# The Completion of Covariance Kernels

# ERRATA

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### Contents



## 1. Errata

Example 3.4 (Brownian Motion) of [2] contains a mistake. The expression used for evaluating the RKHS inner product in the space $\mathcal{H}(K_{[1/3,2/3]})$ was

$$\langle f, g\rangle_{\mathcal{H}(K_{[1/3,2/3]})} = \frac{1}{(1/3)}\int_{1/3}^{2/3} f'(u)g'(u)\ du.$$

However, this formula is incorrect and evaluates to 0 for $f = K(s,\cdot)$ and $g = K(t,\cdot)$, instead of $t$ as should be the case for $t < 1/3 < 2/3 < s$. The correct expression for the RKHS inner product is actually given by

$$\langle f, g\rangle_{\mathcal{H}(K_{[1/3,2/3]})} = \frac{1}{(1/3)} f(1/3)g(1/3) + \int_{1/3}^{2/3} f'(u)g'(u)\ du, \tag{1}$$

which evaluates to $3\cdot(s\wedge 1/3)(t\wedge 1/3) + \int_{1/3}^{2/3} 0\ du = 3\cdot(1/3)\cdot t = t$ as was supposed.

To see why this formula is correct, notice that the inner product of the space $\mathcal{H}(K_{[0,1]})$ is given by

$$\langle \tilde{f}, \tilde{g}\rangle_{\mathcal{H}(K_{[0,1]})} = \int_0^1 \tilde{f}'(u)\tilde{g}'(u)\ du \tag{2}$$

(see [1, Theorem 1.6]). Using the subspace isometry (see [1, Theorem 2.8]), we can evaluate the inner product between $f$ and $g$ as follows: Define the functions

$$\tilde{f}(u) = \begin{cases} f(1/3)\left[\frac{u}{(1/3)}\right] & 0\le u < 1/3 \\ f(u) & 1/3 \le u \le 2/3 \\ f(2/3) & 2/3 < u \le 1 \end{cases}$$

$$\tilde{g}(u) = \begin{cases} g(1/3)\left[\frac{u}{(1/3)}\right] & 0\le u < 1/3 \\ g(u) & 1/3 \le u \le 2/3 \\ g(2/3) & 2/3 < u \le 1 \end{cases}$$

in the space $\mathcal{H}(K_{[0,1]})$. Notice that

$$\tilde{f} = \operatorname*{argmin}_{h|_{[1/3,2/3]}=f} \|h\|_{\mathcal{H}(K_{[0,1]})} \text{ and } \tilde{g} = \operatorname*{argmin}_{h|_{[1/3,2/3]}=g} \|h\|_{\mathcal{H}(K_{[0,1]})}.$$

It follows that

$$\langle f, g\rangle_{\mathcal{H}(K_{[1/3,2/3]})} = \langle \tilde{f}, \tilde{g}\rangle_{\mathcal{H}(K_{[0,1]})}.$$

which evaluates to (1) due to (2).

## References


[1] Saitoh, S. and Sawano, Y., 2016. *Theory of reproducing kernels and applications*. Singapore: Springer Singapore.

[2] Waghmare, K.G. and Panaretos, V.M., 2021. *The Completion of Covariance Kernels.*