Hashtables

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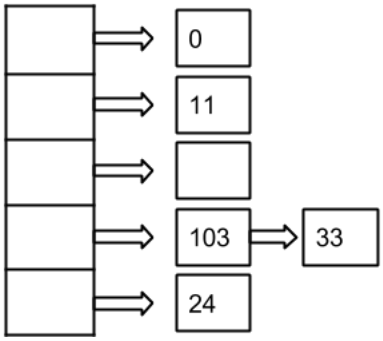
Say we wanted to store items in an array like structure, where our key has some correlation with it's index; however, what if our key is not an integer, say a String, or Animal? Or what if we have integers with huge differences, like 1 and 100000000. The first case would be impossible to achieve in an organized fashion because we would have no idea where keys would be. The second case is possible; however, the memory that would be taken up would be enormous, and it would be unfeasible to use so much space for only 2 items. This is the motivation behind hashing.

Hashing is the process of creating a number that represents some key. We can hash names, numbers, food- the possibilities are endless! Usually, ***hashmaps*** are represented by arrays, which means that when we hash some key, we should get some index in the array- we will refer to the amount of indices in the array to be buckets. Let's start with a basic example where our key is an integers and M is the amount of buckets. [1]

1 key%M

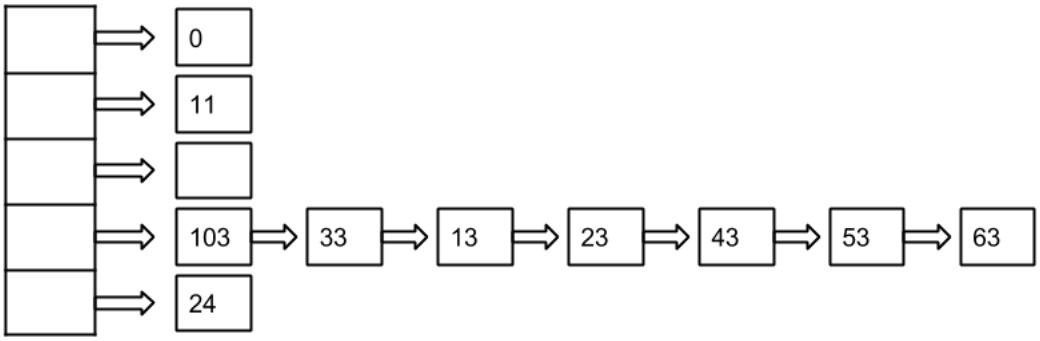
This hash function will always return some integer between 0 and M - 1. This means that numbers will map to some index in the array. Let's try it out on an array of size 5, so M =5, with the calls in the following order: 0,11, 24, 103, 33.

At this point, we have inserted 0, 11, 24, and 104. But wait there's a problem now, 33 and 103 hash to the same index in the array. How can we fix this? Well instead of storing 1 element within the array, we can instead store some other data structure, such as linkedlist at each index, this will allow us to store more than 1 element per index.



Now that we can take care of more than 1 element, we should be good to go right? Let's try inserting the following keys: 13, 23, 43,53,63.

[1] Note usually in hashing functions, we perform some arithmetic on the key before moding it, so that our encryption is more secure.



Do you see the problem? If we keep inserting items that have a remainder of 3 when divided by 5, our runtime will be no better than a linkedlist, we would just be using more memory because we have an array too. In order to fix this, while inserting items, we should have some clause that if $\frac{N}{M} >= somenumber$ we resize our array. This basically means if the average amount of items per bucket is greater than or equal to some number we resize the array. After resizing our array, we rehash all of our items since the amount of buckets has changed. Then , we should get a more even distribution. The question now is how should we increase our buckets? Let's consider the following options:

```
1 M = M * 2
2 M = M + 1000
```

When considering which resizing factor we should choose, we want to make sure that we do not need to resize too frequently because resizing is a relatively expensive operation. At first glance $M = M + 1000$ may seem tempting; however, it is important to realize that the number of buckets being added is not increasing with as N grows larger. This means that, in the long run, the 1000 will not be substantial enough to make the resizing factor negligible. As a result, the best resizing option we have is $M = M * 2$.

Let's analyze the runtime of Hashmaps now. It seems that in amortized time, Hashmaps run fairly quickly. Get, and put takes $\Theta(1)$ time. However, in worst case, for these operations, we would need to resize the array and rehash all the items, taking $\Theta(N)$ time. There is also another case for which worst case run time can occur. If we have a bad hashing function, it could be exploited so that, with a series of inserts, everything hashes to the same bucket. This is why it is imperative that a good hashing function is used in a hashmap.

To use hashmaps, we need to use the following functions.

```
1 public int hashcode(){
2     ......
3     return some hashcode;
4     }
5 public boolean equals(Object obj){
6     ....
7     return if obj equals your current object based off a factor of your choosing
8 }
```

One important key idea to remember when analyzing hashing functions is that any 2 items that are equal should hash to same number. This means that if *a.equals(b)* then *a.hashcode()* must equal *b.hashcode()* and vice versa. This means that if you override the equals method, you must also overwrite the hashcode method. Another very important idea is that, 2 elements. Another important characteristic of hash functions is that they should provide a relatively even distribution meaning that one bucket should not be hashed to a disproportionate amount compared to other buckets.