AUTOMATIC CONTROL

KTH

EL2520 Control Theory and Practice - Advanced Course

Exam 08.00–13.00 May 29, 2018

Aid:

Course book *Glad and Ljung, Control Theory / Reglerteori* **or** *Skogestad and Postlethwaite, Multivariable Control*, basic control course book *Glad and Ljung, Reglerteknik*, or equivalent if approved by examiner beforehand, copies of slides from this years lectures, this years published lecture notes, mathematical tables, pocket calculator (graphing, not symbolic). Any notes related to solutions of problems are not allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3-5 weeks after the exam at "My Pages" (the delay is due to KTH replacing the current reporting system in June).

Responsible: Elling W. Jacobsen 0703722244

Good Luck!

1. (a) Consider the system

$$G(s) = \frac{1}{s+2} \begin{pmatrix} s+1 & 1 \\ 4 & 2 \end{pmatrix}$$

(i) Determine the poles and zeros of the system and for those in the RHP the corresponding output direction. (4p)

(ii) What is the minimum number of states needed in a state-space realization of the system? (1p)

(b) Compute the singular values and $\mathcal{H}_\infty$-norm for the system

$$G(s) = \frac{1}{2s+1} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(3p)

(c) A discrete time system

$$y_{k+1} = 2y_k + u_k$$

is controlled with a P-controller $u_k = -Ky_k$. For what values of the controller gain K is the closed-loop stable? (2p)

2. (a) To determine a controller for a multivariable mechanical system with 3 inputs and 3 outputs, someone has proposed to use $\mathcal{H}_\infty$-optimization to shape the sensitivity function S and the complementary sensitivity function T

$$\min_{F_y} \left\| \begin{matrix} W_S S \\ W_T T \end{matrix} \right\|_\infty$$

The scalar weights are given by

$$W_S = \frac{0.5s+1}{s} \; ; \quad W_T = 10(s+0.05)$$

The amplitudes of the weights are shown in Figure 1. The aim is to obtain $\|W_S S\|_\infty < 1$ and $\|W_T T\|_\infty < 1$. Is this feasible, that is, does there exist any stabilizing controller that will satisfy this? Motivate your answer. (3p)

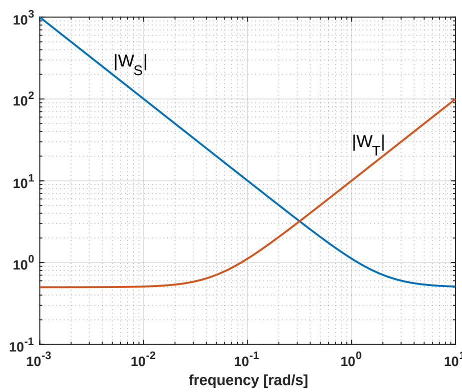

Figure 1: Weights in problem 2a.

(b) Consider the system

$$Y(s) = \frac{1}{(s+2)(s-1)} \begin{pmatrix} -2 & 3 \\ s-3 & 6 \end{pmatrix} U(s) + \frac{1}{s+1} \begin{pmatrix} 1 \\ 2 \end{pmatrix} D(s)$$

An engineer, who is not an expert on control, has proposed a controller that yields the sensitivity function

$$S(s) = \frac{s}{s+2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Explain why this is a bad proposal. (3p)

(c) We shall consider control of the product compositions in a chemical reactor. The model of the reactor is given by

$$Y(s) = \frac{1}{10s+1} \begin{pmatrix} s+1 & 1 \\ s+2 & 1.5 \end{pmatrix} U(s) + \frac{1}{10s+1} \begin{pmatrix} 2 \\ 3 \end{pmatrix} D(s)$$

where $y = [c_A \; c_B]$ are the concentrations of component A and B, respectively, $u = [F \; Q_c]$ are the feed flow rate and cooling rate, respectively, and the disturbance $d = c_{Ai}$ is the composition of A in the feed flow. Assume the model has been scaled so that the aim is to keep $|y| < 1$ for all frequencies when the disturbance magnitude $|d| < 1$. You can assume there are no constraints on the inputs. Is the control objective feasible, i.e., does there exist a stabilizing controller that yields $|y| < 1$ when $|d| < 1$ for all frequencies? (4p)

3. (a) Consider the unstable system

$$Y(s) = \frac{1}{s-1}U(s)$$

The control objective is to bound the complementary sensitivity function as follows

$$\|W_T T\|_\infty < 1 \tag{1}$$

where the weight W_T is given by

$$W_T(s) = \frac{2}{3}(s+1)$$

(i) Let us first try with a simple P-controller

$$U(s) = -K_p Y(s)$$

What is the minimum value of $\|W_T T\|_\infty$ that can be achieved with a P-controller? (3p)

(ii) Assume we instead use $\mathcal{H}_\infty$-optimization to find a controller, is it then possible to satisfy the bound in (1). (1p)

(b) Consider the MIMO system

$$Y(s) = G(s)(U(s) + W_U(s))$$

where u is the control input and w_u is a disturbance at the input. We want to determine a controller that satisfies the following specifications

I. Input disturbances w_u should be attenuated in the output y up to frequency 0.5 rad/s, and be completely rejected at frequency 0 rad/s.

II. The closed-loop system should be robustly stable for 20% uncertainty in the inputs, i.e., 20% relative uncertainty at the input side of G.

(i) Formulate an $\mathcal{H}_\infty$-optimal control problem that reflects the specifications given above. (3p)

(ii) Determine the inputs w_e and outputs z_e of an extended system $z_e = G_{ec}w_e$ such that solving the problem

$$\min_{F_y} \sup_{w_e} \frac{\|z_e\|_2}{\|w_e\|_2}$$

corresponds to solving the problem formulated in (i). (3p)

4. (a) The system

$$Y(s) = \frac{2}{s}U(s) \tag{2}$$

is to be controlled with a P-controller

$$U = -K_pY$$

(i) Determine the controller gain K_p that minimizes the objective function

$$\int_0^\infty y(t)^2 + 0.1u(t)^2 dt \tag{3}$$

(2p)

(ii) It turns out that the measurement y_m of y is noisy

$$y_m = y + n \ ; \ R_n = 0.1$$

where R_n is the variance of the noise n. Determine the Kalman filter for the optimal estimate of y. How will the use of the observer affect the optimal controller gain in (i)? (3p)

(b) We shall consider Model Predictive Control (MPC) of the system in (a), assuming the input is constrained as $|u| < 1 \ \forall t$.

(i) Consider the sampling time $T = 0.5s$ and determine the discrete time state space model for the system (2), assuming zero order hold on the input. (1p)

If you did not solve (i), you can use $y_{k+1} = 2y_k + u_k$ in (ii) below.

(ii) The MPC optimization problem is

$$\min_u \left[\sum_{i=k}^{i=k+N} Q_y y_k^2 + \sum_{i=k}^{i=k+N} u_k^2 \right] \tag{4}$$

subject to $|u_k| < 1 \ \forall k$. Consider now the prediction horizon $N = 1$ and translate the MPC problem into a Quadratic Programming (QP) problem

$$\min_u u^T H u + h^T u \ ; \quad Lu \leq b$$

That is, determine H, h, L and b for the problem. (4p)

5. To analyze robust stability, we can consider perturbations of a nominal model, e.g.,

$$G_p(s) = G(s) + \Delta(s)$$

where $G_p(s)$ is the perturbed model, $G(s)$ the nominal model and $\Delta(s)$ a perturbation which is assumed to be stable and norm-bounded. Robust stability is then defined as closed-loop stability with all perturbed models $G_p(s)$.

(a) Explain briefly why we usually assume that the perturbation $\Delta(s)$ is stable. (1p)

A common uncertainty description is

$$G_p(s) = G(s)(I + W_I(s)\Delta_I(s)) \; ; \quad \|\Delta_I\|_\infty < 1 \tag{5}$$

corresponding to relative uncertainty at the input side. The corresponding robust stability condition is then

$$\|W_I T_I\|_\infty < 1$$

However, since $\Delta_I(s)$ is restricted to be stable, the perturbation can not change the number of RHP poles in the perturbed model, i.e., $G(s)$ and $G_p(s)$ must have the same number of RHP poles. An alternative uncertainty description, allowing for poles of $G_p(s)$ to cross the imagnary axis even if the perturbation is stable, is

$$G_p(s) = G(s)(I + W_{iI}(s)\Delta_{iI}(s))^{-1} \; ; \quad \|\Delta_{iI}\|_\infty < 1 \tag{6}$$

(b) Derive a robust stability condition for a closed-loop MIMO system when the model of the system is assumed to belong to the uncertainty set in (6). (3p)

Consider now control of the uncertain system

$$G_p(s) = \frac{k}{s + \alpha}$$

where the parameters k and α are uncertain. The controller is designed based on the nominal model

$$G(s) = \frac{1}{s + 1}$$

We assume a P-controller

$$F_y = K_p$$

(c) Assume first that $\alpha = 1$, but that k can be in the interval $k \in [0.25,\ 1.25]$. Determine which of the two uncertainty descriptions (5) and (6) that give the least conservative robust stability result for this case. For what values of the controller gain K can stability be guaranteed in the two cases? (3p)

(d) Assume now that $k = 1$, but that α can be in the interval $\alpha \in [0.25,\ 2]$. Which uncertainty description gives the least conservative result in this case? For what values of the controller gain K can stability be guaranteed in the two cases? (3p)