Topics: Normal distribution, Functions of Random Variables

1. **The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
   - **A. 0.3875**
   - **B. 0.2676**
   - **C. 0.5**
   - **D. 0.6987**

Ans - We have a normal distribution with μ = 45 and σ= 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X -μ) / = (X - 45)/8.0

Thus Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be

= 100-73.4 = 26.6% or 0.2676

2. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.**
   - **A. More employees at the processing center are older than 44 than between 38 and 44.**
   - **B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

Ans - A) Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44 = (44 - 38)/6 = 1 => 84.13 %

People above 44 age= 100 - 84.13 = 15.87% ≈ 63 out of 400

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing centre are older than 44 than between 38 and 44. is F**ALSE**

B) Z score for 30 = (30 - 38)/6 = -1.33 = 9.15 % ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the centre would be expected to attract about 36 employees - **TRUE**

3. **If X_1 ~ $N(\mu, \sigma^2)$ and X_2 ~ $N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.**

Ans - The difference between $2X_1$ and $X_1 + X_2$ is $N(\mu, \sigma^2)$.

According to the **Central Limit Theorem**, any **large sum** of **independent**, **identically distributed** random variables is approximately **Normal**.

The **Normal distribution** is defined by two parameters, the **mean**, μ, and the **variance**, σ^2 and written as $X \sim N(\mu, \sigma^2)$.

Given $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are two independent identically distributed random variables.

From the properties of **normal random variables**,

if $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent identically distributed random variables then

- the **sum** of normal random variables is given by

  X+Y $\sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$,

- and the **difference** of normal random variables is given by

  X-Y $\sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$,

- When Z=aX, the **product** of X is given by

  Z$\sim N(a\mu_1, \alpha^2\sigma_1^2)$

- When Z = aX+bY, the **linear combination** of X and Y is given by

  Z$\sim N(a\mu_1 + b\mu_2, \alpha^2\sigma_1^2 + b^2\sigma_2^2)$

Given to find, $2X_1$

Thus, following the property of multiplication, we get

$2X_1 \sim N(2\mu, 2^2\sigma^2) \gg 2X_1 \sim N(2\mu, 4\sigma^2)$

and following the property of addition,

$X_1 + X_2 \sim \mathrm{N}(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$

And the difference between the two is given by

$2X_1 - (X_1 + X_2) \sim N(2\mu - 2\mu, 2\sigma_1^2 + 4\sigma_2^2) \sim N(0, 6\sigma^2)$

The mean of$2X_1$ and $X_1 + X_2$ is same but the var(σ^2) of $2X_1$ is 2 times more than the variance of $X_1 + X_2$.

The difference between the two says that the two given variables are **identically** and **independently** distributed.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Ans - Given: p(a<x<b) = 0.99 , mean =100,standardDeviation = 20

To Find:

Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details, we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z (0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z (99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20*2.576) + 100= 48.5

b = (20*2.576) +100= 151.5

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5].

5. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit$_1$ ~ N(5, 3^2) and Profit$_2$ ~ N(7, 4^2) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
    A. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
    B. **Specify the 5th percentile of profit (in Rupees) for the company**
    C. **Which of the two divisions has a larger probability of making a loss in a given year?**

Ans - Given that:

$1 = Rs. 45

$PROFIT_1 \sim N(5, 3^2)$

$PROFIT_2 \sim N(7, 4^2)$

Company's profit:

$P \sim N(5 + 7, 3^2 + 4^2) = N(12, 5^2)$

A) 95% of the **probability lies** between 1.96 **standard deviations** of the **mean**.

Thus **range** is:

= (12-1.96*5,12+1.96*5)

= ($2.2M,$22.8M)

= Rs. 99 m, Rs. 1026 m

B) Fifth percentile is calculated as:

P(Z $\leq \frac{P-12}{5}$) = 0.05

From p **values** of z **score table**, we get:

$$\frac{P-12}{5} = -1.644$$

P = 12 – 8.22

= 3.78

Thus at $3.78M **dollars**, or Rs. 170.1M **amount**, 5th **percentile** of **profit lies**.

Or 5th **percentile** of **profit** is Rs. 170.1 Million.

C) Loss is when profit < 0

Thus: p < 0

The first **division** of **company**, thus have **larger probability** of making a loss in a given year.