Assignment on Propositional Logic

1. Let p stand for the proposition "I bought a lottery ticket "and q for "I won the jackpot".

   Express the following as natural English sentences:

   (a) $\neg p$

   (b) $p \vee q$

   (c) $p \wedge q$

   (d) $p \Rightarrow q$

   (e) $\neg p \Rightarrow \neg q$

   (f) $\neg p \vee (p \wedge q)$

2. Formalize the following in terms of atomic propositions r, b, and w, first making clear how they correspond to the English text.
   a) Berries are ripe along the path, but rabbits have not been seen in the area.
   b) Rabbits have not been seen in the area, and walking on the path is safe, but berries are ripe along the path.
   c) If berries are ripe along the path, then walking is safe if and only if rabbits have not been seen in the area.
   d) It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.
   e) For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
   f) Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.

3. Let A ="Aldo is Italian" and B ="Bob is English". Formalize the following sentences:
   a) "Aldo isn't Italian"
   b) "Aldo is Italian while Bob is English"
   c) "If Aldo is Italian then Bob is not English"
   d) "Aldo is Italian or if Aldo isn't Italian then Bob is English"
   e) "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"

4. For each of the following propositions, construct a truth table

   (a) $p \wedge \neg p$

   (b) $p \vee \neg p$

   (c) $(p \vee \neg q) \Rightarrow q$

   (d) $(p \vee q) \Rightarrow (p \wedge q)$

   (e) $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$

   (f) $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$

5. For each of the following propositions, construct a truth table
   (a) $p \Rightarrow (\neg q \vee r)$
   (b) $\neg p \Rightarrow (q \Rightarrow r)$
   (c) $(p \Rightarrow q) \vee (\neg p \Rightarrow r)$
   (d) $(p \Rightarrow q) \wedge (\neg p \Rightarrow r)$
   (e) $(p \Leftrightarrow q) \vee (\neg q \Leftrightarrow r)$
   (f) $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (q \Leftrightarrow r)$
6. Use truth tables to determine which of the following are equivalent to each other:
   (a) P
   (b) $\neg P$
   (c) $P \Rightarrow F$
   (d) $P \Rightarrow T$
   (e) $F \Rightarrow P$
   (f) $T \Rightarrow P$
   (g) $\neg\neg P$
7. Use truth tables to determine which of the following are equivalent to each other:
   (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
   (b) $\neg P \vee Q$
   (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$
   (d) $\neg(P \vee Q)$
   (e) $(Q \wedge P) \vee \neg P$
8. Let's consider a propositional language where
   • A="Angelo comes to the party",
   • B="Bruno comes to the party",
   • C ="Carlo comes to the party",
   • D="David comes to the party".
   Formalize the following sentences:
   a) "If David comes to the party then Bruno and Carlo come too"
   b) "Carlo comes to the party only if Angelo and Bruno do not come"
   c) "David comes to the party if and only if Carlo comes and Angelo doesn't come"
   d) "If David comes to the party, then, if Carlo doesn't come then Angelo comes"
   e) "Carlo comes to the party provided that David doesn't come, but, if David comes, then Bruno doesn't come"
   f) "A necessary condition for Angelo coming to the party, is that, if Bruno and Carlo aren't coming, David comes"
   g) "Angelo, Bruno and Carlo come to the party if and only if David doesn't come, but, if neither Angelo nor Bruno comes, then David comes only if Carlo comes"