AUTOMATIC CONTROL

KTH

<u>EL2520 Control Theory and Practice - Advanced Course</u>

Exam 14.00–19.00 August 19, 2016

Aid:

Course book *Glad and Ljung, Control Theory* or *Reglerteori*, basic control course book *Glad and Ljung, Reglerteknik* or equivalent if approved by examiner beforehand, copies of slides from this years (2016) lectures, mathematical tables, calculator.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: $\geq$ 43, Grade B: $\geq$ 38

Grade C: $\geq$ 33, Grade D: $\geq$ 28

Grade E: $\geq$ 23, Grade Fx: $\geq$ 21

Results: The results will be available about 3 weeks after the exam at "My Pages".

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. (a) Given the multivariable system

$$G(s) = \frac{1}{s+1} \begin{pmatrix} s+1 & -2 \\ 1 & s-1 \end{pmatrix}$$

Determine the system poles and zeros (with multiplicities). (4p)

(b) A feedback controller has been designed for the system

$$G(s) = \frac{-s+1}{2s+1}$$

The resulting complementary sensitivity function is

$$T(s) = \frac{1}{s+1}$$

Determine if the system is internally stable. (3p).

(c) Given the system

$$G(s) = \frac{1}{10s+1} \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$$

Determine a feedback controller $u = F(s)(r - y)$ so that the closed-loop system becomes

$$Y = \frac{1}{\lambda s + 1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} R$$

(3p)

2. (a) Consider the system

$$Y(s) = \frac{1}{s+1} \begin{pmatrix} s+1 & s+1 \\ s-2 & -s-1 \end{pmatrix} U(s)$$

   (i) Determine if the system has any fundamental limitations on what can be achieved with feedback control. (3p)

   (ii) The system is to be controlled with a decentralized controller with bandwidth around 0.1 rad/s. Use the RGA to determine an appropriate pairing of inputs and outputs. (2p)

(b) For a multivariable system the following specifications shall be met using feedback

   1. The impact of disturbances d on the output shall be attenuated by at least a factor 10 for frequencies lower than 0.1 rad/s and at least by a factor 100 at steady-state (frequency 0).
   2. Measurement noise shall be attenuated by at least a factor 10 for frequencies above 10 rad/s.

   (i) Formulate the above requirements as requirements on the sensitivity function S and complimentary sensitivity function T.

   (ii) Translate the requirements on S and T into requirements on the loop-gain $L = GF_y$. Draw a simple figure that shows the "forbidden" areas for the singular values of L as functions of frequency.

   (iii) Can it prove hard to meet the specifications, even if the system is minimum-phase and open-loop stable? Motivate!

(5p)

3. We shall consider the ability to achieve acceptable performance, using feedback control, for a given linear system. The scaled system is given by

$$y = G_1 u_1 + G_2 u_2 + G_d d$$

and

$$G_1(s) = \frac{50e^{-2s}}{10s+1} \; ; \quad G_2(s) = \frac{20}{10s+1} \; ; \quad G_d(s) = \frac{5}{5s+1}$$

in which u_1, u_2 and d all $\in [-1, 1]$ and acceptable control corresponds to keeping $y \in [-1, 1]$.

(a) Determine if acceptable control, in theory, can be achieved using only one of the two inputs u_1 and u_2. Which input should be used? (4p)

(b) Determine a controller that satisfies the performance requirement with the input chosen in (a). *Tip: determine the simplest required closed-loop and compute the controller from this.* (3p)

(c) Assume we want to design a controller by solving the optimization problem

$$\min_{F_y} J$$

Determine the objective function J that reflects the performance specification for y in the presence of the disturbance d while keeping the control input within its constraints. (3p)

4. Given the multivariable system on state space form

$$\dot{x} = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} u(t)$$

$$y = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} x(t)$$

(a) Determine the transfer-matrix $G(s)$. (2p)

(b) We shall determine a state feedback controller that minimizes the criterion

$$J = \int_0^\infty (x^T Q_1 x + u^T Q_2 u)$$

Use $Q_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $Q_2 = I$ and determine the resuling controller and the corresponding closed-loop poles. (4p)

(c) Determine the complimentary sensitivity function at the input, T_I, when the controller from (b) is used. (2p)

(d) Based on the result in (c) and the robustness criterion, which input can we tolerate the largest uncertainty in before getting problems with instability? (2p)

5. A closed loop system, based on a feedback controller with one degree of freedom

$$u = F(s)(r - y)$$

has the sensitivity function

$$S = \frac{1}{s+1}\begin{pmatrix} s+0.1 & -0.1 \\ -0.1 & s+0.1 \end{pmatrix}$$

(a) What is the largest relative output uncertainty that is allowed if we are to guarantee robust stability according to the robustness criterion? (5p)

(b) What combination of step changes in the setpoints, with the constraint $|r| < 1$, will give the largest and smallest steady-state control error, respectively? (5p)