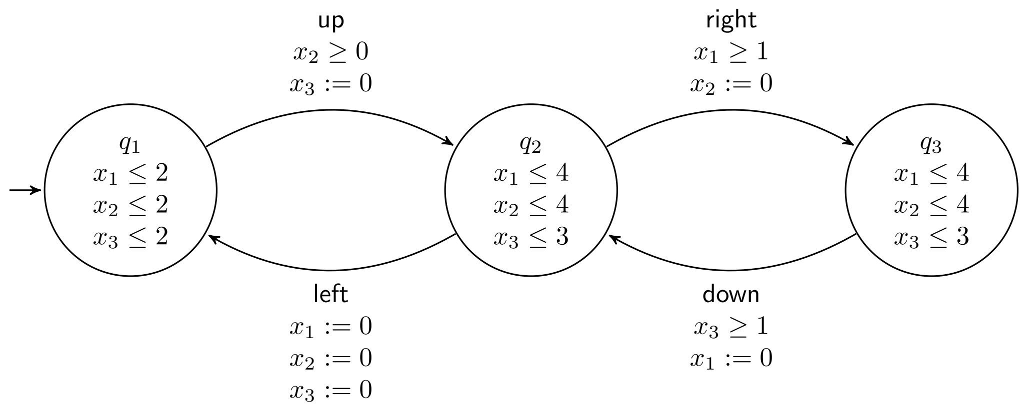

Figure 1: The timed automaton $\mathcal{T}$.

1. Consider the Timed Automaton $\mathcal{T}$ depicted in Figure 1.

   (a) [3p] Model formally the Timed Automaton as a Hybrid Automaton:

   $$\mathcal{H} = (Q, X, \mathsf{Init}, f, \mathsf{Act}, D, E, G, R).$$

   (b) [3p] Model formally the Timed Automaton as a Transition System:

   $$\mathcal{TS} = (S, S_0, \Sigma, \rightarrow).$$

   (c) [3p] Decide whether the following states are reachable from the state $(q_1, 0, 0, 0)$:
   (i) $(q_3, 3, 1, 2)$
   (ii) $(q_3, 0, 0, 3)$
   (iii) $(q_3, 1.5, 0, 1.5)$
   (iv) $(q_2, 1, 1, 3)$.

   (d) [1p] Describe a procedure under which the reachability of a Timed Automaton can be performed.

1. (a) The required sets are given as:
   - $Q = \{q_1, q_2, q_3\}$
   - $X = \mathbb{R}^3_+$
   - $\text{Init} = (q_1, 0, 0, 0)$
   - $f(q_1, x_1, x_2, x_3) = f(q_2, x_1, x_2, x_3) = f(q_3, x_1, x_2, x_3) = (1, 1, 1)$
   - $\text{Act} = \{\text{up}, \text{right}, \text{down}, \text{left}\}$
   - $D(q_1) = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \leq 2, x_2 \leq 2, x_3 \leq 2\}$,
     $D(q_2) = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \leq 4, x_2 \leq 4, x_3 \leq 3\}$,
     $D(q_3) = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \leq 4, x_2 \leq 4, x_3 \leq 3\}$
   - $E = \{(q_1, \text{up}, \mathrm{q_2}), (\mathrm{q_2}, \text{right}, \mathrm{q_3}), (\mathrm{q_3}, \text{down}, \mathrm{q_2}), (\mathrm{q_2}, \text{left}, \mathrm{q_1})\}$
   - $G(q_1, \text{up}, \mathrm{q_2}) = \{(\mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) \in \mathbb{R}^3 : \mathrm{x_2} \geq 0\}$,
     $G(q_2, \text{right}, \mathrm{q_3}) = \{(\mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) \in \mathbb{R}^3 : \mathrm{x_1} \geq 0\}$,
     $G(q_3, \text{down}, \mathrm{q_2}) = \{(\mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) \in \mathbb{R}^3 : \mathrm{x_3} \geq 0\}$,
     $G(q_2, \text{left}, \mathrm{q_1}) = \emptyset$
   - $R((q_1, \text{up}, \mathrm{q_2}); \mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) = (\mathrm{x_1}, \mathrm{x_2}, 0)$,
     $R((q_1, \text{up}, \mathrm{q_2}); \mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) = (\mathrm{x_1}, 0, \mathrm{x_3})$,
     $R((q_1, \text{up}, \mathrm{q_2}); \mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) = (0, \mathrm{x_2}, \mathrm{x_3})$,
     $R((q_1, \text{up}, \mathrm{q_2}); \mathrm{x_1}, \mathrm{x_2}, \mathrm{x_3}) = (0, 0, 0)$

   (b) The required set are given as:
   - $S = \{q_1, q_2, q_3\} \times \mathbb{R}^3$
   - $S_0 = \text{Init} = (\mathrm{q_1}, 0, 0, 0)$
   - $\Sigma = \{\text{up}, \text{right}, \text{left}, \text{down}\} \cup \text{Time}$
   - The transition relation $\rightarrow$ can be:
     (a) *Event-based transitions :*
     $(q_1, x_1, x_2, x_3) \xrightarrow{\text{up}} (q_2, x_1, x_2, x_3)$ if $x \models G(q_1, \text{up}, \mathrm{q_2})$, $\{y\} = R((q_1,\ \text{up},\ q_2),\ x_1 4,\ x_2,\ 0)$ and $y \models D(q_2)$. Similarly, we write the form of the other 3 event-based transitions for the actions $\{\text{down}, \text{left}, \text{right}\}$.
     (b) *Time-based transitions :*
     $(q_1, x_1, x_2, x_3) \xrightarrow{\text{Time}} (q_1, x_1', x_2', x_3')$ if $x_i' = x_i + \text{Time}$ and $x_i' \models D(q_1)$ for every $i \in \{1, 2, 3\}$. Similary, we write the form of the other 2 time-based transitions for the states q_2 and q_3.

(c) (i) The state is reachable under the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\overset{Time=1}{\longrightarrow}(q_1, 1, 1, 1)\\ &\overset{Action=up}{\longrightarrow}(q_2, 1, 1, 0)\\ &\overset{Time=1}{\longrightarrow}(q_1, 2, 2, 1)\\ &\overset{Action=right}{\longrightarrow}(q_3, 2, 0, 1)\\ &\overset{Time=1}{\longrightarrow}(q_3, 3, 1, 2)\end{aligned}$$

(ii) The state is reachable under the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\overset{Action=up}{\longrightarrow}(q_2, 0, 0, 0)\\ &\overset{Time=3}{\longrightarrow}(q_2, 3, 3, 3)\\ &\overset{Action=right}{\longrightarrow}(q_3, 3, 0, 3)\\ &\overset{Action=down}{\longrightarrow}(q_2, 0, 0, 3)\end{aligned}$$

(iii) The state is reachable under the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\overset{Ation=up}{\longrightarrow}(q_2, 0, 0, 0)\\ &\overset{Time=1.5}{\longrightarrow}(q_2, 1.5, 1.5, 1.5)\\ &\overset{Action=right}{\longrightarrow}(q_3, 1.5, 0, 1.5)\end{aligned}$$

(iv) The state is reachable under the following sequence of transitions:

$$\begin{aligned}(q_1, 0, 0, 0) &\overset{Time=2}{\longrightarrow}(q_1, 2, 2, 2)\\ &\overset{Action=up}{\longrightarrow}(q_2, 2, 2, 0)\\ &\overset{Time=2}{\longrightarrow}(q_3, 4, 4, 2)\\ &\overset{Action=right}{\longrightarrow}(q_3, 4, 0, 2)\\ &\overset{Action=down}{\longrightarrow}(q_3, 0, 0, 2)\\ &\overset{Time=1}{\longrightarrow}(q_2, 1, 1, 3)\end{aligned}$$

(d) In general cases, the reachability can be performed by using equivalent regions and region automata.