Income Prediction using Multiple Regression

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***Abstract*—This is a report explaining how the income of an individual can be predicted using multiple regression.**

Keywords— multiple regression

I. Introduction

Regression is a method/statistical way to explain a relationship between a dependent variable and one or more predictors/independent variables using an equation.

The Multiple regression or multiple linear regression creates an equation between the dependent variable (Y) and more than one independent variable ($X_1 \ldots X_n$) so that we can predict the Y by providing the X(s).

II. Methodology

Figure 1: - Methodology Diagram [1]

The Methodology is referenced from [1] but is re-designed in this project.

A. Preparation Phase

A.1. Exploratory Data Analysis (EDA)

The Exploratory Data Analysis is an initial analysis done on the whole raw data to get information about the nature and characteristics of the variables/features present in the data set. It also tells how the different features are distributed in the dataset.

How does it help us? based on this analysis we can make a list of ordinals, non-ordinals, temporal, discrete and continuous features. We can use this list to transform features based on their type and distribution.

"Skewness refers to a distortion or asymmetry that deviates from the symmetrical bell curve, or normal distribution, in a set of data. If the curve is shifted to the left or the right, it is said to be skewed. Skewness can be quantified as a representation of the extent to which a given distribution varies from a normal distribution."[2]

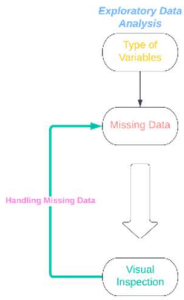


Figure 2: Steps in EDA

Step 1: Types of Variables

Table 1 - Type of Variables

Feature	Type	Sub-Type
edcat	Categorical	Non-Ordinal
default	Categorical	Binary
jobsat	Categorical	Ordinal
homeown	Categorical	Binary
cars	Discrete	
yrsed	Discrete	
yrsempl	Discrete	
address	Discrete	
age	Discrete	
income	Discrete	
creddebt	Continuous	
othdebt	Continuous	
carvalue	Continuous	

Step 2: Checking Missing Values

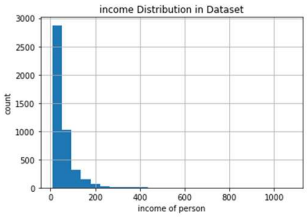
The below code snippet returns a list of features containing missing values: -

```
features_miss_val= [features for features in dataset.columns if dataset[features].isnull().sum()>1]
print(features_miss_val)
```

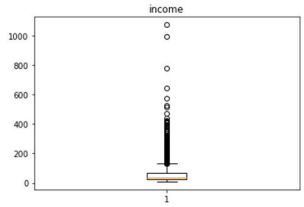
The List returned is empty, hence we can conclude that data contains no missing values.

Step 3: Visual Inspection

a. Income: -



Plot 1: - Skewness in Income

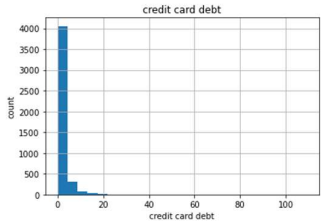


Plot 2: - Outliers in Income

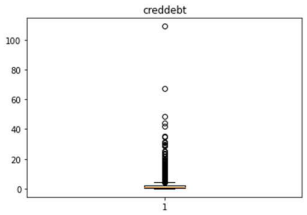
Result of inspection: -

1. Highly Right Skewed
2. Skewness - 5.233689457953158
3. A large Number of Outliers

b. Credit card debt (in thousands) creddebt: -



Plot 3: - Skewness id creddebt

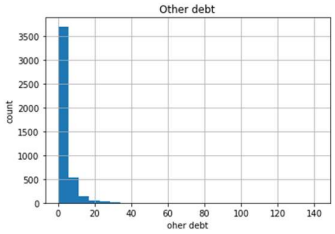


Plot 4: - outliers in creddebt

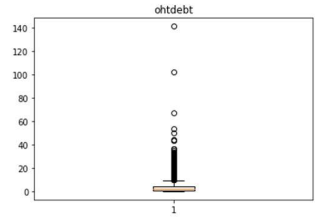
Result of inspection: -

1. Highly Right Skewed
2. Skewness - 10.962120273419949
3. A large Number of Outliers

c. Other Debt (in thousands) othdebt: -



Plot 5: - Skewness in othdebt

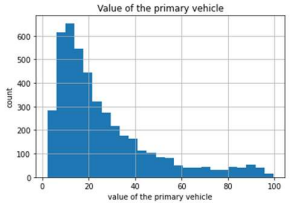


Plot 6: - outliers in othdebt

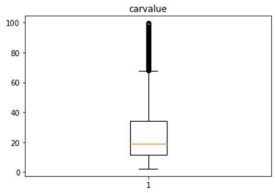
Result of inspection: -

1. Highly Right Skewed
2. Skewness - 7.69338390729621
3. A large Number of Outliers

d. Value of the primary Vehicle (carvalue): -



Plot 7: - Skewness in carvalue



Plot 8: - outliers in carvalue

Result of inspection: -

1. Highly Right Skewed
2. Skewness - 1.5301894027033907
3. A large Number of Outliers

Summary of the Data: -

Table 2: - Statistic Summary of Dataset

Descriptive Statistics

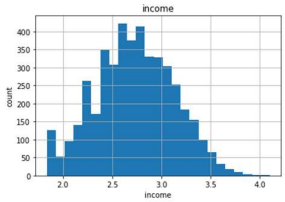
	N	Minimum	Maximum	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
age	4508	18	79	46.93	17.665	.094	.036	-1.175	.073
yrsed	4508	6	23	14.53	3.286	.020	.036	-.606	.073
edcat	4508	1	5	2.67	1.214	.257	.036	-.993	.073
yrsempl	4508	0	52	9.72	9.651	1.249	.036	1.066	.073
income	4508	9	1073	55.41	56.514	5.234	.036	57.644	.073
creddebt	4508	.000000	109.072596	1.89786636	3.542646400	10.962	.036	237.337	.073
othdebt	4508	.000000	141.459150	3.69144686	5.378583009	7.693	.036	133.643	.073
default	4508	0	1	.24	.426	1.225	.036	-.500	.073
jobsat	4508	1	5	2.96	1.377	.031	.036	-1.231	.073
homeown	4508	0	1	.63	.483	-.532	.036	-1.718	.073
address	4508	0	57	16.37	12.368	.717	.036	-.183	.073
cars	4508	1	8	2.37	1.158	.875	.036	.799	.073
carvalue	4508	2.2	99.6	26.082	20.8626	1.530	.036	1.884	.073
Valid N (listwise)	4508								

A.2. Transformation and Scaling

From EDA, we can conclude that the variables are skewed and have a high number of outliers.

In this step, we'll be doing Box-Cox transformation on the dependent variable (income). "We won't be doing transformation on the independent variables, as Linear Regression Model is not affected by the distribution of the predictors." [4]

"A box-cox transformation is a commonly used method for transforming a non-normally distributed dataset into a more normally distributed one." [3]



Plot 9: - Income After Transformation

Now, we'll be handling the outliers.

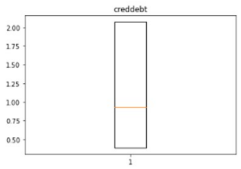
"An *outlier* is an observation that lies an abnormal distance from other values in a random sample from a population. In a sense, this definition leaves it up to the analyst (or a consensus

process) to decide what will be considered abnormal. Before abnormal observations can be singled out, it is necessary to characterize normal observations." [5]

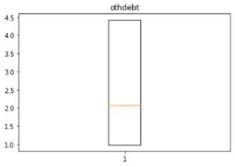
If a data point is less than the value of the 1st Quartile or is greater than the value of the 3rd Quartile, then we consider that data point as an Outlier. The outliers need to handle as they can cause a drastic shift in the mean.

Consideration while handling outliers: -

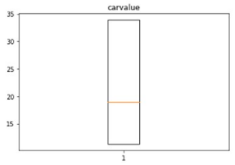
1. Check if that outlier is necessary or not,
   E.g., Age of person = 180 years is an outlier in the income dataset.
2. We can trim the insignificant outliers.
3. We can perform Quantile-based Flooring and Capping [6]



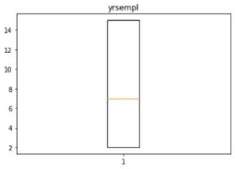
Plot 10: - creddebt after handling outliers



Plot 11: - othdebt after handling outliers



Plot 12: - carvalue after handling outliers



Plot 13: - yrsempl after handling outliers

How Quantile-based Flooring and Capping [6] was applied?

1. For each variable the Q1, IQR and Q3 were calculated.
2. The Data points greater than Q3 or less than Q1 was replaced by the Q3 & Q1 respectively.

The Encoding is required for the level of education (edcat) as it is a non-Ordinal Categorical variable. The best encoding, in this case, will be One-Hot [7] encoding.
The One-Hot [7] encoding takes every value from the variable and creates a binary data column for each type
E.g., if x has values (1,2,3), we'll have 2 new columns with binary data in it, like this (for 1 all columns will be 0): -

Table 3: -One-Hot Encoding Example

X	2	3
1	0	0
2	1	0
3	0	1

B. *Testing Phase (Before Model Creation)*

B.1. Assumptions

1. Correlation between the variables: -

Null Hypothesis 1: - The Independent features are Highly correlated with each other
Null Hypothesis 2: - The Independent features are multi-Correlated

Table 4: -Correlation Matrix

	yrsempl	income	creddebt	othdebt	default	jobsat	homeown	address	cars	carvalue	2	3	4	5
yrsempl	1.000000	0.311253	0.218660	0.245371	-0.363322	0.503632	0.024256	0.611117	0.006647	0.368083	0.019866	-0.045318	-0.094877	-0.077902
income	0.311253	1.000000	0.570192	0.645215	-0.033584	0.250681	0.156806	0.196321	0.056096	0.889347	-0.103246	0.022602	0.114828	0.123884
creddebt	0.218660	0.570192	1.000000	0.600492	0.160191	0.180391	0.070693	0.135896	0.022499	0.530280	-0.031156	-0.005258	0.075184	0.060650
othdebt	0.245371	0.645215	0.600492	1.000000	0.094991	0.197541	0.093721	0.176155	0.041540	0.604885	-0.068500	-0.004209	0.094432	0.082026
default	-0.363322	-0.033584	0.160191	0.094991	1.000000	-0.203444	-0.027361	-0.357760	0.013890	-0.070019	-0.001450	0.010645	0.064227	0.040252
jobsat	0.503632	0.250681	0.180391	0.197541	-0.203444	1.000000	0.022143	0.350661	0.013886	0.272611	-0.004409	-0.011538	-0.039797	-0.032055
homeown	0.024256	0.156806	0.070693	0.093721	-0.027361	0.022143	1.000000	0.137432	0.017491	0.140499	-0.023626	0.023809	0.036212	0.001901
address	0.611117	0.196321	0.135896	0.176155	-0.357760	0.350661	0.137432	1.000000	-0.000072	0.252117	-0.051968	-0.023364	-0.008046	-0.000447
cars	0.006647	0.056096	0.022499	0.041540	0.013890	0.013886	0.017491	-0.000072	1.000000	0.055792	0.000054	0.013717	0.008527	-0.017404
carvalue	0.368083	0.889347	0.530280	0.604885	-0.070019	0.272611	0.140499	0.252117	0.055792	1.000000	-0.094809	0.005893	0.121886	0.103692
2	0.019866	-0.103246	-0.031156	-0.068500	-0.001450	-0.004409	-0.023626	-0.051968	0.000054	-0.094809	1.000000	-0.342529	-0.359345	-0.188303
3	-0.045318	0.022602	-0.005258	-0.004209	0.010645	-0.011538	0.023809	-0.023364	0.013717	0.005893	-0.342529	1.000000	-0.268046	-0.140461
4	-0.094877	0.114828	0.075184	0.094432	0.064227	-0.039797	0.036212	-0.008046	0.008527	0.121886	-0.359345	-0.268046	1.000000	-0.147356
5	-0.077902	0.123884	0.060650	0.082026	0.040252	-0.032055	0.001901	-0.000447	-0.017404	0.103692	-0.188303	-0.140461	-0.147356	1.000000

Please Note: - 2, 3, 4, 5 are the new columns created after One-Hot encoding of the edcat (Level of education) and the edcat is dropped from the dataset.

The variables with Pearson's correlation value >=0.5 between them are considered highly correlated and we can either remove one of them or we can aggregate them during model training. For those variables (except *vs income*) our assumption of *Multicorrelation* among the independent variables *Fails*.

(The Assumption of Multicorrelation states "there is no correlation between the Independent Variables")

Since the Assumption of Multicorrelation failed, therefore we fail to reject the Null Hypothesis 1.

Though income and carvalue depict a very high correlation, since income is the dependent variable, we can say that carvalue is essential in predicting income.

To check for *Null Hypothesis 2,* we'll look for VIF (Variance Inflation Factor). The table below only shows features with VIF>=10

Table 5: -VIF of the features

Feature	VIF
age	32.790978
yrsed	57.512400
carvalue	12.639053

The feature with VIF >=10 should be considered for the removal from the dataset as these features are not much independent.
Before considering a feature for the removal, we checked how much it is good in predicting the income. To decide this, we compared the correlation of each feature with high VIF with income and found age and yrsed *(years of education)* have the least correlation of 0.1 approx. Therefore, these features (age & yrsed *(years of education)*) were removed from the dataset.

Table 6: -Correlation of Income with High VIF features

Feature vs Income	correlation
income & age	0.12833155470874583
income & yrsed	0.19798343138251495
income & carvalue	0.8893470860662386

C. *Creation Phase*

C.1. Feature Selection and Model Training

In Feature Selection we try to select only the most relevant feature for our model, this process helps in avoiding

the overfitting and underfitting of the linear regression model.
In Model Training we use the combination of the selected features, to create a highly accurate model.

We combined the Feature selection and Model training phase to generate a model with the best subset of features.
For this, we applied Subset Algorithm [8]

Subset Algorithm for Model Training and Selection: -

1. The Dataset is split into Test and Training data
2. The Train Data is used to train various models based on different combinations of the features.
3. The Test Data is used to test these models in every iteration
4. All the models with their respective R^2 and RSS value are stored in the list.
5. The model with the respective subset that has the highest R^2 is selected.

Total no. of combination on which models are created and tested is 2^n
Where n = no. of features, therefore we had 2^{14} = 16384 combinations

Out of these combinations, only 2 models fitting the criteria (given below) were selected with an approx. accuracy of 81.9%.
One of the models was dropped as it contained more features than the other.

The selected model is then tested for various assumptions of the linear regression model, if the selected model is satisfactorily satisfying the assumptions, we consider the relationship is accurate enough for prediction.

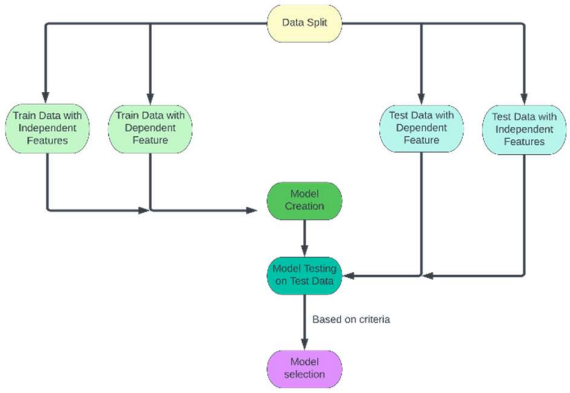


Figure 2: -Model Training and Testing

The criteria based on which the model is selected: -

1. The model with accuracy >70% (R^2 >0.70)
2. The model with the least features
3. The model with the least skewness {*in Testing Phase (After model creation)*}

Equation of the selected model: -
*income = 1.8627 + credebt *0.0520 + othdebt *0.0380 – default *0.0210 + homeown *0.0297 – address *0.0014 + cars *0.0026 + carvalue *0.0331 + edcat(level-3) *0.0202 + edcat(level-4) *0.0062 + edcat(level-5) *0.0511*

D. Testing Phase(After model creation)

D.1. Assumptions of Linear Regression Model

The Gauss Markov Theorem [9]

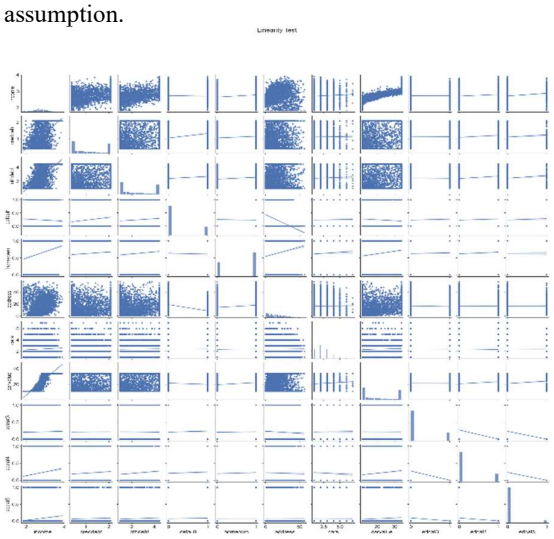
Some important information regarding Gauss-Markov Theorem

- We say that an estimator is linear if it is a linear function of y1, ..., yn. the OLS estimators b1, b2 are linear estimators.
- We say that an unbiased estimator is more efficient than another unbiased estimator if it has a smaller variance
- We say that an estimator is Blue (Best Linear Unbiased Estimator) if it is linear and unbiased and more efficient than any other linear and unbiased estimator.

"The Gauss-Markov theorem states that if your linear regression model satisfies the first six classical assumptions, then ordinary least squares (OLS) regression produces unbiased estimates that have the smallest variance of all possible linear estimators"[9].

The Assumptions of Linear Regression: -

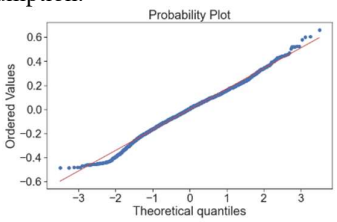
1. *Linear Relationship*
   Aims at finding a linear relationship between the independent and dependent variables.
   Scatter plots are used to visually determine this assumption.



Plot 14: - Scatter-plots

From the above plot, we can conclude that the income is satisfactorily linearly related to the independent variables

2. *Variables follow a normal distribution*
   This assumption ensures that for each value of the independent variable, the dependent variable is a random variable following a normal distribution and its mean lies on the regression line.
   Quantile-Quantile plot is a visual way to inspect this assumption.



Plot 15: -Q-Q plot

From the above plot, we can infer that Assumption 2 is almost satisfied

3. *Little or no multicollinearity*
It tests the correlation between the independent variables.
If multicollinearity exists between them, they are no longer independent.
We checked the VIF of each independent variable and the correlation between each independent variable.

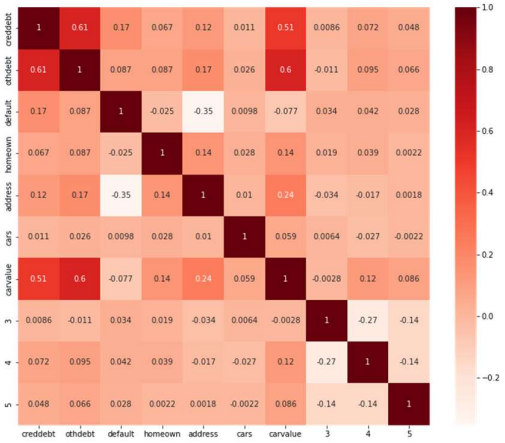


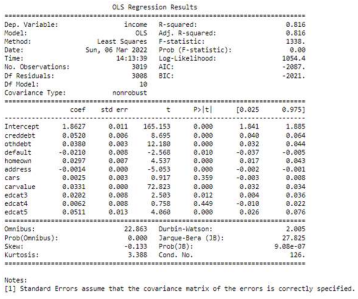
Table 7: -Correlation Matrix

By observing the above table, we can conclude that there are no signs of multicollinearity as no correlation is >=0.8.

4. *Little or no Autocorrelation*
This assumption is like the above assumption, only the exception is, it applies to the residuals of the linear regression model.

We can test the assumption with the Durbin-Watson test.

Values from the Durbin-Watson test are in the range 0-4 where if d = 2, we accept that there is no autocorrelation.



OLS Regression Results

Dep. Variable:	income	R-squared:	0.816
Model:	OLS	Adj. R-squared:	0.816
Method:	Least Squares	F-statistic:	1338.
Date:	Sun, 06 Mar 2022	Prob (F-statistic):	0.00
Time:	14:13:39	Log-Likelihood:	1054.4
No. Observations:	3019	AIC:	-2087.
Df Residuals:	3008	BIC:	-2021.
Df Model:	10		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	1.8627	0.011	165.153	0.000	1.841	1.885
creddebt	0.0528	0.006	8.695	0.000	0.040	0.064
othdebt	0.0380	0.003	12.180	0.000	0.032	0.044
default	-0.0210	0.008	-2.560	0.010	-0.037	-0.005
homeown	0.0297	0.007	4.537	0.000	0.017	0.043
address	-0.0014	0.000	-5.053	0.000	-0.002	-0.001
cars	0.0025	0.003	0.917	0.359	-0.003	0.008
carvalue	0.0331	0.000	72.823	0.000	0.032	0.034
edcat3	0.0202	0.008	2.503	0.012	0.004	0.036
edcat4	0.0062	0.008	0.758	0.449	-0.010	0.022
edcat5	0.0511	0.013	4.060	0.000	0.026	0.076

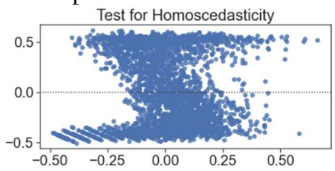
Omnibus:	22.863	Durbin-Watson:	2.005
Prob(Omnibus):	0.000	Jarque-Bera (JB):	27.825
Skew:	-0.133	Prob(JB):	9.08e-07
Kurtosis:	3.388	Cond. No.	126.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Figure 3: -Model Summary

By observing the value from the Durbin-Watson test(d=2.005), we can conclude that there is no autocorrelation.

5. *Data is homoscedastic*
According to this assumption, the error terms along the regression line are equal.
It is also applied to the residuals of the linear regression model.
This assumption can be tested visually using a scatter plot of the residuals.



Plot 16: -Scatter plot for residuals

The above plot fails to provide any signs of heteroscedastic pattern in residual; therefore, we can conclude that the data is Homoscedastic.

Since all the Assumptions of Linear Regression Model satisfied to an extent where we can say our model follows Gauss-Markov theorem with an accuracy of 81.6% in predicting the Income of an Individual.

E. *Abbreviations and Acronyms*

1. LRM - Linear Regression Model
2. E.g. - For Example
3. VIF - Variance Inflation Factor
4. Corr - Correlation

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