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1(a) If we can get a uniform distribution on a circle, centered at origin, then we can scale it along x-axis to get the required ellipse.

For circle,

if we can get a distribution where pdf(r) $\propto r$, then we can ~~distribute~~ model it as pdf of r such that it denotes probability of points occuring at r distance from centre.

And then we can further distribute it over the periphery uniformly which makes the whole distribution uniform in circle.

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~~Lets~~ say
Now we have uniform random variable U on $[0,1]$

$$pdf(u) = \begin{cases} 1 & 0 \le u \le 1 \\ 0 & \text{otherwise} \end{cases}$$

~~Let~~ R model our variable such that,

$$pdf(r) = kr \quad \forall\ r \in [0,1]$$
$$= 0 \quad \text{otherwise}$$

$$\therefore \int_0^1 kr\,dr = 1$$

$$\cancel{k = 2} \quad \frac{k}{2} = 1$$

$$(k = 2)$$

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$\therefore$ We have to get transformation function g such that

$$R = g(U)$$

Let $h(x) = g^{-1}(x)$.

$$\therefore \quad pdf_R(x) = pdf_U(g^{-1}(x)) \times \left|\frac{d}{dx} g^{-1}(x)\right|$$

- As $\frac{d}{dx}(pdf_R(x)) > 0$

$$\therefore \quad \frac{d}{dx} g^{-1}(x) > 0$$

Let $pdf_R = q$

& $pdf_U = p$

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$\therefore\ q(x) = p(h(x))\,\dfrac{dh(x)}{dx}$

for $x \in [0, 1]$

$p(h(x)) = \cancel{h(x)}\ 1$

& $q(x) = 2x$

$\therefore\ 2x = \dfrac{dh(x)}{dx}$

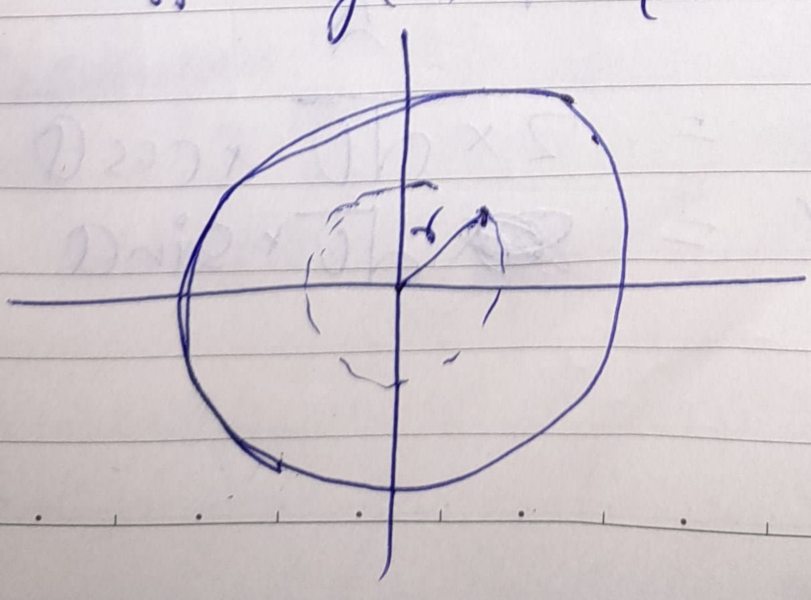
$h(x) = x^2 + C$

$h(0) = 0$ & $h(1) = 1$

$\boxed{C = 0}$

$\therefore\ h(x) = x^2$

or $g(x) = \sqrt{x}$



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∴ If $g(x) = \sqrt{x}$

then $g^{-1}(y) = y^2$

∴ $pdf_R(y) = pdf_U(y^2) \left| \frac{d}{dy} y^2 \right|$

$pdf_R(y) = 2y$

If we distribute θ uniformly over $[0, 2\pi]$,

then $X = R \cos\theta$

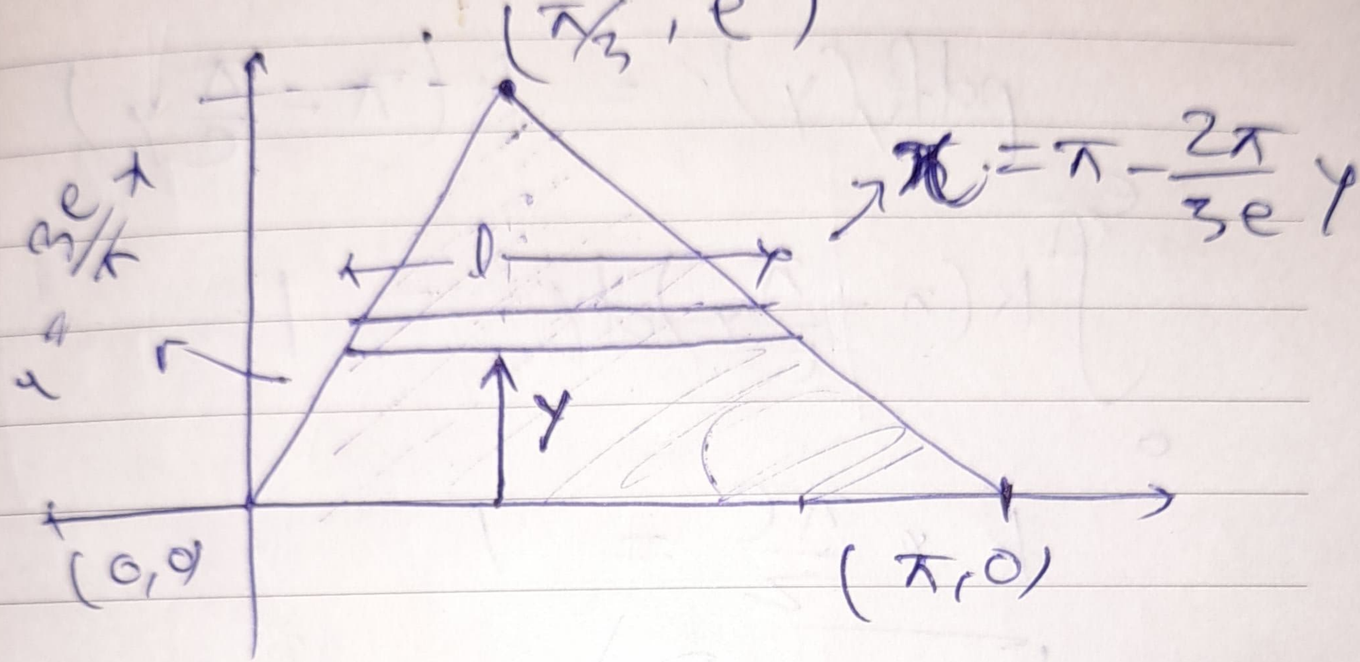
$Y = R \sin\theta$

For ellipse of major axis 2 & minor axis 1

$X = 2 \times \overset{R}{\sqrt{U}} \times \cos\theta$

$Y = \sqrt{U} \times \sin\theta$

1.(a) Our distribution should be like



$$l = \pi - \frac{2\pi}{3e} y - \frac{\pi y}{3e}$$

$$l = \pi - \frac{\pi}{e} y$$

if distribution of $y \propto l$, then we can distribute x ~~between~~ in $\left[\frac{\pi y}{3e}, \pi - \frac{2\pi}{3e} y\right]$ uniformly.

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Let $pdf_Y(y) = kd$

$$pdf_Y(y) = k\left(\pi - \frac{\pi}{e} y\right)$$

$$\int_0^e k\left(\pi - \frac{\pi}{e} y\right) dy = 1$$

$$k \times \frac{\pi e}{2} = 1$$

$$\boxed{k = \frac{2}{\pi e}}$$

Let $pdf_Y(y) = q(y)$

$$\therefore \quad q(y) = \frac{2}{e}\left[1 - \frac{y}{e}\right]$$

We have to get transformation function such that

$$Y = g(U)$$

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$\therefore \; g(y) = e\left[1-\sqrt{1-y}\right]$

$g^{-1}(y) = 1-\left(1-\frac{y}{e}\right)^2$

$\therefore \; \frac{d}{dy} g^{-1}(y) = +2\left(1-\frac{y}{e}\right)\left(\frac{+1}{e}\right)$

$\therefore \; q(y) = p(g^{-1}(y)) \times \left|\frac{d}{dy} g^{-1}(y)\right|$

$\qquad\qquad\qquad \scriptstyle{1}$

$= \frac{2}{e}\left[1-\frac{y}{e}\right]$

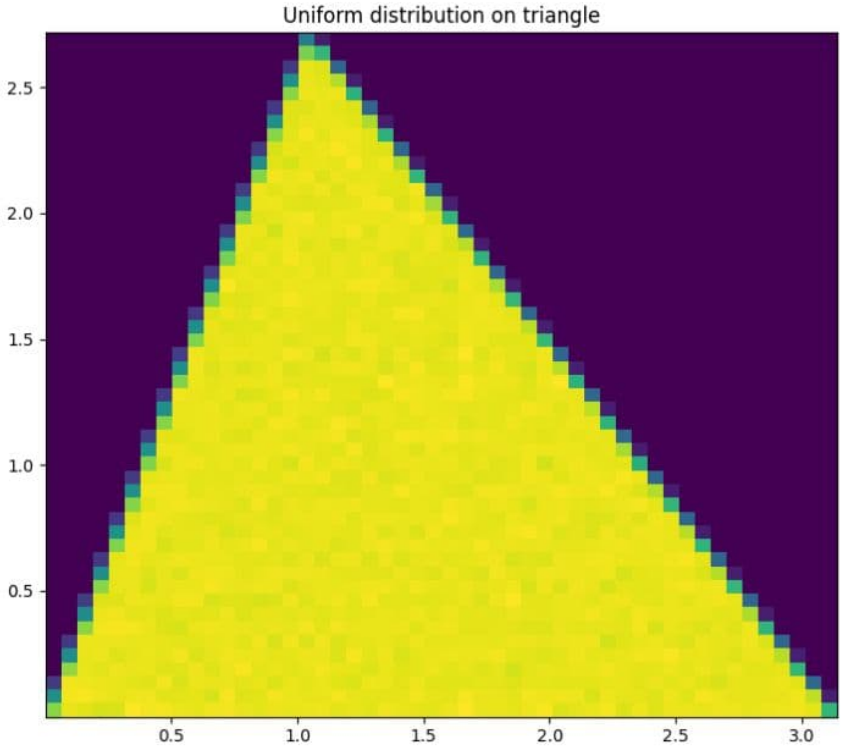
As required

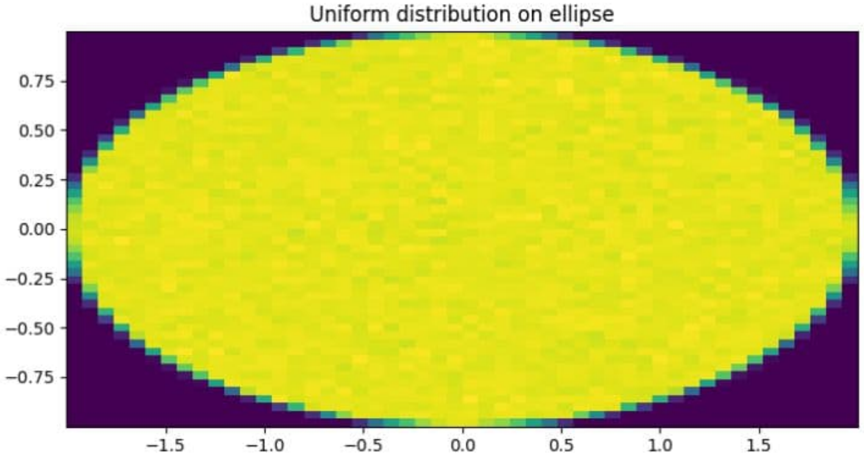
$\therefore$ First we get uniform random variable in $[0,1]$

then

$Y = g(u)$

& then for X we use the interval $\left[\frac{\pi Y}{3e}, \pi - \frac{2\pi Y}{3e}\right]$ uniformly distributed.


Uniform distribution on triangle
2.5
2.0
1.5
1.0
0.5
0.5
1.0
1.5
2.0
2.5
3.0


Uniform distribution on ellipse
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5