

18/04/2022

STATISTICS EXAM

(P1)

Q1. ANOVA - Analysis of Var.

$$\sum V_1 = 500, \sum V_2 = 530, \sum V_3 = 440, n = 30$$

$$\sum V_1^2 = 26742, \sum V_2^2 = 28586, \sum V_3^2 = 20422$$

$$\text{Correction term: } C_x = \frac{\sum (x)^2}{n} = \frac{(500 + 530 + 440)^2}{30} = \frac{(1470)^2}{30}$$

$$\Rightarrow \frac{2160900}{30} = 72030$$

a. Sum of Squares total - SST

$$SST = \sum x^2 - C_x = (26742 + 28586 + 20422) - 72030$$

$$\Rightarrow 75750 - 72030$$

$$SST \Rightarrow 3720$$

b. Sum of Squares among grp - SSA

$$SSA = \left(\frac{\sum x^2}{n} \right) - C_x = \left(\frac{500^2}{10} + \frac{530^2}{10} + \frac{440^2}{10} \right) - 72030$$

$$\Rightarrow \left(\frac{250000}{10} + \frac{280900}{10} + \frac{193600}{10} \right) - 72030$$

$$\Rightarrow (25000 + 28090 + 19360) - 72030$$

$$\Rightarrow 72450 - 72030$$

$$SSA \Rightarrow 420$$

(c) Sum of square within the group.

$$\begin{aligned} SS_w &= SST - SSA \\ &= 3720 - 420 \\ &= 3300 \end{aligned}$$

(d) mean of Sum of Sqr. among group.

$$\begin{aligned} \mu_{SSA} &= \frac{SSA}{k-1} = \frac{420}{2-1} = \frac{420}{2} & \left| \begin{array}{l} k = \text{Category of} \\ \text{Table} \end{array} \right. \\ &= 210 \end{aligned}$$

(e) mean of Sum of Sqr. within group

$$\mu_{SSw} = \frac{SSw}{N-k} = \frac{3300}{30-3} \Rightarrow \frac{3300}{27} = 122.22$$

$$F\text{-Ratio} = \frac{\mu_{SSA}}{\mu_{SSw}} = \frac{210}{122.22} = 1.718$$

Comparison of F-Ratio Vs F-value (@ 0.05 Sign. level)

Source of Variance	df	SS	μ_{SS}	F-Ratio	F-Value
Among grp	2	420	210	1.718	3.354
within grp	27	3300	122.22		

$\Rightarrow 1.718 < 3.354 \Rightarrow F\text{-Ratio} < F\text{-Value}$
We accept the null hypothesis.

Q2) Determining ANOVA with Stock Market Example.

Financial (x), Energy (y), Utilities (z)

$$\sum x = 92.68, \sum y = 110.77, \sum z = 71.3, n = 24$$

$$\sum x^2 = 1257.48, \sum y^2 = 1699.56, \sum z^2 = 779.13$$

$$\begin{aligned} \text{Correction term: } C_x &= \frac{(\sum x)^2}{n} = \frac{(92.68 + 110.77 + 71.3)^2}{24} \\ &= \frac{(274.75)^2}{24} = \frac{75487.56}{24} = 3145.31 \end{aligned}$$

$$\begin{aligned} \text{Sum of Squ. total} &= SST = \sum x^2 - C_x \\ &= (1257.48 + 1699.56 + 779.13) - 3145.31 \\ &= 3736.16 - 3145.31 \\ SST &= 590.85 \end{aligned}$$

$$\begin{aligned} \text{Sum of Squ. among Grp} - SSA &= \left(\frac{\sum x^2}{n} \right) - C_x \\ &= \left(\frac{92.68^2 + 110.77^2 + 71.3^2}{24} \right) - 3145.31 \\ &= \left(\frac{8589.58 + 12269.99 + 5083.69}{24} \right) - 3145.31 \\ &= \left(\frac{35843.26}{24} \right) - 3145.31 \\ &= 1493.47 - 3145.31 \\ &= -2651.84 \end{aligned}$$

(P4)

$$= (1073.69 + 1533.74 + 635.46) - 3145.31$$

$$= 3242.90 - 3145.31$$

$$SS_A = 97.59$$

Sum of Squares within the group:-

$$SS_W = SS_T - SS_A$$

$$= 590.85 - 97.59$$

$$= 493.26$$

$$\mu_{SSA} = \text{Sum of Sq. among group} = \frac{SS_A}{K-1}$$

$$\mu_{SSA} = \frac{97.59}{3-1} = \frac{97.59}{2} = 48.79$$

Mean of Sum of Sq. within group.

$$\mu_{SSW} = \frac{SS_W}{N-K} = \frac{493.26}{24-3} = \frac{493.26}{21} = 23.48$$

$$F\text{-Ratio} = \mu_{SSA} / \mu_{SSW} = 48.79 / 23.48 = 2.077$$

Compare.

Source of Variance	df	μ_{SS}	SS	F-Ratio	F-Value (0.05)
Among grp	2	48.79	97.59	2.077	3.466
Within grp	21	23.48	493.26		

F-Ratio \neq F-Value.

So there is a difference in rate of return on \downarrow Industries

(Q3) Hyp. test for 2 Sample Variables.

H_0 = There is no significant diff. between Machine 1 and 2 on thickness.

H_1 = There is significant diff. b/w machine 1 and 2 on thickness.

$\alpha = 0.05$, Two Tail

$$F\text{-dist} = F = \frac{S_1^2}{S_2^2} = \frac{\text{est. } \sigma_1^2}{\text{est. } \sigma_2^2}$$

$$\frac{S_1^2}{S_2^2} = \frac{0.797}{0.688} =$$

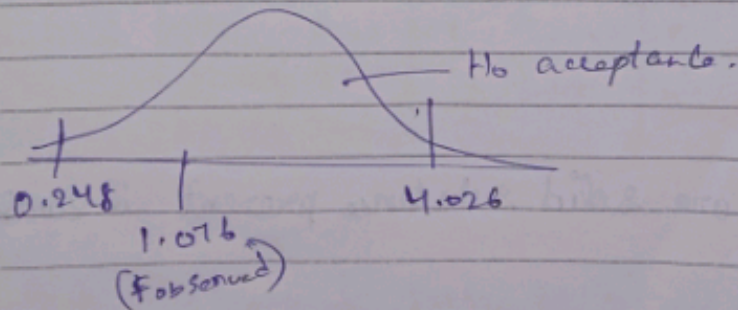
$$\frac{G_1}{G_2} = \frac{0.893}{0.829} = 1.076 = F_{\text{observed}}$$

F-table with df of Samples $\alpha = 0.025$

$$F_{0.25, 9, 9} = 4.026$$

For Two-tail.

$$F_{0.975} = \frac{1}{F_{0.025}} = \frac{1}{4.026} = 0.248$$



Since $F_{observed}$ is falling under Critical Region H_0 is accepted.

So there is no significant difference in thickness between machine 1 and machine 2.

Q4. t -dist. and C.I. for Can Speed.

$$t_{dist} = \bar{X} \pm t \times S/\sqrt{n}, \quad t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$\Rightarrow \bar{X} = 100.83$$

$$\Rightarrow \mu = 100$$

$$\Rightarrow S = 5.55$$

$$\Rightarrow \sqrt{n} = 3.16$$

$$\Rightarrow t = \frac{100.83 - 100}{5.55(3.16)}$$

$$= \frac{0.83}{17.57} = 0.0472$$

Sample en.

$$= 5.55/3.16$$

$$S/\sqrt{n} = 1.75$$

$$\Rightarrow t \times S/\sqrt{n} = 0.0472 \times 1.75$$

$$\text{margin of error} = 0.083$$

$$t_{dist} = 100.83 \pm 0.083 < \mu < 100.83 + 0.083$$

$$= 100.913 < \mu < 100.747$$

So the interval of the true mean will fall under above 95% Confidence Interval.

Q5) T-test of book Sales.

H_0 = There is no significant diff. in book sales after the book enhancement.

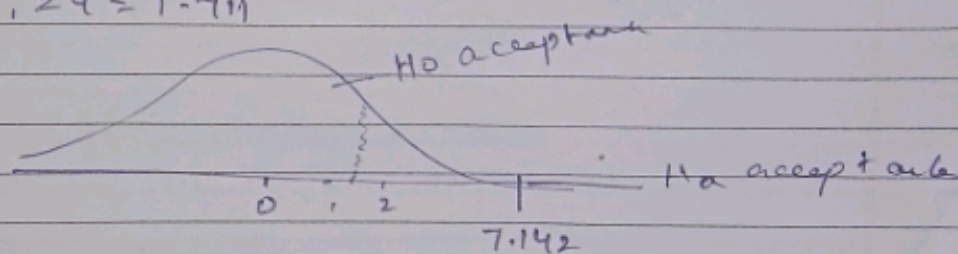
H_a = There is significant diff. in book sales after book enhancement.

$$T\text{-test} = \frac{\bar{x} - \mu}{s/\sqrt{n}} \Rightarrow \mu = 105, \bar{x} = 125, s = 14, n = 25$$

$\alpha = 0.05$, One-tailed.

$$= \frac{125 - 105}{14/\sqrt{25}} = \frac{20}{2.8} = 7.142$$

$$t_{0.05, 24} = 1.711$$



Reject H_0

Since the value of 7.142 falls in rejection area, it is highly likely that mean sale is greater and the book enhancement is probably a success.

Q6) Z-test for Mean distance.

H_0 = The mean distance of U.S. workers is equal to the mean distance of workers in New York

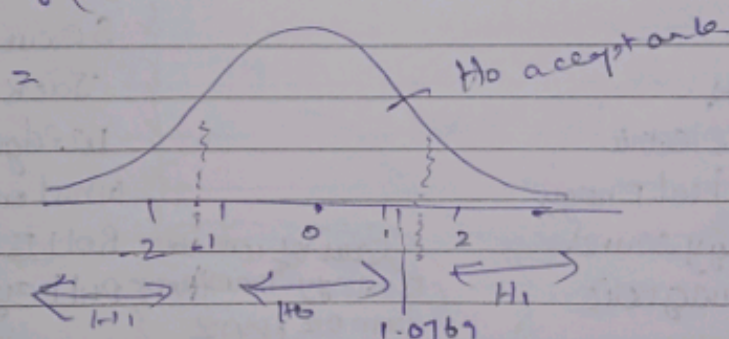
H_0 = The mean distance is not same.

$$Z\text{-test} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow \bar{x} = 16, \mu = 15, \sigma = 14, n = 169.$$

$\alpha = 0.10$, Two tailed.

$$= \frac{16 - 15}{14/\sqrt{13}} = \frac{1}{0.923} = 1.0769.$$

$$Z_{\text{score}}(\alpha/2) \Rightarrow Z_{0.05} = 1.645$$



Since the Z score is falling in acceptance region H_0 is accepted.

Interpretation:- The national average distribution does describe the mean commuting distance of all workers in New York area.