Homework 1 in EL2450 Hybrid and Embedded Control Systems

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Task 1

The tap gain is set as 0 because the outflow of water from the upper tank is only via a small hole connected to the lower tank and not via the other pipe to which the tap is connected. It can be modeled as a zero gain as there is zero outflow of water through it.

Task 2

The code for the transfer functions of upper tank, lower tank and the overall system is as follows:

```
numUT = [k_tank]; % Numerator of the upper tank
denUT = [Tau 1]; % Denominator of the upper tank
numLT = [gamma_tank]; % Numerator of the lower tank
denLT = [(gamma_tank*Tau) 1]; % Denominator of the lower tank

uppertank = tf(numUT,denUT); % Transfer function for upper tank
lowertank = tf(numLT,denLT); % Transfer function for lower tank
G = uppertank*lowertank; % Transfer function from input to lower tank level
```

Task 3

The reference signal is a step signal with amplitude 10 and time shifted to the right by 25 sec.

$$u(t) := unitstepfunction$$
$$reference\ signal = 10 * u(t - 25)$$

U_{ss} is the Steady-State Input, which means that U_{ss} is the input to the pump when the 2 tank system is in equilibrium.
Similarly, Y_{ss} is the Steady-State Output of the system i.e. it is the height of the lower tank at equilibrium (here, considered as 40).

Task 4

The PID Controller parameters were derived by using the following constants:

$$\zeta = 0.4705; \omega_0 = 0.3147; \chi = 0.1 \tag{1}$$

Thus,

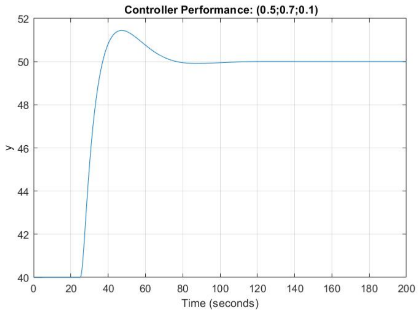
$$K_{\mathrm{pid}} = 3.7193; T_{\mathrm{i}} = 17.8170; T_{\mathrm{d}} = 3.1297; N = 0.3353 \tag{2}$$

```
% PID transfer function
n1 = (K*Ti)*((N*Td)+1);
n2 = K*(1+(N*Ti));
n3 = K*N;
numF = [n1 n2 n3]; % Numerator of PID
denF = [Ti N*Ti 0]; % Denominator of PID
F = tf(numF,denF);
```

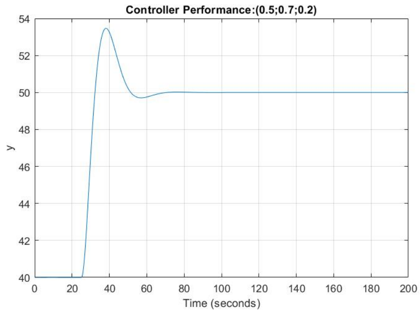
Task 5

χ	ζ	ω	T_{r}	M	T_{set}
0.5	0.7	0.1	$8.8sec$	2.9%	$32sec$
0.5	0.7	0.2	$4.86sec$	6.94%	$21.78sec$
0.5	0.8	0.2	$5.02sec$	6.34%	$21.7sec$

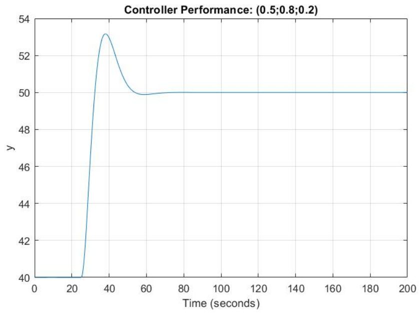
Figure (1) shows the simulation results from which the above time-response characteristics were calculated



(a) Controller Performance for Case 1



(b) Controller Performance for Case 2



(c) Controller Performance for Case 3

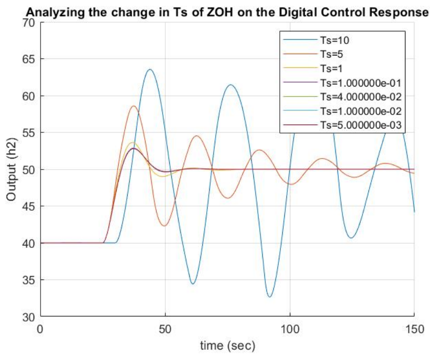
Figure 1: **Simulation Results**

The system has some step response characteristics requirements on the closed loop system such as $T_{\text{r}} < 6s; M < 35\%; T_{\text{set}} < 30s$
Thus, from the above table it is evident that the 3^{rd} parameter setting of $\chi = 0.5; \zeta = 0.8; \omega_0 = 0.2$ gives the best control performance as a control system with less overshoot, small rise-time and small settling-time is desirable.

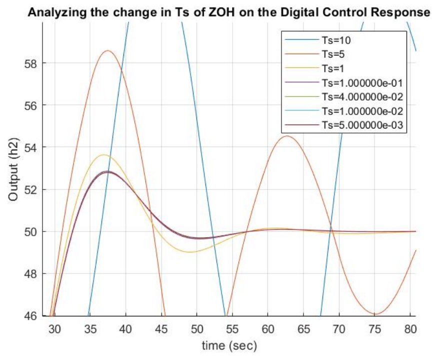
Task 6

The gain cross over frequency of the open loop system is 0.2622 rad/sec with Phase Margin of 46.0314°In order to calculate it, we need to analyze the Bode Plot. The frequency at which the open loop gain first reaches 1 is the gain cross over frequency(ω_{gc}) i.e it is the frequency at which the Phase Margin is calculated.

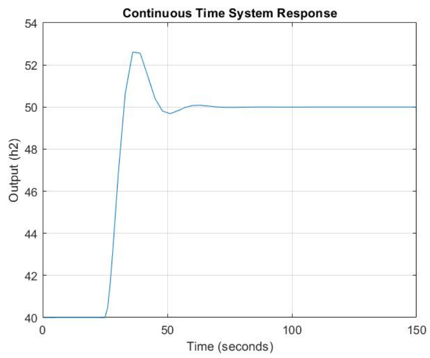
Task 7



(a) System Response with ZOH (Various Ts)



(b) Peaks of the System Response with ZOH

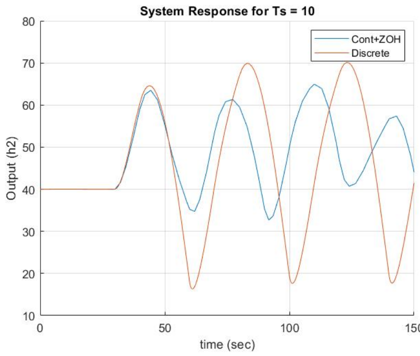


(c) Continuous Time System Response

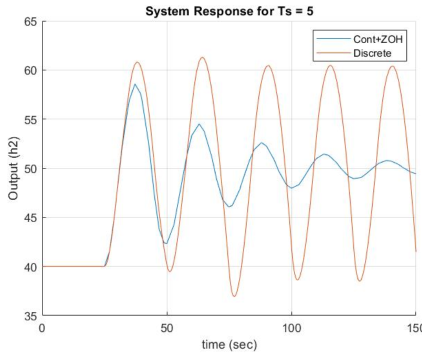
Figure 2: **Differences in Control Performance**

From figure (2), we can see that the system time responses for various sampling times : $T_s = \{10, 5, 1, 0.1, 0.04, 0.01, 0.005\}$ are plotted. It has been observed that for very high sampling time of the ZOH, the time response is very oscillatory with a lot of overshoot . As we further reduce the sampling time, the damping increases leading to reduction in the peak overshoot and the settling time. Around a T $_s$ of 0.005 sec or less, the system starts behaving similar to the continuous time close-loop system but with some more overshoot and delayed settling time mainly due to the delay introduced by the Zero-order Hold.

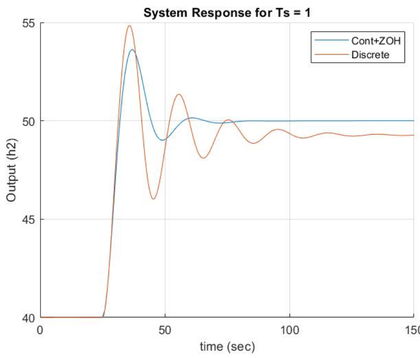
Task 8



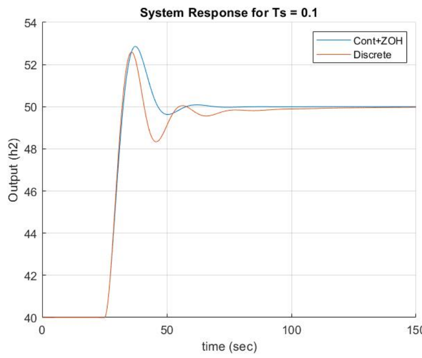
(a) System Response for $T_s = 10$ sec



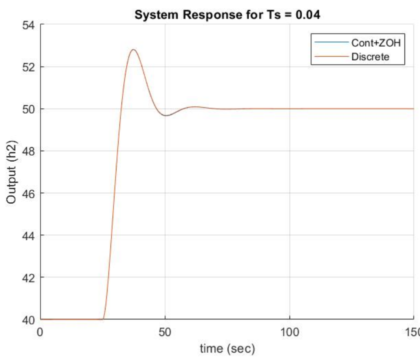
(b) System Response for $T_s = 5$ sec



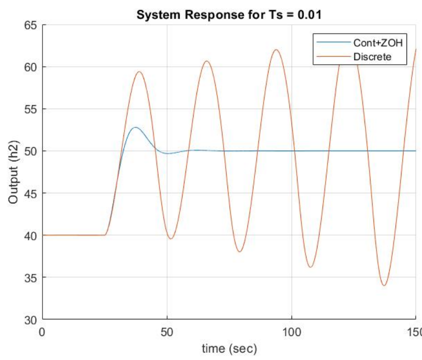
(c) System Response for $T_s = 1$ sec



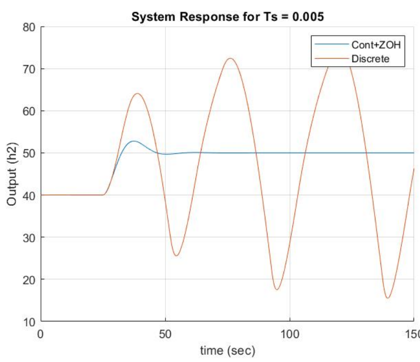
(d) System Response for $T_s = 0.1$ sec



(e) System Response for $T_s = 0.04$ sec



(f) System Response for $T_s = 0.01$ sec



(g) System Response for $T_s = 0.005$ sec

Figure 3: **Differences in Control Performance**

From the simulation results (*see fig. (3)*), we see that for larger sampling time, there are even more oscillations in the Discretized Controller system as compared to the system with Continuous controller and ZOH. For $T_s = 0.1$ sec, the system performance for both the controllers was found to be almost similar with the discretized controller system having a larger settling time. But, for $T_s = 0.04$ sec, the response of both the systems overlapped each other exactly. Further decreasing the Sampling time deteriorated the performance of the discretized controller system and the system started to diverge as increasing the sampling rate can actually push the poles of the closed loop system outside the unit circle of stability.

Task 9

When implementing a continuous controller digitally, we should choose the T_s such that

$$\begin{aligned} T_s * \omega_c &\approx 0.05 \; to \; 0.14 \\ \omega_c &= 0.2622 \; rad/sec \\ T_s &= 0.1 \div 0.2622 \\ \implies T_s &= 0.3814 \; sec \end{aligned}$$

Task 10

The maximum possible sampling time without affecting the control performance of a system with discretized controller is observed to be around 0.5 sec. The calculated Sampling time in the above task came out to be 0.3814 sec. which is between the minimum possible sampling time of 0.04 sec and the maximum possible sampling time of 0.5 sec.

Task 11

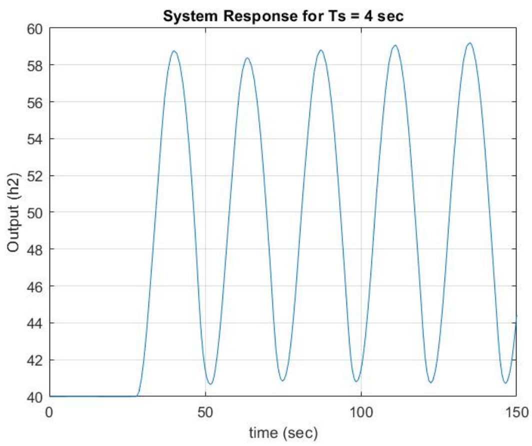


Figure 4: **Control Performance for T= 4 sec**

For the mentioned sampling time of 4 sec, the control performance is highly sinusoidal with negligible damping.

Task 12

The State-Space form of the Continuous system was discretized with $T_s = 4$ sec and the Discrete-time State Space matrices were derived as follows:

$$\phi = \begin{bmatrix} 0.4916 & -0.0188 \\ 2.8993 & 0.9581 \end{bmatrix}$$

$$\gamma = \begin{bmatrix} 2.8993 \\ 6.4741 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.0142 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Task 13

The observability and reachability of the discrete-time plant model was checked using MATLAB.

```
% Observability and reachability
Wo = obsv(Phi,C)
isObsv = length(Phi) - rank(Wo)
Wc = ctrb(Phi,Gamma)
isCtrb = length(Phi) - rank(Wc)
```

After executing the above code snippet, we got

$$W_o = \begin{bmatrix} 0 & 0.0142 \\ 0.0142 & 0.0136 \end{bmatrix}$$

$$W_c = \begin{bmatrix} 2.8993 & 1.3037 \\ 6.4741 & 14.6090 \end{bmatrix}$$

Also,

$$isObsv = 0$$
$$isCtrb = 0$$

which means that the Observability matrix and Reachability matrix has full rank and the discrete-time plant is **Observable** and **Reachable**.

Task 14

The reference gain l_r is necessary in order to ensure that the steady state output $y_e = r$ i.e it ensures that we are able to design the dynamics of the system to satisfy our goal.

Task 15

According to the Separation Principle, both the Control and Estimation parts can be designed independently. Estimator poles should be farther away than the control poles in order to ensure fast dynamic response. So consider the poles for the State Feedback Controller to be denoted by P_{SFC} and that of the Dynamic Observer to be P_{DO}.

Thus,

$$P_{\text{SFC}} = [0.5, -0.3]$$
$$P_{\text{DO}} = [-0.6, -0.7]$$

Now, using the `acker` function, **K** and **L** were derived to be

$$K = [30.3667, 194.2497]$$
$$L = [0.3214, 0.0491]$$

And the A_a and B_a came out to be:

$$A_{\text{a}} = \begin{bmatrix} 0.4916 & -0.0188 & -0.5494 & -0.1832 \\ 2.8993 & 0.9581 & -1.2268 & -0.4091 \\ 0 & -0.0078 & -0.0578 & -0.1942 \\ 0 & 0.1093 & 1.6726 & 0.4397 \end{bmatrix}$$

$$B_{\text{a}} = \begin{bmatrix} 14.2677 \\ 31.8595 \\ 14.2677 \\ 31.8595 \end{bmatrix}$$

Task 16

The closed loop system is denoted by

$$z(k+1) = \begin{bmatrix} \phi - \gamma L & \gamma L \\ 0 & \phi - KC \end{bmatrix} z(k) + \begin{bmatrix} \gamma l_r \\ 0 \end{bmatrix}$$

Now if we try to find the eigen values of the above system, we get

$$[sI - (\phi - \gamma L)][sI - (\phi - KC)] = 0 \tag{3}$$

Thus pole placement for both can be done independently, which shows that Separation Principle holds.

Task 17

The poles of the continuous time closed loop models were converted into discrete-time using the equation

$$z_{\text{i}} = e^{T_{\text{s}}\ p_{\text{i}}}$$
$$\implies z_{\text{i}} = [0.6658, 0.6746, 0.2456 + 0.4957i, 0.2456 - 0.4957i]$$

Out of these the larger poles were considered for estimator as we want fast response and then, the **K** value was calculated using the `acker` formula. We have also verified that the A_a has the desired poles.

Task 18

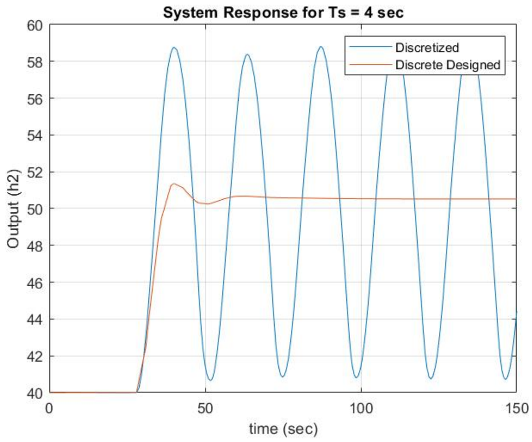


Figure 5: **Control Performance for $\mathbf{T_{=}}$ 4 sec**

From Figure (5), we can see that the performance of the system has improved a lot (oscillations have reduced to a great extent and T_s has also reduced) for the discrete designed controller system. This may be because this controller is specifically designed for the discrete system and has been designed to place the closed loop discrete poles at the desired location by considering State Feedback as well as Estimator design.
The time-response of the discrete system settles down having a steady-state error value of **0.52**. There is a small steady-state error in the output which maybe because of the following reasons

- The type number of the system is zero and hence, for a step input the steady-state error is non-zero.
- As the Observer Gain takes large values, there is a possibility of noise amplification.

Task 19

Quantization with 10 bits is abs$(\frac{0-100}{2^{10}}) = 0.0976$.

Task 20

A Quantization block was successfully constructed in Simulink by connecting a Quantizer block to a Saturation block. The Saturation block is necessary because the only parameter that the Quantizer block has is the Quantization Interval and no upper or lower limit. Thus, in order to limit the signal between the desired limits, we add a Saturation block.

Task 21

Different Quantization levels [12.5, 6.25, 1.56, 0.78125, 0.39, 0.097] corresponding to [3, 4, 6, 7, 8, 10] bits were used to simulate the system.

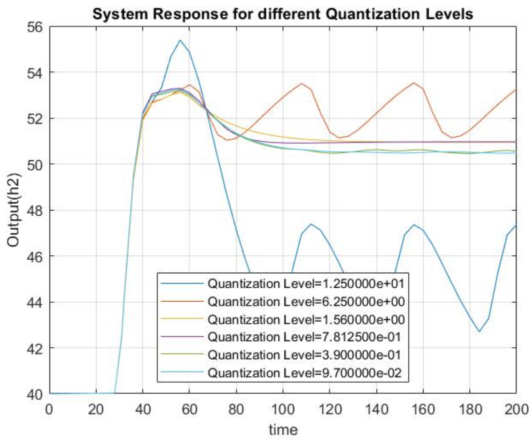


Figure 6: **System Response for different Quantization Levels**

From figure (6), we can observe that for Quantization levels greater than **0.78125** (corresponding to lower than 7 bits), the control performance starts to degrade. The Simulink model with Quantization Subsystem consisting of Quantizer in series with Saturation is shown below:

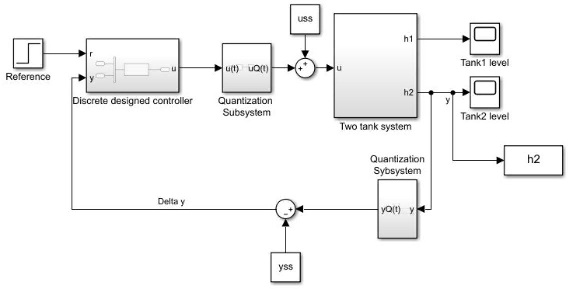


Figure 7: **Simulink Tank Model with Quantization blocks**