Contents

Preface **ix**
A Note to Students **xv**

Chapter 1 Linear Equations in Linear Algebra 1

INTRODUCTORY EXAMPLE: Linear Models in Economics and Engineering **1**
1.1 Systems of Linear Equations **2**
1.2 Row Reduction and Echelon Forms **12**
1.3 Vector Equations **24**
1.4 The Matrix Equation $A\mathbf{x} = \mathbf{b}$ **34**
1.5 Solution Sets of Linear Systems **43**
1.6 Applications of Linear Systems **49**
1.7 Linear Independence **55**
1.8 Introduction to Linear Transformations **62**
1.9 The Matrix of a Linear Transformation **70**
1.10 Linear Models in Business, Science, and Engineering **80**
Supplementary Exercises **88**

Chapter 2 Matrix Algebra 91

INTRODUCTORY EXAMPLE: Computer Models in Aircraft Design **91**
2.1 Matrix Operations **92**
2.2 The Inverse of a Matrix **102**
2.3 Characterizations of Invertible Matrices **111**
2.4 Partitioned Matrices **117**
2.5 Matrix Factorizations **123**
2.6 The Leontief Input–Output Model **132**
2.7 Applications to Computer Graphics **138**
2.8 Subspaces of $\mathbb{R}^n$ **146**
2.9 Dimension and Rank **153**
Supplementary Exercises **160**

Chapter 3 Determinants 163

INTRODUCTORY EXAMPLE: Random Paths and Distortion **163**
3.1 Introduction to Determinants **164**
3.2 Properties of Determinants **169**

3.3 Cramer's Rule, Volume, and Linear Transformations **177**
Supplementary Exercises **185**

Chapter 4 Vector Spaces 189

INTRODUCTORY EXAMPLE: Space Flight and Control Systems **189**
4.1 Vector Spaces and Subspaces **190**
4.2 Null Spaces, Column Spaces, and Linear Transformations **198**
4.3 Linearly Independent Sets; Bases **208**
4.4 Coordinate Systems **216**
4.5 The Dimension of a Vector Space **225**
4.6 Rank **230**
4.7 Change of Basis **239**
4.8 Applications to Difference Equations **244**
4.9 Applications to Markov Chains **253**
Supplementary Exercises **262**

Chapter 5 Eigenvalues and Eigenvectors 265

INTRODUCTORY EXAMPLE: Dynamical Systems and Spotted Owls **265**
5.1 Eigenvectors and Eigenvalues **266**
5.2 The Characteristic Equation **273**
5.3 Diagonalization **281**
5.4 Eigenvectors and Linear Transformations **288**
5.5 Complex Eigenvalues **295**
5.6 Discrete Dynamical Systems **301**
5.7 Applications to Differential Equations **311**
5.8 Iterative Estimates for Eigenvalues **319**
Supplementary Exercises **326**

Chapter 6 Orthogonality and Least Squares 329

INTRODUCTORY EXAMPLE: The North American Datum and GPS Navigation **329**
6.1 Inner Product, Length, and Orthogonality **330**
6.2 Orthogonal Sets **338**
6.3 Orthogonal Projections **347**
6.4 The Gram–Schmidt Process **354**
6.5 Least-Squares Problems **360**
6.6 Applications to Linear Models **368**
6.7 Inner Product Spaces **376**
6.8 Applications of Inner Product Spaces **383**
Supplementary Exercises **390**

Chapter 7 Symmetric Matrices and Quadratic Forms 393

INTRODUCTORY EXAMPLE: Multichannel Image Processing **393**
7.1 Diagonalization of Symmetric Matrices **395**
7.2 Quadratic Forms **401**
7.3 Constrained Optimization **408**
7.4 The Singular Value Decomposition **414**
7.5 Applications to Image Processing and Statistics **424**
Supplementary Exercises **432**

Chapter 8 The Geometry of Vector Spaces 435

INTRODUCTORY EXAMPLE: The Platonic Solids **435**
8.1 Affine Combinations **436**
8.2 Affine Independence **444**
8.3 Convex Combinations **454**
8.4 Hyperplanes **461**
8.5 Polytopes **469**
8.6 Curves and Surfaces **481**

Chapter 9 Optimization (Online)

INTRODUCTORY EXAMPLE: The Berlin Airlift
9.1 Matrix Games
9.2 Linear Programming—Geometric Method
9.3 Linear Programming—Simplex Method
9.4 Duality

Chapter 10 Finite-State Markov Chains (Online)

INTRODUCTORY EXAMPLE: Google and Markov Chains
10.1 Introduction and Examples
10.2 The Steady-State Vector and Google's PageRank
10.3 Communication Classes
10.4 Classification of States and Periodicity
10.5 The Fundamental Matrix
10.6 Markov Chains and Baseball Statistics