac{m}{2}\omega^2 x^2} = \underbrace{\frac{1}{2} m\omega^2 A^2}_{\text{constant}} \Rightarrow$ Straight line



Position at which KE & PE are same

∴ K·E & P·E energy at Point A and B is $(\frac{1}{2} E_T)$

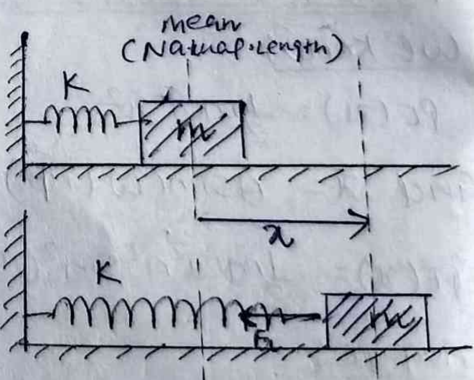
So;

Potential E at A = $\frac{1}{2} E_T$

$\cancel{\frac{1}{2}} m\omega^2 x^2 = \cancel{\frac{1}{2}}(\frac{1}{2} m\omega^2 A^2)$

$\Rightarrow x^2 = \frac{A^2}{2} \Rightarrow \boxed{x = A/\sqrt{2}}$

> TIME PERIOD OF SHM :-



$F_{restoring}(F_r) = Kx$

$F_r = Kx$

$ma_r = Kx \Rightarrow a_r = \left(\frac{K}{m}\right)x$ —— ①

$a \propto x$; It is SHM

So, $a_r = (\omega^2)x$ —— ②

Comparing eq 1 and 2

$\boxed{\omega^2 = \frac{K}{m}} \Rightarrow$ and; $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{M/K}}$

> TIME PERIOD OF SHM (IN Y):-

$$\boxed{Kx_0 = mg}$$

$$\Rightarrow x_0 = \frac{mg}{K} \quad \text{--- (1)}$$

Now;

$$(F_{res.})_{Net} = ma_{res}$$

$$\boxed{K(x + x_0) - mg = ma_{res}}$$

$$Kx + Kx_0 - mg = ma_{res} \quad \text{--- (2)}$$

from eq 1 and 2:-

$$Kx = ma_{res}$$

$$a_{res} = \left(\frac{K}{m}\right)x$$

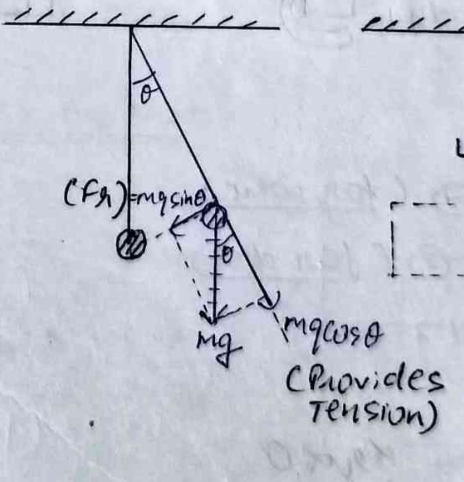
> $a \propto x$ SHM:- $a_{res} = \omega^2 x$

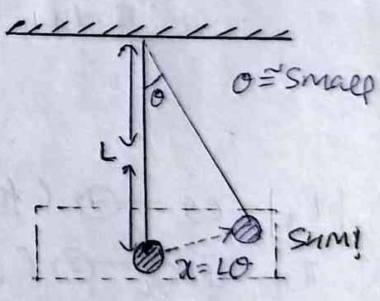
$$\omega^2 x = \left(\frac{K}{m}\right)x \Rightarrow \omega^2 = \frac{K}{m}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/K} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

> TIME PERIOD OF SHM OF SIMPLE PENDULUM:-

> Method-I





∴ from fig-I

$$x = L\theta \Rightarrow \boxed{\theta = x/L}$$

and since; $\theta \simeq$ Small

$$\theta = \sin\theta \quad \text{--- (1)}$$

from eq 2:-

$$F_r = ma_r$$

$$mg\sin\theta = ma_r$$

$$\Rightarrow a_r = g\sin\theta \quad \text{--- (2)}$$

from eq 1 and 2

$$a_r = g\theta = g\left(\frac{x}{L}\right) \Rightarrow a_r = \left(\frac{g}{L}\right)x \Rightarrow a_r \propto x$$

SHM ☺ same

Now in SHM:-

$$a_r = \omega^2 x$$

$$\omega^2 x = \left(\frac{g}{L}\right)x$$

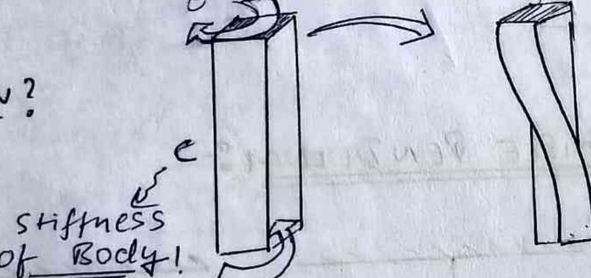
$$\Rightarrow \omega = \sqrt{g/L}$$

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{L/g}}$$

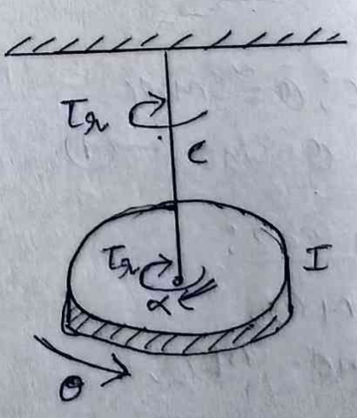
>M-II

> TIME PERIOD OF A TORSIONAL PENDULUM :-

> TORSION?



tension!
$U = \frac{1}{2}C\theta^2$
∵ (Just like Spring $K \equiv C$, $\theta \equiv x$)



$|\tau_r| = C\theta$ —① (for wire)

$\tau_r = I\alpha_r$ —② (for disc)

from eq 1 and 2

$C\theta = I\alpha_r$

$\alpha_r = \left(\frac{C}{I}\right)\theta \Rightarrow \underbrace{\alpha_r \propto \theta}_{\text{ROTATIONAL SHM}}$

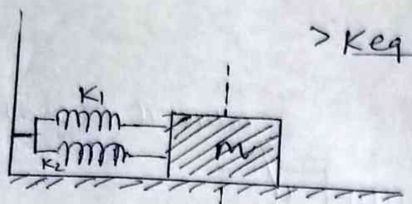
In Rotational SHM:-

$\alpha_r = \omega^2\theta = \left(\frac{C}{I}\right)\theta$

$\Rightarrow \boxed{\omega^2 = \frac{C}{I}}$ NOW, $T = \frac{2\pi}{\omega} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{C}}}$

> COMBINATION OF SPRING IN SHM :-

(I)

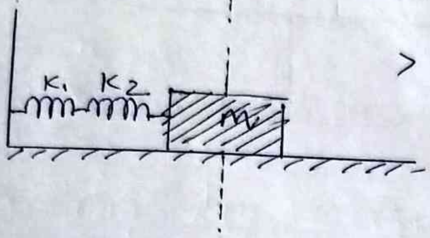


> K_{eq} in Parallel Combination :-

$$\underline{K_{Eq} = K_1 + K_2}$$

$$\boxed{T = 2\pi\sqrt{\frac{M}{K_{eq}}}}$$

(II)



> K_{eq} in series combination :-

$$\underline{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}}$$

$$\boxed{T = 2\pi\sqrt{\frac{M}{K_{eq}}}}$$

> EFFECT OF CONSTANT EXT. FORCE ON SHM :-

> TIME PERIOD OF SHM BY ENERGY METHOD :-

$$\underline{E_i = E_f}$$

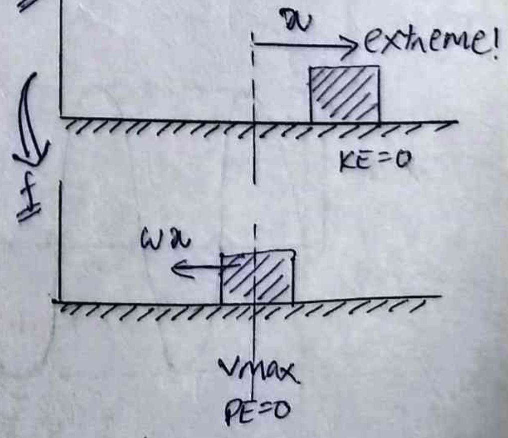
$$\frac{1}{2}Kx^2 = \frac{1}{2}m(\omega x)^2$$

$$\cancel{\frac{1}{2}}K\cancel{x^2} = \cancel{\frac{1}{2}}m\omega^2\cancel{x^2}$$

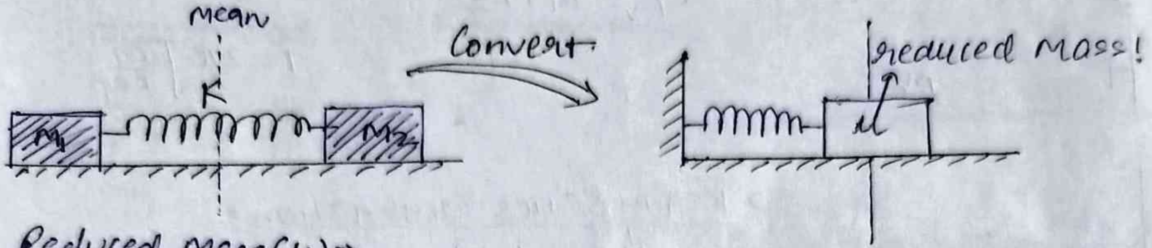
$$\omega = \sqrt{K/m}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/K}$$

$$\therefore \boxed{T = 2\pi\sqrt{m/K}}$$



>TIME PERIOD :- (😊)



Reduced Mass (μ) ⇒

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

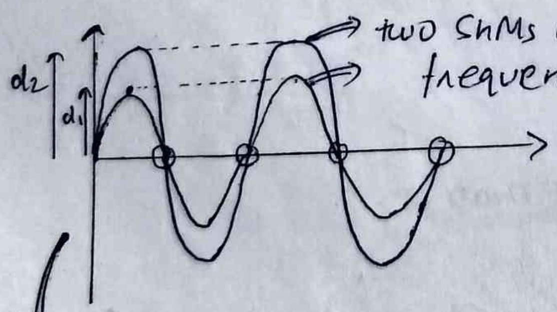
$$\mu = \left(\frac{m_1 m_2}{m_1 + m_2}\right)$$

$$T = 2\pi\sqrt{\frac{\mu}{K}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m_1 m_2}{K(m_1+m_2)}}}$$

> Composition of multiple SHMs :-

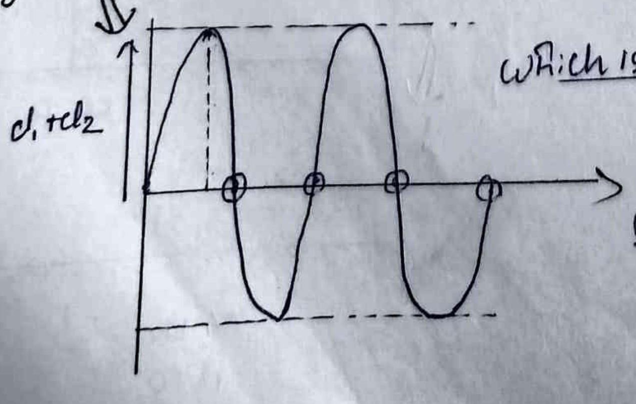
$\because y_1 = A_1 \sin(\omega t + \phi_1)$
$\because y_2 = A_2 \sin(\omega t + \phi_2)$

} two SHMs of same frequency.



→ two SHMs of same frequency If superimposed Gives a New SHM

SUPER IMPOSITION 😊



Which is also a New SHM
$y = A\sin(\omega t + \phi)$
of same ~~diff~~ frequency
But diff Amplitude
and initial phase 😊

> to find ; A and ϕ of superimposed SHM :-

∵ PHASOR METHOD :-

$y_1 = A_1 \sin(\omega t) = A_1 \sin(\omega t + 0)$ ← Initial phase —— [SHM 1]

$y_2 = A_2 \sin(\omega t + \phi) = A_2 \sin(\omega t + \phi)$ ← Initial phase —— [SHM 2]

> Put their Amplitude vector at an angle of their Initial phase with X-axis and resultant of these vectors with Gives Amplitude of Superposition SHM.

> The Angle of Resultant vector with X-axis Given initial phase of superposition vectors.