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Q. Consider the following set of functional dependencies on a relational schema R(W,X,Y,Z)

$F = \{X \rightarrow W,\ WZ \rightarrow XY,\ Y \rightarrow WXZ\}$

Find the irreducable equivalent for this set F.

Ans Let the irreducable set be F_c

Initialization : $F_c = F$
$= \{X \rightarrow W,\ WZ \rightarrow XY,\ Y \rightarrow WXZ\}$

Step-1 : Check if union rule can be applied.
As the left hand side of each FD is unique, so union rule cann't be applied.

Step-2 : Check for extraneous attribute
Consider the first FD
$X \rightarrow W$
Here extraneous attribute is not possible in both LHS or RHS, so we'll keep it as it is.

Now Let's consider the 2nd FD
$WZ \rightarrow XY$
In LHS, neither W nor Z can be extraneous as W^+ in $F = \{W\}$ } they are not able to determine XY indivisually
and Z^+ in $F = \{Z\}$

In RHS, Let's consider X as extraneous
So we have to check WZ^+ on a modified set of FD F'

$F' = \{X \rightarrow W,\ WZ \rightarrow Y,\ Y \rightarrow WXZ\}$

$WZ^+ = \{W, Z, Y, X\}$

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As WZ^+ is correctly determining X on F', we can conclude that X is actually extraneous in $WZ \rightarrow XY$, So the canonical cover of F is

$$F_c = \{X \rightarrow W,\ WZ \rightarrow Y,\ Y \rightarrow WXZ\}$$

Now let's consider the 3rd FD

$$Y \rightarrow WXZ$$

In LHS, there is no extraneous attribute, so let's check in RHS.
In RHS, let's consider W as extraneous. So modified FD set is

$$F' = \{X \rightarrow W,\ WZ \rightarrow Y,\ Y \rightarrow XZ\}$$

Now, Y^+ on F' is

$$Y^+ = \{Y, X, Z, W\}$$

as Y is still able determine W, so we have correctly eliminated W from $Y \rightarrow WXZ$, and W is extraneous here.

So $F_c = \{X \rightarrow W,\ WZ \rightarrow Y,\ Y \rightarrow XZ\}$

Similarly, if we'll check for X or Z, neither of them will be extraneous in $Y \rightarrow XZ$, so the final irreducible set of FD is

$$F_c = \{X \rightarrow W,\ WZ \rightarrow Y,\ Y \rightarrow XZ\}$$