# Game Theoretic approach for lending to poor

## BTP Mid-Semester Evaluation

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# Outline

# Understanding a related optimisation problem

# Stability, efficiency, and contentedness of social storage networks

Data backup is an important necessity in today's world.
Existing solutions for which include:

1. Local backup
2. centralised online backup
3. decentralised Peer-to-Peer backup
4. Social storage or Friend-to-Friend backup system

# Stability, efficiency, and contentedness of social storage networks

Social storage issues addressed in this paper :

1. Modelling social storage system as an endogenous network formation game.
2. Stability, efficiency and contentedness of the storage system.

# Model - Variables and Constraints

| | |
|---|---|
| $\mathfrak{g}$ | social storage network. |
| $\mathbf{A}$ | set of agents (or vertices). |
| $N$ | number of agents in $\mathfrak{g}$ (that is, $N$ is the number of elements in the set $\mathbf{A}$). |
| $\mathbf{L}$ | set of links (or edges). |
| $\langle ij \rangle$ | link between agents $i$ and $j$. |
| $a_{ij}$ | indicator for data backup partnership between agents $i$ and $j$. |
| $c$ | cost incurred by an agent to maintain a link. |
| $\beta_i$ | worth (or value) that agent $i$ has for its data. |
| $s_i$ | amount of storage available with agent $i$ that it can contribute to other agents. |
| $d_i$ | amount of data that agent $i$ wants to backup. |
| $b_i$ | budget allocated by agent $i$ towards backup partnerships. |
| $\lambda$ | probability of failure of a disk. |
| $\eta_i(\mathfrak{g})$ | neighborhood size of agent $i$ in $\mathfrak{g}$. (Also denotes the set of neighbors of $i$). |
| $\mathfrak{g} + \langle ij \rangle$ | new link $\langle ij \rangle$ is added to $\mathfrak{g}$. |
| $\mathfrak{g} - \langle ij \rangle$ | existing link $\langle ij \rangle$ is deleted from $\mathfrak{g}$. |

# Model - Variables and Constraints

Assumption: all agents trust each other, and thus, anyone can form links with anyone.

**Mutual consent is necessary for adding as well as for deleting links**

# Model - Utility Function

- Each agent incurs a cost c to maintain a link.
- The total cost associated with maintaining the links is $c \times \eta_i(\mathfrak{g})$
- Each agent wants to minimise this cost

# Model - Utility Function

- The expected value of i's backup data is $\beta_i(1-\lambda^{\eta_i(\mathfrak{g})})$

  given the disk failure rate is $\lambda$, $i$ has $\eta_i(\mathfrak{g})$ neighbors and that the local copy of the agent's data has been damaged or lost
- Each agent wants to maximise the expected value of backup data

# Model - Utility Function

Both these goals have been encapsulated in a single utility function $u_i(\mathfrak{g})$ as follows:

$$u_i(\mathfrak{g}) = \beta_i \times (1 - \lambda^{\eta_i(\mathfrak{g})}) - c \times \eta_i(\mathfrak{g})$$

$\forall\ i \in A$, for the given network $\mathfrak{g}$

Each agent $i$ wants to maximise $u_i(\mathfrak{g})$ over all possible values of $\eta_i(\mathfrak{g})$.

# Model - Optimisation Problem

The social optimisation problem is be formulated as:

$$\textbf{max} \sum_{i \in A} (u_i(\mathfrak{g}))$$

$$\textit{such that} \quad \eta_i(\mathfrak{g}) = \sum_{j \in \mathfrak{g}} a_{ij} \quad \textit{and} \quad s_i \geq \sum_{j \in \eta_i(\mathfrak{g})} d_j a_{ij}$$

where,

$$a_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ have a backup agreement} \\ 0 & \text{otherwise} \end{cases}$$

# Model - Solution concept

**Bilateral stability :**

A social storage network $\mathfrak{g}$ is bilaterally stable if and only if

1. $\forall \langle ij \rangle \notin \mathfrak{g}$, if $u_i(\mathfrak{g} + \langle ij \rangle) > u_i(\mathfrak{g})$, then $u_j(\mathfrak{g} + \langle ij \rangle) < u_j(\mathfrak{g})$, and
2. $\forall \langle ij \rangle \in \mathfrak{g}$, if $u_i(\mathfrak{g} - \langle ij \rangle) > u_i(\mathfrak{g})$, then $u_j(\mathfrak{g} - \langle ij \rangle) < u_j(\mathfrak{g})$.

# Stability point

The stability point $\hat{\eta}$ of $\mathfrak{g}$ is defined as the neighborhood size such that no agent in g has any incentive to increase or decrease its neighborhood size than $\hat{\eta}$.
For the simple case of $\mathfrak{g}$ where

1. value associated with backed-up data is the same for all agents in the network.
2. each agent in the given network $\mathfrak{g}$ has as much storage as is required for all other agents in $\mathfrak{g}$.

the stability point $\hat{\eta}$ of $\mathfrak{g}$ is unique and is given by

$$\hat{\eta} = \left\lceil \frac{\left|\ln\left(\frac{c}{\beta(1-\lambda))}\right)\right|}{|\ln\lambda|} \right\rceil = \left\lfloor \frac{\left|\ln\left(\frac{c\lambda}{\beta(1-\lambda))}\right)\right|}{|\ln\lambda|} \right\rfloor$$

# Introduction to our problem

# Background

- Banks don't prefer to give poor people loans as they don't have collateral and they are more likely to default.
- Money lenders give loans to poorer at very high interest rates.
- Microfinance institutes are also willing to give loans to poorer section of society at nominal interest rates. However, they lack a lot of flexibility in their working.

# Problem Statement

We will create a mathematical model around the scenario of how lending happens to poorer section of our society. The four sections of our solution are listed below.

1. Variables
2. Utility Function
3. Solution Concept
4. Stability point

# Understanding the problem

# Players

There are three main players present in our scenario.

1. Borrowers
2. Money Lenders
3. Microfinance Institutes

# Borrowers

- The borrowers in our case are poor people.
- Poor people generally don't have any collateral against which they can take out loans.
- Hence, they prefer borrowing money from those who do not ask for collateral.

# Money Lenders

- Money Lenders are the major source for loans for poor peoples.
- They have to regularly monitor the borrowers. Their monitoring and screening costs are covered by the interests they charge.
- Loans from money lenders are flexible
- **Multiplier Effect**: When interest rate goes up, the borrower may try not to repay, increasing monitoring and hence cost of lending. This pushes the interests rate up, necessitating more scrutiny and so on. This skyrockets the interest rate.

# Microfinance Institutes

- **Microfinance**: Microfinance is a banking service provided to unemployed or low-income individuals or groups who otherwise would have no other access to financial services.
- MFI contract involves giving loans to group of borrowers who are liable for each other's loans. Group meetings happen between borrowers each week.
- Borrower have to make repayment at each weekly meeting.
- ROI is low because of low administration costs.
- MFIs threaten to stop future lending to avoid defaults.

# Variables and Utility Functions

# Variables

Important variables to take a note of :

1. **Rate of interest ($R$)**: $R_a$ denotes ROI offered by MFIs and $R_b$ denotes ROI offered by Money Lenders.
2. **Principal amount of loan ($P$)**: $P_i$ for borrower $i$.
3. **Delay Time ($D$)**: Period between two consecutive payments. $D_a$ for MFIs and $D_b$ for Money Lenders.
4. **Number of installments to pay the loan ($n$)**: $n_i$ for borrower $i$
5. **Rate of Default ($\delta$)**
6. **Capital already owned by borrower ($C$)**: $C_i$ for borrower $i$.

# Variables

7. **Administration costs ($\alpha$)**: Includes cost of monitoring and screening borrower, salary of loan officer and other costs of lending. $\alpha_a$ for MFIs and $\alpha_b$ for Money Lenders.
8. **Default Penalty ($\epsilon$)**
9. **Connectivity with other borrowers ($\gamma$)**: $\gamma_{ij}$ denotes how well borrower *i* and *j* are connected with each other in the society.
10. **Emergency Loans ($\mu$)**: $\mu_i$ denotes the expected value of emergency loans required by borrower *i* for which he isn't sure when can he can repay them.

# Utility Functions

What borrowers wants to optimize:

1. Decrease the amount payments in interests
2. Increase flexibility in payment (here Delay Time ($D$))

What lenders(both Money Lenders and MFIs) wants to optimize:

1. Increase the amount collected in interests
2. Decrease administration costs ($\alpha$)

# Utility Functions

The initial utility function which we have come up with is:

- For borrower $i$:

$$u(i) = -P_i((1+R)^{n_i} - 1) + D$$

- For lender (both Money Lender and MFI):

$$u' = \sum_i P_i((1+R)^{n_i} - 1) - \alpha$$

where $i$ denotes all borrowers who have lent from this particular lender

Note1: When $R$ changes to $R_a$ or $R_b$ for borrower when they lend from MFI or Money Lender respectively. Similar for D.
Note2: For lenders, $\alpha$ will change to $\alpha_a$ or $\alpha_b$ when the function is written for MFI or Money Lender respectively.

# Future Work

# Future Works

1. Improving utility functions to make the model more real-world.
2. Designing our solution concept for the model
3. Finding the stability point

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# Thank You

## Questions?