6a) Mean image is as it is, but eigen vetors are shown by first shifting such that minimum is 0 & then scaling such that maximum will be 1 for all entries of matrix.

v_1, v_2, v_3 & v_4 are 4 asked eigenvetors.

Graph for top 10 eigen values is as expected decreasing steeply and (last 2 or 3 doesn't have much difference compared to larger ones.

Image representation of mean and eigenvectors

Graph for top 10 eigenvalues
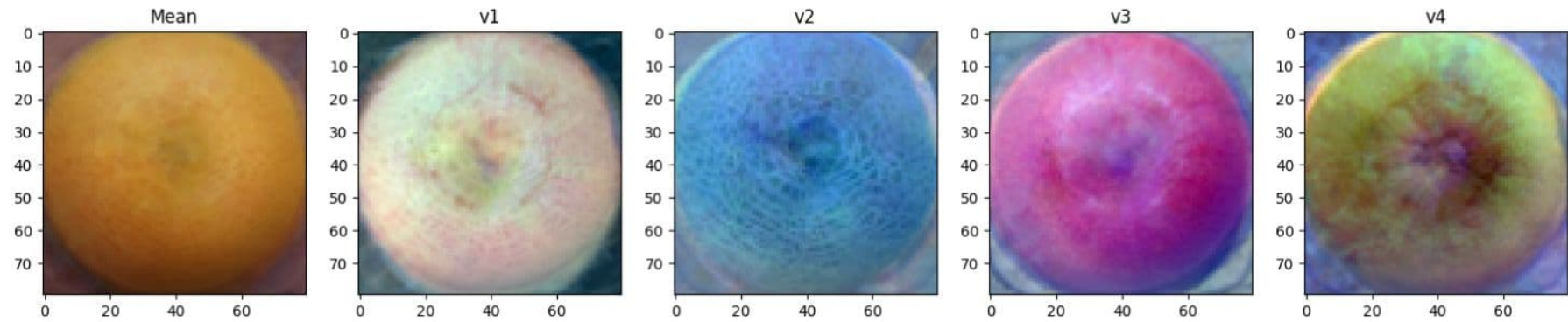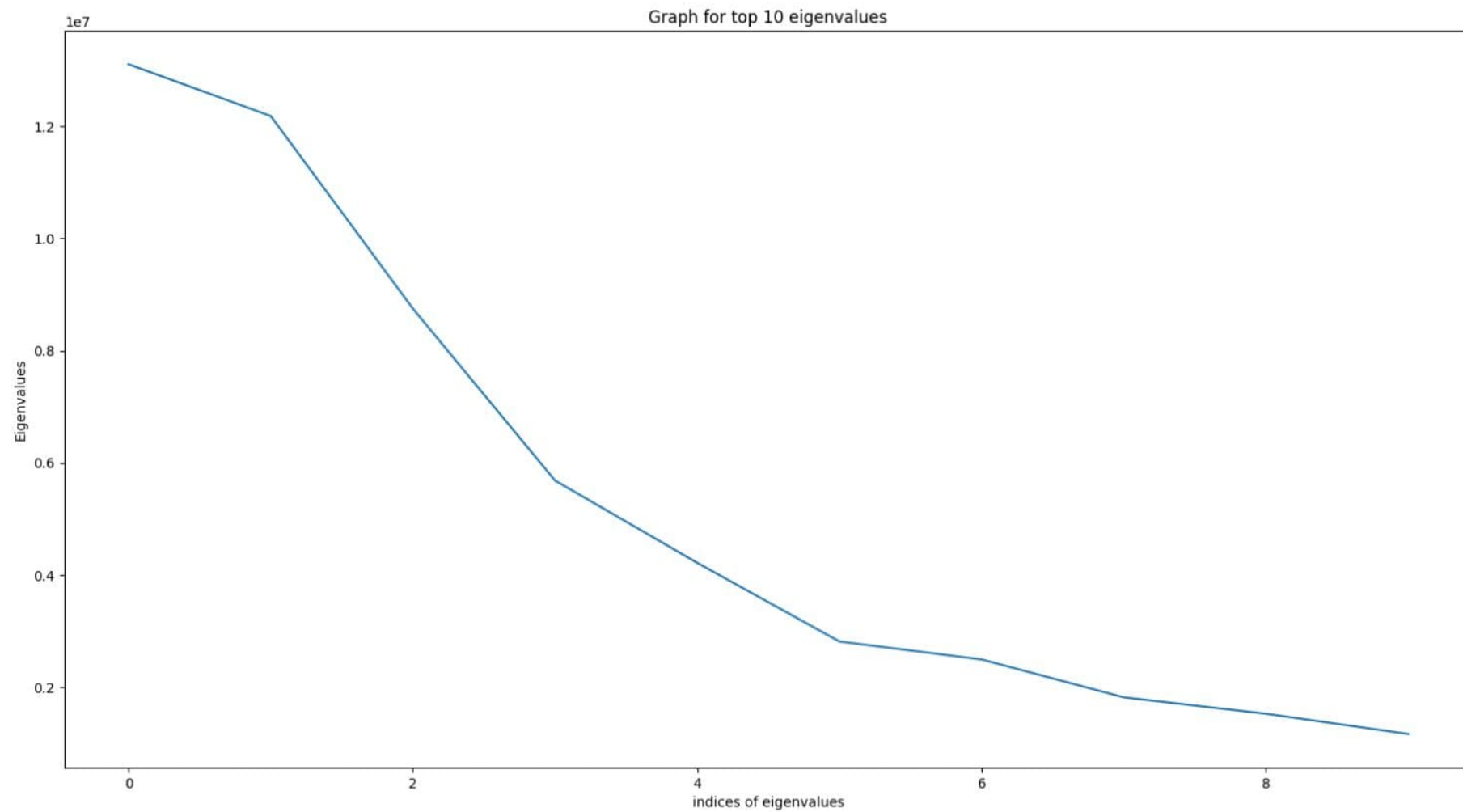
4) We have closest representation by using top 4 eigenvectors as basis similar to Q5 and then representing each fruit as a projection on that 4 dimensional hyperplane.
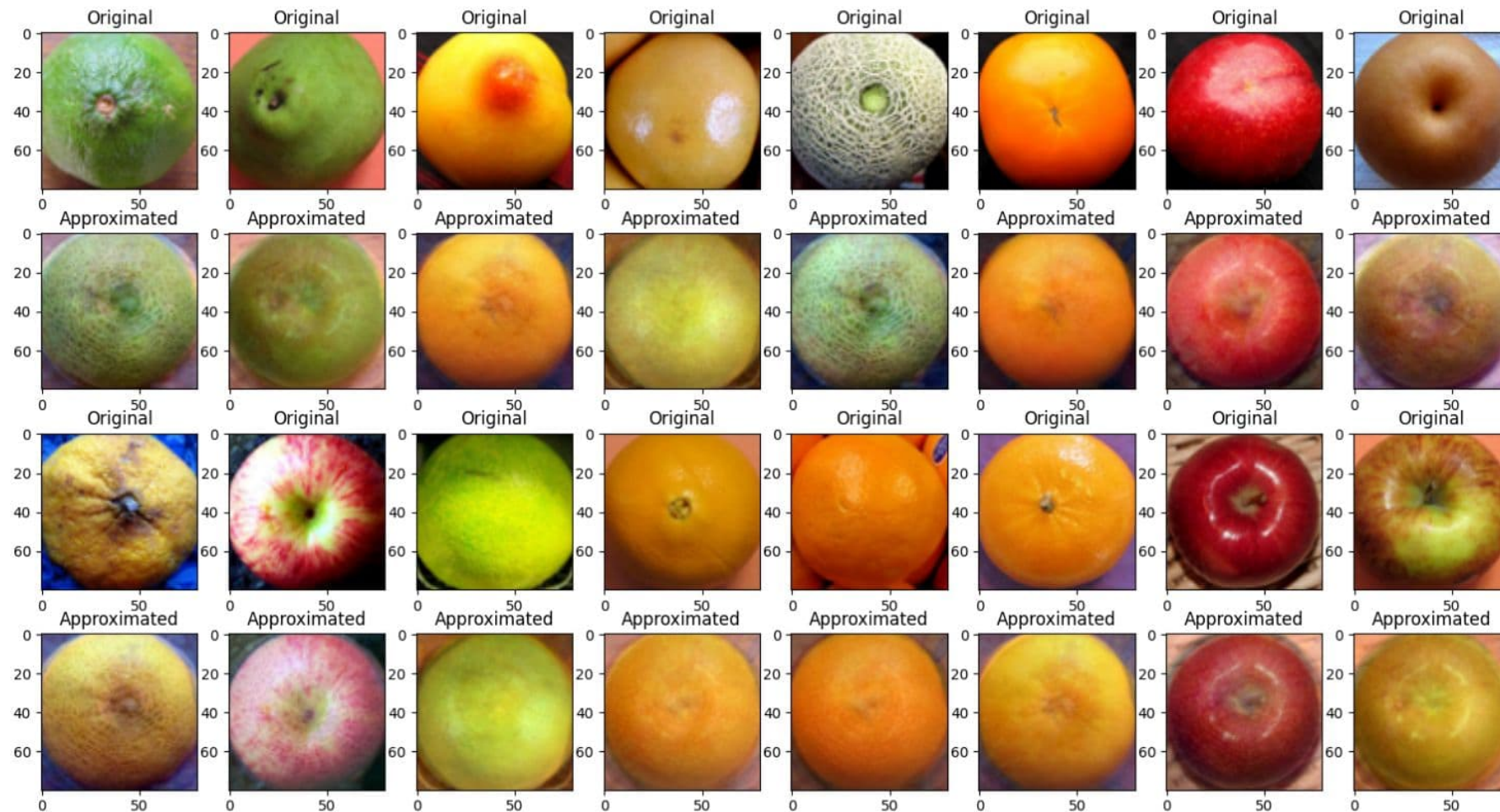
First we calculated the 4 co-ordinates for the basis and let's say c_1, c_2, c_3 & c_4 then approximated image was

$$approx\_image = mean + c_1 \times v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4$$

where v_1, v_2, v_3 & v_4 are orthonormal,

Approximation of fruit images

(c) We have used generated 3 images as mean & linear combination of principal modes of variation around mean each contributing same.

Fruit 1 is the mean of sample data.

~~Fruit 2~~ → lets say μ = mean v_1, v_2, v_3 & x_4 are top 4 eigen vectors.

Fruit 2 = $\mu + \frac{1}{2}v_1 + \frac{1}{2}v_2 + \frac{1}{2}v_3 + \frac{1}{2}v_4$

Fruit 3 = $\mu - \left[\frac{1}{2}v_1 + \frac{1}{2}v_2 + \frac{1}{2}v_3 + \frac{1}{2}v_4 \right]$

These 3 can be considered as new, as they doesn't match any of original fruits. and are representation of whole data set.

3 new fruits

Fruit1   Fruit2   Fruit3