AUTOMATIC CONTROL
KTH

EL2520 Control Theory and Practice - Advanced Course

Exam 14.00–19.00 August 18, 2017

Aid:

Course book *Glad and Ljung, Control Theory / Reglerteori*, basic control course book *Glad and Ljung, Reglerteknik* or equivalent if approved by examiner beforehand, copies of slides from this years (2017) lectures, mathematical tables, calculator. Any notes related to solutions of problems are not allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3 weeks after the exam at "My Pages".

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. (a) For the 2×2 system

$$G(s) = \begin{pmatrix} \frac{s-1}{s+1} & \frac{2(s-1)}{s+1} \\ \frac{s+1}{s-1} & \frac{-(s+1)}{s-1} \end{pmatrix}$$

Determine the poles and zeros of the system. How many states are needed in a minimal state space realization of $G(s)$? (4p)

(b) Compute the H_∞ norm of

$$G(s) = \frac{1}{(s+1)^2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

(2p)

(c) A feedback controller has been designed for the system

$$G(s) = \frac{2s+1}{s-1}$$

such that the closed-loop sensitivity function is

$$S(s) = \frac{s}{s+1}$$

Is the closed-loop system internally stable? Motivate! (4p)

2. (a) We shall consider control of a 2×2 system with transfer matrix

$$G(s) = \frac{1}{s+1} \begin{pmatrix} 1 & \frac{1}{s+1} \\ 2 & 1 \end{pmatrix}$$

The aim is to have a bandwidth for the closed-loop sensitivity around $w_B = 0.5\ rad/min$.

(i) Consider first decentralized control and use the RGA to determine the most suitable pairing of inputs and outputs. Would you recommend use of decentralized control for this system? Motivate! (3p)

(ii) A consultant has proposed to remove all interactions between the outputs by employing a decoupler $W(s)$ such that

$$G(s)W(s) = \frac{1}{s+1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Explain why this is a bad proposal! (2p)

(iii) Propose a decoupler $W(s)$ that provides the desired decoupling att all frequencies while ensuring internal stability. (3p)

(b) The discrete time system

$$x_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & -0.1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k$$

is controlled using the state feedback

$$u_k = -\begin{bmatrix} 1 & 1 \end{bmatrix} x_k$$

Determine if the closed-loop is stable. (2p)

3. Given the scaled linear system

$$y = G_1(s)u_1 + G_2(s)u_2 + G_d(s)d$$

where

$$G_1(s) = \frac{s-4}{10s+1} \;;\quad G_2(s) = \frac{e^{-2s}}{10s+1} \;;\quad G_d(s) = 3\frac{e^{-s}}{10s+1}$$

and the inputs u_1, u_2 and the disturbance d are all $\in [-1, 1]$. The aim is to keep the output $y \in [-1, 1]$ in the presence of disturbance d.

(a) Determine if it, in theory, is possible to achieve acceptable control performance according to the above specifications, and if this can be done using only one of the two available inputs. Which input should be used? (3p)

(b) Determine a feedback controller, based on measuring y, that satisifies the performance requirements with the chosen input from (a). (2p)

(c) Formulate an H_∞-optimal control problem that reflects the performance requirement for disturbance attenuation, the limited control input u and a requirement of robust stability in the presence of 20% uncertainty at the output. You should also determine the inputs and outputs of an extended system that reflects the objective. (5p)

4. (a) Consider the system

$$y = \frac{10}{s+1}\begin{pmatrix} 2 & 2s \\ 1 & 1 \end{pmatrix} u + \frac{3}{s+1}\begin{pmatrix} 1 \\ 2 \end{pmatrix} d_1 + \frac{2}{s+1}\begin{pmatrix} 4 \\ 2 \end{pmatrix} d_2$$

Which of the two disturbances d_1 and d_2 do you expect to be most difficult to attenuate? Motivate! (4p)

(b) An open-loop unstable system is controlled by a feedback controller that yields the closed-loop sensitivity function

$$S(s) = \frac{1}{s+1}\begin{pmatrix} s+0.1 & -0.1 \\ -0.1 & s+0.1 \end{pmatrix}$$

Is the closed-loop system stable? Motivate! (2p)

(c) Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t) \\ y(t) &= \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) \end{aligned}$$

Determine a state feedback that minimizes the criterion

$$J = \int_0^\infty y^2(t) + 2u^2(t)dt$$

for any initial condition $x(0) = x_0$. (4p)

5. We shall consider control of a 2×2 system for which the following model has been derived

$$G(s) = \frac{1}{23s+1}\begin{pmatrix} 2 & \delta_1 \\ \delta_2 & 1.5 \end{pmatrix}$$

where δ_1, δ_2 are real numbers.

(a) For the control design, one decides to neglect the off-diagonal terms in $G(s)$ and hence base the design on the simplified model

$$G_0(s) = \frac{1}{23s+1}\begin{pmatrix} 2 & 0 \\ 0 & 1.5 \end{pmatrix}$$

Determine the relative model uncertainty Δ_G at the input when $G_0(s)$ is used as the nominal model and the true model $G(s)$ should be within the model set. (2p)

(b) For the control design one employs the diagonal PI-controller

$$F(s) = \begin{pmatrix} K_1\frac{\tau_{I1}s+1}{\tau_{I1}s} & 0 \\ 0 & K_2\frac{\tau_{I2}s+1}{\tau_{I2}s} \end{pmatrix}$$

Determine the controller parameters such that the closed-loop, with the simplified model $G_0(s)$, has two poles at $s = -2$. (2p)

(c) Derive a robust stability criterion based on the small gain theorem and determine for what values of δ_1, δ_2 the closed-loop with the controller from (b) will be guaranteed to be stable under the uncertainty from (a). (4p)

(d) For what values of δ_1, δ_2 will the closed-loop system be stable? Compare with the results from (c) and explain possible differences. (2p)