Divide and Conquer / Sorting Basic: Recurrence Relations

Directly solve, by unrolling the recurrence, the following relations to find their exact solutions.

1. $T(n) = T(n-1) + n$

2. $T(n) = T(\frac{n}{2}) + 1$

Use induction to show bounds on the following recurrence relations.

3. Show that $T(n) = 2T(\frac{n}{2}) + n \in \Omega(nlog(n))$. *Note: We are using big-omega here, so your inequality will use* $T(n) \geq c * n * log(n)$.

4. Show that $T(n) = 2T(\sqrt{n}) + log(n) \in O(log(n) * log(log(n)))$. *Hint: Try creating a new variable m and substituting the equation for m to make it look like a common recurrence we've seen before. Then solve the easier recurrence and substitute n back in for m at the end.*

5. Show that $T(n) = 4T(\frac{n}{2}) + n \in \Theta(n^2)$. You'll need to subtract off a lower-order term to make the induction work here. *Note: we are using big-theta here, so you'll need to prove the upper AND lower bound.*

6. Show that $T(n) = 4T(\frac{n}{3}) + n \in \Theta(n^{log_3(4)})$. You'll need to subtract off a lower-order term to make the induction work here. *Note: we are using big-theta here, so you'll need to prove the upper AND lower bound.*

Use the master theorem (or main recurrence theorem if applicable) to solve the following recurrence relations. State which case of the theorem you are using and why.

7. $T(n) = 2T(\frac{n}{4}) + 1$

8. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

9. $T(n) = 2T(\frac{n}{4}) + n$

10. $T(n) = 2T(\frac{n}{4}) + n^2$