AUTOMATIC CONTROL

KTH

EL2520 Control Theory and Practice - Advanced Course

Exam 14.00–19.00 June 5, 2017

Aid:

Course book *Glad and Ljung, Control Theory / Reglerteori*, basic control course book *Glad and Ljung, Reglerteknik* or equivalent if approved by examiner beforehand, copies of slides from this years (2017) lectures, mathematical tables, calculator. Any notes related to solutions of problems are not allowed.

Note that separate notes, exercise material and old exams etc are NOT allowed.

Observandum:

- Name and social security number (*personnummer*) on every page.
- Only one solution per page.
- Do only write on one side per sheet.
- Each answer has to be motivated.
- Specify the total number of handed in pages on the cover.
- The exam consists of five problems worth a total of 50 credits

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results: The results will be available about 3 weeks after the exam at ”My Pages”.

Responsible: Elling W. Jacobsen 070 372 22 44

Good Luck!

1. (a) Consider the system

$$G(s) = \frac{1}{5s+1}\begin{pmatrix} 1 & -s+1 \\ 1 & 1 \end{pmatrix}$$

(i) Determine the poles and zeros of the system (3p)

(ii) Is it possible to control the two outputs independently at steady-state? Motivate! (2p)

(b) The system

$$G(s) = \frac{s-1}{s}$$

is to be controlled with the performance objective

$$\|w_P S\|_\infty \leq 1 \ , \quad w_P = 0.5\frac{s+1}{s}$$

(i) Consider first using a P-controller $F_y = K$. Determine the controller gain K that minimizes $\|w_P S\|_\infty$. Is it possible to achieve acceptable performance with a P-controller? (3p)
Hint: note that the peak amplitude of a transfer-function $(as+1)/(bs+1)$ *will occur at either* $\omega = 0$ *or* $\omega = \infty$*, depending on* a/b*.*

(ii) Assume we change the objective to

$$\|w_P S\|_\infty \leq 1 \ , \quad w_P = \frac{2}{3}\frac{s+1}{s}$$

Is it possible to satisfy this performance objective with a more advanced controller? Motivate! (2p)

2. (a) We shall consider control of a 2×2 system with transfer matrix

$$G(s) = \frac{1}{5s+1} \begin{pmatrix} 1 & -1.1 \\ 2 & \frac{-2}{s+1} \end{pmatrix}$$

The aim is to have a bandwidth (for the sensitivity) at around $\omega_B = 1$. Use the RGA to determine the most suitable pairing of inputs and outputs. Would you recommend use of decentralized control for this system? Motivate! (3p)

(b) The dynamics of a mechanical system with two inputs and two outputs is described by the model

$$G(s) = \frac{1}{2.2s+1} \begin{pmatrix} 1 & -s+2 \\ 1 & 1 \end{pmatrix}$$

A consultant has proposed a feedback controller that gives the sensitivity function

$$S = \frac{10s}{10s+1} \begin{pmatrix} 1 & 0.1 \\ 0 & 1 \end{pmatrix}$$

Explain/show why this is a bad proposal. (3p)

(c) Consider now stabilization of an unstable 3×3 system. The same consultant as above has proposed a controller that gives the sensitivity function

$$S = \frac{s}{s+1} I_{3\times 3}$$

Explain/show why this is bad proposal (you do not need to know the model of the system to answer this problem). (4p)

3. Given the scalar system

$$\hat{y} = \hat{G}\hat{u} + \hat{G}_d\hat{d}$$

where

$$\hat{G} = \frac{1}{(2s+1)}e^{-4s} \ ; \quad \hat{G}_d = \frac{8}{(2s+1)(5s+1)}e^{-2s}$$

and $\hat{u} \in [-3, 3]$, $\hat{d} \in [-0.1, 0.1]$. Assume the nominal operating point is $\hat{u} = 0, \hat{d} = 0, \hat{y} = 0$ and that the control objective is to keep $\hat{y} \in [-0.5, 0.5]$.

(a) Determine the scaled system

$$y = Gu + G_d d$$

such that u and $d \in [-1, 1]$ and acceptable control performance corresponds to $y \in [-1, 1]$. (2p)

(b) Determine if it is possible to achieve acceptable disturbance attenuation using feedback control only. (3p)

(c) Formulate an H_∞-optimal control problem that reflects the performance requirement for disturbance attenuation, the limited control input u and a requirement of robust stability in the presence of 30% uncertainty at the output. You should also determine the inputs and outputs of an extended system that reflects the objective. (5p)

4. (a) Consider the system

$$y = \frac{1}{5s+1}\begin{pmatrix} 1 & -s+2 \\ 1 & 1 \end{pmatrix} u + \frac{1}{s+1}\begin{pmatrix} 10 \\ 10 \end{pmatrix} d_1 + \frac{1}{s+1}\begin{pmatrix} 3 \\ -3 \end{pmatrix} d_2$$

The aim is to attenuate disturbances $d_1 \in [-1, 1]$ and $d_2 \in [-1, 1]$ such that $|y| < 1$ at all frequencies. There is no limit on the control input u. Which of the two disturbances do you expect to be most difficult to attenuate? Motivate! (3p)

(b) We shall consider Model Predictive Control for robotic motion planning. Such planning often focus on relatively simple high-level dynamics, such as

$$\dot{x}(t) = u(t)$$

Here we assume $x, u \in \mathcal{R}^1$, i.e., we consider actuation in one dimension only.

(i) Assume the system is sampled with a sample time T and zero-order hold input. Determine the corresponding discrete time model

$$x_{k+1} = Fx_k + Gu_k$$

Discuss briefly the connection between stability of the original system and stability of the discretized system. (2p)

(ii) We shall now employ (unconstrained) MPC for tracking a desired trajectory $r(k)$. The cost function to be minimized is

$$J = \sum_{i=k_0+1}^{k_0+N} q_1(x(i) - r(i))^2 + q_2 u^2(i-1)$$

where $q_1, q_2 \geq 0$ are design parameters and k_0 is the current time. Note that k_0 is increased by 1 after each iteration of the MPC. Determine the explicit solution in terms of a feedback control law that minimizes the criterion above for the case with $N = 1$. Also determine the stability of the closed-loop system, and discuss what happens as $q_1 \to \infty$. (5p)

5. We shall consider robust stabilization of a system with uncertain zeros and poles

$$G(s) = 2\frac{(s+1)(s+z)}{(5s+1)^2(s+p)} \ , \quad z \in [1,3], \quad p \in [-1,3] \tag{1}$$

The aim is to find a controller that guarantees robust stability for all z and p within the given intervals. For this purpose we shall employ robust control theory based on uncertainty sets and associated robust stability criteria.

(a) Explain why an uncertainty set corresponding to the "standard" multiplicative uncertainty

$$\Pi_I = \left\{ G_p(s) = G(s)(1 + W_I(s)\Delta_I(s)) \ | \|\Delta_I\|_\infty \leq 1 \right\}$$

is not suitable in this case. (2p)

(b) Propose an adequate uncertainty description and derive a corresponding robust stability criterion. (5p)
Hint: note that feedback around a stable perturbation can provide an unstable perturbation.

(c) For system (1), what is the bandwidth requirement imposed by the robust stability criterion derived in (b)? You can assume $z = 1$ and $p = 1$ in the nominal model. (3p)