Comparision between Collapse Models

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Abstract

In section 1, the mathematical form of two collapse models is explained briefly. The next section presents the problems and inconsistencies in the recent model proposed by Naveen Kumar and Apoorva Patel, while the third section contains the possible modifications in the present model. I will ignore H in the entire article, as it can be easily added later without any trouble.

1 The Collapse Models

We will discuss the generalsied version of Continous Spontaneous Localisation model (hereby referred as CSL model) and the recent collapse model proposed by Naveen Kumar and Apoorva Patel, hereby referred as NKAP model.

1.1 CSL model

p Assume a linear stochastic operator for time evolution resulting in a new, non-normalised wavefunction

$$d\left|\phi\right\rangle = (Cdt + \vec{A}\cdot d\vec{B})\left|\phi\right\rangle \tag{1}$$

where A is a vector of operators, and dB is a n dimensional Weiner Process. For the rest of the paper, $n = 1$.

We normalise the above equation to get

$$d\left|\phi\right\rangle = \{\frac{\gamma}{2}[-(A^\dagger - R)A + (A - R)R]dt + (A - R)dB\}\left|\psi\right\rangle \tag{2}$$

where $R = \left\langle\psi\right| A \left|\psi\right\rangle$, $\overline{dB} = 0$ and $\overline{dB \cdot dB} = \gamma dt$ for some probability distribution of dB.
in terms of density matrix,

$$\rho = \gamma(A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\})dt + (A - R)dB\rho + \rho(A^\dagger - R)dB \tag{3}$$

where Poisson notation is used.

for $A = A^\dagger$,

$$d\left|\phi\right\rangle = \{\frac{-\gamma}{2}(A-R)^2 dt + (A-R)dB\}\left|\psi\right\rangle \tag{4}$$

and

$$\rho = \gamma(A\rho A - \frac{1}{2}\{A^2, \rho\})dt + \{(A-R), \rho\}dB \tag{5}$$

If A has eigenvectors $\left|phi_i\right\rangle$ having eigenvalues α_i, then in the eigenbasis of A

$$d\rho_{jk} = \frac{-\rho_{jk}\gamma}{2}(\alpha_j - \alpha_k)^2 dt + \rho_{jk}(\alpha_j + \alpha_k - 2\sum_i \alpha_i \rho_{ii})dB \tag{6}$$

Taking average,

$$\frac{d\overline{\rho_{jk}}}{dt} = \frac{-\rho_{jk}\gamma}{2}(\alpha_j - \alpha_k)^2 \tag{7}$$

Hence, on an average, off diagonal terms decay, while diagonal terms do not change.
The model is consistent, and the only problem is with assigning intuitive meaning to eq 2. It is difficult to assign intuitive meaning to eq 1 as well since the equation results in a non-normalised wave function. It seems that luckily eq 1 fits our description.

1.2 NKAP model

Consider a wavefunction $\left|\psi\right\rangle$ drifting towards one of the eigenfunction $\left|\phi_i\right\rangle$

We assume that operator O_i is the infinitesimal drift operator, resulting in the non- normalised wavefunction.

$$O_i = (1-ds)I + dsP_i \tag{8}$$

where s is a parameter of evolution. Hence the equation of evolution resulting in normalised wavefunction is

$$d\left|\psi\right\rangle = \frac{O_i\left|\psi\right\rangle}{\sqrt{(\langle\psi O_i|)(|O_i\psi\rangle)}} \tag{9}$$

$$d\left|\psi\right\rangle = ds(P_i - \left\langle\psi\right| P_i \left|\psi\right\rangle)\left|\psi\right\rangle \tag{10}$$

where P_i is the usual projection operator. We expect that the parameter s is proportional to time, and hence assume $ds = gdt$. Since the wavefunction drifts towards various eigenfunction with different probability w_i, Hence

$$\frac{d\left|\phi\right\rangle}{dt} = \sum_j w_j g(P_j - \left\langle\psi\right| P_j \left|\psi\right\rangle)\left|\psi\right\rangle \tag{11}$$

which gives

$$\frac{d\rho}{dt} = \sum_j w_j g(P_j\rho + \rho P_j - 2\rho Tr(P_j\rho)) \tag{12}$$

In the chosen Eigenbasis, we get

$$\frac{d\rho_{jk}}{dt} = g\rho_{jk}(w_j + w_k - 2\sum_i w_i\rho_{ii}) \tag{13}$$

Also, observe that the equation results is

$$2\frac{d\rho_{jk}}{\rho_{jk}} = \frac{d\rho_{jk}}{\rho_{jk}} + \frac{d\rho_{kk}}{\rho_{kk}} \tag{14}$$

Since initially $\rho_{jk} = c_j c_k^*$ for a pure ensemble, the purity will be maintained.

The model proposes that the weight(or probability) of moving towards a particular eigenfunction is equal to ρ_{ii} and some noise term. Since we want $\sum w_i = 1$, there can be $n-1$ independent noise terms. Also, I it natural to want noise such that the noise term in $w_i - w_j$ have same rms value independent of choice of distinct i and j. Fortunately, there exist an algorithm to generate such w_i.

$$\sum_{i=1}^{k-1} w_i - w_k = \sum_{i=1}^{k-1} \rho_{ii} - \rho_{kk} + \sqrt{\frac{k(k-1)S}{2}}\xi_k \tag{15}$$

for $k = 1, 2, 3..., n$. If you define noise as $\eta_{ij} = (w_i - w_j) - (\rho_{ii} - \rho_{jj})$ for distinct i and j, then we will have following two conditions:

$$\overline{\eta_{ij}} = 0 \tag{16}$$

$$\overline{\eta_{ij}\eta_{ik}} = S\delta_k^j \tag{17}$$

2 Problems with NKAP model

2.1 Conversion to diagonal Matrix

By substituting $k = j$ in eq 13, we get

$$\frac{d\rho_{jj}}{dt} = 2g\rho_{jj}(w_j - \sum_i w_i\rho_{ii}) \tag{18}$$

Taking average values,

$$\frac{d\overline{\rho_{jj}}}{dt} = 2g\rho_{jj}(\overline{w_j} - \sum_i \overline{w_i}\rho_{ii}) \tag{19}$$

Since we want that on an average, diagonal elements do not change, we will need $\overline{w_j} = \sum_i \overline{w_i}\rho_{ii}$ for $j = 1, 2....n$. This can only be true only if $w_j = 1/n$, which is not the case according to this model.

Solution(by NKAP model): The problem can be resolved if we (arbitrarily) assume that the given equation is in stratonovich form, which must be converted to ito form. This gives an equation resulting in collapse in case of 2 dimensional system.

2.2 Non Linearity and Non-Zero off diagonal elements

Suppose ρ_1 denotes a pure ensemble consisting of state $|\psi\rangle$ and ρ_2 denotes a pure ensemble consisting of state $|\phi\rangle$. If ρ denotes ensemble consisting of states $|\psi\rangle$ and $|\phi\rangle$ with weight λ and $1-\lambda$ respectively, then we must have

$$\rho = \lambda |\psi\rangle \langle\psi| + (1-\lambda) |\phi\rangle \langle\phi| = \lambda\rho_1 + (1-\lambda)\rho_2 \tag{20}$$

Since the evolution of 2 systems is independent, the equation must hold true at all times, resulting in

$$d\rho = \lambda d\rho_1 + (1-\lambda)d\rho_2 \tag{21}$$

This suggests that the equation of eolution of rho must be linear, which is not true in NKAP model. Also, I performed explicit calculation in 2d system and concluded that the linearity implied by eq 20 is not obeyed by the given model.

According to equation 14,

$$\rho_{jk}(t) = \rho_{jk}(0)\sqrt{\frac{\rho_{jj}(t)\rho_{kk}(t)}{\rho_{jj}(0)\rho_{kk}(0)}} \tag{22}$$

This equation suggests that after collapse, if two diagonal terms are non-zero, we must have a non-zero off diagonal element. Assuming an ensemble of wavefunctions collapsing in different eigenstates, there will necessarily be more than one non-zero off diagonal terms, and hence non-zero off-diagonal terms. The statement in itself is incositent in the sense that since collapse has occurred, the there is no correlation and hence off-diagonal terms must be zero.

Solution: Both of these problems can be solved if we change the definition of density matrix so that it only represent a single wavefunction, and not an ensemble. But this will then not be required since we already have an equation of evolution of a single wavefunction, and will obviously create problems in description of ensemble under this model.

2.3 Stochastic Calculus Inconsistency

In stochastic calculus,

$$d(fg) = fdg + gdf + \overline{(df)(dg)} \tag{23}$$

Since eq 11 contains w_i, which are stochastic, it should be converted into a stochastic differential equation and then equation 23 should have been used, but in NKAP model the stochastic terms are treated in normal terms, and ruled of ordinary Calculus have been used. Hence, instead of

$$d\rho = (d\left|\psi\right\rangle)\left\langle psi\right| + \left|\psi\right\rangle(d\left\langle\psi\right|) \tag{24}$$

the correct equation that should have been used should be

$$d\rho = (d\left|\psi\right\rangle)\left\langle psi\right| + \left|\psi\right\rangle(d\left\langle\psi\right|) + \overline{(d\left|\psi\right\rangle)(d\left\langle\psi\right|)} \tag{25}$$

where the third term should be non-zero because of presence of stochastic part as $\overline{\xi dt * \xi dt} = dt$

2.4 Validity of higher dimensional systems

Eq 12 along with definitions of w_i according to eq 15 results in collapse of 2-dimensional system. Working with arbitrary dimensional system is difficult, and hence the calculation was carried out for a 3-d system. The calculation shows that the equation does not result in collapse.

Possible Explanation: Since I am not well versed in Stochastic calculus, there is a possibility that some mistake was made during conversion from stratonovich form to ito form, or the subsequent calculations.

3 Possible Modification

In the NKAP model, the evolution equation is more intuitive. Unlike CSL model, NKAP model assigns intuitive meaning to the terms present in the evolution equation. However, also unlike the CSL model, NKAP model is highly inconsistent.

There can be few possible modification of NKAP model which might resolve some of the inconsistencies present in the model. This section will be finished later.