Sync - An Emergent Property of a System of Fireflies

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***Abstract*— In this article the natural system of fireflies is examined. These autonomous agents(fireflies) glow through a biological chemical reaction, periodically. However, things become interesting when each of these agents adjusts their light cycle upon receiving stimuli from the flashes from other agents in the system. Eventually these interactions lead to inevitable emergence of synchronous flashing of all the agents almost inevitably. The article covers this exhibit of natural marvel from it's historic roots from how it intrigued the scientific community, till how understanding such systems has helped engineers solve problems pertaining internet traffic. This article also examines a mathematical model that was developed to stimulate such behavior. Further, a stimulation is based on this model accompanying this article. Also, systems in nature that exhibit synchrony are briefly visited while attempts to understand the rationality of such a natural system to achieve the feat of synchrony has also been discussed. The goal of this article has been to examine the aforementioned theories from the perspective of the fundamental concepts of Multi Agent Systems like agency, message passing, invariants, emergent global properties and asynchronous clocks.**

I. Fascinating mystery of fireflies

The Great Smoky Mountains in Tennessee, Congaree National Park in South Carolina and the Allegheny National Forest, Pennsylvania have something in common. These places attract many tourists across the world during the summers to watch a natural light show. In this show, thousands of fireflies flash off in perfect sync exhibiting nature's wonder. For centuries, the scientific community debated the mystery of synchrony emerging among thousands of these natural agents(fireflies). As early as 1917, this phenomenon was disregarded to be scientific. In an article published in 1917, in the Science journal[1], the following was quoted,

> The apparent phenomenon(of fireflies synchrony) was caused by twitching or sudden raising of my eyelids. Many times in the past twenty years I have proved that my solution is right.
>
> *Philip Laurent*

However, through observations made by various people from the scientific community in the years to come changed the perception of this phenomenon of natural synchrony. The significant historical events that transformed our understanding towards synchronous behaviour of fireflies, have been picked from the book[6] written by Strogatz. This article is an attempt to look at natural synchrony through the lens of Multi Agents Systems.

The first significant breakthrough in solving the mystery of synchronous fireflies came in late 1960s, when scientists took note of rhythm apart from synchrony. This meant that the whole act of flashing of fireflies was not just synchronous but was also periodically occurring. Further, if a firefly was isolated, it would continue to glow in a rhythm in correspondence to an intrinsic clock. This observation from the perspective of Multi Agent systems can be understood as autonomy exhibited by each of the fireflies. The sense of internal clock within each of the fireflies depicts free will that the firefly has on it's light cycle, which in turn reflects the notion of agency of the firefly.

The next crucial observation came from the experiments conducted by the biologist John Buck. When a bunch of fireflies were let open in a dark room, it was observed that the fireflies initially fired as per their intrinsic clocks. However, they soon formed small pods where they started to glow in unison, by adjusting their clock cycles as per the stimulus received from their neighbouring agents. This suggested that the intrinsic clock of each of the agents was *adjustable*, and was directed by some interactions of the agents in the system. This also signifies that each firefly is showing certain rationality by adjusting something that is under its control based on the stimulus received from it's neighbouring fireflies. How could these interactions be defined? and, What is the rationality of an autonomous firefly? are questions that are addressed in later sections.

Thus, we have a natural system of autonomous agents where the agents interactions lead to an global emergent property of synchronous firing. It is also important to notice that there is no notion of a global clock here, we can't comment which firefly's light cycle is ahead of another. But that's not the problem under consideration instead we seek answers to how these complex(not complicated) interactions between these natural agents, lead to a resilient global property of synchronous firing.

II. SYNC IN NATURAL SYSTEMS

Synchronous behaviour occurs in many natural systems. It is due to the occurrence of the similar phenomenon across different fields of study, that a diverse set of minds aid each other's understanding to come up with mathematical models that explain synchronous behaviour. The following are few natural systems where sync emerges in nature that are referred from the work[7] by Kevin et. al. :

- Pacemaker cells : These cells fire synchronously in a periodic manner indicating the heart when to beat. This system is very similar to a system of fireflies.
- Laser : Trillions of atoms emit photons of same phase and frequency, enabling a laser to travel large distances without attenuation.
- Menstrual cycle : Women who spend majority of the time together sync their menstrual cycles.
- Epilepsy : Infamous example, where millions of neurons fire in rhythm causing seizures. However, understanding how to prevent sync can also be of great use.

III. MATHEMATICAL MODEL OF SYSTEM OF FIREFLIES

A. Peskin's abstract model for pacemaker cells

Peskin in his work[9] abstracted the complexities involved in the system of cardiac pacemaker cells, to form a simplified mathematical model that depicted emergence of sync. This model can be extended to the system of fireflies as well.

Every agent, be it a firefly or a pacemaker cell in the system was abstracted as an oscillator. A RC circuit was considered as the physical realisation of this natural oscillator. Whenever the voltage crosses a threshold(the red dot in Fig. 1) the oscillator fires and then voltage is reset(the dotted line in Fig. 1 to 0.

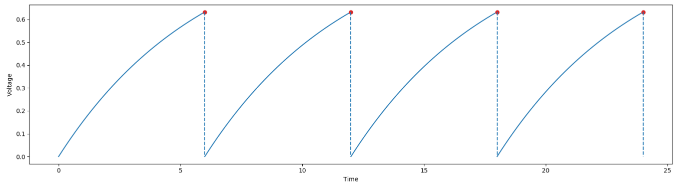

Fig. 1. Voltage vs. Time of the physical oscillator

The communication between agents happens as follows, whenever an agent fires(oscillator reaches the threshold), then instantly the voltage of all other agents is increased by a constant amount. This is how the agents communicate with each other in the framework of this model whose emergent property is synchrony of oscillators. The intuition for the proof of emergence of sync is developed in the next section.

B. Mathematical proof

The proof presented in this section is developed from the understandings from the book[6] by Strogatz. A core idea involved in this mathematical model is that of *Absorption*. If one oscillator kicks another oscillator over threshold they will remain synchronized forever. This can also be observed in the stimulation that accompanies this article, which depicts when a pod of fireflies attains sync, they always remains in sync.

Let us consider the abstract quantity that oscillates(which was abstracted as Voltage of RC circuit by Peskin) in the case of fireflies by a neural fluid. Once this neural fluid crosses a threshold the firefly glows, and it gets reset to zero. Now getting to the proof, suppose we are examining a system of n agents(fireflies). Then a $n-tuple$ describes the state of the system, where the i^{th} entry in the tuple, represents the amount of neural fluid in the i^{th} agent(firefly) in that state. If at any point of time we have a tuple of all zeros, then sync has emerged. The states of the system can be realised as a point in $n-dimensional$ hypercube. If we plot the state of the system at every time instant then we can observe how the whole system evolves.

Then, there is a need to understand two qualitatively different points in this hypercube. The good points, are the ones from where the system eventually reaches the origin, and thus sync would emerge from there on. The bad points, are the ones that get stuck in nasty cycles and never reach the origin. The work [5] by Strogatz and Mirollo proved that the probability of starting at such bad point is zero, by the following two arguments[1]:

- If we start off at bad point, then each subsequent point in the n dimensional space we visit will be a bad point. Now suppose you collected all the bad points into a set, then on applying the transformation(going one timestamp ahead as per current state) you will still end up in the same set. This set never increases in it's size.
- If we take any set of n dimensional points and apply the transformation, the size of the set will strictly increase. The function of the abstract quantity that varies has to be a non-decreasing concave function(refer the function in Fig. 1 for instance), for this condition to hold.

The above two conditions can be consistent only if the hyper volume occupied by the set of all bad points is zero, only then even after transformation the set of points does get larger and yet remains of same size. While the argument might seem tricky, the essence of the proof does establish that sync not emerging is an event of almost zero probability.

IV. Stimulation of fireflies

Now that the mathematical basis for the model has been laid out, a stimulation was built to see emergence of sync. In this stimulation the intrinsic clock of the firefly directs glow at the fourth second and reaches back to it's non-luminescent state by the sixth second. For consistence with the previous section, the neural fluid needs to be modelled as a non-decreasing concave function. Thus, neural fluid as per the stimulation is a function of time as follows,

$$fluid(t) = tanh(t/6) \tag{1}$$

[1]The work by Strogatz and Mirollo has been groundbreaking in mathematical biology with over 2500 citations. For rigorous mathematical proof, the reader may refer to the original work.

Each agent starts off with a random time on it's intrinsic clock. However, the firing of one of the agents increases the neural fluid in every other agent by constant amount, and during this if in any agent it crosses the threshold then as per the *absorption* phenomenon these agents would sync up.

The emergence of sync has been observed as invariant property for 2,10,50,100,500,1000 agents through stimulation. Indeed, through stimulation that sync seems inevitable. The code for the stimulation can be accessed at the following, [Github] repository. The file sync.html can be opened in a browser to observe the emergence of sync after a while, starting with random initialization of individual light cycle.

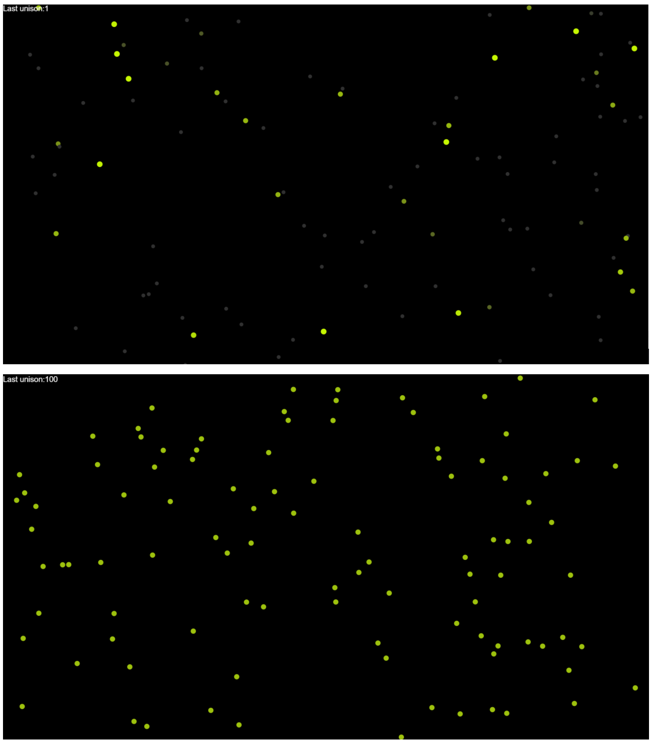

Fig. 2. A system of 100 fireflies, before and after sync emerges

The Fig.1 show to snapshots of a system with 100 agents, before and after sync had emerged in the stimulation.

V. Rationality behind achieving sync

There have been multiple hypothesis presented by biologists to explaining the need for fireflies syncing. Understanding these hypothesis will help appreciate the rational thought behind why a natural system would go through the trouble of achieving sync.

In 1966, John Buck and Elisabeth Buck in their work[2] proposed the Beacon hypothesis. It was an already known fact that it was the male fireflies that synchronised their flashes. It was hypothesized that, male fireflies sync their flashes to become visible to their female counterparts miles apart in the dense forests and attract them for mating. While an individual male firefly might not able to make an impact of it's presence by just the intensity of it's own biolumniscence, syncing up their light cycles can produce a collective light of great intensity.

Recent study[8] suggests that the chemical *lucibufagin* that is responsible for biolumniscence, is distasteful and sometimes even toxic for a predator that may feed on fireflies. Thus predators like bats that feed on insects are in a sense warned from far off from the intense synchronous light to stay away from the natural system of fireflies.

Did you wonder why would an individual alter it's clock cycle by moving it towards the point where it could glow? The work[4] by Vencl and Carlson hypotheses the rationality of individual firefly. The female firefly gets attracted to it's male counterpart that glows first. Thus, when a male firefly glows the other male fireflies adjust their light cycles to glow quicker in order to compete with the male firefly that glowed earlier. In this way the manner in which the interactions in the earlier mathematical model between the agents seem to be more logical. Also, suggesting that there is rational decision making at different levels(individual and collective) that results into a global emergent behaviour of sync.

VI. Natural systems solve engineering problems

To understand why would someone want to even study natural systems, one has to look back at instances where this understanding has helped to solve real life engineering problems. Early 1994, while the internet had started to grow, an unusual spontaneous pulsation of internet traffic was observed. This is undesired as high demand of resources on limited infrastructure at certain times would can cause long delays. Yet, no one formally understood this periodic pulsation that occurred in network traffic. To bring the problem into the frame of Multi Agent systems, we zoom into routers. In the network of systems connected by internet, routers are the agents that route the data by deciding the best possible paths for data flow. A router communicates with it's neighbouring router whenever it needs to transmit some data, however after a request it waits for some fixed time before further processing. This is how the agents here communicate, they have some periodic communication going on between them. It is worthy to note that unlike fireflies, routers can only handle request from another router at a time. The engineers who handled the internet took it for granted that the traffic on the internet will be more or less uniform because the requests from the various sources occur randomly, they felt some pattern can't be a byproduct of randomness.

However the work[3] by Sally Floyd and Van Jacobson highlighted that the processing demands of the routers can sync up causing high delays of data transmission[2]. They could stimulate such behaviour with the understanding of mathematical models of the synchronous fireflies that has been talked all along in this article. This work also contributed by suggesting one could strategically insert noise into such networks to avoid such traffic congestion. Thus, although understanding natural systems might not practically sound appealing, by connecting the dots backwards, we realise that they help immensely in solving problems of humanity.

References

[1] P. Laurent. "The supposed synchronal flashing of fireflies." In: *Science* 1150.45 (1917), pp. 44–44.

[2] Buck E. Buck J. "Synchronous fireflies." In: *Scientific American* 235.5 (1976), pp. 74–85. URL: http://www.jstor.org/stable/24950352.

[3] S. Floyd and V. Jacobson. "The synchronization of periodic routing messages." In: *IEEE/ACM transactions on networking* 2.2 (1994), pp. 122–136. DOI: 10.1109/90.298431.

[4] Carlson A.D. Vencl F.V. "Proximate Mechanisms of Sexual Selection in the Firefly Photinus pyralis (Coleoptera: Lampyridae)". In: *Journal of Insect Behavior* 11 (1998), pp. 191–207. DOI: 10.1023/A:1021091806472.

[2]The readers can further explore the above work, to understand how a decentralised solution was developed for the aforementioned problem of internet traffic for another mandate contribution.

[5] Mirollo Renato E. and Steven H. Strogatz. "Synchronization of pulse-coupled biological oscillators." In: *SIAM Journal on Applied Mathematics* 50.6 (1999), pp. 1645–1662. DOI: 10.1137/0150098.

[6] Steven H. Strogartz. "Sync : The Emerging Science of Spontaneous Order". In: 1st ed. Hyperion, 2003. Chap. 1.

[7] Hong H. O'Keeffe Kevin.P. and Strogatz S.H. "Oscillators that sync and swarm". In: *Nat Commun* 8.1504 (2017). DOI: 10.1038/s41467-017-01190-3.

[8] Andrew Cruickshank. *Research shows a new reason for why fireflies glow*. URL: https://cottagelife.com/outdoors/research-shows-a-new-reason-for-why-fireflies-glow/.

[9] Charlie Peskin. *Principles of Electrophysiology with Special Reference to the Heart*. URL: https://www.math.nyu.edu/~peskin/heartnotes/CLN-Peskin1975-7.pdf.