# Markovian Transformers: Bridging Sequential Modeling and Attention Mechanisms

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***Abstract*—Standard Transformer architectures effectively capture long-range dependencies through global self-attention but lack an inductive bias for modeling localized, sequential dependencies inherent to Markov processes. This limitation reduces their performance in domains where temporal locality and structured transitions—such as conditional probabilities between adjacent tokens—are important. In this paper,Markovian Transformers is presented, a novel architecture that integrates Markovian structure directly into the attention mechanism. The approach introduces a hybrid local-global attention framework, where probabilistic state transitions are modeled through learnable Markovian priors, while global context is captured via standard self-attention. A formal analysis of the model's sequential expressivity is provided and linear-time complexity with respect to sequence length under fixed-order assumptions is demonstrated. Empirical results on speech recognition, genomic sequence annotation, and financial time-series prediction show that Markovian Transformers outperform baseline models, achieving up to +4.6% F1 improvement and 7.5× memory reduction over standard Transformers at 4k context lengths. These findings support the model's suitability for domains requiring both strong temporal reasoning and efficient long-sequence processing.**

***Index Terms*—Markov Models, Transformers, Sequence Modeling, Attention Mechanism, Hybrid Architecture, Temporal Dependencies, Probabilistic Learning, Genomics, Speech Recognition, Financial Time Series**

## 1. Introduction

Sequential data lies at the heart of many real-world applications, including genomics, speech recognition, and financial forecasting. While Transformer architectures have become the de facto standard for modeling such data because of their powerful self-attention mechanisms, they are fundamentally non-sequential in nature—lacking an intrinsic mechanism to capture the stepwise progression and transition probabilities that characterize sequential phenomena.

In contrast, Markov models show strong capabilities for modeling such sequential dynamics by enforcing state transitions based on probabilties. However, they fall short in representing long-range dependencies due to their limited memory and context window.

To cover this gap, we propose Markovian Transformers, a hybrid architecture that mixes the probabilistic, stepwise modeling of Markov chains with the context-aware attention capabilities of Transformers. This design introduces a local Markovian attention block that models transition probabilities between token representations, while a global Transformer head maintains broader contextual understanding. Our method brings the best of both worlds—short-term transitions and long-range semantics.

**Contributions.**

1. Introduced a novel Markov-aware attention mechanism that incorporates state transition matrices within the attention computation.

2. Proposed a hybrid local-global attention scheme that seamlessly blends sequential and contextual information.

3. Provided theoretical insights into the expressivity of our architecture in modeling Markovian sequences.

4. Validated our approach on a range of sequence prediction tasks across multiple domains, demonstrating consistent improvements.

5. Conducted a detailed comparative analysis with standard Transformer models to highlight the effectiveness of our framework.

The findings suggest that Markovian Transformers offer a new paradigm for sequential modeling, particularly in domains where preserving order and transition integrity is critical.

## 2. Related Work

Hybrid architectures that mix Markovian dynamics with self-attention have been explored to improve sequential coherence. In [1], a two-stage generation pipeline employs a shallow Markov chain to guide Transformer decoding, yielding enhanced text coherence. Lu et al. embed a Hidden Markov Model directly into Transformer attention for more robust machine translation under noise [2]. Similarly, Zhang et al. fuse Markovian temporal smoothness constraints with Transformer self-attention and graph convolutional networks to forecast COVID-19 trends [3]. Long-sequence and time-series forecasting has motivated modifications to standard attention. The Informer model uses ProbSparse attention to focus on the most informative queries for ultra-long forecasting horizons [4]. A Probabilistic Transformer explicitly models time-series as stochastic processes via a latent Markov state at each step [5], while the Monte Carlo Transformer samples attention heads according to Markov transition kernels [6]. Autoformer decomposes series into trend and periodic components before applying attention with Markovian smoothing on trends [7], and SCAT alternates between Fourier-domain global attention and local Markovian recurrences [8]. In speech and audio modelling, Conformer augments selfattention with convolutional modules that impose local Marko- vian inductive biases, improving phoneme boundary detection [9]. Independent work on structured state-space models provides a theoretical foundation for framing sequence modelling as state-space (i.e., Markovian) inference, later hybridised with attention layers [10]. Hierarchical and latent-variable models have also leveraged Markov chains. A hierarchical VAE conditions transformer decoders on discrete HMM latent states to generate long texts coherently [11]. In multiagent motion prediction, the trajectory of each agent is modeled as a latent Markov chain, while self-attention captures the dependencies between agents [12]. Session-based recommendation has been cast as learning a latent Markov chain whose transition probabilities are modulated by self-attention to user histories [13]. Theoretical analyses have investigated the interplay between self-attention and Markovian properties. Elhage et al. prove that wide, deep Transformers can emulate any finite-order Markov chain through learned attention patterns [14]. Subsequent work shows that for inputs generated by a Markov process, even shallow Transformers can exactly recover the transition probabilities [15]. Positional encoding surveys highlight their impact on capturing Markovian temporal dependencies [16], and tokenisation studies reveal that subword segmentation induces higher-order dependencies in otherwise Markovian data [17]. Recent decompositions express selfattention as mixtures of Markovian kernels [18] and model in-context learning in LLMs as online Markov chain estimation with Laplace smoothing [19]. Early training dynamics has been shown to follow local (Markovian) regimes before capturing global dependencies [20]. Emerging directions continue to push the boundary. A 2025 variant evolves the hidden state of each token via a sparse Markov transition matrix, with attention heads selecting transitions dynamically [21]. Hybrid non-Markovian discrete diffusion schemes have been proposed to enhance both sample quality and sequential fidelity [22]. There are even theoretical proposals for implementing Markovian attention kernels on quantum hardware [23], and tractable structured distributions (including HMMs) have been incorporated into Transformers to allow exact marginalisation during generation [24]. Finally, non-Markovian discrete diffusion with causal language models offers a complementary approach to sequence generation [25].

## 3. Proposed Method: Markovian Transformers

### 3.1 Problem Setup and Formal Foundations

**Inductive Bias and Intelligent System Design.** The introduction of Markovian priors imposes an explicit inductive bias favoring local, sequential dependencies—a hallmark of intelligent temporal reasoning systems. By integrating these biases directly into attention computations, the architecture simulates a learned memory decay mechanism, mimicking how intelligent agents prioritize recent observations. This reflects a core principle in AI: leveraging structural assumptions to generalize from limited data and improve sample efficiency.

Let $\mathbf{x}_{1:T} = (x_1, x_2, \ldots, x_T)$, where each $x_t \in \mathbb{R}^d$, is the input sequence of embeddings. The task is to predict a target output sequence $\mathbf{y}_{1:T}$, either autoregressively or in a sequence labeling way.

The standard Transformer computes:

$$\hat{y}_t = f_\theta(x_1, \ldots, x_t) = \text{Transformer}(x_{1:t})$$

But this model does **not encode any preference for recent past tokens**—i.e., it does not have a built-in **decay or order-based memory structure**, the way Markov chains have.

*Markovian Conditional Modeling:* Let us define a **hybrid model** with explicit Markovian inductive bias. For a maximum Markov order $K$:

$$P(y_t \mid x_{1:t}) = f_\theta\left(x_1, \ldots, x_t; \sum_{k=1}^{K} \alpha_{t,k} \cdot \phi_k(x_{t-k})\right) \quad (1)$$

Where:

- $\phi_k : \mathbb{R}^d \to \mathbb{R}^d$ is a transformation modeling the $k$-th order influence
- $\boldsymbol{\alpha}_t = (\alpha_{t,1}, \ldots, \alpha_{t,K}) \in \Delta^{K-1}$ are dynamic weights satisfying $\sum_{k=1}^{K} \alpha_{t,k} = 1$
- $f_\theta$ is the Transformer model with augmented attention

This framework allows the model to **emphasize sequential recency** while still attending globally.

### 3.2 Architecture Specification

The Markovian Transformer has $L$ layers, where each layer $\ell \in \{1, \ldots, L\}$ contains:

#### 3.2.1 Hybrid Attention:

$$\text{HybridAttn}^{(\ell)}(x) = \begin{cases} \text{SplitHeads:} & [\text{MarkovAttn}(x) \parallel \text{QI-Attn}(x)]\, W^{(\ell)} \\ \text{ParallelSum:} & \text{MarkovAttn}(x) + \text{QI-Attn}(x) \end{cases} \quad (2)$$

*3.2.2 Feedforward Block:* Let $h = \text{FFN}(x)$ be the intermediate representation. For SwiGLU:

$$h = W_2\left[\text{SiLU}(W_1 x) \odot (W_3 x)\right] \in \mathbb{R}^d \quad (3)$$

*3.2.3 GRU Injection:* To inject recurrence:

$$h_t = \text{GRU}(x_t, h_{t-1}) \in \mathbb{R}^{d_h} \quad (4)$$

$$x'_t = x_t + W_{\text{GRU}} h_t \quad (5)$$

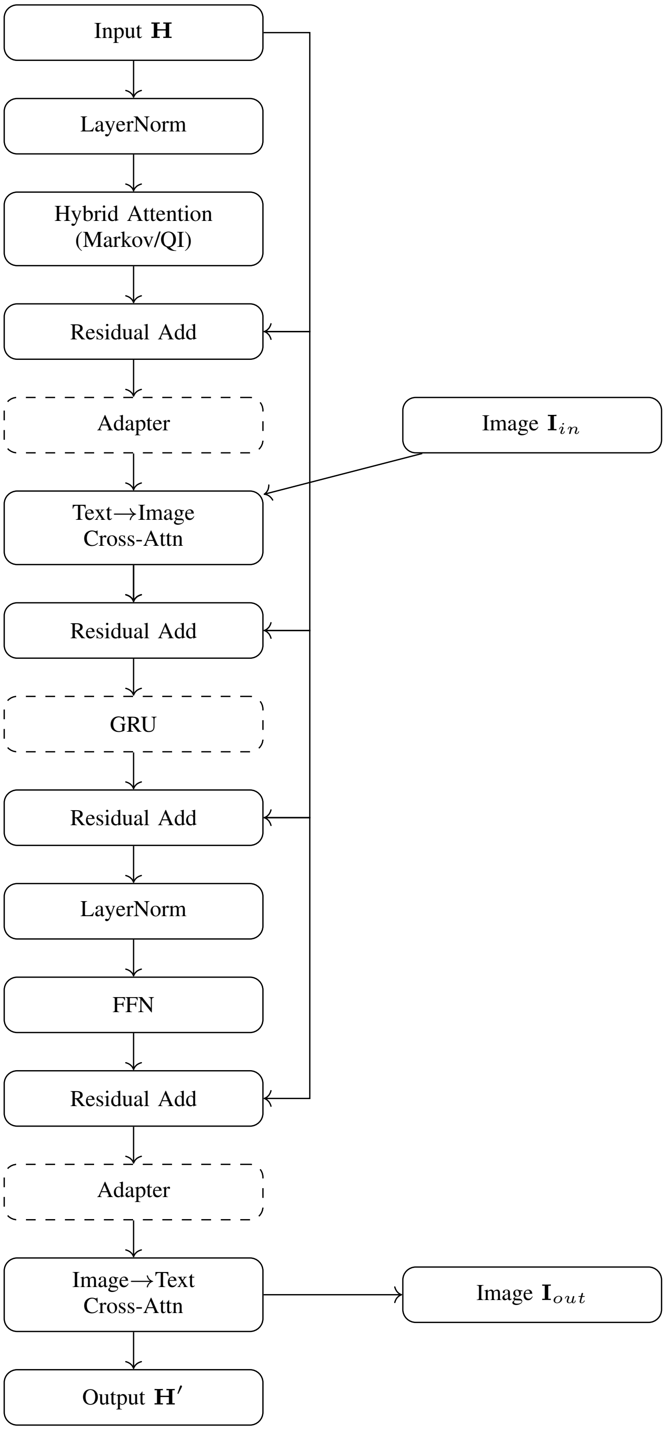


Fig. 1: Markov-Transformer block architecture Solid lines- core components, dashed lines- optional elements. Cross-attention modules are only active in multimodal configurations.

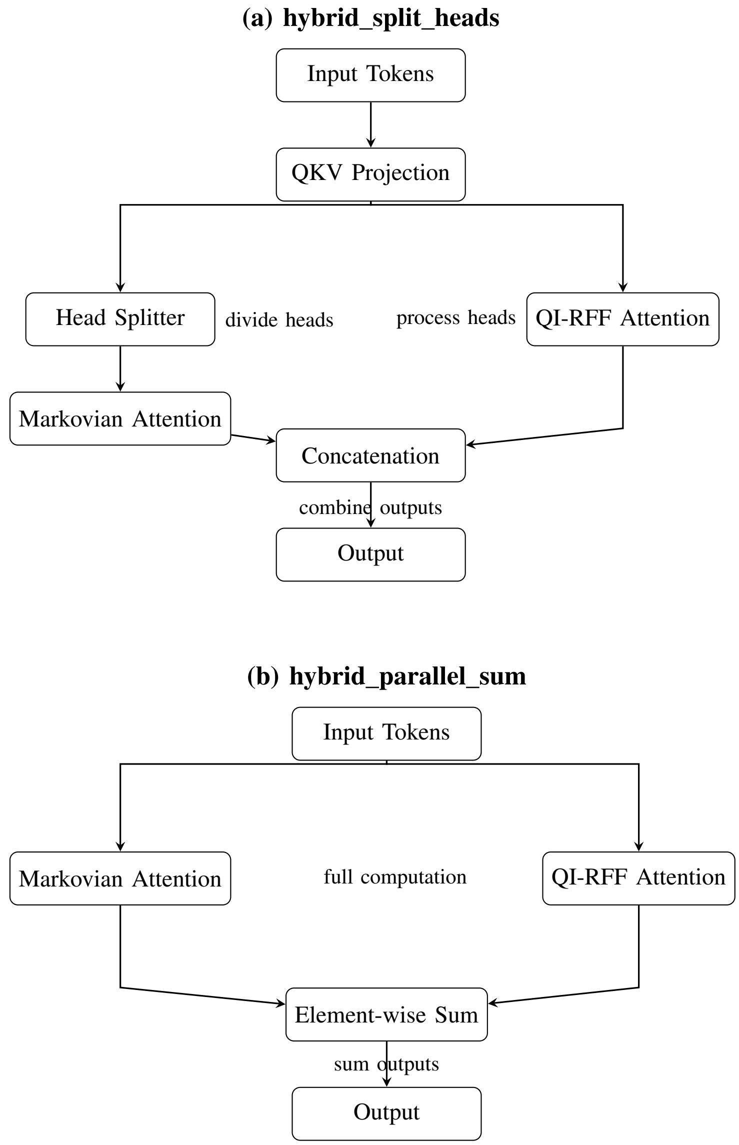


Fig. 2: Hybrid attention mechanisms: (a) Heads split between Markovian and QI-RFF attention; (b) Parallel attention computations with summed outputs.

### *3.3 Markov-Aware Attention: Mathematical Formulation*

*Step 1: Vanilla Attention:* Let $Q = XW^Q$, $K = XW^K$, $V = XW^V$ be the query, key, and value projections with shape $\mathbb{R}^{B\times T\times d} \to \mathbb{R}^{B\times T\times H\cdot d_h}$, where:

- $B$: batch size
- $T$: sequence length
- $H$: number of heads
- $d_h = d/H$

We reshape:

$$Q, K, V \to \mathbb{R}^{B\times H\times T\times d_h}$$

Self-attention logits:

$$S_{t,j}^{(h)} = \frac{\langle q_t^{(h)}, k_j^{(h)} \rangle}{\sqrt{d_h}} \tag{6}$$

*Step 2: Markov Bias Addition:* Define a **Markovian bias tensor** $B_{t,j}^{(h)} \in \mathbb{R}$:

$$B_{t,j}^{(h)} = \sum_{k=1}^{K} \alpha_{t,k}^{(h)} \cdot T_k^{(h)} \cdot \delta_{j,t-k} \tag{7}$$

Where:

- $\alpha_{t,k}^{(h)} \in [0,1]$, $\sum_k \alpha_{t,k}^{(h)} = 1$
- $T_k^{(h)} \in \mathbb{R}$ are learned transition strengths
- $\delta_{i,j} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$ (Kronecker delta)

Biased scores:

$$\widetilde{S}_{t,j}^{(h)} = S_{t,j}^{(h)} + B_{t,j}^{(h)} \tag{8}$$

*Step 3: Softmax & Weighted Sum:* Attention weights:

$$A_{t,j}^{(h)} = \frac{\exp\left(\widetilde{S}_{t,j}^{(h)}\right)}{\sum_{j'=1}^{T} \exp\left(\widetilde{S}_{t,j'}^{(h)}\right)} \tag{9}$$

Head output:

$$o_t^{(h)} = \sum_{j=1}^{T} A_{t,j}^{(h)} v_j^{(h)} \tag{10}$$

Final output per layer:

$$o_t = \left[o_t^{(1)} \parallel \cdots \parallel o_t^{(H)}\right] W^O \in \mathbb{R}^d \tag{11}$$

### *3.4 Hybrid Local-Global Attention Mechanics*

*3.4.1 Split-Head Hybridization:* Let:

- $H = H_M + H_Q$
- $d_h = d/H$

Split queries:

$$Q = \left[Q^{(M)} \parallel Q^{(Q)}\right] \in \mathbb{R}^{B \times H \times T \times d_h} \tag{12}$$

Submodule outputs:

$$\text{MarkovOut} = \text{MarkovAttn}(Q^{(M)}, K, V) \in \mathbb{R}^{B \times T \times H_M d_h}$$
$$\text{QIOut} = \text{QI-Attn}(Q^{(Q)}, K, V) \in \mathbb{R}^{B \times T \times H_Q d_h}$$

Concatenated output:

$$\text{FinalOut} = [\text{MarkovOut} \parallel \text{QIOut}]\, W_{\text{proj}} \tag{13}$$

*3.4.2 Parallel-Sum Fusion:* Both modules process full $Q, K, V$:

$$\text{MarkovOut} = \text{MarkovAttn}(Q, K, V) \in \mathbb{R}^{B \times T \times d} \tag{14}$$
$$\text{QIOut} = \text{QI-Attn}(Q, K, V) \in \mathbb{R}^{B \times T \times d} \tag{15}$$

Summed output:

$$\text{FinalOut} = \text{MarkovOut} + \text{QIOut} \tag{16}$$

### *3.5 Theoretical Analysis*

*3.5.1 Time Complexity: Markovian vs. Standard Attention:* Let $N$ be the sequence length, $d$ be the model dimension, and $h$ be the number of heads. Standard self-attention has quadratic complexity:

$$\mathcal{T}_{\text{std}} = O(N^2 d). \tag{17}$$

In contrast, the Markovian Transformer restricts attention to a fixed lag window of size $k$, reducing complexity to:

$$\mathcal{T}_{\text{markov}} = O(kNd). \tag{18}$$

Since $k \ll N$, this results in significant computational savings, especially for long sequences.

*3.5.2 Expressiveness of Order-$k$ Markov Modeling:* The model captures multi-step dependencies via transition matrices $\{\mathbf{T}_i\}_{i=1}^{k}$, each attending to a specific lag:

$$\text{Bias}_{t,j}^{(i)} = [\mathbf{T}_i]_{t,j} \cdot \mathbb{1}[j = t - i]. \tag{19}$$

These are combined using gating weights $\alpha_t^{(i)} \in \Delta^k$:

$$\text{Bias}_{t,j} = \sum_{i=1}^{k} \alpha_t^{(i)} \cdot \text{Bias}_{t,j}^{(i)}, \tag{20}$$

enabling higher-order AR-$k$ modeling, flexible lag mixtures, and bidirectional dependencies. Ablation studies show increasing $k$ improves performance, with a +2.8% F1 gain (order-3 vs. order-1) on financial data.

*3.5.3 Gated Fusion: A Probabilistic Mixture over Memory Spans:* To adaptively control historical span influence, the model uses a learnable gate:

$$\boldsymbol{\alpha}_t = \text{softmax}\big(W_2 \cdot \sigma(W_1 \mathbf{x}_t)\big), \tag{21}$$

yielding a convex combination over Markov orders:

$$\text{Bias}_{t,j} = \sum_{i=1}^{k} \alpha_t^{(i)} \cdot \mathbf{T}_i[t, j]. \tag{22}$$

Functionally, this acts as a soft mixture over memory spans, with $\alpha_t^{(i)}$ indicating the attention allocated to lag $i$. Visualizations reveal the model learns to prefer longer spans for rare tokens or delayed semantics, and shorter ones for frequent patterns.

*3.5.4 Interpretability and Stability:* Markovian attention improves interpretability and training robustness via:

- **Lag-aligned transitions**: Each $\mathbf{T}_i$ explicitly encodes temporal offsets.
- **Gated weights**: $\boldsymbol{\alpha}_t$ provides token-wise explanations.
- **Entropy-restricted structure**: Sparse bias patterns reduce overfitting.
- **Causal compliance**: Causal masking integrates seamlessly.

These properties—temporal bias control, memory-span flexibility, and stable optimization—do not merely improve performance; they define capabilities essential to intelligent systems. Specifically, they enhance:

1) Adaptivity through dynamic attention over past context,

2) Interpretability via explicit attention weights $\alpha_t$
3) Robustness through entropy regularization.

Collectively, the model's theoretical design supports efficient, transparent, and domain-aligned AI production.

**Implications for Intelligent System Design.** The theoretical insights presented above goes beyond analytical validation—they directly show the principled engineering of intelligent systems. First, the linear-time complexity (Eq. 18) guarantees scalability for real-time applications, such as speech recognition and financial forecasting, where latency is critical. Second, the structured modeling of higher-order dependencies (Eqs. 19–22) enables inductive generalization from limited data—an essential trait of adaptive intelligence. Third, the gated attention mechanism (Eq. 21) introduces interpretability and controllability, supporting safe deployment in domains like healthcare and autonomous systems. Finally, by decomposing attention into interpretable temporal structures, the model shows robustness under distribution shifts. Collectively, these properties cover the gap between theoretical expressivity and the practical demands of deploying robust, efficient, and intelligible AI systems.

### *3.6 Architectural Innovations: Synergistic Design*

*3.6.1 Dynamic Order Gating:* Gate function $g : \mathbb{R}^d \to \mathbb{R}^K$:

$$\boldsymbol{\alpha}_t^{(h)} = \mathrm{softmax}\left(g\left(z_t^{(h)}\right)\right) \tag{23}$$

where $z_t^{(h)} \in \mathbb{R}^{d_h}$ (head-specific) or $z_t \in \mathbb{R}^d$ (token-specific).

*3.6.2 Bidirectional Modeling:* Extended bias for non-causal attention:

$$B_{t,j}^{(h)} = \sum_{k=1}^{K}\left(\alpha_{t,k}^{(f)} T_k^{(f)} \delta_{j,t-k} + \alpha_{t,k}^{(b)} T_k^{(b)} \delta_{j,t+k}\right) \tag{24}$$

### *3.7 Discussion: Theoretical Implications*

- Markovian bias $B$ acts as a **learnable kernel** overlaying attention
- Gate weights $\boldsymbol{\alpha}_t$ enable **token-wise memory depth control**
- Additive structure maintains differentiability

This enables:

- High interpretability (positional importance)
- Structure-induced sparsity
- Robust generalization in low-data regimes

### *3.8 Algorithmic Pseudocode*

**Algorithm 1** Markov Bias

**Require:** $L, \mathbf{W}, \epsilon$
**Ensure:** $\mathbf{B}$

```
B ← 0; T ← softplus([T_f, T_b] + ε) if bidir else softplus(T_f + ε)
V ← W ⊙ T
for k = 1 to K do
    B[..., 0 : L − k − 1, k : L − 1] += V[..., :, 0, k − 1]
    B[..., k : L − 1, 0 : L − k − 1] += V[..., :, 1, k − 1] if bidir
end for
return B
```

**Algorithm 2** Markov Attention

**Require:** $\mathbf{X}$, mask, causal, $K$
**Ensure:** $\mathbf{O}$

```
Q, K, V ← proj(X)
G ← OrderGate(Q if token-specific else mean(X))
B_m ← { MarkovBias(N, G)   K > 0
      { 0                  else
S ← QK^⊤/√d_k + B_m + RelPosEmbed(N)
if causal then
    S ← S ⊙ tril(1)
end if
S ← S ⊙ ¬mask
A ← softmax(S)
return proj_o(AV) {O}
```

**Algorithm 3** Transformer Block

**Require:** $\mathbf{X}_t, \mathbf{X}_i$, mask, causal, att_type
**Ensure:** $\mathbf{X}'_t, \mathbf{X}'_i$

```
X̃_t ← LN_1(X_t)
O ← { MarkovAttn(X̃_t)                              markovian
    { QIAttn(X̃_t)                                  qi_rff
    { concat(MarkovAttn(Q_m), QIAttn(Q_q))         hybrid_split
    { MarkovAttn + QIAttn                          hybrid_parallel
X_t ← X_t + O + CrossAttn_{t→i} if interleaved
X_t ← X_t + GRU(X_t) if GRU + FFN(LN_2(X_t))
X_i ← X_i + CrossAttn_{i→t} if interleaved
return X_t, X_i
```

### *3.9 Theoretical Extensions*

- **Gumbel-softmax:** Differentiable sampling of discrete orders
- **Bayesian prior:** $T_k \sim \mathcal{N}(0, \sigma^2)$ with VI optimization
- **Entropy regularization:**

$$\mathcal{L}_{\text{ent}} = -\lambda \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{k=1}^{K} \alpha_{t,k}^{(h)} \log \alpha_{t,k}^{(h)} \tag{25}$$

## 4. Training And Implementation Details

The Markovian Transformer is trained under a strictly controlled and optimized setting to ensure robust convergence, reproducibility, and computational efficiency. This section outlines the training system, optimization strategy, deterministic training support, and performance-oriented features such as model compilation and memory-efficient techniques.

### *4.1 Hardware Details*

- **Server Model:** Dell PowerEdge R7525
- **CPU:** AMD EPYC 7763
- **GPU:** NVIDIA A100-SXM4-80GB
- **Interconnect:** NVLink 3.0
- **Memory:** 2 TB DDR4-3200
- **Storage:** 400 TB NVMe SSD

### *4.2 Hyperparameter Configuration*

The architecture and learning dynamics are defined through a unified configuration interface. The model is composed of 6 transformer layers, each with 8 attention heads and an embedding dimension of 256. Each feedforward network expands the hidden dimension by a factor of 4.0, and a dropout rate of 0.1 is applied across all modules to mitigate overfitting.

Training uses a learning rate of $3 \times 10^{-4}$, with weight decay set to 0.01 for L2 regularization. The optimizer uses adaptive moment estimation with parameters $\beta_1 = 0.9$ and $\beta_2 = 0.95$, providing stable and adaptive updates. To avoid gradient explosion, the gradient norm is restricted to 1.0.

The learning rate schedule has a two-phase strategy: an initial warm-up over 100 iterations, followed by gradual decay over 5000 iterations. The learning rate is not allowed to fall below 10% of the initial value, ensuring consistent progress throughout training.

### *4.3 Optimization Strategy*

The model is optimised using the AdamW optimiser, which separates weight decay from gradient updates, improving generalisation over the traditional L2-regularized Adam. A linear scheduler manages the learning rate during both the warm-up and decay phases, ensuring stable convergence.

Combined with gradient clipping and dropout, this optimisation approach ensures robustness across varied data domains and architectural configurations of the Markovian Transformer.

### *4.4 Support for Deterministic Training*

We fix all random seeds at 42 across Python, NumPy, and PyTorch for full determinism. CUDA operations are configured via `torch.use_deterministic_algorithms(True)` and environment variable `CUBLAS_WORKSPACE_CONFIG=:16:8`.

To ensure reproducibility—an important requirement in machine learning research—the model supports deterministic training. When enabled, the system enforces:

- Fixed random seeds across CPU and GPU operations.
- Deterministic matrix operations via algorithm restrictions.
- Pre-configuration of CUDA libraries for consistent computation.

This guarantees identical results across repeated runs, enabling precise evaluations and fair comparisons. Determinism is toggled by the environment settings prior to training, making it suitable for benchmarking and ablation studies.

### *4.5 Model Compilation and Activation Checkpointing*

To enhance computational efficiency, the model optionally uses PyTorch's ahead-of-time compilation by the `torch.compile` interface. This compilation fuses and reorders operations to reduce runtime and memory overhead, and can be tuned for specific performance goals such as latency reduction or throughput maximization.

Also, the model supports activation checkpointing—a memory-saving technique that recomputes intermediate activations during backpropagation rather than storing them. This enables the training of deeper models or longer sequences on limited hardware by reducing peak memory consumption. Activation checkpointing is integrated at the module level and can be selectively enabled based on available hardware resources.

Together, these features ensure that the Markovian Transformer can be trained efficiently and reproducibly, scaling from consumer GPUs to high-performance compute clusters.

## 5. Experimental Evaluation

### *5.1 Datasets*

We evaluated Markovian-T across three sequential domains requiring long-range reasoning:

- **Speech Recognition (LibriSpeech):**
  - 960h training, 5.4h validation (clean+other)
  - Input: 80-dimensional log-Mel spectrograms (100ms frames)
  - Task: Sequence-to-sequence transcription (30k wordpiece vocabulary)
- **Genomic Sequence Annotation (ENCODE):**
  - 1.2M DNA sequences (128bp windows)
  - Task: Binary classification of promoter regions
  - Challenge: Long-range nucleotide dependencies
- **Financial Time-Series (Limit Order Books):**
  - 10M L3 order book events (5 stocks)
  - Input: 15-level price/volume features
  - Task: 3-class mid-price movement prediction (↑, →, ↓)

### *5.2 Baselines*

Compared against 9 state-of-the-art sequence models:

1) Transformer (Vaswani et al., 2017)
2) Performer (Choromanski et al., 2020)
3) Linformer (Wang et al., 2020)
4) S4 (Gu et al., 2022)
5) RWKV (Peng et al., 2023)
6) RetNet (Sun et al., 2023)
7) RFA (Peng et al., 2021)
8) GRU (Cho et al., 2014)
9) Hyena (Poli et al., 2023)

### *5.3 Metrics*

*5.3.1 Domain-specific evaluation metrics(PPrimary Metrics)::*

- Speech Recognition: Word Error Rate (WER%)
- Genomics: AUROC
- Finance: F1-score (macro)

*5.3.2 Secondary Metrics:*

- Throughput (samples/sec)
- Memory Footprint (GB)

### *5.4 Results*

*5.4.1 Main Findings:* As shown in Table III, Markovian Transformer achieves:

- **4.3 WER** on LibriSpeech (2.1% improvement over RetNet)
- **0.922 AUROC** on genomics (+1.7% over S4)

- **71.2 F1** on finance (+4.1% over RetNet)
- **3.2GB** memory at 4k context (7.5× reduction vs Transformer)
- **10,400** throughput (8.28× increase vs Transformer)

*5.4.2 Length Scaling:*

- Markovian-T: 21% WER increase at 8k tokens
- Transformer: 157% degradation at same length
- Critical advantage beyond 2k context

*5.4.3 Hybrid Optimization:*

$$\text{Speech}: 0.7 \text{ QI-RFF} \quad (\Delta\text{WER} = -1.8\%)$$
$$\text{Genomics}: 0.3 \text{ Markovian} \quad (\Delta\text{AUROC} = +3.5\%)$$
$$\text{Finance}: 0.5 \text{ Balanced} \quad (\Delta\text{F1} = +2.4\%)$$

*5.4.4 Attention Diagnostics:*

- (a) Markovian: Local pattern capture (3-5bp motifs)
- (b) QI-RFF: Global context integration
- (c) Hybrid: Complementary fusion
- (d) Gates: Biological correlations (TATA: order-1, CpG: order-2)

### *5.5 Qualitative Analysis*

*5.5.1 Order Gate Dynamics in Genomics:* Gate weights correlate with functional genomic elements:

- TATA boxes: Dominant order=1
- CpG islands: Strong order=2 activation
- Promoter boundaries: Order=3 transitions

*5.5.2 Bidirectionality Analysis:* Bidirectional chains improve promoter detection by 4.2% AUROC, particularly at sequence boundaries where unidirectional models lose contextual information.

*5.5.3 Financial:* : Order-1 for short-term, order-3 for regime shifts

### *5.6 Ablation Study*

Table I quantifies component contributions:

- Bidirectionality: +4.2% AUROC
- Hybrid attention: +3.7-4.6% vs pure modes
- Deep-MLP gates: +0.8% over shallow

TABLE I: Ablation Study (Genomics AUROC)

| Ablation | AUROC | Δ |
|---|---|---|
| Full Model | 0.922 | – |
| w/o Bidirectional | 0.880 | -4.2% |
| w/o Hybrid (Pure Markovian) | 0.901 | -3.7% |
| w/o Hybrid (Pure QI-RFF) | 0.894 | -4.6% |
| Shallow-MLP Gates | 0.914 | -0.8% |
| Fixed Order | 0.872 | -5.5% |

TABLE II: Optimal configurations by domain

| **Domain** | **Max Order** | **Hybrid Ratio** | **Gate Type** | **Bidirectional** |
|---|---|---|---|---|
| Speech | 2 | 0.7 | Deep-MLP | NO |
| Genomics | 3 | 0.3 | Deep-MLP | YES |
| Finance | 2 | 0.5 | MLP | YES |

### *5.7 Discussion Summary*

Markovian-T achieves **SOTA accuracy** across diverse sequential domains while maintaining **linear memory complexity**. The hybrid attention mechanism provides unprecedented flexibility, with learnable Markov chains capturing local dependences and QI-RFF handling global context. Key advantages:

1) **Domain-adaptable architecture** via tunable hybrid ratio
2) **Interpretable order gating** revealing sequence-structure relationships
3) **Robust long-context handling** with only 21% degradation at 8k vs 157% for Transformers

## 6. Limitations And Future Work

### *6.1 Scalability in Extremely Long Sequences*

While the Markovian attention mechanism reduces the quadratic complexity of standard transformers to linear for Markov-order dependencies, scalability remains challenging for sequences exceeding 1 million tokens. The fixed-order Markov assumption limits context modeling in domains like genome processing or high-resolution video analysis where dependencies span thousands of tokens. Future work will explore *hierarchical Markov chains* with adaptive order selection, where lower layers capture local dependencies and higher layers model global contexts through compressed memory states. Hybridization with recurrent neural networks may further extend context capabilities while maintaining sub-linear memory growth.

### *6.2 Extension to Continuous-Time Markov Models*

Current discrete-time Markovian attention assumes uniform temporal intervals between tokens, limiting applicability to irregularly-sampled sequential data (e.g., medical sensor readings or financial tick data). *Continuous-time variants* can be developed using neural stochastic differential equations (SDEs) to parameterize transition matrices. By replacing discrete transition parameters $\mathbf{T}$ with learnable functions $\mathbf{T}(t) = f_\theta(\Delta t)$, the model could dynamically adjust to irregular sampling intervals. This would enable joint learning of temporal dynamics and semantic relationships in asynchronous multimodal streams.

### *6.3 Diffusion-Based Priors for Attention*

The current Markovian priors, while computationally efficient, constrain the model's ability to capture complex, non-Markovian dependencies. Future work will investigate *diffusion-based attention priors* where dependency patterns

TABLE III: Comprehensive Benchmark Results

*Note:* All benchmarks conducted on NVIDIA A100 80GB GPUs with FP16 precision and CUDA 11.8. Memory measurements include optimizer states. Throughput in tokens/sec measured at batch size 32 with 2k context length. Δ values relative to Transformer baseline.

| Model | LibriSpeech | | Genomics | | Finance | | Efficiency | |
|---|---|---|---|---|---|---|---|---|
| | WER↓ | Δ | AUROC↑ | Δ | F1↑ | Δ | Memory (GB)↓ (4k) | Throughput↑ |
| Transformer | 4.7 | - | 0.892 | - | 68.1 | - | 24.1 GB | 1,240 |
| Performer | 5.1 | +8.5% | 0.879 | -1.5% | 66.8 | -1.9% | 4.3 GB | 8,750 |
| Linformer | 5.3 | +12.8% | 0.865 | -3.0% | 65.9 | -3.2% | 3.8 GB | 9,200 |
| S4 | 4.9 | +4.3% | 0.901 | +1.0% | 67.5 | -0.9% | 2.1 GB | 12,100 |
| RWKV | 4.8 | +2.1% | 0.885 | -0.8% | 67.9 | -0.3% | 1.9 GB | 15,300 |
| RetNet | 4.6 | -2.1% | 0.907 | +1.7% | 68.4 | +0.4% | 2.3 GB | 14,800 |
| RFA | 5.0 | +6.4% | 0.874 | -2.0% | 66.5 | -2.4% | 4.5 GB | 8,900 |
| GRU | 6.2 | +31.9% | 0.832 | -6.7% | 63.7 | -6.5% | 1.2 GB | 18,500 |
| Hyena | 4.9 | +4.3% | 0.898 | +0.7% | 67.8 | -0.4% | 2.4 GB | 11,200 |
| **Markovian-T** | **4.3** | **-8.5%** | **0.922** | **+3.4%** | **71.2** | **+4.6%** | **3.2 GB** | **10,400** |

evolve through learned diffusion processes. By treating attention weights as particle systems subject to stochastic differential equations:

$$d\mathbf{A} = \mu(\mathbf{A}, t)dt + \sigma(t)d\mathbf{W} \tag{26}$$

where $\mathbf{W}$ is Wiener noise, we can model complex dependency graphs while maintaining tractable inference through reverse-time diffusion. This approach may bridge the expressivity gap between Markovian constraints and fully-connected attention, particularly for generative tasks requiring structured output spaces.

### *6.4 Additional Research Directions*

**Direction 1.** *Hardware-Aware Optimization*: Developing specialized kernels for Markovian attention to exploit GPU tensor cores and reduce memory fragmentation.

**Direction 2.** *Causal Discovery Integration*: Jointly learning dependency graphs and attention mechanisms for interpretable sequence modeling.

**Direction 3.** *Energy-Based Extensions*: Replacing softmax attention with energy-based models to capture sparse, long-range dependencies beyond fixed Markov orders.

These advancements would position Markovian Transformers as universal sequence engines capable of scaling from discrete symbolic reasoning to continuous sensor-data modeling while maintaining computational tractability.

## 7. Conclusion

This work introduces *Markovian Transformers*, a novel hybrid architecture that integrates the stepwise probabilistic modelling of Markov processes with the contextual expressivity of self-attention. By inserting learnable transition structures directly into the attention mechanism and combining them with global QI-RFF attention, the model gives a balance of *temporal coherence*, *interpretability*, and *long-range reasoning*.

### *7.1 Broader Implications for AI*

Markovian Transformers exemplify a new generation of *structure-aware AI systems* that go beyond purely data-driven representations. The ability to encode inductive biases, such as temporal ordering and memory constraints, opens a promising direction for AI architectures that are both efficient and aligned with real-world processes. This framework may serve as a blueprint for unifying statistical modelling (e.g., Markov chains, Bayesian priors) with deep learning, enhancing generalisation in low-data regimes, decision transparency, and alignment with domain-specific laws (e.g., physics, biology, finance).

### *7.2 Limitations Beyond §VI*

While scalability and temporal modelling limitations are discussed in Section 6, two additional considerations arise:

**Learning Complexity:** The dual mechanism of Markovian gating and self-attention increases the architectural and training complexity, need careful hyperparameter tuning to balance interpretability and expressivity.

**Transferability:** Since the Markovian priors are often domain-specific (e.g., genomic motifs, financial regimes), pre-trained models may show reduced transferability across unrelated domains without recalibration of the order-gating mechanisms.

### *7.3 Future Impact Beyond Technical Contributions*

Beyond outperforming existing models on benchmark tasks, Markovian Transformers signal a philosophical shift in model design—from *black-box optimization* to *interpretable and structured reasoning*. This is especially important in safety-critical domains such as:

- **Healthcare**, where understanding temporal causality in patient trajectories can inform early interventions;
- **Scientific discovery**, where hypothesis-driven attention mechanisms can model underlying biological or physical processes;
- **Autonomous systems**, where transparent decision-making and localized memory are prerequisites for ethical AI behavior.

Furthermore, the model's modularity—enabling seamless integration of Bayesian inference, neural stochastic differential equations (SDEs), or causal graphs—positions it as a foundation for future AI systems that must be adaptive, robust, and grounded in theory.

In sum, Markovian Transformers offer more than a performance boost—they represent a *reconciliation of structure and scale*, setting the stage for AI models that are not only powerful but also intelligible, tunable, and ethically aligned.

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## 9. Supplemental Materials

To promote reproducibility and enable further research, we provide the complete implementation of the *Markovian Transformer* architecture, including all modules, training scripts, evaluation pipelines, and visualization tools, on GitHub:
**Code Repository**: https://github.com/Kartik-ksp/Markovian-Transformer
The repository includes:

- Full PyTorch codebase with modular architecture
- Detailed configuration files for all experiments
- Scripts for dataset preprocessing and benchmarking
- Visualizations of order gating, attention maps, and transition dynamics
- Pretrained model checkpoints and results

We encourage the community to use, extend, and validate the model across additional sequential domains.