EL2520 - Control Theory and Practice - Advanced Project Lab : The Four Tank Process

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Abstract — **This document is a report for the Project Lab conducted under the EL2520 course. This project was divided into 2 distinct sessions. The first one involved deriving a physical model of a four-tank process for minimum phase and non-minimum phase configurations via experimentation and investigating the coupling between the tanks. Emphasis was also given to manually controlling the process to understand the performance limitations due to the non-minimum dynamics. The second instance was dedicated to testing the model-based decentralized PI controller and the robust Glover-McFarlane method.**

I. Modelling

Here, the nonlinear differential equations describing the plant will be derived.
The rate of change of volume in each tank is given as:

$$A\frac{dh}{dt} = q_{\text{in}} - q_{\text{out}}$$

From Bernoulli's law,

$$q_{\text{out}} = a\sqrt{2gh} \quad \forall g = 981cm/s^2$$

Now, as there was a pump used, its flowrate is goverened as follows:

$$q_L = \gamma ku,\ q_U = (1-\gamma)ku \ \ \forall \gamma \in [0,1]$$

$\forall$ q_L denotes the lower tank and q_U denotes the upper tank From the above equations, the non-linear system can be derived as follows:

$$\begin{aligned}
A_1\frac{dh_1}{dt} &= -q_{out,1} + q_{out,3} + q_{L,1}\\
A_2\frac{dh_2}{dt} &= -q_{out,2} + q_{out,4} + q_{L,2}\\
A_3\frac{dh_3}{dt} &= -q_{out,3} + q_{U,2}\\
A_4\frac{dh_4}{dt} &= -q_{out,4} + q_{U,1}
\end{aligned}$$

Thus, the final system of equations turns out to be:

$$\begin{aligned}
\frac{dh_1}{dt} &= \frac{-a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1\\
\frac{dh_2}{dt} &= \frac{-a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2\\
\frac{dh_3}{dt} &= \frac{-a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2\\
\frac{dh_4}{dt} &= \frac{-a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1
\end{aligned} \tag{1}$$

A. Equilibrium Equations

For the equilibrium condition, rate of change of height was taken as 0 to get the following set of equations:

$$\begin{aligned}
\frac{-a_1}{A_1}\sqrt{2gh_1^0} + \frac{a_3}{A_1}\sqrt{2gh_3^0} + \frac{\gamma_1 k_1}{A_1}u_1^0 &= 0\\
\frac{-a_2}{A_2}\sqrt{2gh_2^0} + \frac{a_4}{A_2}\sqrt{2gh_4^0} + \frac{\gamma_2 k_2}{A_2}u_2^0 &= 0\\
\frac{-a_3}{A_3}\sqrt{2gh_3^0} + \frac{(1-\gamma_2)k_2}{A_3}u_2^0 &= 0\\
\frac{-a_4}{A_4}\sqrt{2gh_4^0} + \frac{(1-\gamma_1)k_1}{A_4}u_1^0 &= 0\\
y_i^0 = k_c h_i^0 \ \ \forall i &= \{1,2,3,4\}
\end{aligned}$$

where $h_i^0,\ u_i^0,\ y_i^0$ denote the steady state values.

B. Linearization

Let $\Delta u_i = u_i - u_i^0$, $\Delta h_i = h_i - h_i^0$ and $\Delta y_i = y_i - y_i^0$ denote the deviations from the equilibrium and also let

$$u = \begin{bmatrix}\Delta u_1\\ \Delta u_2\end{bmatrix}, x = \begin{bmatrix}\Delta h_1\\ \Delta h_2\\ \Delta h_3\\ \Delta h_4\end{bmatrix}, y = \begin{bmatrix}\Delta y_1\\ \Delta y_2\end{bmatrix}$$

For Linearizing the above-derived system of non-linear equation [1] around its equilibrium points, Taylor Series of Expansion (neglecting HOTs) was used to obtain the following matrices:

$$A = \begin{bmatrix} \frac{\partial \Delta h_1}{\partial h_1} & \frac{\partial \Delta h_1}{\partial h_2} & \frac{\partial \Delta h_1}{\partial h_3} & \frac{\partial \Delta h_1}{\partial h_4} \\ \frac{\partial \Delta h_2}{\partial h_1} & \frac{\partial \Delta h_2}{\partial h_2} & \frac{\partial \Delta h_2}{\partial h_3} & \frac{\partial \Delta h_2}{\partial h_4} \\ \frac{\partial \Delta h_3}{\partial h_1} & \frac{\partial \Delta h_3}{\partial h_2} & \frac{\partial \Delta h_3}{\partial h_3} & \frac{\partial \Delta h_3}{\partial h_4} \\ \frac{\partial \Delta h_4}{\partial h_1} & \frac{\partial \Delta h_4}{\partial h_2} & \frac{\partial \Delta h_4}{\partial h_3} & \frac{\partial \Delta h_4}{\partial h_4} \end{bmatrix} \Bigg|_{h_i^0, u_i^0}$$

$$B = \begin{bmatrix} \frac{\partial \Delta h_1}{\partial u_1} & \frac{\partial \Delta h_1}{\partial u_2} \\ \frac{\partial \Delta h_2}{\partial u_1} & \frac{\partial \Delta h_2}{\partial u_2} \\ \frac{\partial \Delta h_3}{\partial u_1} & \frac{\partial \Delta h_3}{\partial u_2} \\ \frac{\partial \Delta h_4}{\partial u_1} & \frac{\partial \Delta h_4}{\partial u_2} \end{bmatrix} \Bigg|_{h_i^0, u_i^0}$$

$$C = \begin{bmatrix} \frac{\partial \Delta y_1}{\partial h_1} & \frac{\partial \Delta y_1}{\partial h_2} & \frac{\partial \Delta y_1}{\partial h_3} & \frac{\partial \Delta y_1}{\partial h_4} \\ \frac{\partial \Delta y_2}{\partial h_1} & \frac{\partial \Delta y_2}{\partial h_2} & \frac{\partial \Delta y_2}{\partial h_3} & \frac{\partial \Delta y_2}{\partial h_4} \end{bmatrix} \Bigg|_{h_i^0, u_i^0}$$

$$D = \begin{bmatrix} \frac{\partial \Delta y_1}{\partial u_1} & \frac{\partial \Delta y_1}{\partial u_2} \\ \frac{\partial \Delta y_2}{\partial u_1} & \frac{\partial \Delta y_2}{\partial u_2} \end{bmatrix} \Bigg|_{h_i^0, u_i^0}$$

After solving the above matrices, the result was as follows:

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix}$$

and D = 0, with $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$.

C. Transfer Matrix

In order to obtain the Transfer Matrix,the following formula was used-

$$G(s) = C(sI - A)^{-1}B$$

i.e. G(s) =

$$\begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{T_1}{1+sT_1} & 0 & \frac{\frac{A_3 T_1}{A_1}}{(1+sT_1)(1+sT_3)} & 0 \\ 0 & \frac{T_2}{1+sT_2} & 0 & \frac{\frac{A_4 T_2}{A_2}}{(1+sT_2)(1+sT_4)} \\ 0 & 0 & \frac{T_3}{1+sT_3} & 0 \\ 0 & 0 & 0 & \frac{T_4}{1+sT_4} \end{bmatrix}$$

$$* \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\gamma_1 k_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)k_2 c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)k_1 c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 k_2 c_2}{1+sT_2} \end{bmatrix}$$

D. Zeros of Transfer Matrix

Zeros of the Transfer matrix G(s) are given by the numerator of the det(G(s)). In other words, zeros of the Transfer matrix can be obtained by solving the equation :

$$\gamma_1\gamma_2 T_3 T_4 s^2 + \gamma_1\gamma_2(T_3+T_4)s + (\gamma_1+\gamma_2-1) = 0$$
$$\implies T_3 T_4 s^2 + (T_3+T_4)s + \frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2} = 0$$

The solution of the above quadratic equation is:

$$s_1 = -\frac{T_3+T_4}{2T_3T_4} + \frac{1}{2T_3T_4}\sqrt{(T_3+T_4)^2 - 4\frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2}}$$
$$s_2 = -\frac{T_3+T_4}{2T_3T_4} - \frac{1}{2T_3T_4}\sqrt{(T_3+T_4)^2 - 4\frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2}}$$

Here, as s_2 has both terms negative, the zero is on the Left Half of the s-plane. But when it comes to s_1, if the term $\frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2}$ is positive, value of the square root term will be less than T_3+T_4, which makes the zero negative. Thus, for the **Minimum Phase case**, $(\gamma_1+\gamma_2-1) > 0$ i.e $\gamma_1+\gamma_2 > 1$ Now as $\gamma_1, \gamma_2 \in [0,1]$, $\gamma_1+\gamma_2 \leq 2$

$$\therefore\ 1 < \gamma_1+\gamma_2 \leq 2$$

For the **Non-minimum phase case**, one of the zeros is positive which means $(\gamma_1+\gamma_2-1) < 0$. Also, as $\gamma_1, \gamma_2 \in [0,1]$, $\gamma_1+\gamma_2 > 0$.

$$\therefore\ 0 < \gamma_1+\gamma_2 \leq 1$$

E. RGA Analysis

RGA(G(0)) $= G(0). * G(0)^{-1^T}$ Thus,

$$G(0) = \begin{bmatrix} \gamma_1 k_1 c_1 & (1-\gamma_2)k_2 c_1 \\ (1-\gamma_1)k_1 c_2 & \gamma_2 k_2 c_2 \end{bmatrix}$$

$$G(0)^{-1^T} = \left(\frac{1}{\gamma_1\gamma_2 k_1 k_2 c_1 c_2 - (1-\gamma_1)(1-\gamma_2)k_1 k_2 c_1 c_2}\right)$$

$$\begin{bmatrix} \gamma_2 k_2 c_2 & -(1-\gamma_1)k_1 c_2 \\ -(1-\gamma_2)k_2 c_1 & \gamma_1 k_1 c_1 \end{bmatrix}$$

Hence, the RGA of $G(0)$ is given by

$$\begin{bmatrix} \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} & \dfrac{\gamma_1+\gamma_2-1-\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \\[2ex] \dfrac{\gamma_1+\gamma_2-1-\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} & \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \end{bmatrix}$$

Now, let $\lambda = \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}$. Thus,

$$RGA(G(0)) = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

For **Minimum Phase case** :
$\lambda_1 = 0.625 = \lambda_2$. Thus,

$$RGA(G_{mp}(0) = \begin{bmatrix} 1.5625 & -0.5625 \\ -0.5625 & 1.5625 \end{bmatrix}$$

For **Non-Minimum Phase case** :
$\lambda_1 = 0.375 = \lambda_2$. Thus,

$$RGA(G_{nmp}(0) = \begin{bmatrix} -0.5625 & 1.5625 \\ 1.5625 & -0.5625 \end{bmatrix}$$

F. Determination of k_1 and k_2

For determining the values of k_1 and k_2, the rate at which the pump fills up a tank needs to be measured. Thus, for k_1, tank 1 was observed and the following experiment was conducted. The outflow from tank 1 was stopped and it was ensured that only the pump input was allowed into the tank. This helped in obtaining the value of k_1 using the following equation:

$$\frac{dh_1}{dt} = -\cancelto{0}{\frac{a_1}{A_1}\sqrt{2gh_1}} + \cancelto{0}{\frac{a_3}{A_1}\sqrt{2gh_3}} + \frac{\gamma_1 k_1}{A_1}u_1$$

The same process was repeated with the tank 2 for obtaining the value of k_2.
After multiple readings using different voltages, the following results were obtained:

$$k_1 = 4.4247 cm^3/sV$$
$$k_2 = 4.1092 cm^3/sV$$

G. Determination of areas of holes

Before proceeding with the procedure, some assumptions used are as follows:

- The cross-sectional areas of the tanks are the same i.e A = 15.52 cm^2
- The areas a_1 and a_2 remain the same for both Minimum and non-minimum phase cases
- The areas a_3 and a_4 will vary for Minimum and non-minimum phase cases

So, for calculating the areas, tank 3 and tank 4 were considered as they just had a single input and single output making the calculation for a_3 and a_4 simple. Then the upper 2 tanks were allowed to settle at their equilibrium points h_3^0 and h_4^0 by keeping $u_1^0 = 7.5V$ and $u_2^0 = 7.5V$.

$$\therefore\ a_3 = \frac{(1-\gamma_2)k_2}{\sqrt{2gh_3^0}}u_2^0$$
$$and\ \ a_4 = \frac{(1-\gamma_1)k_1}{\sqrt{2gh_4^0}}u_1^0$$

Now, the entire system was driven to equilibrium and all the steady-state heights(h_i^0) were measured. Since there are 2 equations(one each for tank 1 and tank 2) and 2 unknown, the areas of holes 1 and 2 were calculated by solving the 2*2 linear system as follows:

$$a_1 = \frac{a_3}{\sqrt{2gh_1^0}}\sqrt{2gh_3^0} + \frac{\gamma_1 k_1}{\sqrt{2gh_1^0}}u_1^0$$
$$a_2 = \frac{a_4}{\sqrt{2gh_2^0}}\sqrt{2gh_4^0} + \frac{\gamma_2 k_2}{\sqrt{2gh_2^0}}u_2^0$$

For **Minimum Phase Case**

$$a_1 = 0.2250\ cm^2$$
$$a_2 = 0.2389\ cm^2$$
$$a_3 = 0.0633\ cm^2$$
$$a_4 = 0.0888\ cm^2$$

For **Non-Minimum Phase Case**

$$a_1 = 0.2250\ cm^2$$
$$a_2 = 0.2389\ cm^2$$
$$a_3 = 0.115\ cm^2$$
$$a_4 = 0.213\ cm^2$$

II. Manual Control

In this section, manual control in both minimum and non-minimum phase is explained.

A. Minimum Phase

Table 1 depicted below shows the values observed while performing the experiment and the calculated values from the equations. It can be seen, level of the tanks are almost in accordance with the calculated values with slight variation. While performing the experiment, value of input was kept as u = 50 % of maximum voltage (7.5 V).

Minimum Phase		
Tanks	Observed Value	Calculated Value
Tank 1	19 cm	21.50 cm
Tank 2	17 cm	18.9 cm
Tank 3	16 cm	16.98 cm
Tank 4	11.5 cm	10 cm

Table 1: Observed Value and Calculated Value at Equilibrium for Minimum Phase

The figure 1 below is the output of the system when u1 = 7.5V and u2 = 0V. It was observed that when input 1 acts, the heights of tank 1 and tank 4 are significantly affected. Also, when u1 = 0V and u2 = 7.5V, tank 2 and tank 3 are coupled. Thus, it can be said that system is coupled. Also, from the above $RGA(G_{mp}(0))$ matrix, it can be seen that the input 1 should be coupled with output 1 i.e u_1 -> h_1 which is exactly what was found experimentally.

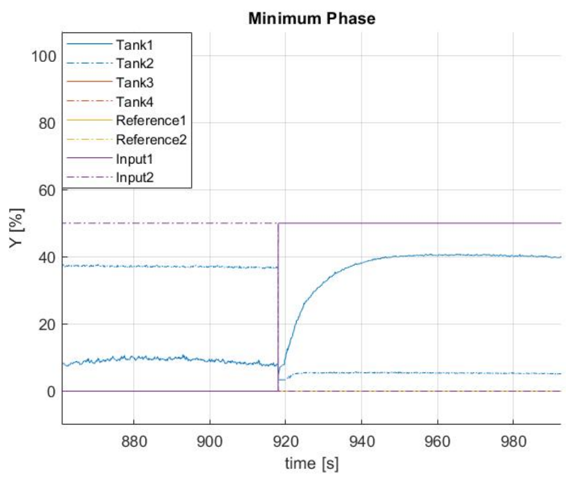

Figure 1: Step responses (two outputs) from one input at a time, tank 1 and tank 4 coupled

The figure 2 presents the reference level for the lower two tanks, in this case it was 62%. The input values u1 and u2 were set manually as u1 = 6V (40 % of maximum voltage, 15V) and u2 = 6.75V (45 % of maximum voltage, 15V). And, calculated the rise time for this plot as 31.13 sec.

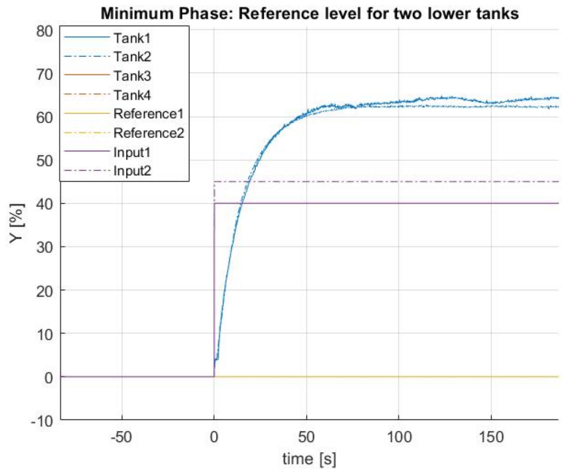

Figure 2: Reference level for lower two tanks in minimum phase

B. *Non-Minimum Phase*

Table 2 depicted below shows the values observed while performing the experiment and the calculated values from the equations. It can be seen, level of the tanks are almost in accordance with the calculated values with slight variation. While performing the experiment, the value of input was kept as u = 50 % of maximum voltage (7.5 V).

Non-Minimum Phase		
Tanks	Observed Value	Calculated Value
Tank 1	17 cm	15.11 cm
Tank 2	20 cm	22.5 cm
Tank 3	7.5 cm	14.29 cm
Tank 4	7 cm	5 cm

Table 2: Observed Value and Calculated Value at equilibrium for Non-Minimum Phase

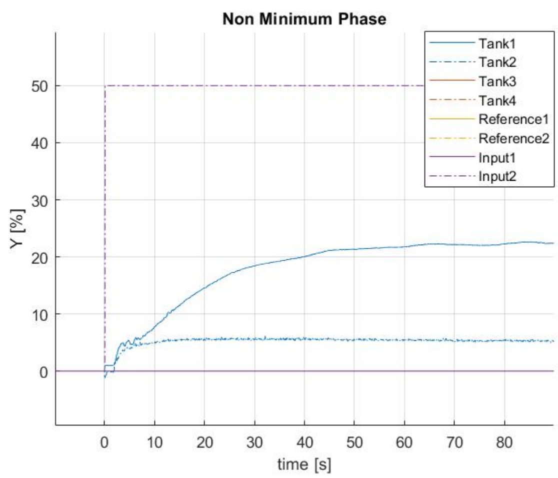

Figure 3: Step responses (two outputs) from one input at a time, tank 1 and tank 3 coupling

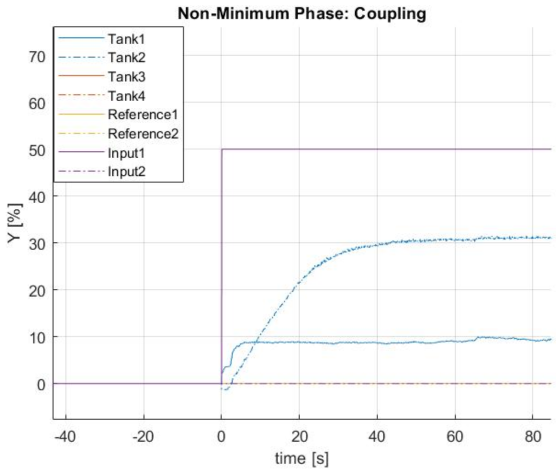

Figure 4: Step responses (two outputs) from one input at a time, tank 2 and tank 4 coupling

The figure 3 is the output of the system when, u1 = 0V and u2 = 7.5V. It is observed that when input acts, the heights of tank 1 and tank 3 are significantly affected. Also, when, u1 = 7.5V and u2 = 0V, found out that tank 2 and tank 4 are coupled. Thus, it can be said that system is coupled. Also, from the above $RGA(G_{nmp}(0))$ matrix, it can be seen that cross-coupling is suggested which means the input 2 affects output 1 i.e u_2 -> h_1 which is exactly what was found experimentally.

The figure 5 presents the reference level for the lower two tanks, in this case it was 60%. The input values u1 and u2 are set manually as u1 = 5.25V (35 % of maximum voltage, 15V) and u2 = 7.5V (50 % of maximum voltage, 15V). And, calculated the rise time for this plot as 25.28 sec.

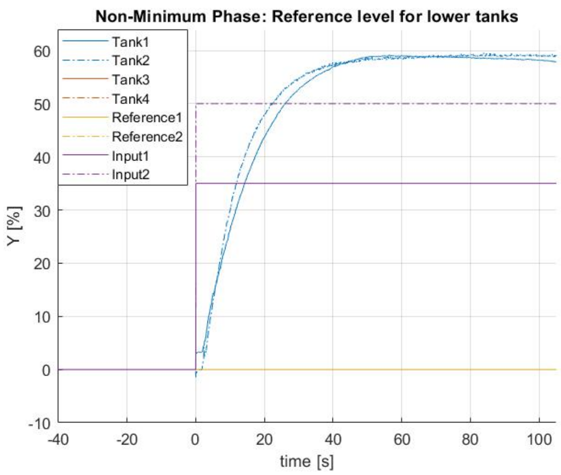

Figure 5: Reference level for lower two tanks in non-minimum phase

C. *Differences*

While working on manual control for minimum and non minimum phase, it could be concluded that the major difference was as follows. In case of non-minimum phase it took many iterations to set u1 and u2 to get the desired performance as compared to the minimum phase, where the intended performance quite quickly. Also, as per the manual instructions, tanks configuration was different for both minimum and non-minimum phase.

III. Calculation of Controllers

By using the value of k_1, k_2, and the respective areas for the holes in the minimum and non-minimum phase as derived above, the controllers were calculated. These values were modified in the files minphase.m and nonminphase.m and the files from the computer exercise 4 were run to obtain the necessary controllers. Both, the decentralized controller and the Glover-McFarlane controller were calculated for both minimum and non minimum Phase cases.

IV. Decentralized control

In this section, decentralized controllers for minimum phase and non-minimum phase is discussed.

A. *Minimum Phase*

The figure [6], shows the response of the system when a decentralized controller was used. To check the effectiveness of the controller, the reference values were modified to the tune of atleast 5 percentage points.

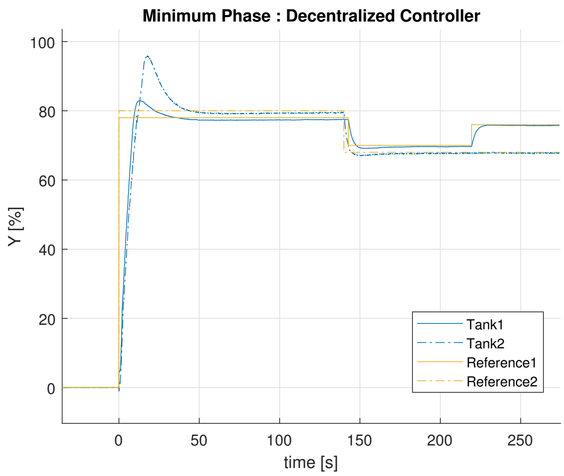

Figure 6: Step Response of decentralized controller in minimum phase for variation in input signal.

The variation in the reference values were done as follows :

Minimum Phase		
Variation	Tank 1	Tank 2
Case 1	78%	80%
Case 2	70%	68%
Case 3	76%	68%

Table 3: Variation in Tank reference values

All the variations were done simultaneously as and when the system became stationary. And, in all three situations, it can be seen clearly that the controller is highly effective and corrects itself with the respective changes.
The rise time for tank 1 is seen to be 6.82 secs and for tank 2 to be 8.89 secs. The overshoot for tank 2 is noted to be slightly higher (19.875%) here as compared to that of tank 1 (7.46%).

The figure [7] shows the system response when external load disturbances were added to the system.

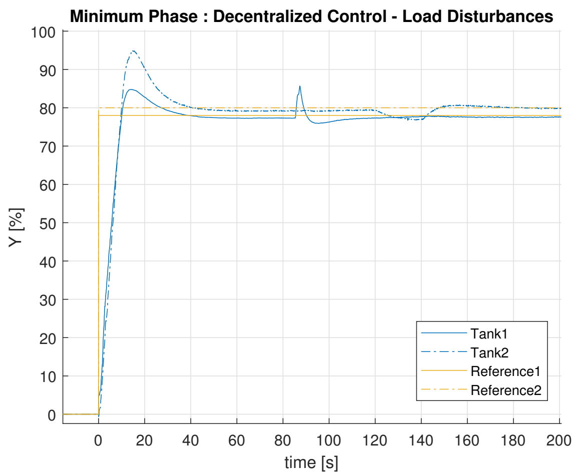

Figure 7: Step Response of decentralized controller in minimum phase for load disturbances.

The performance of the system was good in different load disturbance scenarios, first when an extra cup of water was added to tank 1 at t = *85 sec* and second when an extra outlet was opened on tank 4 at t = *120 sec*. The system was able to correct itself and eliminate the load disturbances in 25 to 30 secs time interval for both the scenarios as can be seen in figure [7].

B. Non - Minimum Phase

The non minimum phase controller was successfully run on the real time system but the controller was found to be comparatively very slow. It took considerable amount of time to reach the equilibrium levels. The observations are as follows :
The figure [8], shows the response of the system when a decentralized controller was used. To check the effectiveness of the controller, the reference values were modified to the tune of at least 5 percentage points.

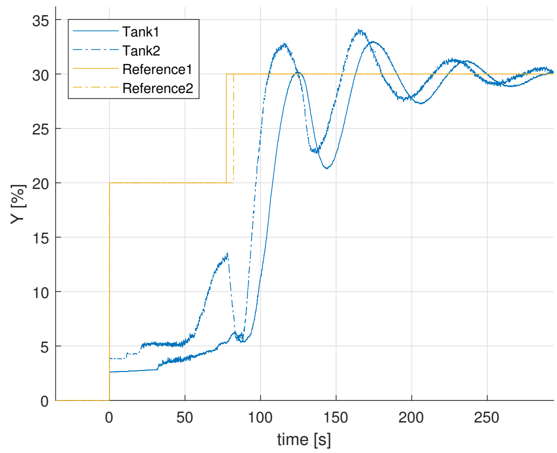

Figure 8: Step Response of decentralized controller in non-minimum phase for variation in input signal.

As it was difficult to make to system track its equilibrium points or any higher reference value, small step wise input changes were given to the reference level so that controller performs as needed. The final reference levels was set at 30% for both the tanks.
It can be seen that the controller settles over time but takes a lot of time to settle i.e. around 207 sec for tank 1 and 200 sec for tank 2. The rise time was around 83 seconds for tank 1 and around 100 seconds for tank 2. The maximum overshoot for tank 2 was 9.2% while for tank 1 it was 9.7% on the second oscillation.
The figure [9] shows the system response when load disturbances were added to the system.

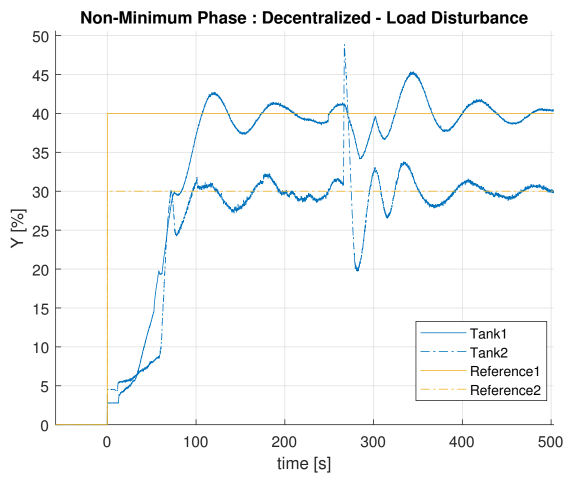

Figure 9: Step Response of decentralized controller in non-minimum phase for load disturbances.

The system performed alright and as expected in different load disturbance scenarios, first when an extra cup of water(w_1) was added to tank 2 at t = *264 sec* and second when an extra outlet was opened on tank 3(w_2) at t = *270 sec*. The system was able to correct itself and eliminate the load disturbance w_1 in 59.8 sec and the disturbance w_2 in 110.6 sec as can be seen in figure [9].

The decentralized controller designed for the non-minimum phase was also run on the simulator before performing the experiment on the real time system which helped us to assess its performance and the figure [10] shows the response.

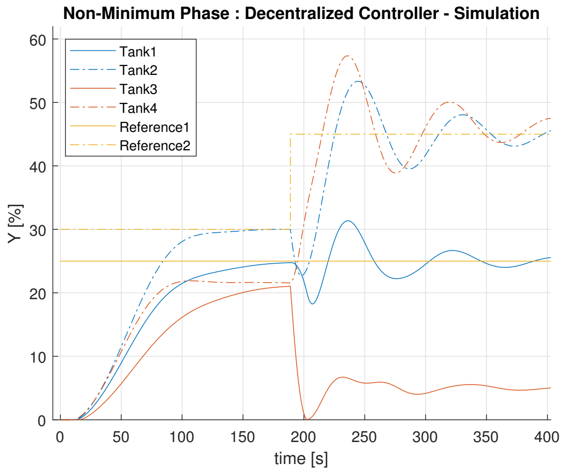

Figure 10: Step Response of decentralized controller in non-minimum phase on simulator.

In the figure [10], it can be seen that the rise time even in the simulation was over 100 seconds for both the tanks. It can also be seen that the settling time for both tanks, 1 and 2 is very high. These observations were verified by the results which were obtained in the real time situation for the controller.

C. *Differences between Minimum and Non-Minimum Phase of Decentralized Controller*

The main difference between the minimum phase and non-minimum phase was the settling time. The controller for the minimum phase had a considerably smaller settling time where as in the non-minimum phase, the system took a significant amount of time to reach the reference levels. Even the rise time in non minimum were higher than those of minimum phase but the overshoot in non-minimum was seen to be smaller. Refer figure [6] and figure [8] for the same.

The minimum phase performed better when it came to load disturbances and eliminated the disturbance faster than non-minimum phase. Again due to the high settling time, the correction of the response took a lot of time in the non-minimum phase as compared to the that of minimum phase Refer figure [7] and figure [9] for the same.
Also it is important to note that, implementation of the non-Minimum Phase decentralized controller on the real time apparatus is a typical task, which does not provide with consistent results which can be erratic at certain points. This is primarily due to its high settling time as any small disturbance affects the system which leads to increase in time to reach the steady state.

V. Robust Control

In this section, robust control via GLover-McFarlane controller for minimum phase and non-minimum phase is discussed.

A. *Minimum Phase*

The figure [11], shows the response of the system when a Glover-McFarlane controller is used. To check the effectiveness of the controller, the reference values were modified to the tune of atleast 5 percentage points.

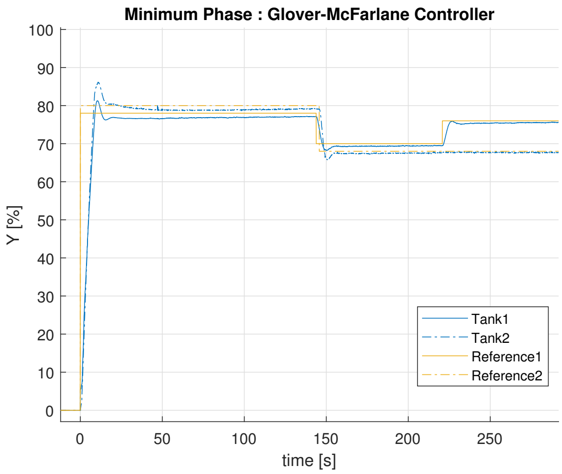

Figure 11: Step Response of Glover-McFarlane controller in minimum phase for variation in input signal.

The variation in the reference values were done as follows :

Minimum Phase		
Variation	Tank 1	Tank 2
Case 1	78%	80%
Case 2	70%	68%
Case 3	76%	68%

Table 4: Variation in Tank reference values

All the variations were done simultaneously as and when the system became stationary. And, in all three situations, it can be seen clearly that the controller is highly effective and corrects itself with the respective changes.
The rise time for tank 1 is seen to be 6.62 secs and for tank 2 to be 5.82 secs. The overshoot for tank 1 is 4.11% and 7.62% for tank 2 .

The figure [12] shows the system response when load disturbances are done to the system.

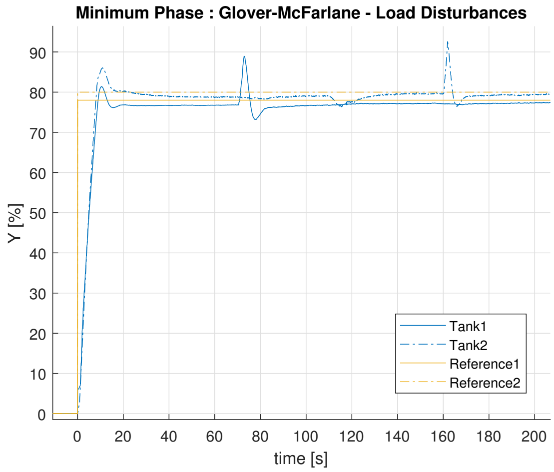

Figure 12: Step Response of Glover-McFarlane controller in minimum phase for load disturbances.

The system performed well in different load disturbance scenarios, first when an extra cup of water was added(w_1) to tank 1 at t = *70 sec* and second when an extra outlet was opened on tank 4(w_2) at t = *111.7 sec*. A third load disturbance(w_3) was also added to the system here, a cup of water to tank 2 at t = *157.4 sec*. The system corrected itself and eliminated the load disturbances in 18 to 25 secs time interval for all three scenarios as can be seen in figure [12].

B. Non - Minimum Phase

The non minimum phase controller was successfully run on the real time system but the controller was found to be very slow. It took considerable amount of time to reach equilibrium levels. The observations were as follows :
The figure [13], shows the response of the system when a Glover-McFarlane controller is used. To check the effectiveness of the controller, the reference values are modified to the tune of atleast 5 percentage points.

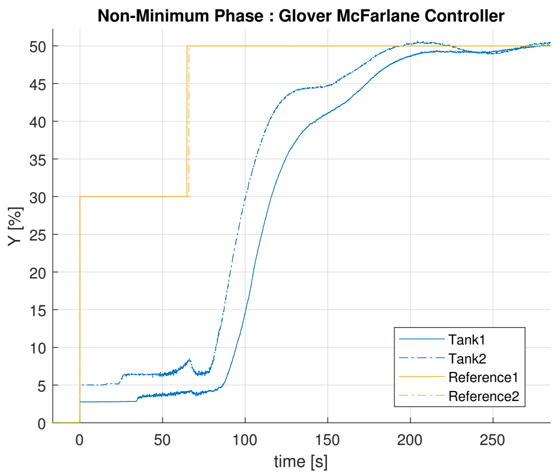

Figure 13: Step Response of Glover-McFarlane controller in non-minimum phase for variation in input signal.

A small step wise input change was given to the reference levels so as the controller performs as needed. When direct values were given for the reference levels, the controller took a very long time to reach the reference levels. The reference levels were set at 50% for both the tanks.
It can be seen that the controller settles over time but is slow in nature, the rise time is around 150 sec for tank 2 and is about 170 sec for tank 1. The over shoot is very low for tank 2 i.e. 1% and zero for tank 1.

The figure [14] shows the system response when load disturbances were added to the system.

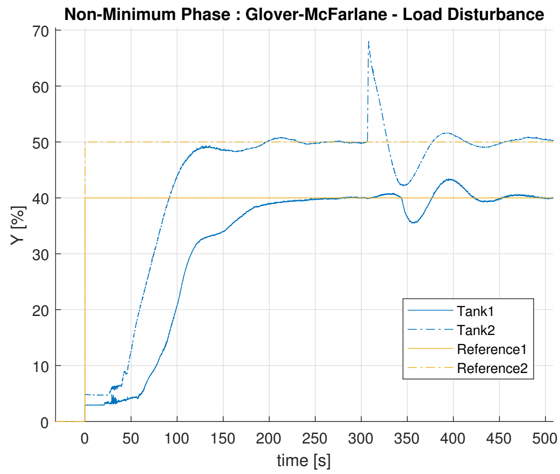

Figure 14: Step Response of Glover-McFarlane controller in non-minimum phase for load disturbances.

The system performed reasonably well in different load disturbance scenarios, first when an extra cup of water was added to tank 2(w_1) at t = *305 sec* and second when an extra outlet was opened on tank 3(w_2) at t = *342 sec*. The system was able to correct itself and eliminate the load disturbances but the time taken is very high, almost 75 seconds for w_1 and 85 sec for w_2 as can be seen in figure [14].

C. Difference between Decentralized and GM Controller

As it can be seen from the Figures [6], [8], [11] and [13], the Glover McFarlane controller is a more robust and efficient controller than the decentralized controller, in both minimum and non minimum scenarios. The rise time is smaller with reduced overshoot and decreased settling time in all cases.
Even the elimination of the load disturbances in Glover-McFarlane is faster than it is in decentralized controller. The Figures [7] and [14] clearly show that GM has better ability to compensate disturbances.

D. Differences between Minimum and Non-Minimum Phase of GM Controller

The main difference between the minimum phase and non-minimum phase was the settling time. The controller for the minimum phase had a considerably smaller settling time where as in the non-minimum phase, the system took a significant amount of time to reach the reference levels. Even the rise time in non minimum were higher than those of minimum phase but the overshoot in non-minimum was seen to be smaller. Refer figure [11] and figure [13] for the same.

The minimum phase performed better when it came to load disturbances and eliminated the disturbance faster than non-minimum phase. Again due to the high settling time, the correction of the response took a lot of time in the non-minimum phase as compared to the that of minimum phase Refer figure [12] and figure [14] for the same.

VI. Conclusion

This report was a summary of our experience in the project lab of EL2520 course and contained our observations and results. In this laboratory project, we performed System Modelling, Manual Analysis of the Four-tank process, Designing of different controllers and real-time testing of performance of controllers for both Minimum phase and Non-minimum phase cases. This helped our team to gain insights on various aspects of Process control and the differences between theoretical and real-time control.