1. Introduction

1.1. Introduction to the problem

Why do we choose quantum physics as a topic? In short, it is because quantum physics is an extremely important and influential physics theory, and because teaching and learning quantum physics is a challenging task for both lecturers and students. In this report we will elaborate on this, in order to make the importance of quantum physics education research clear. Although we are aware that each researcher probably considers her or his own research field as particularly important, we cannot help feeling that we are particularly favoured our choice of research field, since the necessity of quantum physics education research is dramatically obvious.

1.2. The importance of quantum physics

I believe that most physicists if asked by a non-physicist whether quantum physics is important, would start to smile. Some would do this out of sheer politeness. A few would possibly do it while thinking "how can anyone ask such a stupid question?" But most, I believe, would do it because they feel that they finally have a physics question that they can answer in a way that laypeople would understand.

To a physicist, the importance of quantum physics is self-evident. She, or he, knows that quantum physics is the theory to use when it comes to microscopic phenomena: no other theory has been able to describe and predict, for example, atomic behaviour nearly as accurately as quantum physics has been able to. A physicist would also know that microscopic phenomena are extremely important when it comes to understanding matter at a larger scale. For example, by understanding the atomic structure you can explain why leaves are green, why certain plastics bend when others break, why metals conduct electricity, why it takes so much energy to heat up water in

relation to equal amounts of many other substances, and why chemical reactions take place the way they do.

A physicist might also tell you that quantum physics is also a remarkable predictor. In fact, quantum physics has produced one of the most detailed predictions verified so far, with the so-called fine structure constant, describing the strength of electromagnetic interaction in the cosmos. The fine structure constant has so far been measured to a precision of twelve digits (Gabrielse et al. 2006). For comparison, a twelve digit precision can be exemplified by measuring the circumference of Earth with a precision of a hair width (or a twentieth of a millimetre). Visit **https://gtbit-it2-aoqp.firebaseapp.com/** to know about the biggest contributors of QUANTUM PHYSICS.

A physicist will probably also start talking about string theory, physics' best attempt so far for a grand unified theory – a theory of everything. It so happens that string theory is anchored in quantum physics, along with the general theory of relativity. It is likely that some physicists would also go on to talk about quarks, radiation, anti-particles and other aspects of quantum physics, but at this point the listener is likely to get tired of examples from physics: OK, I get the point – quantum physics is an important theory in physics.

Influence of quantum physics outside physics

However, I am not completely satisfied with the conclusion that quantum physics is important in physics. Yes, quantum physics is an extremely important theory in physics, but the importance of quantum physics goes well beyond physics theories. To make this clear, we will also ask a few other imaginary representatives from other professions.

An engineer familiar with quantum physics would tell us that if it was not for quantum physics, we would not be able to make semiconductors in the way we do today. This means, for example, that we would not have cell phones, LCD displays, computers, light emitting diodes, and basically all other electronic equipment. A medical doctor would add that we, among other things, would not have magnetic resonance imaging; a powerful tool used for imaging the inside of our body. Also, the medical doctor would agree with the

molecular biologist that quantum physics has made it possible to simulate how medical substances interact with the proteins of our body – an efficient and safe first step in testing new medical substances.

If we would go on to ask a science fiction writer about quantum physics, she or he would probably get excited. The writer would talk about quantum computers that are immensely more powerful than our ordinary computers; about quantum teleportation, creating an exact replica of whatever is teleported and at the same time destroying the original; or about quantum cryptography, a way of transmitting information without even a theoretical possibility of eavesdropping. This may seem a bit far-fetched, and indeed, quantum computers and quantum teleportation still have a long way to go before they can leave the laboratory environment. But quantum cryptography is actually commercially available, even if the range of communication is limited.

Finally, we turn to a philosopher, to ask about the significance of quantum physics. Assuming that the philosopher knows quantum physics well – and there are definitely some who do – she or he would tell us that quantum physics has had a profound impact on what we mean by space and time, and possibly more importantly, cause and effect. Quantum physics has also shown that our world cannot be described by a so-called local realistic theory, which is basically that every part of the world is in itself a determined reality that can be observed. Instead, the world must either be described by a non-realistic theory – that the world is not determined before we observe it; or a non-local realistic theory – basically saying that what we perceive as two different places in space are in some aspects actually the same place (or are in direct contact).

1.3. Introduction to quantum physics

This section briefly introduces one way of looking at quantum physics. The description follows the so-called modal interpretation of quantum physics, although this will not be particularly important at this introductory level.

It is, however, important to note that this is one way of looking at quantum physics. I have chosen this view because I believe it to be relevant for a non-physicist reader. It introduces quantum mechanical concepts in a way that (hopefully) can be understood without prior knowledge of quantum physics or sophisticated mathematics.

Some limitations of this particular view are discussed in the end of this section.

1.3.1.   The context of quantum physics

Before introducing any technical aspects of quantum physics, it is important to know the contexts where quantum physics is used.

Although quantum physics can be used to describe the motion of billiard balls and pressure of gases, the main realm of quantum physics is the atomic level. At sizes visible to our eyes, the differences between quantum physics and classical physics is so small that, in almost all cases, quantum physics provides but an insignificant correction to classical physics – just as the theory of relativity is irrelevant when dealing with velocities much smaller than the speed of light.

However, quantum physics should not be mistaken as a theory that only applies to nano-scale environments, which becomes "correct" only when sizes are sufficiently small. You could rather say that in nano-scale environments, classical physics is no longer a useful approximation of quantum physics. Still, there are examples of quantum mechanical phenomena visible in our everyday environment, such as diffraction patterns of light.

1.3.2. States, eigenstates and probabilities

One absolutely crucial aspect of quantum physics is that particles may have undetermined values for properties such as energy, velocity and angular momentum (rotation). This means that, for example, an electron may have several values of its potential energy. However, in the event of measuring the electrons potential energy, the potential energy will change into one of the possible values. This is what is often referred to as the "collapse of the wave function", and is an important example, that, according to quantum physics, measurement is not something passive, but an active process that may affect a particle.

An electron which only has one possible value for potential energy – such as an electron whose potential energy has just been measured – is said to be in an eigenstate of potential energy. An electron that is not in an eigenstate of potential energy can always be written as a sum of eigenstates, for example Ψ = 0.1 E1 + 0.5 E2 + 0.2 E3 + … (where Ψ represents the non-eigenstate of the electron, and Ei represents the different eigenstates of potential energy).

The coefficients in front of the different eigenstates may be used for calculating the probabilities for the specific potential energy associated to that eigenstate. This is done by squaring the coefficient (or actually by taking the absolute value of the square, since the coefficients may be complex-valued). Thus, if we were to measure the potential energy of the electron above, we have a 0.01 probability of obtaining the potential energy associated to the eigenstate E1 and a 0.25 chance of obtaining the E2 potential energy.

In the example above we are using potential energy, but the same reasoning holds for any measurable property of a particle. Thus, the state of an electron may be described as a sum of eigenstates of potential energy, but also another set of eigenstates of velocity, angular momentum, and so on.

1.3.3. Spatial distribution

In particular, the reasoning in the section above may be applied to the property position, which is a measurable property of a particle.

This means that a particle does not, in general, have a definite position in space. By measuring the position, however, the particle is forced into assuming one of the possible positions.

That a particle does not have a well-defined position contradicts how we normally conceptualize particles – as a point in space, with mass but no spatial size. It is tempting to visualize the quantum mechanical particle as a localised point-particle, which we do not know where it is, but that still has a pre-determined position even before we measure it. However, it turns out that this view is counterproductive when trying to understand quantum physics. Properties are not determined until they are measured, and there have actually been quite a number of experiments proving this.

As with other properties, the state of a particle may be described as a sum of eigenstates of position. Since position most often is a continuous variable, the coefficients are not described through discrete sets of numbers, but through a function describing a distribution. For every possible position, the function has a particular value. Taking the absolute value squared will yield the probability density of detecting the particle at that particular point in space.

This function, describing a distribution in space, is usually called the wave function of a particle, and is commonly denoted $\Psi(x, t)$, where x denotes a position (in one or more dimensions), and t denotes time. For example, $\Psi(0, t)$ would give us information of the probability of detecting a particle at the origin, at a certain time t0.
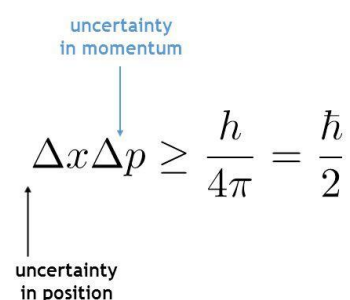
The wave function is a central part of quantum physics. Often, especially in practical situations, the probability distribution (being the absolute value of the wave function squared) is more useful than the wave function itself, not least

since the probability distribution may be empirically tested through repeated measurements. Still, it should be noted that the spatial expansion of a particle's state is only one of several possible ways of describing the state.

1.3.4. Heisenberg Uncertainty Principle

1.3.5. Heisenberg uncertainty principle is perhaps the best known result of the wave particle duality, i.e., the concept of waves or wave packet associated with a moving particle. According to Heisenberg uncertainty principle it is impossible to determine simultaneously the exact position and momentum (or velocity) of a small moving particle like electron. Visit **https://gtbit-it2-aoqp.firebaseapp.com/** and look for **Heisenberg Uncertainty Principle** under **CONCEPT** section for more information.

Heisenberg's Uncertainty Principle

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in momentum

uncertainty in position

The more accurately you know the position (i.e., the smaller Δx is), the less accurately you know the momentum (i.e., the larger Δp is); and vice versa

1.3.5.1. Explaining Heisenberg Uncertainty Principle with an example

Electromagnetic radiations and microscopic matter waves exhibit a dual nature of mass/ momentum and wave character. Position and velocity/momentum of macroscopic matter waves can be determined accurately, simultaneously. For example, the location and speed of a moving car can be determined at the same time, with minimum error. But, in microscopic particles, it will not be possible to fix the position and

measure the velocity/momentum of the particle simultaneously.

An electron in an atom has a mass of 9.91×10^{-31} Kg. Naked eyes will not see such small particles. A powerful light may collide with the electron and illuminate it. Illumination helps in identifying and measuring the position of the electron.
The collision of the powerful light source, while helping in identification increases the momentum of the electron and makes it move away from the initial position. Thus, when fixing the position, the velocity /momentum of the particle would have changed from the original value. Hence, when the position is exact the error occurs in the measurement of velocity or momentum. In the same way, the measurement of momentum accurately will change the position.

Hence, at any point in time, either position or momentum can only be measured accurately.

Simultaneous measurement of both of them will have an error in both position and momentum. Heisenberg quantified the error in the measurement of both position and momentum at the same time.

1.3.5. **The Schrödinger equation**

The Schrödinger equation is a linear partial differential equation that governs the wave function of quantum mechanical system via the wave function .The trajectory, the positioning, and the energy of these systems can be retrieved by solving the Schrödinger equation. All of the information for a subatomic particle is encoded within a wave function. The wave function will satisfy and can be solved by using the Schrodinger equation. The Schrodinger equation is one of the fundamental axioms that are introduced in undergraduate physics. It is also increasingly common to find the Schrödinger equation being introduced within the electrical

engineering syllabus in universities as it is applicable to semiconductors. Visit **https://gtbit-it2-aoqp.firebaseapp.com/** and look for **The Schrödinger equation** under **CONCEPT** section for more information.

The Schrödinger equation describes how a quantum mechanical system changes with time:

*Figure 1. The Schrödinger equation. ∂ represents a derivative and **Ψ** represents the wave function. i is the imaginary unit (where i2 = -1) and h is Planck's constant (where the bar denotes that it is divided by 2π).*

The H with hat is an operator, acting on the wave function. The left side of this equation is the time derivative of the wave function, describing the rate of change in the wave function as time changes. The right side of the equation consists of a complex factor, and then an operator corresponding to the total energy of the particle. The operator, called the Hamilton operator, involves kinetic energy, potential energy, and sometimes more terms relating, for example, to attraction or repulsion between particles. Thus, the Hamilton operator depends on the environment of a particle, and so does the time development of the particle.

  A full explanation of the Schrödinger equation is beyond the scope of this brief introduction, but it should still be noted that in the case of a particle in an eigenstate of total energy, the time evolution of the system becomes a trivial problem. Because of this, it is often very informative to analyse the eigenstates of the total energy. Indeed, many very important quantum mechanical applications deal solely with finding or approximating these Eigenstates.

1.3.5.1. Time Independent Schrödinger equation

Schrodinger's time-independent wave equation describes the standing waves. Sometimes the potential energy of the particle does not depend upon time, and the potential energy is only the function of position. In such cases, the behaviour of the particle is expressed in terms of *Schrodinger's time-independent wave equation*.

Consider a system of stationary waves associated with a moving particle. The waves are said to be stationary w.r.t the particle. If the position coordinates of the particle are (x, y, z) and ψ be the periodic displacement for the matter waves at any instant of time t, then we can represent the motion of the wave by a differential equation as follows.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \qquad \text{(i)}$$

where u is the velocity of wave associated with the particle. The solution of Eq. (i) gives ψ as a periodic displacement in terms of time, i.e.,

$$\psi(x, y, z, t) = \psi_0(x, y, z)e^{-i\omega t} \qquad \text{(ii)}$$

where ψ_0 is the amplitude of the particle wave at the point (x, y, z) which is independent of time (t). It is a function of (x, y, z). i.e., the position r and not of time t, Here.

$$r = x\hat{i} + y\hat{j} + z\hat{k} \qquad \text{(iii)}$$

Eq. (ii) may be expressed as

$$\psi(r, t) = \psi_0(r)e^{-i\omega t} \qquad \text{(iv)}$$

Differentiating Eq. (iv) twice with respect to t, we get

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0(r)e^{-i\omega t}$$

or

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \qquad \text{(v)}$$

Substituting the value of $\dfrac{\partial^2 \psi}{\partial t^2}$ from this equation in Eq. (i), we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\omega^2}{u^2}\psi = 0 \qquad \text{(vi)}$$

$$\text{where } \omega = 2\pi\nu = 2\pi(u/\lambda) \qquad \qquad [\text{as } u = \lambda\nu]$$

Also from the deBroglie wave concept

$$\lambda = \frac{h}{mv}$$

Using this relation in Eq. (ix) gives

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \qquad (x)$$

Here it can be noted that the velocity of particle v has been introduced in the wave equation.

If E and V are respectively the total energy and potential energy of the particle then its kinetic energy is given by

$$\frac{1}{2} mv^2 = E - V$$

$$m^2 v^2 = 2m(E - V) \qquad (xi)$$

The use of Eq. (xi) in Eq. (x) gives rise to

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V)\psi = 0$$

or $\qquad\qquad \nabla^2 \psi + \frac{2m}{\hbar^2} (E - V)\psi = 0 \qquad (xii)$

This is the time independent Schrödinger equation, where the quantity ψ is known as *wave function*.

For a freely moving or free particle $V = 0$. Therefore, Eq. (xii) becomes

$$\nabla^2 \psi + \frac{2mE}{\hbar^2} \psi = 0 \qquad (xiii)$$

This is called time independent Schrödinger equation for a free particle.

1.3.5.2 Time Dependent Schrödinger equation

In order to obtain a time dependent Schrödinger equation, we eliminate the total energy E from time independent Schrödinger equation. For this we differentiate Eq. (iv) w.r.t. t and obtain

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0(r)e^{-i\omega t}$$

$$= i(2\pi v)\psi_0(r)e^{-i\omega t}$$

$$= -2\pi vi\psi = -2\pi i\frac{E}{h}\psi = -\frac{iE}{h} \times \frac{i}{i}\psi$$

$$\Rightarrow \qquad\qquad \frac{\partial \psi}{\partial t} = \frac{E\psi}{i\hbar}$$

or $\qquad\qquad E\psi = i\hbar \frac{\partial \psi}{\partial t} \qquad (xiv)$

Substituting the value of $E\psi$ from Eq. (xiv) in Eq. (xii), we have

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$

or

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right]$$

(xv)

or

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

This equation is known as *Schrödinger's time dependent wave equation*. The operator $\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is called *Hamilitonian operator* and is represented by H. If we see the RHS of Eq. (xv) and keep in mind Eq. (xiv), we notice that the operator $i\hbar\frac{\partial}{\partial t}$ operating on ψ gives E. Hence, Schrödinger equation can be written in operator form, as below

$$H\psi = E\psi$$

1.3.6. Applications of Schrödinger equation

In classical physics, based on Newton's second law of motion (F=ma) we make a mathematical prediction of the path, a given system will take following set of known initial conditions. The analogue of Newton's law is Schrödinger equation in quantum physics for a quantum system such as atoms, molecules and subatomic particles. The subatomic particles may be free, bound or localized. Schrödinger equation describes the time evolution of the system's wave function.

 Schrödinger's equation is extremely useful for investigating various quantum mechanical problems. With the help of this equation and boundary conditions, the expression for the wave function is obtained. Then the probability of finding the particle is calculated by using the wave function. In the following subsections, we discuss different quantum mechanical problems, viz. particle in a box, one-dimensional harmonic oscillator, step potential and step barrier.

1.3.6.1. Particle in a box (Infinite Potential Well)

The simplest quantum mechanical problem is that of a particle trapped in a box with infinitely hard walls. Infinitely hard walls means the particle does not loose energy when it collides with such walls, i.e., its total energy remains constant. A physical example of this problem could be a molecule which is strictly confined in a box.

Let us consider a particle restricted to move along the x-axis between $x = 0$ and $x = L$, by ideally reflecting, infinitely high walls of the infinite potential well, as shown in Fig. 16.2. Suppose that the potential energy V of the particle is zero inside the box, but rises to infinity outside, that is,
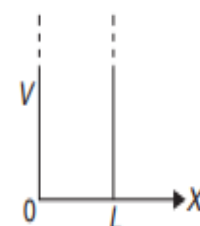
$$V = 0 \quad \text{for} \quad 0 \le x \le L$$
$$V = \infty \quad \text{for} \quad x < 0 \quad \text{and} \quad x > L$$

In such a case, the particle is said to be moving in an infinitely deep potential well. In order to evaluate the wave function ψ in the potential well, Schoredinger equation for the particle within the well ($V = 0$) is written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2} \psi = 0 \tag{i}$$

we put $\dfrac{8\pi^2 mE}{h^2} = k^2$ in the above equation for getting

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \tag{ii}$$

The general solution of this differential equation is

$$\psi(x) = A \sin kx + B \cos kx \tag{iii}$$

where A and B are constants.

Applying the boundary condition $\psi(x) = 0$ at $x = 0$, which means the probability of finding particle at the wall $x = 0$ is zero, we obtain

$$A \sin(0) + B \cos(0) = 0 \qquad \Rightarrow \qquad B = 0$$

Again, we have $\psi(x) = 0$ at $x = L$, then

$$A \sin kL + B \cos kL = 0 \qquad \Rightarrow \qquad A \sin kL = 0$$

The above equation is satisfied when

$$kL = n\pi$$

or $$k = \frac{n\pi}{L} \qquad \text{where } n = 1, 2, 3, \dots$$

or $$k^2 = \frac{n^2 \pi^2}{L^2} \tag{iv}$$

or
$$\frac{8\pi^2 mE}{h^2} = \frac{n^2 \pi^2}{L^2}$$
(v)

or in general we can write Eq. (v) as

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad \text{where } n = 1, 2, 3, \dots$$

Thus, it can be concluded that in an infinite potential well the particle cannot have an arbitrary energy, but can take only certain discrete energy values corresponding to $n=1, 2, 3, \dots$ These are called the *eigen values* of the particle in the well and constitutes the energy levels of the system. The integer n corresponding to the energy level E_n is called its *quantum number*, as shown in Fig. 16.3.

We can also calculate the momentum p of the particle or the eigen values of the momentum, as follows,

$$\text{Since } k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{p}{\hbar}$$

$$p = \hbar k = \frac{n\pi\hbar}{L}$$

The wave function (or eigen function) is given by Eq. (iii) along with the use of expression for k.

$$\psi_n(x) = A \sin\frac{n\pi x}{L}$$

To find the value of A, we use the normalisation condition.

$$\int_{-\infty}^{\infty} |\psi_n(x)|^2\, dx = 1$$

As mentioned earlier, the above expression simply says that the probability of finding the particle is 1. In the present case, the particle is within the box i.e., between $0 < x < L$. So the normalisation condition becomes

$$A^2 \int_{0}^{L} \sin^2 \frac{n\pi x}{L}\, dx = 1$$

$$A^2 \left(\frac{L}{2}\right) = 1 \quad \text{or} \quad A = \sqrt{\frac{2}{L}}$$

The normalised eigen wave function of the particle is, therefore, given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$$
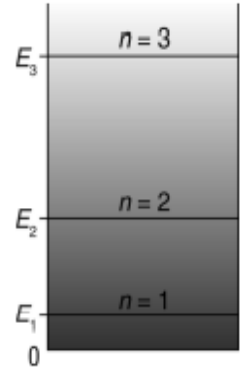
1.3.6.2. One Dimensional Harmonic Oscillator

A physical example of this quantum mechanical problem can be thought as an atom of vibrating diatomic molecule. In general, a particle undergoing simple harmonic motion in one dimension is called one dimensional harmonic oscillator. The potential and total energy of such a system is shown in Fig. _ where the probability density is also shown. In such a motion, the restoring force F is proportional to the particle's displacement x from the equilibrium position, i.e.,

$$F = -kx \qquad\qquad\qquad (i)$$

where k is force constant. The potential energy V can be written as

$$V = \frac{1}{2}kx^2$$

Then, the Schrödinger's equation for the oscillator with $V = \frac{1}{2}kx^2$ is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - \frac{1}{2}kx^2\right]\psi = 0$$

Putting $\hbar = \dfrac{h}{2\pi}$, $\dfrac{8\pi^2 mE}{h^2} = \alpha$ and $\left(\dfrac{4\pi^2 mk}{h^2}\right)^{1/2} = \beta$ in the above equation, we obtain

$$\frac{d^2\psi}{dx^2} + (\alpha - \beta^2 x^2)\psi = 0 \qquad\qquad (ii)$$

Now we introduce a dimension less independent variable as $\xi = \sqrt{\beta}x$. Thus Eq. (ii) becomes.

$$\beta\frac{d^2\psi}{d\xi^2} + \left[\alpha - \beta^2\frac{\xi^2}{\beta}\right]\psi = 0$$

$$\frac{d^2\psi}{d\xi^2} + \left[\frac{\alpha}{\beta} - \xi^2\right]\psi - 0 \qquad\qquad (iii)$$

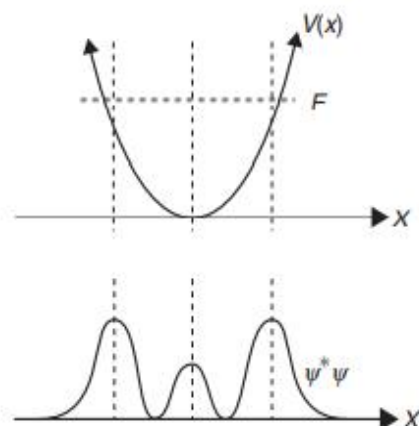The solution of this equation is

$$\psi = CUe^{-\xi^2/2} \qquad\qquad\qquad (iv)$$

where U is a function of ξ. Then Eq. (iii) takes the form

$$\frac{d^2U}{d\xi^2} - 2\xi\frac{dU}{d\xi} + \left[\frac{\alpha}{\beta} - 1\right]U = 0$$

If we replace $\dfrac{\alpha}{\beta} - 1$ by $2n$, this equation becomes Hermite differential equation. Then function $U(\xi)$ may be replaced with Hermite polynomial H. So, we get

$$\frac{d^2H}{d\xi^2} - 2\zeta\frac{dH}{d\xi} + 2nH = 0$$

Thus, the solution of Eq. (iii) is obtained by replacing U by Hermite polynomial H in Eq. (iv). Hence, we get

$$\psi = CHe^{-\xi^2/2}$$

In general, $\psi_n(\xi) = CH_n(\xi)e^{-\xi^2/2}$, where $n = 0, 1, 2,...$

Eigen values of energy

Eigen Values of Energy

Since $\dfrac{\alpha}{\beta} - 1 = 2n$

$$\Rightarrow \qquad \frac{\alpha}{\beta} = 2n+1 \quad \Rightarrow \quad \alpha = (2n+1)\beta \quad \Rightarrow \quad \frac{8\pi^2 mE}{h^2} = (2n+1)\sqrt{\frac{4\pi^2 mk}{h^2}}$$

This restriction gives a corresponding restriction on E, i.e.,

$$E = \left(n + \frac{1}{2}\right)\frac{h}{2\pi}\sqrt{\frac{k}{m}}$$

But $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = v$ is the frequency of oscillations. Hence, the energy can be written in terms of v as

$$E = \left(n + \frac{1}{2}\right)hv$$

Thus, in general, the oscillator has finite, unambiguous and continuous solutions at values of E given by

$$E_n = \left(n + \frac{1}{2}\right)hv \qquad \text{(v)}$$

Following conclusions can be drawn from equation (v)

The particle executing simple harmonic motion can have only

(i) discrete energy levels that are equidistant and are separated by hv, as shown in Fig. 16.8

(ii) The energy levels are non-degenerate.

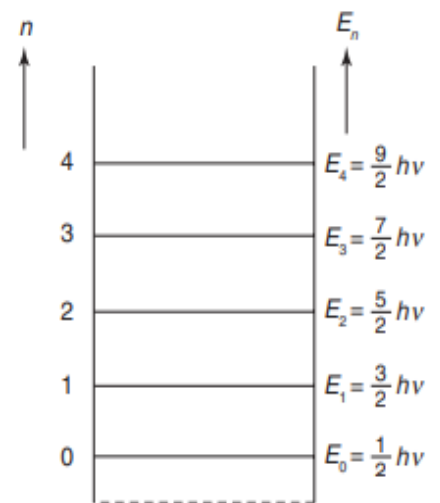(iii) For $n = 0$, $E_0 = \dfrac{1}{2}hv$. It means the minimum energy is not zero.

FIGURE 16.8

About the Website

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