AUTOMATIC CONTROL

KTH

EL2520 Control Theory and Practice - Advanced Course

Exam 14.00–19.00 June 2, 2016

Aid:

Course book *Glad and Ljung, Control Theory* or *Reglerteori*, basic control course book *Glad and Ljung, Reglerteknik* or equivalent if approved by examiner beforehand, copies of slides from this years (2016) lectures, mathematical tables, calculator.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3 weeks after the exam at ”My Pages”.

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. (a) Compute the 2-norm $\|z\|_2$ of the signal

$$z(t) = \begin{pmatrix} e^{-t} \\ e^{-2t} \end{pmatrix} \cdot 1_{t>0}$$

where $1_{t>0}$ is 1 when $t > 0$ and 0 otherwise. (2p)

(b) The transfer-function $G_d(s)$ from a disturbance d to an output z is given by

$$G_d(s) = \frac{1}{s+2} \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}$$

What is the worst case amplification of a disturbance when both disturbance and output are measured in the 2-norm? Also give the corresponding directions of the disturbance and the output. (4p)

(c) Given the system

$$G(s) = \begin{pmatrix} \frac{1}{s+1} & \frac{1}{s} \\ \frac{1}{s+1} & \frac{1}{2s+1} \end{pmatrix}$$

Compute the poles and zeros, and determine how many states are required in a minimal state space realization of $G(s)$. (4p)

2. (a) The system

$$G(s) = \frac{1-2s}{1+2s}$$

is to be controlled with a PI-controller

$$F_y(s) = K\frac{2s+1}{s}$$

(i) Determine $\|S\|_\infty$ and $\|T\|_\infty$ when the controller is tuned so that the system has a phase margin of 45^o. (3p)

(ii) How does the the closed-loop system with the controller from (i) handle low-frequency disturbances on the output and high frequency measurement noise, respectively? (2p)

(b) (i) The system

$$\dot{x} = x(t) + \sqrt{2}u(t)$$

is feedback controlled with $u(t) = -lx(t)$. Determine the l that minimizes

$$\int_0^\infty (\eta x^2(t) + u^2(t))dt$$

What happens to the closed-loop pole as $\eta \to 0$? (2p)

(ii) Assume now that the system instead is described by

$$\begin{aligned} \dot{x}(t) &= x(t) + \sqrt{2}u(t) + v(t) \\ y(t) &= \sqrt{2}x(t) + e(t) \end{aligned}$$

where $v(t)$ and $e(t)$ are uncorrelated normally distributed disturbances with variances ρ and 1, respectively. The system is controlled by $u = -F_y(s)y$. Determine $F_y(s)$ so that

$$E\left\{\int_0^\infty (\eta x^2(t) + u^2(t))dt\right\}$$

is minimized. (3p)

3. (a) Given the system

$$Y(s) = \frac{-s+1}{10s+1}U(s) + \frac{1}{10s+1}D(s)$$

Assume the disturbance is bounded by $-4 < d < 4$ and that the output is to be kept within the bounds $-0.5 < y < 0.5$. We assume there is no limitation in the allowable input u.

(i) Within what frequency range is it necessary to attenuate disturbances so as to keep y within the specified bounds? (3p)

(ii) Is it possible to achieve acceptable performance using linear feedback control? Motivate! (2p)

(b) For a multivariable system the following specifications are given

- disturbances acting on the output should be attenutated by a factor at least 20 for frequencies below 0.1 rad/s and by a factor at least 100 for constant (zero frequency) disturbances.
- measurement noise should be dampened by a factor at least 10 for frequencies above 1 rad/s.
- the system should be robustly stable to multiplicative uncertainty at the output $G_p = (I + \Delta_G)G$ where the maximum singular value of Δ_G is 0.1 at frequencies up to 1 rad/s and then increases to 2.0 at higher frequencies.

(i) Formulate requirements on the sensitivity function S and the complementary sensitivity function T that ensures that the above specifications are satisfied.

(ii) Translate the requirements in (i) into requirements on the loop transferfunction $L = GF_y$. Draw a simple figure which shows the "forbidden" areas for the singular values of $L(i\omega)$.

(iii) Can it be difficult to satisfy all the requirements, even if the system is minimum phase and has no unstable poles? Motivate!

(5p)

4. (a) We shall consider control of a system with 2 inputs and 2 outputs. The transfer-function from input to output has been identified as

$$G(s) = \frac{1}{s+1} \begin{pmatrix} 1 & \frac{-1.1}{0.2s+1} \\ 1 & -1 \end{pmatrix}$$

(i) Consider first decentralized control. Use the RGA to determine the best pairing of inputs and outputs and also make a judgement whether it is reasonable to use decentralized control of this system when we aim for a bandwidth of 1 rad/s. (3p)

(ii) Someone has suggested to use a pre-compensator $W(s)$ to decouple the system so that

$$G(s)W(s) = \frac{1}{s+1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Explain why this is not a good idea! (2p)

(iii) Determine a pre-compensator $W(s)$ that provides dynamic decoupling while ensuring internal stability. (3p)

(b) The discrete time system

$$x_{k+1} = \begin{bmatrix} -2 & 1 \\ 0 & 0.5 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k$$

is controlled by the state feedback controller

$$u_k = -\begin{bmatrix} 1 & 1 \end{bmatrix} x_k$$

Is the closed loop stable? (2p)

5. We shall consider control of a simple domestic water heating system with inputs corresponding to the heating effect and inflow of water, and the outputs being temperature and volume. Through experiments, one has determined two different models of the system

$$G_1(s) = \frac{1}{10s+1}\begin{pmatrix} 2 & -2.5 \\ 0 & 1 \end{pmatrix} \quad ; \quad G_2(s) = \frac{1}{10s+1}\begin{pmatrix} 2.5 & -2 \\ 0 & 0.8 \end{pmatrix}$$

It is decided to base the control design on model $G_1(s)$. However, a requirement is that the closed-loop should be robustly stable for an uncertainty description that includes also model $G_2(s)$.

(a) Determine the relative output uncertainty Δ_{Go} in the uncertainty description

$$G_p = (I + \Delta_{Go})G$$

when model G_1 is used as nominal model and model G_2 is to be within the uncertainty set G_p. (1p)

(b) Assume that the decoupling controller

$$F_y(s) = \frac{1}{s}G_1^{-1}(s)$$

is used. Determine if the closed-loop is robustly stable with the uncertainty in (a). (3p)

(c) Assume that we instead model the uncertainty at the input

$$G_p = G(I + \Delta_{Gi})$$

Use the small gain theorem to derive a robust stability condition for this case. (4p)

(d) With the controller from (b) and the uncertainty modelled at the input, so that model $G_2(s)$ is included in the uncertainty set, is the closed loop robustly stable? Explain any differences to the result in (b). (2p)