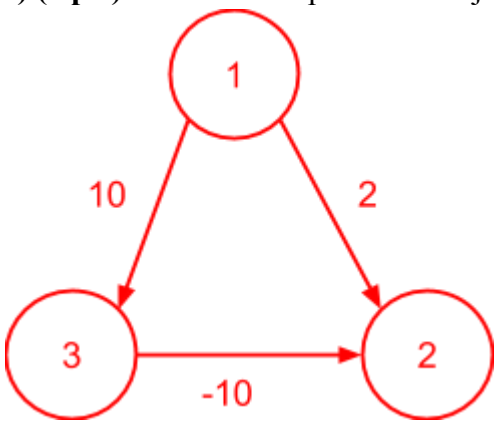
Practice Final- Solutions

This test has 13 questions worth a total of 200 points, and is to be completed in 180 minutes. The exam is closed book, except that you are allowed to use one double sided written cheat sheet (front and back). No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. Write the statement out below in the blank provided and sign. You may do this before the exam begins.

#	Points	#	Points
1	16	8	0
2	12	9	10
3	9	10	15
4	25	11	20
5	8	12	17
6	25	13	30
7	9		
		Total	200

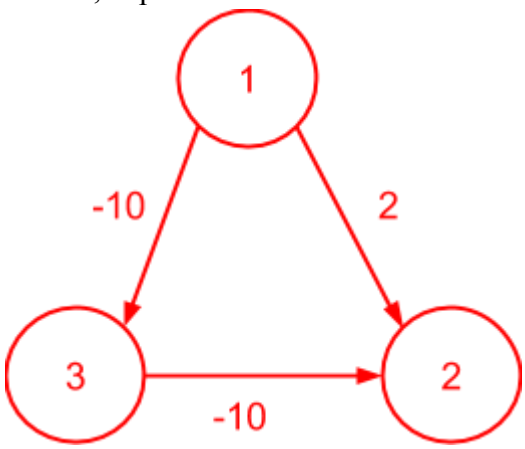
1. Starting this Test on a Positive Note (15 pts).

a) (4 pts) Give an example where Dijkstra's would fail because of a negative edge weight



In this example, we would go from node 1 to node 2. Then we go from node 1 to node 3. But we see that we have shorter path to 2 than we initially had- a contradiction.

b) (4 pts) Give an example where Dijkstra's **does not** fail because of a negative edge weight. State assumption that you must make in order for this to work. If you have some method to make it work, explain the method.



If we start from the node 1, we will get the shortest path because of the fact that the negative edge weights can be reached from the start vertex by solely traversing negative edges. If there was any point where there is a positive edge weight between a negative edge weight and the start vertex, it is possible that Dijkstra's would screw up. This only works if the graph has no cycle formed, otherwise we would get an in infinite loop.

c) (4 pts) What is the least amount of negative edges (>0) that a graph can have and still have Dijkstra's work? What is the most amount of negative edges that we can have? State assumptions that must be made

1 edge is the least amount of negative edges for which dijkstra's algorithm works. This will only work for directed graphs and if the shortest path to the vertex the edge goes to would be the shortest path if the negative edge was weighted 0.

The maximum amount of negative edges we have relies on the fact that the edges are directed. If this is the case, the amount of edges we can have is equal to n-1 (a tree). This is because if we had anymore then there would need to be a cycle. In which case Dijkstra's wouldn't work

d) (4 pts) What is the smallest weight we can have and guarantee that Dijkstra's will always work?
0. Dijkstra's is guaranteed to work on edges with no negative edges.

2. Sorting and Fun (12 pts). We worked with a few various sorts. For each sort, provide the runtime and give a brief description of what it does (how it sorts). Each questions is worth 4 points

Ω(N) O(N^2):
This algorithm swaps 2 elements if they are in the wrong order.
In the best case, the list is almost entirely sorted. In this case, you go through the list once, don't find any (or very few) out of order pairs).
In the worst case, the list is completely inverted (eg {3,2,1}) If this is the case, our counter would actually decrement (i--) and would make the algorithm tend to go to **Θ(N^2)**

```
private static void dwarfSort(int[] ar) {
  int i = 1;
  int N = ar.length;
  while (i < N) {
    if (i == 0 || ar[i - 1] <= ar[i]) {
      i++;
    } else {
      int tmp = ar[i];
      ar[i] = ar[i - 1];
      ar[i--] = tmp;
    }
  }
}
```

Θ(N): This is actually one of the most pointless sort algorithms because all it does is check if the ith element is less than the i-1st element, if so just brute force it to be bigger (adding one to the i-1th element). You go through the array once and you are done "sorting" after 1 iteration

```
private static int[] waffle(int[] arr){
   int[] syrup = arr;
   for(int i = 1; i < arr.length; i++){
      if(arr[i] < arr[i-1]){
        arr[i] = arr[i] + 1;
     }
   }
   return syrup;
}
```

Θ(N^2): In this sort, we are keeping track of some pointer(curr_size) which is initially set to the size of the array. We find the maximum element and then we reverse the array from 0 to the index of the maximum element (so the maximum

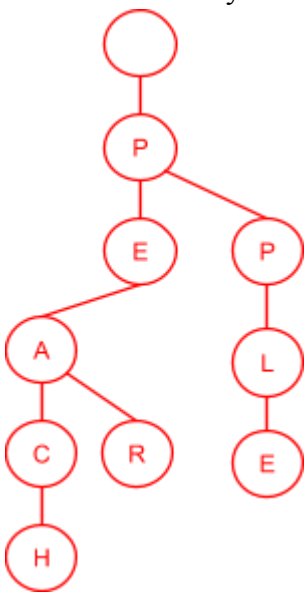
```
//find the runtime of flapJackSort. Note that the
flip function is defined as follows flip(array, index)
and reverses the entire array up until that index.

public static int[] flapJackSort(int arr[], int n){
  for (int curr_size = n; curr_size > 1; curr_size--)
  {
     for(int choco = 0, chip =0; chip < curr_size;
chip++) {
        if (arr[chip] > arr[choco]) {
           choco = chip;
```

element becomes the 0th index). Then we reverse the array from the 0th index to curr_size. We then decrement curr_size so we look at everything before that element. This sort does a total of n "flips"/reversals, so we have a running time of $\Theta(N^2)$

```
            }
            if (choco != curr_size - 1) {
                flip(arr, choco);
                flip(arr, curr_size - 1);
            }
        }
    }
    return arr;
}
```

3. The Fruits of Your Labor (9 pts).

a) (3 pts) You own a tree that can hold many fruits; however, you want to save space so that your neighbor doesn't cut off your branches. Find how you make one modification (change, delete, or add a letter) to one of the following 3 words "Apple", "Peach, "Pear". What modification can you do to save the most space? Draw the resulting trie after your modification.



The modification that was done was removing the "a" from apple. This makes it so that all the items share a prefix of "p". We deleted one node and made one shared among all 3 words.

b) (3 pts) A wealthy CEO wants to encrypt all his secrets; however, he decided that he wants to store all of his secrets within a flashy hashmap. But he doesn't want a plain old hashmap, after all, that could get hacked. To be different, he decides that, for any given day, a secret will hash to different location than it would have the previous day. Is his idea valid? Provide your reasoning.

This IS valid. There is a subtilty in this problem, where we say items hash to a different location. This does not mean that it has a different hashcode. If a secret hashes to a different location then it would have a day ago, that could just mean that the hashmap has been resized. There is no problem with this unless we know that the hashmap retains its size or we know that the hashcode is different for two equivalent secrets.

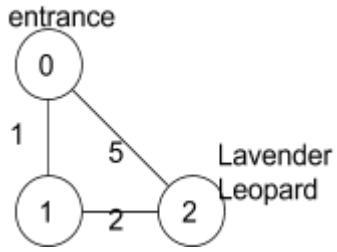
c) (3 pts) 2-3 Trees are known to be fast because they are self balancing and they have a nice low level? Why don't we just use 2-3-4 trees or 2-3-4-5-6 trees?

The main reason we don't have trees like this is because they are mainly fairly difficult to implement. Additionally, there really isn't anything to gain, except for possibly a change in a constant factor. However, if we have larger nodes, we will have to use more memory for each type of node, which leads to more memory overhead to worry about.

4. The Lavender Leopard (20 points).

You are a jewel thief starting at the entrance of a museum where all rooms are connected to each other- that is a room is connected to every room besides itself; however, hall lengths may differ. You are in search of the very expensive gem, the "Lavender Leopard", the only problem is that you have no idea what room it is in.
Your idea is to go to the room closest to the one you are in (excluding ones that you have already visited) and then as soon as you find the gem, you will immediately exit the building, going through the same rooms you did earlier; however, if there was a faster way to a room that is closer to the exit, you will go that way.

a) (20 pts) Fill in the Museum class and following code to find the gem and then escape as quickly as possible. You are guaranteed that the jewel will be in the building.



In the above example, you would travel to 1, then 2, get the gem, and go back to room 1 then the entrance.

```
public class Hall {
  public final Room room1; public final Room room2;
  public final double length;
  public Hall(Room r1, Room r2, double length){
     this.room1 = r1; this.room2 = r2; this.length = length;
  }
  public Room r1(){return room1;}
  public Room r2(){return room2;}
  public double howFar(){return this.length;}
```

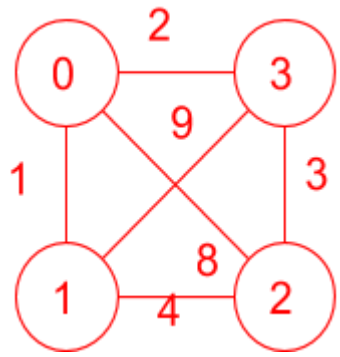
```
public class Room {
  public LinkedList<Hall> adjacentHall;
  boolean beenTo = false;
  public boolean hasGem;
  public int order; //provides a definite ordering for the rooms, does not tell you the
location, but can be used as a reference point.
  public Room(int order, boolean gem, int rooms) {
     this.order = order;
     this.hasGem = gem;
     adjacentHall = new LinkedList<Hall>();
     for(int i = 0; i < rooms; i++){
        adjacentHall.add(null);
}}}
```

```
import java.util.ArrayList; //feel free to use these imports, if not it's okay as well
import java.util.LinkedList;

public class Museum {
public int totalRooms;
public int totalHalls;
public LinkedList<LinkedList<Hall>> adj;
public Room entrance;

//Constructor
public Museum(Room[] rooms) {
  this.totalRooms = rooms.length;
  this.totalHalls = 0;
  adj = new LinkedList();
  for (int room = 0; room < rooms.length; room++) {
     adj.add(room, rooms[room].adjacentHall);
  }
  entrance = rooms[0];
}

public void stealing() {
  ArrayList<Room> visited = new ArrayList<>();
  Room r = goIn(visited);
  goOut(r, visited);
}

//You are sneaking into the museum, attempting to find the jewel, this is the
hardest method of the class just because of sheer length.
public Room goIn(ArrayList<Room> visited) {
  Room r = entrance;
  boolean stolegem = false;
  while (!stolegem) {
     visited.add(r);
     r.beenTo = true;
     if (r.hasGem) {
        stolegem = true;
        break;
     }
     Hall minHall = null;
     for (int i = 0; i < adj.get(r.order).size(); i++) {
        Hall curr = adj.get(r.order).get(i);
        if (minHall == null) {
           if (curr != null && !((curr.r1().beenTo) && curr.r2().beenTo)) {
              minHall = curr;}
        } else if (curr != null && curr.length < minHall.length &&
!((curr.r1().beenTo) && curr.r2().beenTo)) {
           minHall = curr;}}
     if (r.order == minHall.from().order) {
```

```
            r = minHall.to();
        } else {
            r = minHall.from();}}
    return r;}

//you have the gem now you are leaving. You can do this part without having done the earlier method (there are no references to it; however, you should understand what it is doing)
public void goOut(Room r, ArrayList<Room> visited) {
        int curr_lim = visited.size();
        while (r != entrance) {
            Room closestroom = null;
            double dist = Double.MAX_VALUE;
            int currentlimit = curr_lim;
            for (int i = 0; i < currentlimit; i++) {
                Room curr = visited.get(i);
                if (adj.get(r.order).size() < dist) {
                    closestroom = curr;
                    dist = adj.get(r.order).size();
                    curr_lim = i;
                }
            }
            r = closestroom;
        }
    }
        System.out.println("We outtie");
}
```

This accidentally became one of the more coding intensive parts of this test. The key to this problem was seeing that there was an "order" in the Rooms, which allowed us to give them indices or "rightful places". Another thing to realize is that the rooms can be treated as vertices and the halls can be treated as edges (reducing this problem just another graph traversal one).When sneaking into the museum, there was a slightly inelegant solution. This is partially because the edges are not directed, so we have to check to make sure that we did not. The abundance of null checks is mainly due to the fact that we used a LinkedList in our implementation, and as a result, had to "pad" spots. This could have been avoided if you used an array with a set amount of spots.

Leaving the museum is more confusing than the entering. You are only allowed to visit rooms that you've visited already. This means that you cannot just go to the closest room, you must be limited to the list of rooms that we went to. The reason we used an arraylist is so that we could keep track of the ordering and move up more than one space if necessary. An example for when this would be necessary is:



here we have the case where we would find the gem by going from room 0 to room 1 to room 2 and finally reaching room 3 (where the gem is). However, now the alarms have been tripped, and it would take us way too long to get out, instead we could go directly to the entrance.

b) (5 pts) What is the worst case runtime of stealing in terms of rooms (r) and halls (h)? Only keep necessary terms.

$\Theta(2r) = \Theta(r)$. The e is not necessary because we would never travel to one room more than once.

5. Trick or Tree-t (12 pts). Mark true or false for the following problems, a brief justification is required. Each question is worth 3 points

a) The fastest time for any comparison based can ever be is **Ω**(Nlog(N)) regardless of input.
☐ True ■ False
Heapsort, if all the elements are the same, will have a runtime of Θ(N).

b) When looking through a binary search tree, if we are looking for a number of items between 2 elements, the best case runtime is Θ(N) because we will need to go through every single element to see if it fits into our range?
☐ True ■ False
This is false, as this is the definition of pruning, or range finding. The runtime for this process is Θ(logN + R) where R is the total amount of elements that you need to find in your range.

c) All valid left leaning Red Black Trees can become a valid Binary Search Tree by replacing all red nodes with black ones.
■ True ☐ False
By definition, Red Black Trees are self balancing Binary Trees. Essentially, by making all nodes black, you would just be removing the quality of self balancing.

d) 2-3 Trees are faster than Red-Black Trees for most major operations because of the height of the 2-3 tree is smaller than the height of a Red-Black Tree.
☐ True ■ False
This is false, simply because, in a 2-3 tree, instead of traversing down a level like we would in a Red-Black Tree, we would have to make another comparison. This makes the overall runtime the same.

6. The Lavender Leopard Strikes Again (20 pts). You were examining your gem after successfully stealing it only to realize that you had stolen a fake gem, and that the real Lavender Leopard is in a much more secure museum. In this museum, not every room is connected. To avoid ridiculous amounts of backtracking, you have decided to place teleporters in every room you visit. At any point, if there is any path that is closer to the entrance than the one you are taking, you will teleport to the room before and then travel to it. Once you find the gem, you need to get out as soon as possible. To get out, you will teleport in the reverse order that you arrived so you can collect all your teleporters (they aren't that cheap!). Note that Hall and Room have the same definition that they did for #4. Additionally, you **do not** need to redefine class level variables and the constructor (you can if you want to), as they are the same as the Museum class's . You have 3 pages to complete this.
If you cannot write the code, but can write the idea behind the problem, you can get up to % of the points.
Hint 1: Use old algorithms that we've gone over and see how you can modify them to match this problem
Hint 2: Look at other data structures.

```
class Node {
   public Room r;
   public double myPriority;

   public Node(Room r, double priority) {
      this.r = r;
      myPriority = priority;
   }
}
public class TeleportMuseum {
   public int totalRooms;
   public int totalHalls;
   public LinkedList<LinkedList<Hall>> adj;
   public Room entrance;

   public TeleportMuseum(Room[] rooms) {
      this.totalRooms = rooms.length;
      this.totalHalls = 0;
      adj = new LinkedList();
      for (int room = 0; room < rooms.length; room++) {
         adj.add(room, rooms[room].adjacentHall);
      }
      entrance = rooms[0];
   }
 public void teleportInNOut() {
      Stack<Room> s = new Stack<>();
      teleportSteal(s);
      teleportLeave(s);
   }
```

```
public void teleportSteal(Stack s) {
  Hall[] edgeTo = new Hall[totalRooms];
  double[] distTo = new double[totalRooms];
  PriorityQueue pq = new PriorityQueue();
  for (int v = 0; v < totalRooms; v++) {
     distTo[v] = Double.MAX_VALUE;
  }
  distTo[entrance.order] = 0.0;
  pq.insert(entrance, distTo[entrance.order]);
  while (pq.size() != 0) {
     Room temp = pq.removeMin();
     temp.beenTo = true;
     s.push(temp);
     if (temp.hasGem) {
        break;
     }
     for (Hall h : temp.adjacentHall) {
        if (h != null && !(h.from().beenTo && h.to().beenTo())) {
           done(h, temp, distTo, edgeTo, pq);
        }
     }
  }
}
public void done(Hall h, Room temp, double[] distTo, Hall[] edgeTo, PriorityQueue
pq) {
     Room r;
     if (h.from().equals(temp)) {
        r = h.to();
     } else {
        r = h.from();
     }
     if (distTo[r.order] > distTo[temp.order] + h.length) {
        distTo[r.order] = distTo[temp.order] + h.length;
        edgeTo[r.order] = h;
        if (pq.contains(r)) {
           pq.changePriority(r, distTo[r.order]);
        } else {
           pq.insert(r, distTo[r.order]);
        }
     }
  }
}
public void teleportLeave(Stack<Room> s) {
  while (!s.isEmpty()) {
     Room r = s.pop();
  }
  System.out.println("Out again");
}
```

The initial part of the TeleportMuseum class is very similar to problem 4.

The Node class was created as a wrapper for the room and priority so we could use it within a priority queue.

Inside of the teleportInNOut method, we have a stack, which helps us keep track of the order we go to rooms, and we have 2 calls, one to teleportSteal and one following teleportLeave.

The teleportSteal method is extremely similar to Dijkstra's Algorithm. It pretty much runs the exact same, except that each time we remove an element from our priority queue, we add it to our stack.

In the teleportLeave method, all we do is pop our stack until it's empty. The stack naturally serves as the data structure for which we will have "reverse" order because it is FILO (First In Last Out).

This question was pretty coding intensive, more so than #4, but it required less thinking if you knew what the algorithms were. The key was understanding how to use data structures in conjunction with a graph algorithm.

*There was no need to write the class level variables and the constructor as it is identical to #4. It was only put on the answer key for completeness sake.

7. D-Arranged Lab (9 pts). You are viewing a research group, unfortunately, this research group is full of some bumbling buffoons who happens to make a lot of mistakes. Each part is worth 3 points.

a) While putting his chemicals into test tubes, one of the scientists realized that he accidentally made a duplicate of one of his n chemicals, he just doesn't know which one. However, he does know that the duplicate element will be, at most, 10 test tubes away. What sort should he use to quickly find the duplicate? Assume that all the chemicals are in their natural order, except for the duplicate.
This is very similar to finding a sort that has the fewest amount of inversions. Thus, we should use insertion sort

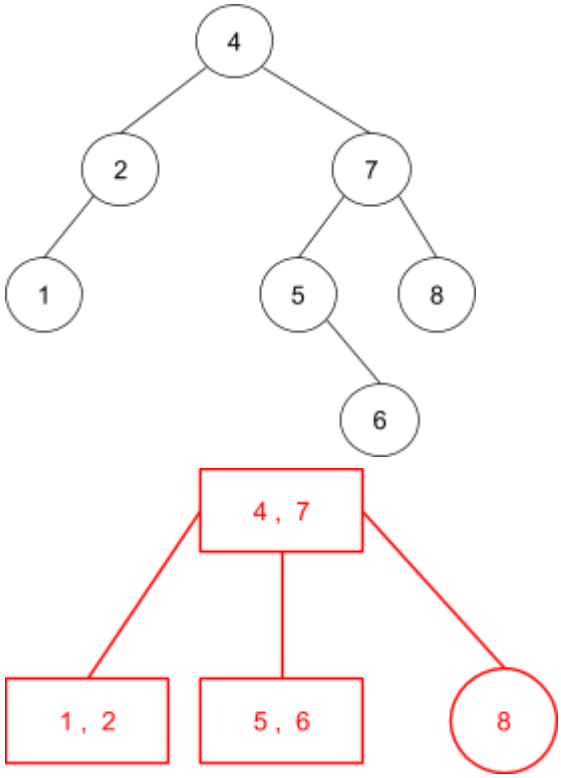
b) You have separated each of your n test tubes into k sections (so test tube 1 is now in seperated into k test tubes and so on). The only problem is that while they were all getting analyzed, you mixed them up again. Luckily, you have the name of the chemical and the time a chemical was put into a test tube. You want to sort your test tubes by chemical and then by amount of time it has been oxidizing. What sort can you use to ensure this is done as quickly as possible?
We will use merge sort. The main reason why is that it takes Θ(nlogn) time to sort as well as it is stable. Because it is stable, the relative ordering, which in this case is the order in which the k test tubes were oxidized.

c) Now the scientists are going to input the name of their chemicals into a spreadsheet to see if they are dealing with your list of hazardous materials. The list is in alphabetical order so to easily cross reference what chemicals are dangerous, the scientists want to put the name of their chemicals in alphabetical order, what sort should they use?
This is a direct application of Radix Sort. The key give away is the "alphabetical order"

8) Riddle me this:
Why is an Orange like a Bell?
They must both be peeled/pealed (pealed means rung)

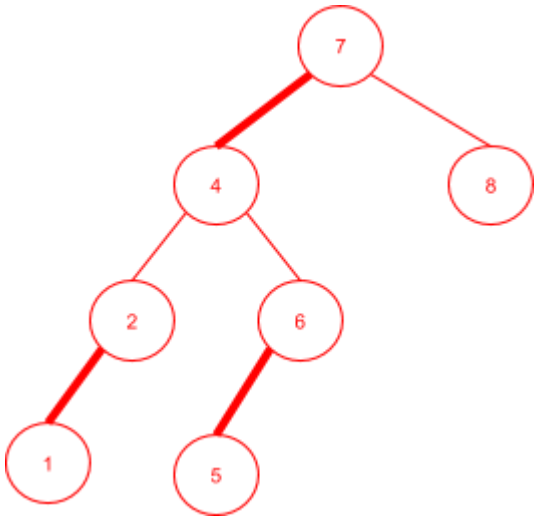
9. Rotations and Aberrations (10 pts).

a) (5 pts) Create the 2-3 tree that corresponds to the below Binary Search Tree.



Something important to note in this problem was the order that the items were inserted in the initial Binary Search Tree. You couldn't just get all the keys and make an arbitrary 2-3 tree. After deriving a set of insertions that work for the initial Binary Search Tree, simply perform the insertions on a 2-3 tree model.

b) (5 pts) Create a Red-Black Tree that corresponds to the 2-3 tree/Binary Search Tree in part a.



Simply convert the above 2-3 tree into a Red-Black Tree using our conversion rules. Another way to do this is look at the order of insertions for the Binary Search Tree and directly insert into the Red-Black Tree.
*The bolded links are red links.

10. JJJJ UNIT (15 pts).
You are starting your rapping career as a part of the prestigious rapping group J Unit. You want to recruit new members so you can have a hip possi. To prove their worth to your group, you have decided that you will test their vocabulary. To do this, you will make a TrieMap. This will make a Trie, but for each word, you also have a note to see how many times it has been used. If any word has been used more than 5 times, the person will not be able to join your group.
You are given the TrieNode class defined as follows:

```
public class TrieNode {
  char c;
  boolean isWord;
  String word;
  TrieNode[] children = new TrieNode[26];
  int count;}
```

Additionally, you are given that the TrieMap has a root class level variable and an **addWord** and **contains** method.
*The LinkedList has an addAll method which adds all items from a collection to the end of the list (e.g. LinkedList A = {A,B,C}, LinkedList B = {D, E,F}. A.addAll(B), A = {A,B,C,D,E,F}

Fill in the below methods

```
public LinkedList getAllWords(TrieNode start) {
  LinkedList list = new LinkedList();
  if (start.isWord) {
     list.add(start);
  }
  TrieNode[] node = start.children;
  TrieNode temp;
  for (int i = 0; i < node.length; i++) {
     temp = node[i];
     if (temp != null) {
        list.addAll(getAllWord(temp));
     }
  }
  return list;
}
```

```
public static boolean testCount(TrieMap t){
  LinkedList<TrieMap.TrieNode> words = t.getAllWords(t.root);
  for(TrieMap.TrieNode node : words){
     if ( node.count < 5){
     return false;
     }
  }
  return true;
}
```

11. Median Heap (20 pts).

We have earlier dealt with Min Heaps and Max Heaps, now we are going to explore a new type of heap, the **Median Heap**. The Median Heap will have 3 methods that you will need to fill in, **median**, which finds the median element, **insert**, which inserts an element into the Median Heap, and **balanceHeap**, which makes the heap balanced. You may need to use some of these methods inside of others. Feel free to use any data structure that we have gone over.

```
public class MedianHeap {
  private MinPQ top = new MinPQ();
  private MaxPQ bottom = new MaxPQ();

  //find the median element, if there are an even amount of elements, take the
average of the 2 elements closest to the middle
  public int median() {
    int minSize = top.size();
    int maxSize = bottom.size();
    if (minSize == 0 && maxSize == 0) {
      return 0;
    }
    if (minSize > maxSize) {
      return top.min();
    }
    if (minSize < maxSize) {
      return bottom.max();
    }
    return (top.min() + bottom.max()) / 2;
  }

  //used to insert elements into the MedianHeap
  public void insert(int element) {
    int median = median();
    if (element > median) {
      top.insert(element);
    } else {
      bottom.insert(element);
    }
    balanceHeap();
  }
```

```
  //Balance the heap
  private void balanceHeap() {
     int minSize = top.size();
     int maxSize = bottom.size();
     int tmp = 0;
     if (minSize > maxSize + 1) {
        tmp = top.delMin();
        bottom.insert(tmp);
     }
     if (maxSize > minSize + 1) {
        tmp = bottom.delMax();
        top.insert(tmp);
     }
  }
}
```

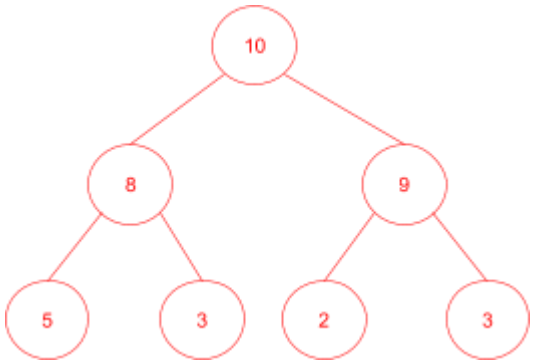
The key to this problem was to realize that you could make a MinHeap with the top 50% of the data inside of it and a MaxHeap with the bottom 50% of the data. To find the median, we need to first check if either of the heaps is bigger than the other (the maximum difference is 1, we will go over why later) if so, you pick the root element from the bigger heap. If they are the same size, you find the average of the roots of both the heaps.

Inserting into the Median Heap is relatively straightforward. You simply check to see if the element is greater than the median, if so you put it in the "top" MinHeap, if not you put it in the "bottom" MaxHeap. After is this call the balanceHeap().

The balanceHeap() method requires you to think about when the median element would not be either the root of one of the heaps or the average if 1 of them. This would only occur if the MinHeap was bigger than the MaxHeap by more than 1 element or vice versa. We make a check for both of these cases and delete the root of the bigger one each time and move it to the other heap.

12. Faster than a Speeding Runtime (17 pts)
Fill in the runtimes for each of the following operations. If there is no tight bound, provide a lower and upper bound. Provide a brief explanation as to why the runtime is what it is.

a) (5 pts) How much time does it take to create a heapify an array of N elements?
O(N). At first glance, it may seem that the answer should be O(NlogN) because of how we learned heapsort. The reason why the runtime is substantially smaller is because we do not have the maximum amount of times an element can sink is proportional to its initial spot in the heap. Keep in mind, that when heapifying, we start from the bottom and move up and that every root must



The bottom is full of singular nodes, all of which are their own heap. Since they are all valid heaps, they cannot sink a total of 0 levels. The 2nd layer to the bottom can sink a total of 1 layer and the top layer can sink a total of 1 layer. There are a total of 7 nodes $(2^3 - 1)$. The total possible amount of sinking operations is equal to $2(1) + 1(2) + 0(4) = 4$.

Now to generalize this, there are a total of N nodes. In a complete heap, approximately $\frac{1}{2}$ of the N nodes will be on the bottom layer, $\frac{1}{4}$ on the layer directly above the bottom one, $\frac{1}{8}$ above that layer and so forth. As we discussed earlier, the nodes on the bottom layer can sink a total of 0 layers, the second bottom layer can sink a total of 1 layer, and so forth. We can rewrite this as

$0(\frac{N}{2}) + 1(\frac{N}{4}) + 2(\frac{N}{8})... = \sum_{i=0}^{log(N)} i(\frac{N}{2^{i+1}}) \leq N$.

b) (3 pts) How much time does it take to find the minimum element in a Binary Search Tree
Ω(1) if it is a spindly Binary Search Tree and it is leaning right
O(N) if it is a spindly Binary Search Tree and it is leaning left

c) (3 pts) How much time does it take to place all the contents of a given MinHeap into an array that is ordered from least to greatest.
Θ(NlogN). You will have to do a total of N deletions, each which will take logN time.

d) (3 pts) How much time does it take to find the minimum value in a MaxHeap.
O(N). You will have to go through the bottom layer, which, as we discovered earlier, contains about $\frac{N}{2}$ nodes. Since we disregard constants, our running time is O(N).

e) (3 pts) Find the runtime of getting all the words from a trie and then using MSD sort on them.
O(W*l), The runtime to find all the words in a trie is O(w*l), where w is the amount of words and l is the length of the longest word. To perform MSD sort on all of them, it takes (w*l). We add the two runtimes and get O(2(w*l)) = O(w*l)

f) (3 pts)

13) Future-Roaming (30 points).
You are a croissant delivery boy in the 31st century. You want to deliver your croissants as quickly as possible; however, because you lost your client's addresses and they live all across the galaxy, so you don't want to go door to door. You have the phone numbers of all the people who ordered croissants from you, so you will attempt to use this to figure out in where you should go. Each phone number is about 10000000000000000000000 digits long and you have a total of N clients. You will use the following information about telephone numbers to help:

1. The sector code (the leftmost 10 digits) will allow you to figure out which sector a client is in. On average, there are a total of $\sqrt{N}$ of households you need to deliver to in the sectors.
2. In the 31st century, telephone numbers have a cool feature where the difference between the last 4 digits of any 2 telephone numbers in the same sector is equal to the distance between their two corresponding households.
3. You know the address of 1 of the houses you need to go in each sector.

You want to go to sectors based off how many of your clients are in them and how close it is . If a sector has 5 clients and is 2 Megadistances away, it is more appealing than a sector that has 1 client and is 1 Megadistance away. If a sector has 6 clients and is 2 Megadistances away, it is less appealing than a sector with 9 clients and 3 Megadistances. If a sector has 4 clients and it is 2 Megadistances away, it is considered equal to a sector that has 2 clients that is 1 MegaDistance away, at this point, you just pick which one to go to randomly . You will deliver to all the households in that sector before going to the next sector. You also know how many megadistances are between each sector, and you can assume that you will start at a location that is equidistant from all the sectors.

Provide an implementation where figuring out which households are in which sector in **Θ(N)** time, figuring out the distance between a house and all the other houses in a sector, must take, on average,**O(** $\sqrt{N}$ **)** time, and delivering croissants to all the households on average, **O(** $N^{\frac{3}{2}} log\sqrt{N}$ **)** time.

Assume that the time it takes you to walk 1 Minidistance (inside of a sector) is the same amount of time it takes your spaceship to travel one Megadistance (between sectors). Justify why your design works in the given time.

What you first want to do is run a special version of MSD on all the telephone numbers. This version of MSD would stop sorting after we get 10 iterations, since we would have sorted by the 10 most significant digits of each telephone number, and have all the numbers sorted by sector.

We will denote the amount of households in a sector by the letter T.

For each sector, we will create a graph with a total of T vertices. Following this, for each household in the sector, we will find the absolute value of the difference between it and every other household in the sector. After finding the absolute value of the difference between the 2 vertices, we would connect these two households with an edge of that weight.
In (somewhat detailed) pseudocode, the above looks like:

```
for(int i = 0; i < number of households; i++){
    for(int k= i + 1; k < number of households; k++){
        int distance = absolute value (last 4 digits of i's telephone # - last 4 digits of k's #);
        addEdge of weight distance between i and k;
     }
}
```

We will denote the amount of sectors with the letter M

Following this, we will create a graph with all the M sectors; however, the "weight" of each edge would be equal to *the Megadistance to the sector / amount of deliveries to make in that sector*, essentially reducing this problem to "which path maximizes the amount of Megadistance/amount of deliveries". We attempt to maximize the value because it makes the weight of the edge larger, essentially making it more costly to travel down that path.

We would perform Dijkstra's from the sector on the graph of sectors and at we would perform Dijkstra's at each sector that we stop in. In the average case, there are a total of $\sqrt{N}$ sectors and $\sqrt{N}$ households per sector. The Dijkstra's between the sectors will take $O(\sqrt{N}\ log(\sqrt{N}))$ and each sector, running Dijkstra's will take $O(\sqrt{N}\ log(\sqrt{N}))$ time. Since, we know that there are a total of $\sqrt{N}$ sectors, we can multiply our sector Dijkstra runtime by $\sqrt{N}$ getting $O(\sqrt{N} * \sqrt{N}\ log(\sqrt{N})) = O(N log(\sqrt{N}))$. We can add the terms to get O($N log(\sqrt{N}) + \sqrt{N}\ log(\sqrt{N})) = O(N log(\sqrt{N}))$ as the average case.

In the worst case we have all the households in one sector, this would result in $O(N * N\ log(\sqrt{N})) = O(N^2\ log(\sqrt{N}))$. In the other worst case, the households are all in different sectors. Since you know the address of one address in each sector, you simply land, drop off the

packet, then leave. However, you have to travel to N different sectors, which means that you will have to take $O(N * N\ log(\sqrt{N})) = O(N^2\ log(\sqrt{N}))$ time.

Easy Mistakes to make:

1. Attempting to perform full MSD or any form of LSD would be incorrect. Our digits are simply too large to perform either one efficiently. The reason why a truncated version of MSD works is because we fit the problem so that only the first digits were considered (essentially making it a constant factor).
2. Constructing a Trie would not work for seeing which numbers contain the same 10 digit prefix because we would have to go all the way down the Trie to see the last few digits that we need.
3. Not figuring out that the weight of an edge should be changed from just the distance or adding up/subtracting the amount of people in a sector and the distance. When adding the 2 values, it is impossible to know if you're dealing with a greater distance or amount of deliveries. When subtracting you could end up with negative edges/cycles. The ratio that we created allows us to properly scale the impact that the amount of people in each sector has on the algorithm.