1. (a) The Hybrid Automaton is as follows:
   - $Q = (q_1, q_2)$
   - $X = \mathbb{R}^2$
   - $\text{Init} \subseteq Q \times X$
   - $f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} A_1 x \\ A_2 x \end{bmatrix}$
   - $D(q_1) = \{x \in X : c^\top x \geq 0\}$, $D(q_2) = \{x \in X : c^\top x < 0\}$
   - $E = \{(q_1, q_2), (q_2, q_1)\}$
   - $G((q_1, q_2)) = \{x \in D_1 : c^\top x < 0\}$, $G((q_2, q_1)) = \{x \in D_2 : c^\top x = 0\}$
   - $R((q_1, q_2), x) = \{x\} = R((q_2, q_1), x)$

   (b) Let a common Lyapunov function $V = x^\top P x$ with $P = \begin{bmatrix} 1 & s \\ s & r \end{bmatrix}$. It must hold $Q_i = -PA_i - A_i^\top P > 0$, $\forall i \in \{1, 2\}$. After calculations,

$$\begin{cases} Q_1 = \det \begin{bmatrix} 2 + 200s & 2s + 100r - 10 \\ 2s + 100r - 10 & -20s + 2r \end{bmatrix} > 0 \\ Q_2 = \det \begin{bmatrix} 2 + 20s & 2s + 10r - 100 \\ 2s + 10r - 100 & 2r - 200s \end{bmatrix} > 0 \end{cases} \Rightarrow$$

$$\begin{cases} -10^4 r^2 + 2004r - 4004s^2 - 100 > 0 \\ -100r^2 + 2004r - 4004s^2 - 10^4 > 0 \end{cases} \Rightarrow$$

$$\begin{cases} \dfrac{\left(r - \dfrac{1002}{10^4}\right)^2}{\left(\dfrac{1002}{10^4}\right)^2 - \dfrac{100}{10^4}} + \dfrac{s^2}{\dfrac{10^4\left(\left(\dfrac{1002}{10^4}\right)^2 - \dfrac{100}{10^4}\right)}{4004}} < 1 \\[2em] \dfrac{\left(r - \dfrac{1002}{10^2}\right)^2}{\left(\dfrac{1002}{10^2}\right)^2 - 10^2} + \dfrac{s^2}{\dfrac{10^2}{4004}\left(\left(\dfrac{1002}{10^2}\right)^2 - 10^2\right)} < 1 \end{cases}$$

The first inequality represents an ellipse centered at $\left(\dfrac{1002}{10^4}, 0\right) = (0.1002, 0)$, with axes length $\sqrt{\left(\dfrac{1002}{10^2}\right)^2 - \dfrac{100}{10^4}} \approx 0.063$ and $\sqrt{\dfrac{10^4\left(\left(\dfrac{1002}{10^4}\right)^2 - \dfrac{100}{10^4}\right)}{4004}} \approx 0.01$.

The second inequality represents an ellipse centered at $\left(\dfrac{1002}{100}, 0\right) = (10.02, 0)$, with axes lengths $\sqrt{\left(\dfrac{1002}{10^2}\right)^2 - 100} = 0.6328$ and $\sqrt{\dfrac{10^2}{4004}\left(\left(\dfrac{1002}{100}\right)^2 - 100\right)} =$

0.1. It is clear that the two ellipses do not intersect, so there's no common Lyapunov function.

(c) The eigenvalues of A_1, A_2 have negative real parts and hence A_1 and A_2 are Hurwitz. Hence, there exist P_1, P_2 such that $-PA_i - A_i^\top P_i > 0$. Hence, the Lyapunov functions are $V_i = x^\top P_i x$, $i \in \{1, 2\}$.

(d) The Lyapunov level surfaces of V_1, V_2 are ellipses centered at (0.0). The switching line $c^\top x = 0$ is a line passing from $(0, 0)$ and hence, in each switching the value of V_i will be smaller.