6

(b)

using the formulas for the above given values: (x_i, y_i)

$\rightarrow K_{b\to\theta} = \frac{K(A)}{\eta \cos\theta'}$ [calculating values using numpy] (for the given set of points)

and $K(A) = \|A\|_2 \|A^+\|_2 = 4823.4753$

$\eta = \frac{\|A\|_2 \|\theta\|_2}{\|A\theta\|_2} = 22.511325$

and $\cos\theta' = \frac{\|y\|}{\|b\|}$ where $y = A\theta$

$= \frac{\|A\theta\|_2}{\|b\|_2} = 0.9999$

$\boxed{\Rightarrow K_{b\to\theta} = 214.26977}$ (using the above formula)

$\rightarrow K_{A\to\theta} = K(A) + \frac{K(A)^2 \tan\theta}{\eta}$

and $\tan\theta = \frac{\|y-b\|_2}{\|y\|_2} = \frac{\|A\theta - b\|_2}{\|A\theta\|_2} = 0.0029785$

then $\boxed{K_{A\to\theta} = 7901.8472}$ (using above formula)

$\Rightarrow$ These values are calculated using linalg module in numpy.

and since these are very big values then we can say that the problem is ill-conditioned.

(6)

(c)

→ To find condition number of θ with respect to vector v containing original data.

we have:

$$K = \frac{\|\delta\theta\|/\|\theta\|}{\|\delta v\|/\|v\|} \quad \text{(using } \infty \text{ norm for the calculations)}$$

$$= \frac{\|\delta\theta\|/\|\theta\|}{\|\delta b\|/\|b\|} \cdot \frac{\|\delta b\|/\|b\|}{\|\delta v\|/\|v\|}$$

$$K = K_{b\to\theta} \cdot \frac{\|\delta b\|/\|b\|}{\|\delta v\|/\|v\|} \quad \text{— (1)}$$

where v is the vector containing $[x_i, y_i]^T$

and b is a vectors that contains $[-(x_i^2+y_i^2)]^T$

⇒ Finding $\frac{\|\delta b\|/\|b\|}{\|\delta v\|/\|v\|}$ using ∞ norm

consider we changed the values x, y by δ

$$\Rightarrow \frac{\|-[(x+\delta)^2 + (y+\delta)^2] + (x^2+y^2)\|}{\|\delta v\|} \cdot \frac{\|v\|}{\|b\|}$$

$$\Rightarrow \frac{\|-(\cancel{x^2}+\delta^2+2x\delta+\cancel{y^2}+\delta^2+2y\delta-\cancel{x^2}-\cancel{y^2})\|}{\delta} \cdot \frac{\|v\|}{\|b\|}$$ (since $\delta v = \delta$ max change in any value $= \delta$)

$$= \frac{|-2\delta(x+y)|}{\delta} \cdot \frac{\|v\|}{\|b\|}$$

(ignoring higher order δ)

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$$\Rightarrow \frac{2\cancel{8}\,|x+y|_2^2}{\cancel{8}} \cdot \frac{\|v\|}{\|b\|}$$

$$\Rightarrow 2|x+y| \cdot \frac{\max(x_i, y_i)}{|x^2+y^2|} \quad \forall x_i, y_i \in V$$

$$= 2\,\frac{|x+y|}{|x^2+y^2|} \max(x_i, y_i) = \frac{2\max(|x_i+y_i|)}{\max(|x_i^2+y_i^2|)} \cdot \max(x_i, y_i)$$

$$\Rightarrow \boxed{K = K_{b\to\theta}\, \frac{2\max(|x_i+y_i|)}{\max(|x_i^2+y_i^2|)} \max(x_i, y_i)} \quad \forall x_i, y_i \in V$$

$$\Rightarrow \boxed{K_{v\to\theta} = K_{b\to\theta} \cdot 2\, \frac{\max(|x_i+y_i|)}{\max(|x_i^2+y_i^2|)} \max(x_i, y_i)} \quad \forall x_i, y_i \in V,$$

and the value of

$$\boxed{K_{v\to\theta} = 516.8348}$$

for the set of data points given.

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