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2.(a) $\mu = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$C = \begin{bmatrix} 1.625 & -1.9486 \\ -1.9486 & 3.875 \end{bmatrix}$$

As C is a real symmetric matrix, hence it is diagonali-
-zable and have both real eigen vectors and are orthogonal.

Let $W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

where w_1 & w_2 are i.i.d. univariate standard normal RVs.

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be point on required bivariate gaussian.

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$\therefore$ We can say that

$$X = AW + \mu$$

where we ~~know~~ have to find A;

As C is diagonalizable

$$\therefore R^{-1} C R = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = K$$

where λ_1 & λ_2 are eigen values of C.

$$R = [\vec{v_1} \quad \vec{v_2}]$$

where $\vec{v_1}$ & $\vec{v_2}$ or orthonormal eigen vetors of C

$$\therefore C\vec{v_i} = \lambda_i \vec{v_i}$$

$$\therefore C = R K R^{-1}$$

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We can write

$$A = RS$$

where S is a diagonal matrix & R is a orthogonal matrix. which represents first scaling of isotropic bivariate & then rotation / reflection.

As $\vec{V_1}$ & $\vec{V_2}$ are orthonormal
R is orthogonal matrix.

$\therefore \; R^{-1} = R^T$

& $C = A A^T$
or $C = RS (RS)^T$

$$RKR^{-1} = RSS^TR^T$$

As S is diagonal matrix,
$S = S^T$

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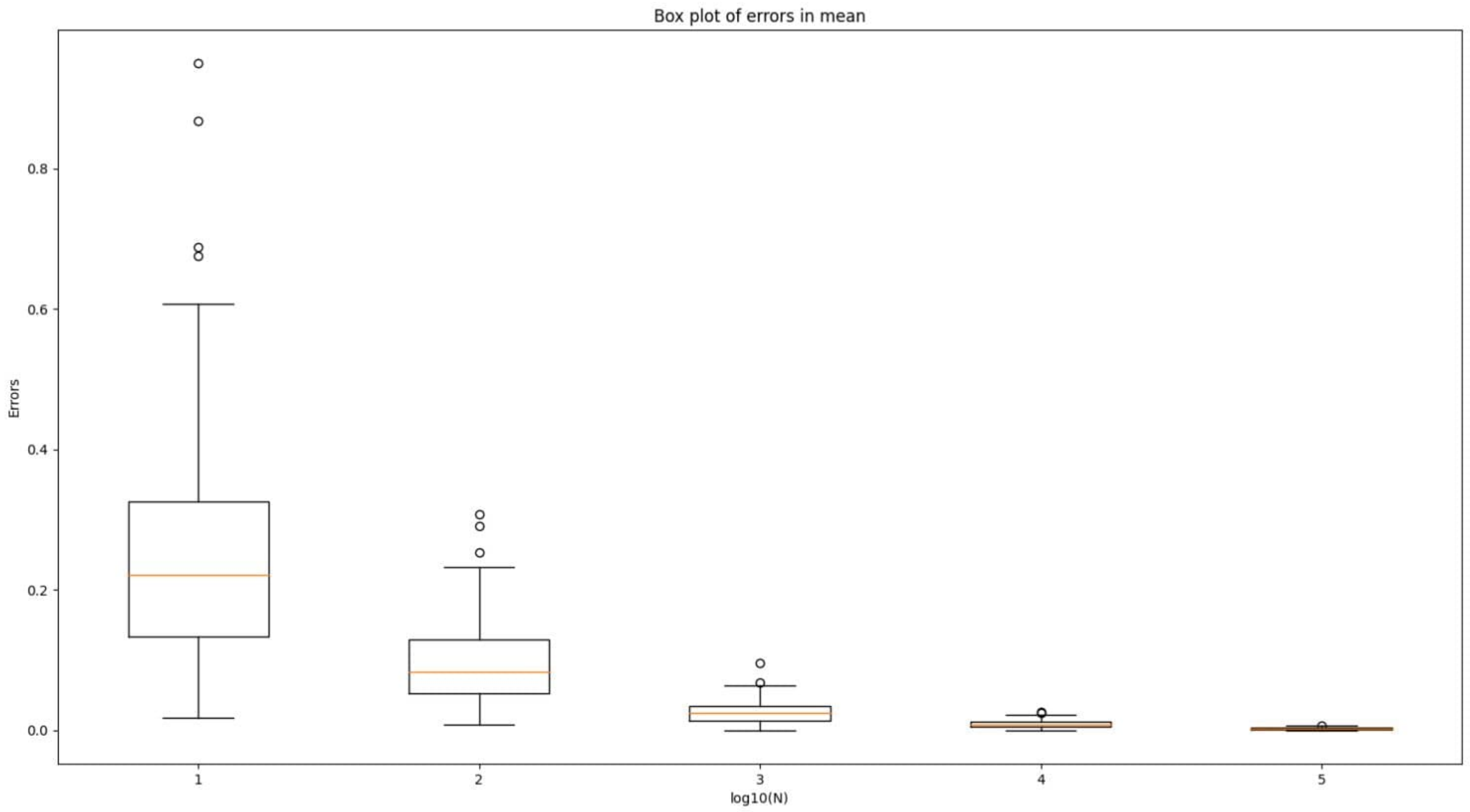
$\therefore RKR^{-1} = RS^2R^{-1}$

So Let $S = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix}$

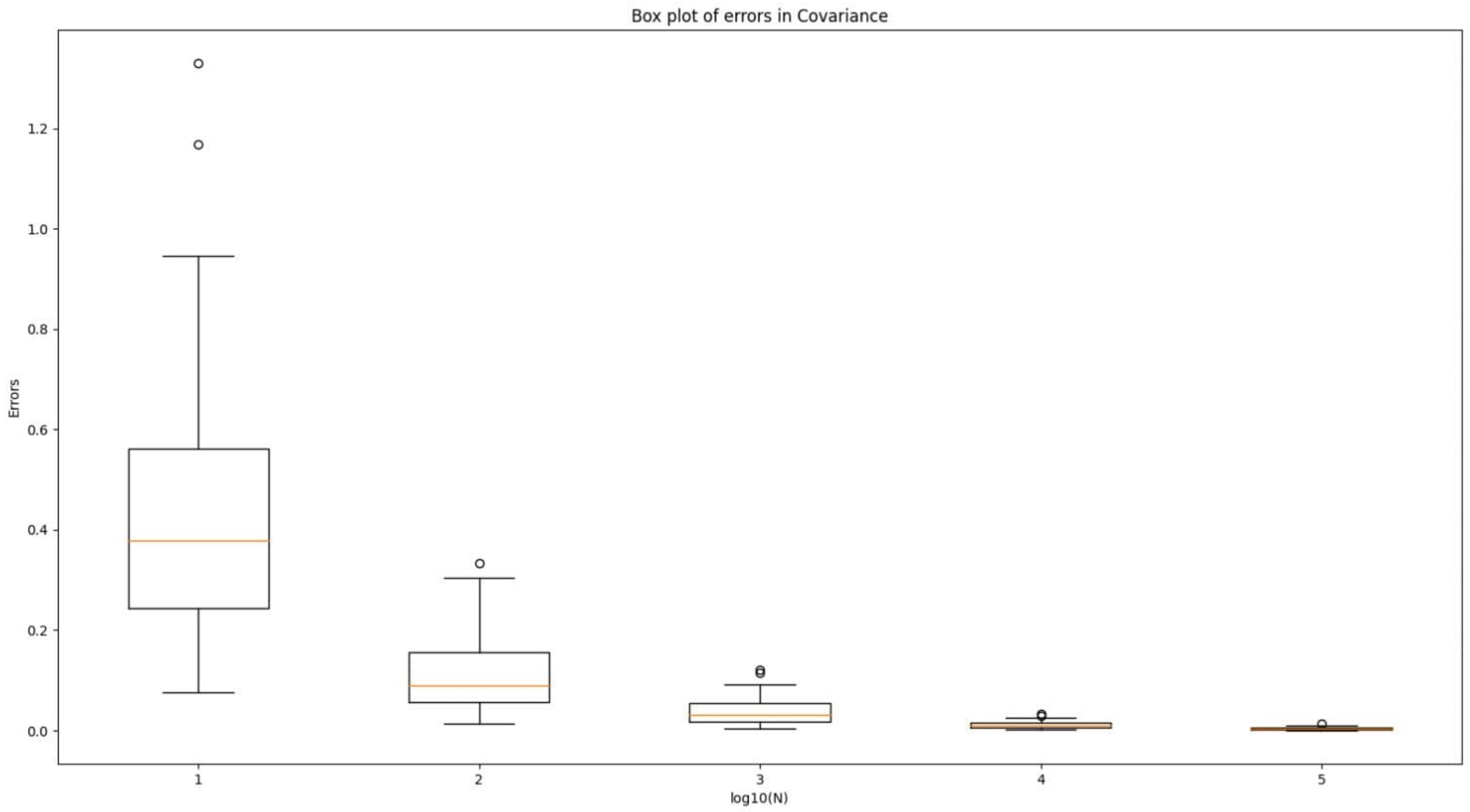
$\therefore$ If $A = RS$,

then $X = AW + \mu$

will provide a point in bivariate gaussian given.



Box plot of errors in mean
Errors
0.0
0.2
0.4
0.6
0.8
log10(N)
1
2
3
4
5



Box plot of errors in Covariance
Errors
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1
2
3
4
5
log10(N)

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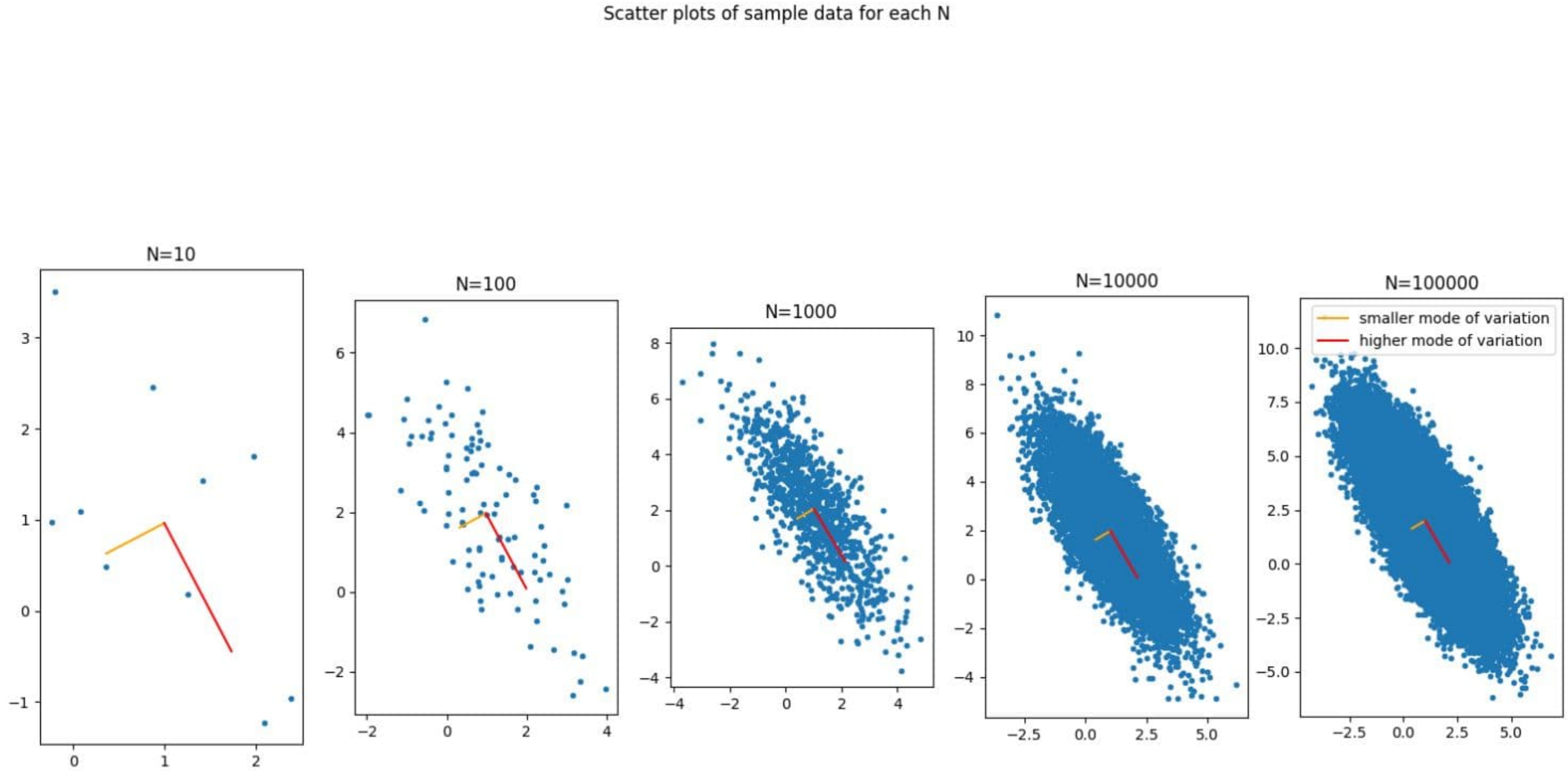
(c)

2b) ~~As~~ we increase sample data,

errors in mean and covariance

both are more and more

localised and also minimizing


Scatter plots of sample data for each N
N=10
N=100
N=1000
N=10000
N=100000
smaller mode of variation
higher mode of variation