AUTOMATIC CONTROL
KTH

EL2520 Control Theory and Practice - Advanced Course

Exam 08.00–13.00 Aug 20, 2018

Aid:

Course book *Glad and Ljung, Control Theory / Reglerteori* **or** *Skogestad and Postlethwaite, Multivariable Control*, basic control course book *Glad and Ljung, Reglerteknik*, or equivalent if approved by examiner beforehand, copies of slides from this years lectures, this years published lecture notes, mathematical tables, pocket calculator (graphing, not symbolic). Any notes related to solutions of problems are not allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3 weeks after the exam at "My Pages".

Responsible: Elling W. Jacobsen 0703722244

Good Luck!

1. (a) Consider the system

$$G(s) = \frac{-s+1}{(s+1)^2} \begin{pmatrix} 1 & s+1 \\ 3 & 1 \end{pmatrix}$$

   (i) Determine the poles and zeros of the system. What is the minimum number of states needed in a state-space realization of this system? (3p)

   (ii) The system is to be controlled with a diagonal controller. Use the RGA to determine a proper pairing of inputs and outputs when the bandwidth should be around $\omega_B = 1\ rad/s$. (3p)

(b) Given the linear system

$$\dot{y} = 5y(t) + u(t)$$

Determine a controller that minimizes the criterion

$$J = \int_0^\infty y^2(t) + Ku^2(t)dt$$

for any arbitrary initial condition $y(0)$. What happens with the closed-loop pole as $K \to 0$ and $K \to \infty$, respectively? (4p)

2. (a) Someone has designed a controller for the system

$$G(s) = \frac{1}{s+1}\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

A plot of what is claimed to be the maximum singular values of the resulting sensitivity function S and complementary sensitivity function T, respectively, is shown below. As an expert in control you immediately see that something is wrong with the plot. What is wrong? (3p)

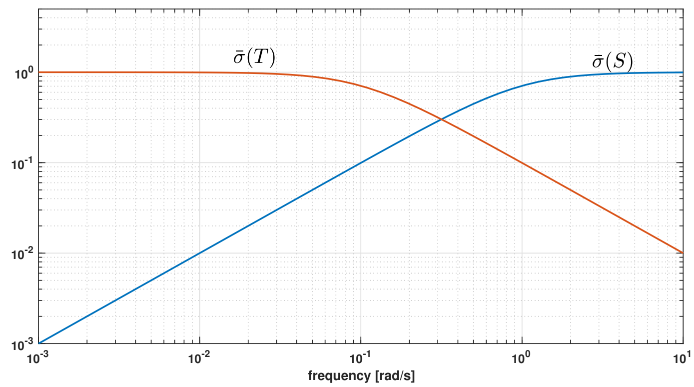


Figure 1: Plot for problem 2a

(b) We shall consider decoupling control of the system

$$G(s) = \frac{1}{10s+1}\begin{pmatrix} 2 & 1 \\ s+1 & 2 \end{pmatrix}$$

(i) One suggestion is to design a one-degree-of-freedom controller $F(s)$ such that the closed-loop transfer-function from setpoint to output becomes

$$Y(s) = \frac{1}{\tau s+1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} R$$

Is the resulting controller an appropriate choice? Motivate! (3p)

(ii) Design a controller such that the closed-loop system is completely decoupled from setpoints to outputs, has the same singular values from R to Y as the transfer-matrix in (i) and, furthermore, is internally stable. (4p)

3. (a) Consider the system

$$y = 5\frac{-4s+1}{10s+1}u + \frac{4}{10s+1}d$$

The aim is to design a controller that keeps $|y| < 1$ despite disturbances $|d| < 1$, while keeping the control input $|u| < 1$. Assume all signals are sinusoidal and determine if acceptable control is possible. If not, what is the maximum magnitude disturbance $|d|$ that can be attenuated in an acceptable way. (4p)

(b) We shall consider control design for a single-input-single-output system. The specifications are that disturbances w on the output z should be attenuated at least by a factor 10 for frequencies up to $\omega = 0.5\ rad/s$ and at least by a factor 100 at steady-state. Measurement noise should be attenuated in the output by at least a factor 5 for frequencies above $2\ rad/s$.

(i) Formulate requirements on the sensitivity S and complementary sensitivity T that satisfy the above requirements.

(ii) Translate the requirements from (i) into (approximate) requirements on the loop gain L. Draw a simple figure showing the constraints imposed on L as a function of frequency.

(iii) Can it be difficult to achieve the specifications? Motivate!

(iv) Determine a weight function W_S so that $\|W_S S\|_\infty < 1$ implies that the specifications on S are satisfied.

(6p)

4. (a) We shall consider Model Predictive Control of a system with a constrained input. The model is

$$\begin{aligned} x_{k+1} &= -x_k + 2u_k \\ y_k &= 2x_k \end{aligned}$$

where $|u_k| < 1 \ \forall k$. The optimization problem solved in the MPC is

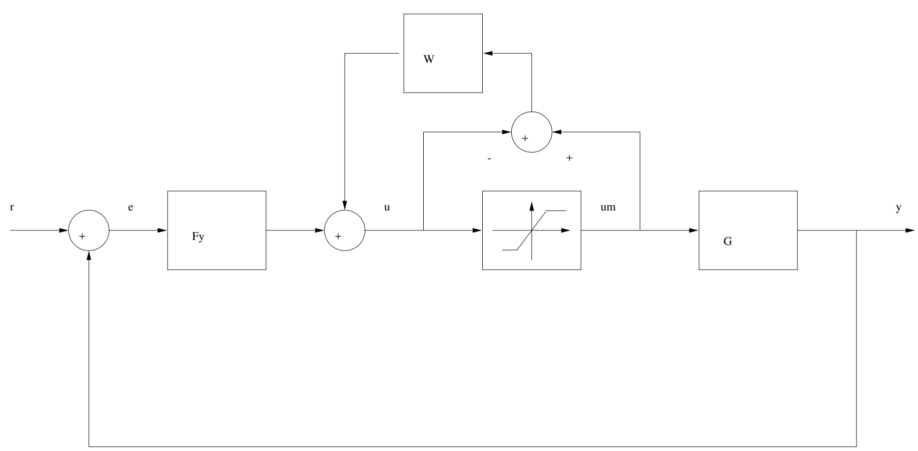
$$\min_u \left[\sum_{t=k}^{t=k+N_P} Q_y y_t^2 + \sum_{t=k}^{t=k+N_P-1} u_t^2 \right]$$

subject to $|u| < 1$. For the prediction and control horizon $N_P = 1$, translate the MPC problem into a Quadratic Programming (QP) problem

$$\min_u u^T H u + h^T u \ : \quad Lu \leq b$$

That is, determine H, h, L and b. (4p)

(b) Consider the block-diagram below for a feedback system with a constrained input and anti reset windup.



(The signs in the summations are such that $u = W(s)(u_p - u) + F_y(s)(r - y)$)

The transfer-functions are

$$G(s) = \frac{1}{s+1}; \quad F_y(s) = K \ ; \quad W(s) = \frac{1}{10s}$$

The constraint is such that $|u_p| < 2$.

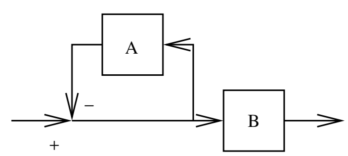
Show that the gain of the saturation is 1 and derive a sufficient condition for stability of the closed-loop system. Explain why the derived condition is not necessary. (6p)

5. We shall consider how various robustness criteria, for multivariable feedback systems, can be derived based on the small gain theorem.

   (a) Consider first the case with additive uncertainty, i.e., the true system is assumed to be given by
   $$G_0(s) = G(s) + \Delta_a(s)$$
   where the uncertainty $\Delta_a(s)$ is assumed stable and satisfy $\|\Delta_a\|_\infty < \delta$. Use the small gain theorem to derive a criterion for robust stability when the system is controlled by a feedback controller $F_y(s)$. State all assumptions you make! (2p)

   (b) The small gain theorem can appear overly conservative, that is, give much stricter bounds for stability than required. Discuss briefly to what extent the small gain theorem is conservative when applied to derive the robustness criterion in (a). (2p)

   (c) The small gain theorem requires the uncertainy block $\Delta(s)$ to be stable. In order to describe an uncertain number of RHP poles in the system one can employ feedback around a stable uncertainty block as illustrated below.



   Here $G_0(s) = G(s)(I + \Delta_I(s))^{-1}$. Derive a criterion for robust stability when the system is in a feedback loop with the controller $F_y(s)$. (3p)

   (d) Given the system
   $$G_0(s) = \frac{1}{s-p}, \quad -1 < p < 1$$
   The system is controlled by a pure P-controller $F_y = K$. For what values of the controller gain K is the system robustly stable according to the criterion derived in (c) when the nominal model $G(s)$ corresponds to $p = 0$. Is the criterion conservative in this case? (3p)