

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-I & II (NEW) EXAMINATION – SUMMER-2019****Subject Code: 3110015****Date: 01/06/2019****Subject Name: Mathematics –2****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		Marks
Q.1	(a) Find the Fourier integral representation of $f(x) = \begin{cases} x & ; x \in (0, a) \\ 0 & ; x \in (a, \infty) \end{cases}$	03
	(b) Define: Unit step function. Use it to find the Laplace transform of $f(t) = \begin{cases} (t-1)^2 & ; t \in (0, 1] \\ 1 & ; t \in (1, \infty) \end{cases}$	04
	(c) Use the method of undetermined coefficients to solve the differential equation $y'' - 2y' + y = x^2 e^x$.	07
Q.2	(a) Evaluate $\oint_C \bar{F} \cdot d\bar{r}$; where $\bar{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ and C is the curve given by the parametric equation $C: r(t) = t^2\hat{i} + t\hat{j}; 0 \leq t \leq 2$.	03
	(b) Apply Green's theorem to find the outward flux of a vector field $\bar{F} = \frac{1}{xy}(x\hat{i} + y\hat{j})$ across the curve bounded by $y = \sqrt{x}$, $2y = 1$ and $x = 1$.	04
	(c) Integrate $f(x, y, z) = x - yz^2$ over the curve $C = C_1 + C_2$, where C_1 is the line segment joining (0,0,1) to (1,1,0) and C_2 is the curve $y=x^2$ joining (1,1,0) to (2,4,0).	07
OR		
	(c) Check whether the vector field $\bar{F} = e^{y+2z}\hat{i} + x e^{y+2z}\hat{j} + 2x e^{y+2z}\hat{k}$ is conservative or not. If yes, find the scalar potential function $\phi(x, y, z)$ such that $\bar{F} = \text{grad } \phi$.	07
Q.3	(a) Write a necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact differential equation. Hence check whether the differential equation $[(x+1)e^x - e^y]dx - xe^y dy = 0$ is exact or not.	03
	(b) Solve the differential equation $(1 + y^2)dx = (e^{-\tan^{-1}y} - x)dy$	04
	(c) By using Laplace transform solve a system of differential equations $\frac{dx}{dt} = 1 - y$, $\frac{dy}{dt} = -x$, where $x(0) = 1, y(0) = 0$.	07
OR		
Q.3	(a) Solve the differential equation $(2x^3 + 4y)dx - xdy = 0$.	03

- (b) Solve: $(x + 1) \frac{dy}{dx} - y = e^{3x}(x + 1)^2$. 04
- (c) By using Laplace transform solve a differential equation $\frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = e^{-t}$, where $y(0) = 0$, $y'(0) = -1$. 07
- Q.4** (a) Find the general solution of the differential equation 03
 $e^{-y} \frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$
- (b) Solve : $\frac{d^3y}{dx^3} - 7 \frac{dy}{dx} + 6y = e^x$ 04
- (c) Find a power series solution of the differential equation $y'' - xy = 0$ near an ordinary point $x=0$. 07
- OR**
- Q.4** (a) Find the general solution of the differential equation 03
 $\frac{dy}{dx} + \frac{y}{x} - \sqrt{y} = 0$.
- (b) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x$ 04
- (c) Find a Frobenius series solution of the differential equation $2x^2y'' + xy' - (x + 1)y = 0$ near a regular-singular point $x=0$. 07
- Q.5** (a) Write Legendre's polynomial $P_n(x)$ of degree- n and hence obtain $P_1(x)$ and $P_2(x)$ in powers of x . 03
- (b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $y'' + xy' = 0$. 04
- (c) Solve the differential equation 07
 $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3 \cos x$
 by using the method of variation of parameters.
- OR**
- Q.5** (a) Write Bessel's function $J_p(x)$ of the first kind of order- p and hence show 03
 that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
- (b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $xy'' + y' = 0$. 04
- (c) Solve the differential equation $y'' + 25y = \sec 5x$ 07
 by using the method of variation of parameters.
