Depth-First Search (DFS)

CS 4102: Algorithms

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Readings

- CLRS:
  - Section 22.3 on DFS
  - Later/eventually:
    - Section 22.4 on Topological Sort
    - Section 22.5 on Strongly Connected Components

DFS: the Strategy in Words

- Depth-first search: Strategy
  - Go as deep as can visiting un-visited nodes
    - Choose any un-visited vertex when you have a choice
  - When stuck at a dead-end, backtrack as little as possible
    - Back up to where you could go to another unvisited vertex
  - Then continue to go on from that point
  - Eventually you'll return to where you started
    - Reach all vertices? Maybe, maybe not

Observations about the DFS Strategy

- Note: we must keep track of what nodes we've visited
- DFS traverses a subset of E (the set of edges)
  - Creates a tree, rooted at the starting point: the Depth-first Search Tree (DFS tree)
  - Each node in the DFS tree has a distance from the start. (We often don't care about this, but we could.)
- At any point, all nodes are either:
  - Un-discovered
  - Finished (you backed up from it), or
  - Discovered (i.e. visited) but not finished
    - On the path from the current node back to the root
    - We might back up to it
  - (Later we'll call these states: white, black and gray respectively)

DFS Strategy 1: Use a stack

- Maintain a Stack (Let's call it S)

- Start at some node 's' (push 's' to S and mark as visited)
- While S not empty
  - Pop a node 'n' from S
  - Process 'n' if necessary (depending on problem you are solving)
  - For each non-visited neighbor of 'n'
    - Mark neighbor as visited
    - Push neighbor onto S
  - Repeat

- Sound familiar? Same as BFS but uses stack instead of queue!
- Or we can implement recursively (see next slide)

DFS Strategy #2

- Use a recursive function to "visit" each node
  - Need a non-recursive function to initialize and make first call

- Before we look at this code… Important!
  - Best to think of DFS is a strategy as well as a single, particular bit of pseudo-code
  - We often add things to DFS code to solve problems

  - "Swiss Army Knife" of graph algorithms?

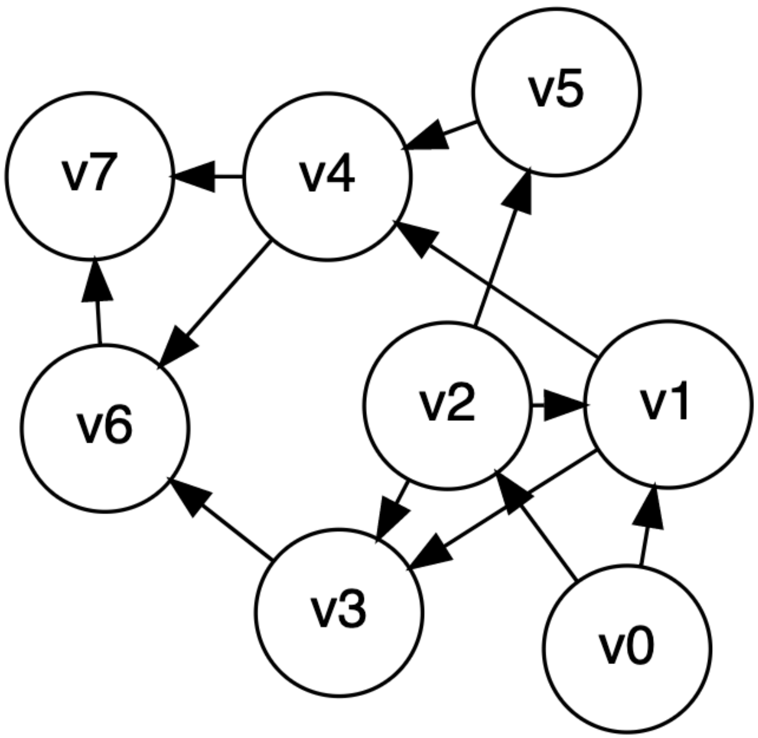
DFS Strategy 2: Recursion

```
def dfs(graph, start):                          //Main loop, inits and calls
    visited = {}
    dfs_recurse(graph, start, visited)


def dfs_recurse(graph, curnode, visited):       //sometimes called dfs_visit()
    visited[curnode] = True
    alist = graph.get_adjlist(curnode)          //get the neighbors of curnode
    for v in alist:
        if v not in visited:
            print "  dfs traversing edge:", curnode, v
            dfs_recurse(graph, v, visited)
    # end for-all adjacent vertices
    return
```

depth-first search, example

- Let's start at V0



DFS to Process all Vertices in a Graph

- Purpose: do all required initializations, then call dfs_recurse() as many times as needed to visit all nodes.
  - May create a DFS forest.
- Can be used to count connected components
  - Could remember which nodes are in each connected component

```
def dfs_sweep(graph, start):
    visited = {}

    # loop repeats DFS on every unvisited node
    for v in graph:
        if v not in visited:
            dfs_recurse(graph, v, visited)
```

Using DFS to Find if a Graphic is Acyclic

- Does a graph have a cycle?
  - DFS is great for this
  - But, slightly harder if graph is undirected

- Use DFS tree: classify edges and nodes as you process them
  - Nodes:
    - White: unvisited
    - Black: done with it, backed up from it (never to return)
    - Gray: Have reached it; exploring its adjacent nodes; but not done with it

CLRS's DFS Algorithm (non-recursive part)

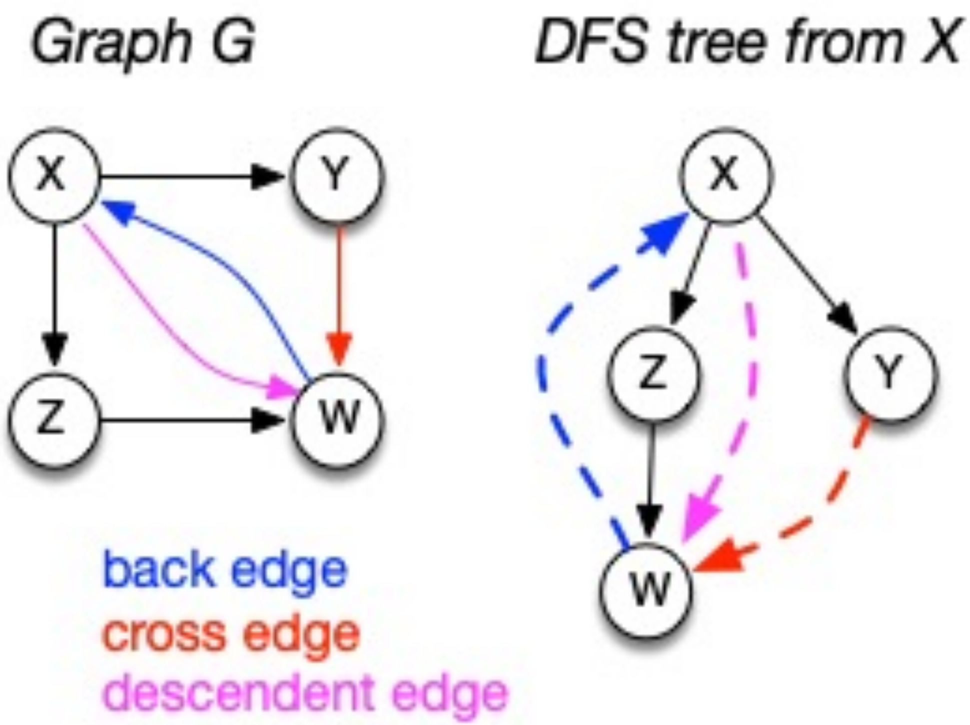
```
DFS(G)
for each vertex u in G.V
    u.color = WHITE
    u.π = NIL
time = 0
for each vertex u in G.V
    if u.color == WHITE  // if unseen
        DFS-VISIT(G, u)  // explore paths out of u
```

CLRS's DFS Algorithm (recursive part)

```
DFS-VISIT(G, u)
time = time + 1  // white vertex u has just been discovered
u.d = time  // discovery time of u
u.color = GRAY  // mark as seen
for each v in G.Adj[u]  // explore edge (u, v)
    if v.color == WHITE   // if unseen
        v.π = u
        DFS-VISIT(G, v)  // explore paths out of v (i.e., go "deeper")
u.color = BLACK  // u is finished
time = time + 1
u.f = time  // finish time of u
```

Depth-first search tree

- As DFS traverses a digraph, edges classified as:
  - tree edge, back edge, descendant edge, or cross edge
  - If graph undirected, do we have all 4 types?



Using Non-Tree Edges to Identify Cycles

- From the previous graph, note that:
- Back edges (indicates a cycle)
  - dfs_recurse() sees a vertex that is gray
  - This back edge goes back up the DFS tree to a vertex that is on the path from the current node to the root
- Cross Edges and Descendant Edges (not cycles)
  - dfs_recurse() sees a vertex that is black
  - Descendant edge: connects current node to a descendant in the DFS tree
  - Cross edge: connects current node to a node in another subtree – not a descendant of current node

Non-tree Edges in DFS

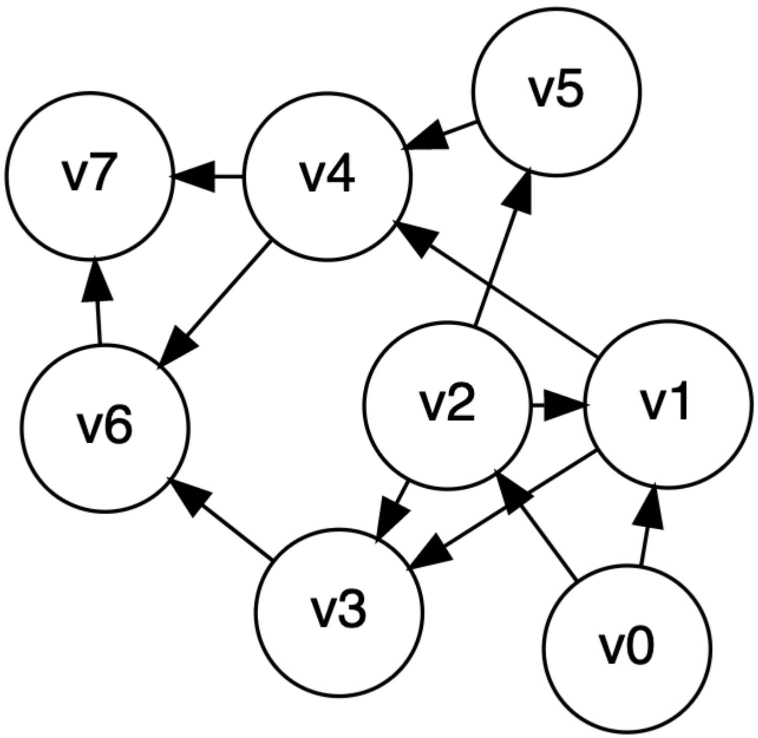
- Question 1: Finding back edges for an undirected graph is not **quite** this simple:
  - The parent node of the current node is gray
  - Not a cycle, is it? It's the same edge you just traversed
  - Question: how would you modify our code to recognize this?
- Question 2:
  - In digraph, how could you modify the code to distinguish cross edges from descendant edges?
  - Need to record the "time" at which a node was discovered (set to "gray") and finished (set to "black")
  - Also, have a "time counter", say, ctr
    - Set d[v] = ctr++ as discovery time
    - Set f[v] = ctr++ as finish time

Time Complexity of DFS

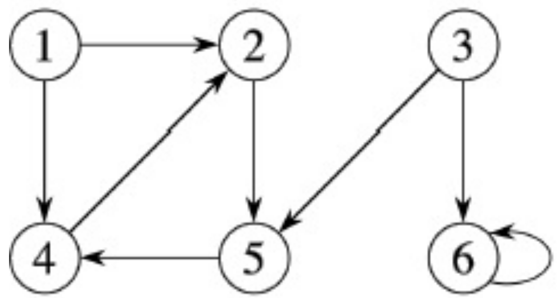
- For a digraph having V vertices and E edges
  - Each edge is processed once in the while loop of dfs_recurse() for a cost of $\theta(E)$
    - Think about adjacency list data structure.
    - Traverse each list exactly once. (Never back up)
    - There are a total of 2*E nodes in all the lists
  - The dfs_sweep() algorithm will do $\theta(V)$ work even if there are no edges in the graph
  - Thus over all time-complexity is $\theta(V+E)$
    - Remember: this means the larger of the two values
    - Note: This is considered “linear” for graphs since there are two size parameters for graphs.
  - Extra space is used for color array.
    - Space complexity is $\theta(V)$

depth-first search, example

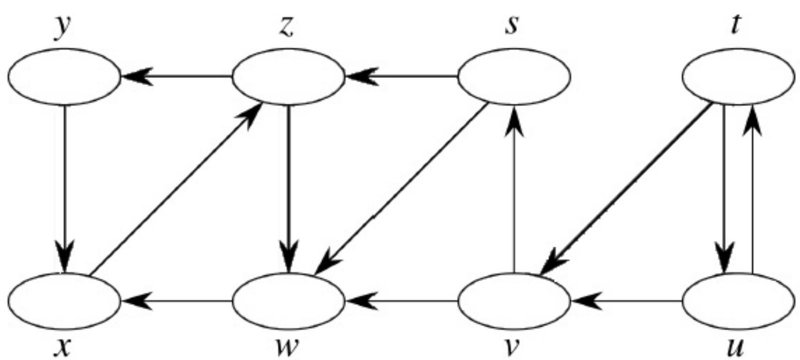
- Let's start at V0



DFS Examples

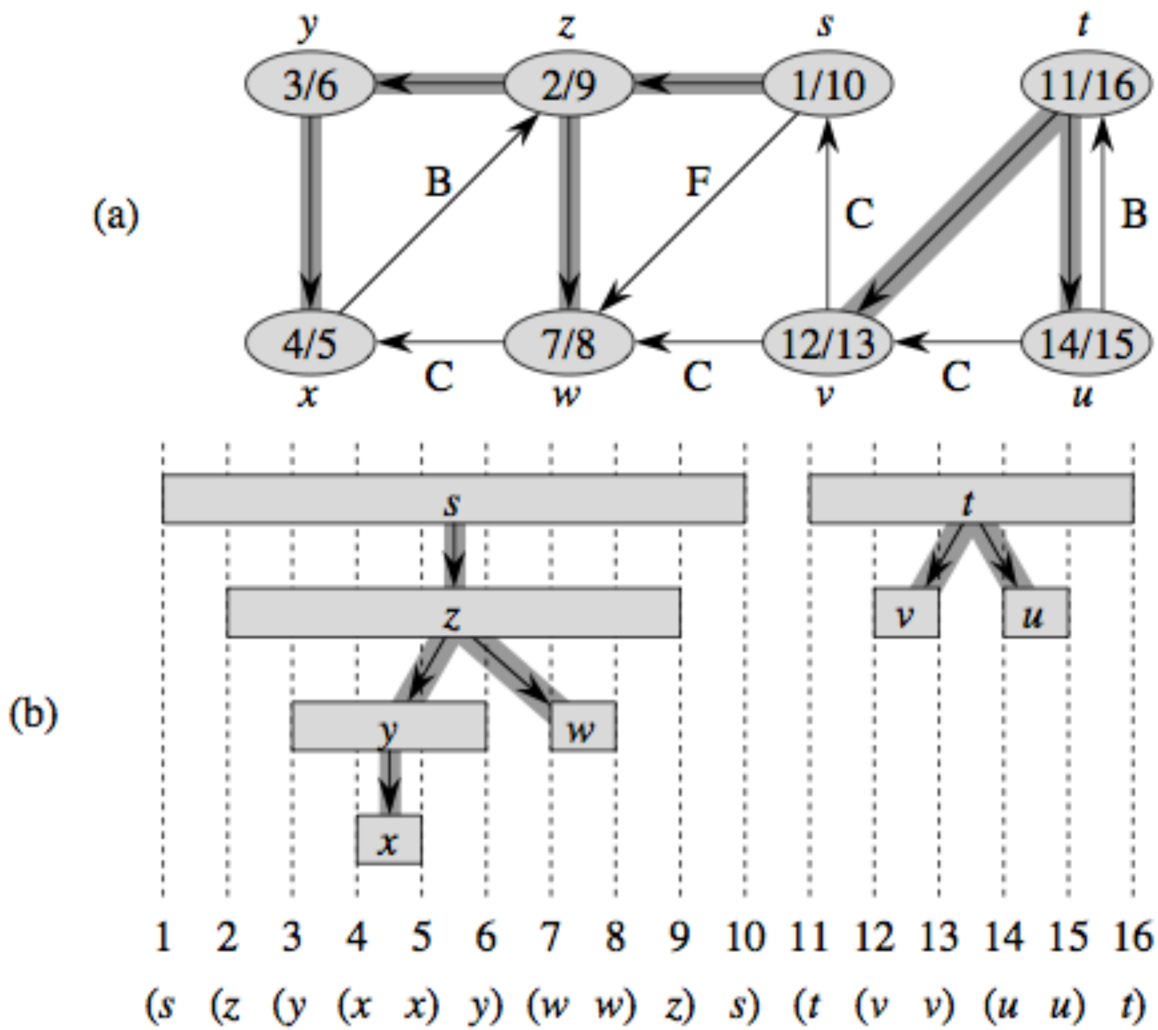


- Source vertex: 1



- Source vertex: s

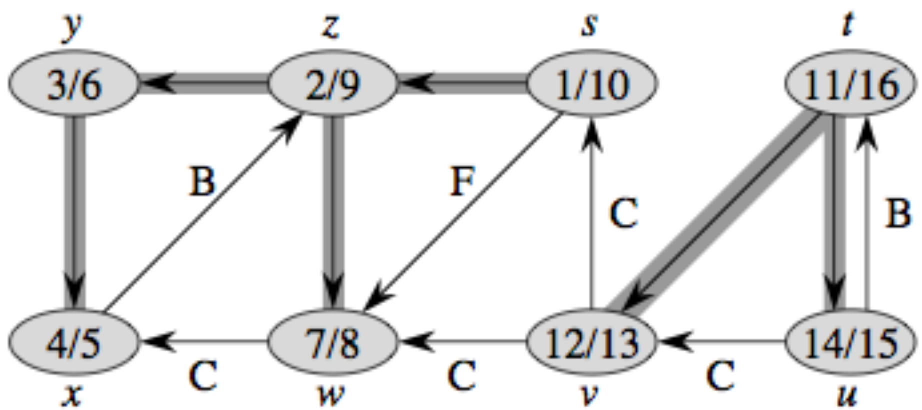
Properties of DFS Search, DFS Trees



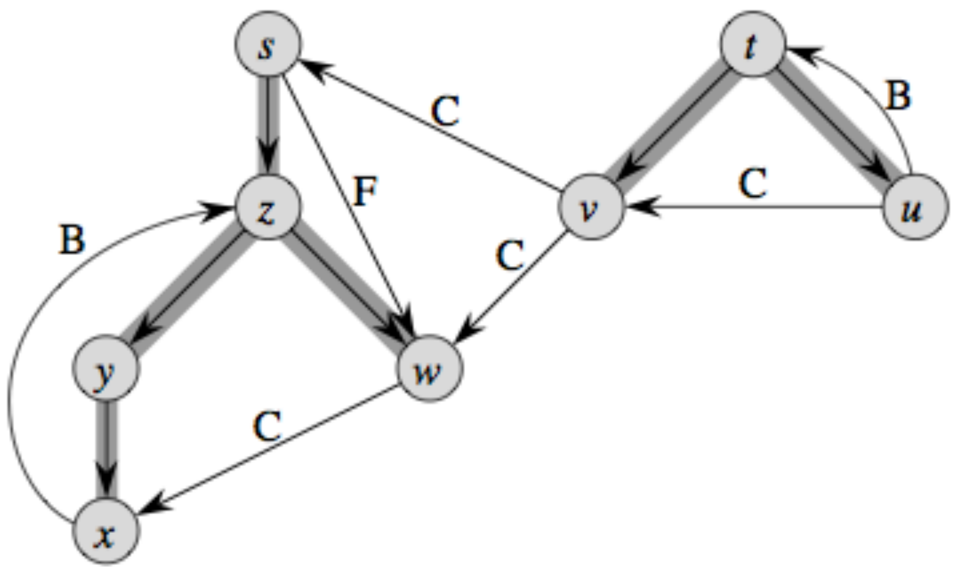
- "Parentheses Structure". See pp. 606-609

Properties of DFS Search, DFS Trees

(a)



(c)



- Edge Classification. See pp. 606-609

Summary

What Did We Learn?

- Traversals of graphs:
  - Breadth-first search
  - Depth-first search
- Coming next – applying these graph algorithms:
  - Topological Sort