Class Notes

BOOLEAN ALGEBRA

➢ Introduction:

- An algebra that deals with binary number system is called "Boolean Algebra".
- It is very power in designing logic circuits used by the processor of computer system.
- The logic gates are the building blocks of all the circuit in a computer.
- Boolean algebra derives its name from the mathematician **George Boole** (1815-1864) who is considered the "**Father of symbolic logic**".
- Boolean algebra deals with truth table TRUE and FALSE.
- It is also called as "**Switching Algebra**".

➢ Binary Valued Quantities – Variable and Constants:

- A variable used in Boolean algebra or Boolean equation can have only one of two variables. The two values are FALSE (0) and TRUE (1)
- A Sentence which can be determined to be TRUE or FALSE are called **logical statements** or **truth functions** and the results TRUE or FALSE is called **Truth values**.
- The variables which can store the truth values are called **logical variables or binary valued variables**. These can store one of the two values 1 or 0.
- The decision which results into either YES (TRUE or 1) or NO (FALSE or 0) is called **Binary decision**.

➢ Truth Table:

- A truth table is a mathematical table used in logic to computer functional values of logical expressions.
- A truth table is a table whose columns are statements and whose rows are possible scenarios.
- Example: Consider the logical expression

Logical Statement: Meals = "Ram prefer rice and roti for the meal"

Y = A AND B (Logical Variables: Y, A, B, Logical Operator AND)

Ram Prefer Rice	Ram Prefer Roti	Meals
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

- If result of any logical statement or expression is always TRUE or 1, it is called **<u>Tautology</u>** and if the result is always FALSE or 0, it is called **<u>Fallacy.</u>**

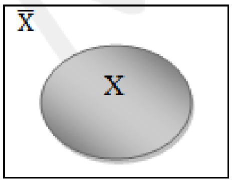
Logical Operators:

- There are three logical operator, NOT, OR and AND.
- These operators are now used in computer construction known as switching circuits.

NOT Operator:

- The Not operator is a unary operator. This operator operates on single variable.
- The operation performed by Not operator is called **<u>complementation</u>**.
- The symbol we use for it is bar.
- $\overline{\mathbf{X}}$ means complementation of X
- If X=1, $\overline{X}$=0   If X=0, $\overline{X}$=1
- The Truth table and the Venn diagram for the NOT operator is:

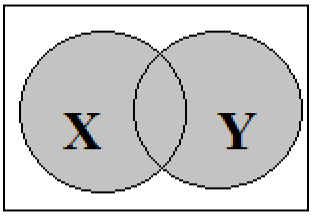
X	$\overline{X}$
1	0
0	1



OR Operator:

- The OR operator is a binary operator. This operator operates on two variables.
- The operation performed by OR operator is called **<u>logical addition</u>**.
- The symbol we use for it is **'+'**.
- Example: X + Y can be read as **X OR Y**
- The Truth table and the Venn diagram for the NOT operator is:

X	Y	X + Y
0	0	0
0	1	1
1	0	1
1	1	1

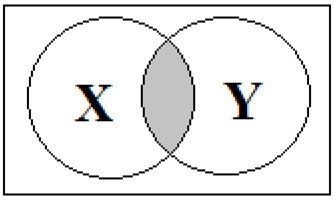


AND Operator:

- The AND operator is a binary operator. This operator operates on two variables.
- The operation performed by AND operator is called **<u>logical multiplication</u>**.
- The symbol we use for it is **'.'**.
- Example: X . Y can be read as **X AND Y**

- The Truth table and the Venn diagram for the NOT operator is:

X	Y	X . Y
0	0	0
0	1	0
1	0	0
1	1	1



➢ Evaluation of Boolean Expression using Truth Table:

- ***To create a truth table, follow the steps given below.***
- Step 1: Determine the number of variables, for n variables create a table with 2^n rows.
    - For two variables i.e. X, Y then truth table will need 2^2 or 4 rows.
    - For three variables i.e. X, Y, Z, then truth table will need 2^3 or 8 rows.
- Step 2: List the variables and every combination of 1 (TRUE) and 0 (FALSE) for the given variables
- Step 3: Create a new column for each term of the statement or argument.
- Step 4: If two statements have the same truth values, then they are equivalent.

➢ Example: Consider the following Boolean Expression F=X+ $\overline{\mathbf{Y}}$

- Step 1: This expression as two variables X and Y, then 2^2 or 4 rows.
- Step 2: List the variables and every combination of X and Y.
- Step 3: Create a new column $\overline{\mathbf{Y}}$ of the statement, and then fill the truth values of Y in that column.
- Step 4: The final column contain the values of **X+** $\overline{\mathbf{Y}}$.

X	Y	$\overline{Y}$	**X+** $\overline{\mathbf{Y}}$
0	0	1	1
0	1	0	0
1	0	1	1
1	1	0	1

➢ Exercise Problems:

1. Prepare a table of combination for the following Boolean algebra expressions.

   a) $\overline{X}\,\overline{Y} + \overline{X}Y$ b) $XY\overline{Z} + \overline{X}\,\overline{Y}Z$

2. Verify using truth table for the following Boolean algebra.

   a) $X + XY = X$ b) $\overline{X+Y} = \overline{X} \,.\, \overline{Y}$

➢ Boolean Postulates:

- The fundamental laws of Boolean algebra are called as the postulates of Boolean algebra.
- These postulates for Boolean algebra originate from the three basic logic functions AND, OR and NOT.
- **Properties of 0 and 1:**

I. If $X \neq 0$ then $X = 1$, and If $X \neq 1$ then $X = 0$

II. OR relation (Logical Addition)

a. $0 + 0 = 0$ c. $1 + 0 = 1$

b. $0 + 1 = 1$ d. $1 + 1 = 1$

III. AND relation (Logical Multiplication)

a. $0 . 0 = 0$ c. $1 . 0 = 0$

b. $0 . 1 = 0$ d. $1 . 1 = 1$

IV. Complement Rules

a. $\overline{0} = 1$ b. $\overline{1} = 0$

➢ Principle of Duality Theorem:

- This is very important principle used in Boolean algebra.
- Principle of Duality states that;
  - Changing each OR sign (+) to an AND sign (.)
  - Changing each AND sign (.) to an OR sign (+)
  - Replacing each 0 by 1 and each 1 by 0.
- The derived relation using duality principle is called dual of original expression.
- Example: Take postulate II, related to logical addition:

1) $0 + 0 = 0$ 2) $0 + 1 = 1$ 3) $1 + 0 = 1$ 4) $1 + 1 = 1$

2. Now working according to above relations, + is changed to . and 0's replaced by 1's

a) $1 . 1 = 1$ b) $1 . 0 = 0$ c) $0 . 1 = 0$ d) $0 . 0 = 0$

- which are nothing but same as that of postulate III related to logical multiplication.
- So 1, 2, 3, 4, are the duals of a, b, c, d.
- Example: Find the duals for the following Boolean Expression

Sl No	Boolean Expression	Duals
1	$X + 0 = X$	$X . 1 = X$
2	$X + 1 = 1$	$X . 0 = 0$
3	$X . \overline{X} = 0$	$X + \overline{X} = 1$
4	$X . (Y + Z)$	$X + (Y . Z)$
5	$X + X.Y = X + Y$	$X.(X + Y) = X.Y$

➢ Boolean Theorems:

- Boolean Theorem can be proved by substituting all possible values of the variable that are 0 and 1.
- This technique of proving theorem is called **Proof by perfect induction.**

Sl No	Theorem	Sl No	Theorem
Properties of 0 and 1		**Associative Law**	
1	$0 + X = X$	12	$X.(Y.Z) = (X.Y).Z$
2	$1 + X = 1$	13	$(X+Y).Z = X+(Y.Z)$
3	$0 . X = 0$	**Distributive Law**	
4	$1 . X = X$	14	$X.(Y+Z) = X.Y + X.Z$
Indempotence Law		15	$X+Y.Z = (X+Y).(X+Z)$
5	$X + X = X$	**Absorption Law**	
6	$X . X = X$	16	$X + XY = X$
Complementary Law		17	$X(X+Y) = X$
7	$X + \overline{X} = 1$	18	$XY + X\overline{Y} = X$
8	$X . \overline{X} = 0$	19	$(X+Y)(X+\overline{Y}) = X$
Involution Law		20	$X+\overline{X}Y = X+Y$
9	$\overline{\overline{X}} = X$	21	$X(\overline{X}+Y) = XY$
Commutative Law			
10	$X + Y = Y + X$		
11	$X . Y = Y . X$		

➢ Theorem 1: $0 + X = X$

<table>
<tr>
<td>Proof: If X = 0
then LHS = 0 + X
= 0 + 0
= 0
= RHS</td>
<td>Proof: If X = 1
then LHS = 0 + X
= 0 + 1
= 1
= RHS</td>
<td>Using Truth Table
<table><tr><th>0</th><th>X</th><th>0+X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr></table></td>
</tr>
</table>

➢ Theorem 2: $1 + X = 1$

<table>
<tr>
<td>Proof: If X = 0
then LHS = 1 + X
= 1 + 0
= 1
= RHS</td>
<td>Proof: If X = 1
then LHS = 1 + X
= 1 + 1
= 1
= RHS</td>
<td>Using Truth Table
<table><tr><th>1</th><th>X</th><th>1+X</th></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td>
</tr>
</table>

- **Theorem 3:** **0 . X = 0**

Proof: If X = 0	Proof: If X = 1
then LHS = 0 . X = 0 . 0 = 0 = RHS	then LHS = 0 . X = 0 . 1 = 0 = RHS

Using Truth Table

0	**X**	**0.X**
0	0	0
0	1	0

- **Theorem 4:** **1 . X = X**

Proof: If X = 0	Proof: If X = 1
then LHS = 1 . X = 1 . 0 = 0 = RHS	then LHS = 1 . X = 1 . 1 = 1 = RHS

Using Truth Table

1	**X**	**0.X**
1	0	0
1	1	1

- **Indempotence Law:** “This law states that when a variable is combines with itself using OR or AND operator, the output is the same variable”.

- **Theorem 5:** **X + X = X**

Proof: If X = 0	Proof: If X = 1
then LHS = X + X = 0 + 0 = 0 = RHS	then LHS = X + X = 1 + 1 = 1 = RHS

Using Truth Table

X	**X**	**X+X**
0	0	0
1	1	1

- **Theorem 6:** **X . X = X**

Proof: If X = 0	Proof: If X = 1
then LHS = X . X = 0 + 0 = 0 = RHS	then LHS = X . X = 1 . 1 = 1 = RHS

Using Truth Table

X	**X**	**X.X**
0	0	0
1	1	1

- **Complementary Law:** “This law states that when a variable is And ed with its complement is equal to 0 and a variable is OR ed with its complement is equal to 1”.

- **Theorem 7:** $\mathbf{X + \bar{X} = 1}$

Proof: If X = 0	Proof: If X = 1
then LHS $= X + \bar{X}$	then LHS $= X + \bar{X}$
$= 0 + 1$	$= 1 + 0$
$= 1$	$= 1$
= RHS	= RHS

Using Truth Table

X	$\bar{X}$	$X + \bar{X}$
0	1	1
1	0	1

- **Theorem 8:** $\mathbf{X . \bar{X} = 0}$

Proof: If X = 0	Proof: If X = 1
then LHS $= X . \bar{X}$	then LHS $= X . \bar{X}$
$= 0 . 1$	$= 1 . 0$
$= 0$	$= 0$
= RHS	= RHS

Using Truth Table

X	$\bar{X}$	$X . \bar{X}$
0	1	0
1	0	0

- **Involution Law:** "This law states that when a variable is inverted twice is equal to the original variable".

- **Theorem 9:** $\mathbf{\bar{\bar{X}} = X}$

Proof: If X = 0, then $\bar{X} = 1$

Take complement again, then $\bar{\bar{X}} = 0$ i.e. X

If X = 1, then $\bar{X} = 0$

Take complement again, then $\bar{\bar{X}} = 1$ i.e. X

Using Truth Table

X	$\bar{X}$	$\bar{\bar{X}}$
0	1	0
1	0	1

- **Commutative Law:** "This law states that the order in which two variable are Or ed or AND ed make no difference".

- **Theorem 10:** $\mathbf{X + Y = Y + X}$

Proof: If Y = 0	Proof: If Y = 1
then LHS $= X + Y$	then LHS $= X + Y$
$= X + 0$	$= X + 1$
$= X$	$= 1$
RHS $= Y + X$	RHS $= Y + X$
$= 0 + X$	$= 1 + X$
$= X$	$= 1$
Therefore LHS = RHS	Therefore LHS = RHS

Using Truth Table

X	Y	X+Y	Y+X
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

- **Theorem 11:** **X . Y = Y . X**

Proof: If Y = 0

then LHS = X . Y
= X . 0
= 0

RHS = Y . X
= 0 . X
= 0

Therefore LHS = RHS

Proof: If Y = 1

then LHS = X . Y
= X . 1
= X

RHS = Y . X
= 1 . X
= X

Therefore LHS = RHS

Using Truth Table

X	**Y**	**X.Y**	**Y.X**
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

- **Associative Law:** "This law allows the removal of brackets from an expression and regrouping of the variables".

- **Theorem 12:** **X.(Y.Z) = (X.Y).Z**

Proof: If Y = 0

LHS = X.(Y.Z)
= X.(0.Z)
= X.0
= 0

RHS = (X.Y).Z
= (X.0).Z
= 0.Z
= 0

Therefore LHS = RHS

Proof: If Y = 1

LHS = X.(Y.Z)
= X.(1.Z)
= X.Z
= XZ

RHS = (X.Y).Z
= (X.1).Z
= X.Z
= XZ

Therefore LHS = RHS

Using Truth Table

X	**Y**	**Z**	**XY**	**YZ**	**X.(Y.Z)**	**(X.Y).Z**
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

- **Theorem 13:** **X+(Y+Z) = (X+Y)+Z**

Proof: If Y = 0

LHS = X+(Y+Z)
= X+(0+Z)
= X+Z

RHS = (X+Y)+Z
= (X+0)+Z
= X+Z

Therefore LHS = RHS

Proof: If Y = 1

LHS = X+(Y+Z)
= X+(1+Z)
= X+1
= 1

RHS = (X+Y)+Z
= (X+1).Z
= 1+Z
= 1

Therefore LHS = RHS

Using Truth Table

X	**Y**	**Z**	**X+Y**	**Y+Z**	**X+(Y+Z)**	**(X+Y)+Z**
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

- **Distributive Law:** "This law allows the multiplying or factoring out an expression".
- **Theorem 14:** **X.(Y+Z) = XY + XZ**

Proof: If X = 0	Proof: If X = 1
LHS = X.(Y+Z) = 0.(Y+Z) = 0 RHS = XY + XZ = 0.Y+0.Z = 0 Therefore LHS = RHS	LHS = X.(Y+Z) = 1.(Y+Z) = Y+Z RHS = XY + XZ = 1.Y+1.Z = Y+Z Therefore LHS = RHS

- **Theorem 15:** $(X+Y)(X+Z) = X+YZ$

LHS: $(X+Y)(X+Z) = XX + XZ + XY + YZ$
$= X + XZ + XY + YZ$
$= X(1+Z) + XY + YZ$
$= X + XY + YZ$
$= X(1+Y) + YZ$
$= X + YZ$
= RHS

Important 2 Marks

- **Absorption Law:** "This law enables a reduction of complicated expression to a simpler one by absorbing common terms".

16) X+XY = X LHS = X + XY = X (1 + Y) = X = RHS	**17) X (X+Y) = X** LHS = X (X+Y) = XX + XY = X + XY = X (1+Y) = X	**18) XY + X$\overline{Y}$ = X** LHS = XY + X$\overline{Y}$ = X(Y+$\overline{Y}$) = X.1 = X = RHS
19) (X+Y) (X+$\overline{Y}$)= X LHS = (X+Y) (X+$\overline{Y}$) = XX + X$\overline{Y}$ + XY +Y$\overline{Y}$ = X + X$\overline{Y}$ + XY + 0 = X (1 + $\overline{Y}$ + Y) = X. 1 = X	**20) X + $\overline{X}$Y = X+Y** LHS = X + $\overline{X}$Y = (X + $\overline{X}$) (X+Y) = 1. (X+Y) = X+Y = RHS	**21) X ($\overline{X}$+Y) = XY** LHS = X ($\overline{X}$+Y) = X. $\overline{X}$ + X.Y = 0 + XY = XY = RHS

➢ DeMorgan's Theorem:

- **DeMorgan's First Theorem:**
  - **Statement: "When the OR sum of two variables is inverted, this is same as inverting each variable individually and then AND ing these inverted variables**"
  - This can be written as $\mathbf{\overline{X+Y} = \overline{X}.\ \overline{Y}}$

Important 5 Marks

  - We can prove the DeMorgan's First theorem by using Truth Table is

X	**Y**	$\mathbf{\overline{X}}$	$\mathbf{\overline{Y}}$	**X+Y**	$\mathbf{\overline{X+Y}}$	$\mathbf{\overline{X}.\ \overline{Y}}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

  - Compare the column $\mathbf{\overline{X+Y}}$ and $\mathbf{\overline{X}.\ \overline{Y}}$**.** Both of these are identical. Hence the DeMorgan's first theorem is proved.

- **DeMorgan's Second Theorem:**
  - **Statement: "When the AND product of two variables is inverted, this is same as inverting each variable individually and then OR ing these inverted variables**"
  - This can be written as $\mathbf{\overline{X.Y} = \overline{X}+\overline{Y}}$
  - We can prove the DeMorgan's Second theorem by using Truth Table is:

X	**Y**	$\mathbf{\overline{X}}$	$\mathbf{\overline{Y}}$	**X.Y**	$\mathbf{\overline{X.Y}}$	$\mathbf{\overline{X}+\overline{Y}}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

  - Compare the column $\mathbf{\overline{X.Y}}$ and $\mathbf{\overline{X}+\overline{Y}}$**.** Both of these are identical. Hence the DeMorgan's Second theorem is proved.

- **Application of DeMorgan's Theorem:**
  - It is used in simplification of Boolean expression.
  - DeMorgan's law commonly apply to text searching using Boolean operators AND, OR and NOT.
  - It is useful in the implementation of the basic gates operations with alternative gates.

➢ Simplification of Boolean Expression:

- Simplification of Boolean expression can be achieved by two popular methods:
  - Algebraic Manipulation
  - Karnaugh Maps
- **Algebraic Manipulation:**

Important
2 Marks

1) $\mathbf{\bar{X}\,\bar{Y}\,Z + \bar{X}\,Y\,Z + X\,\bar{Y}}$ $= \bar{X}\,\bar{Y}\,Z + \bar{X}\,Y\,Z + X\,\bar{Y}$ $= \bar{X}Z\,(\bar{Y} + Y) + X\,\bar{Y}$ $= \bar{X}Z\,(1) + X\,\bar{Y}$ $= \bar{X}Z + X\,\bar{Y}$	2) **XYZ + XYZW + XZ** = XYZ (1 + W) + XZ = XYZ . 1 + XZ = XZ (Y + 1) = XZ
3) **Z(Y+Z)(X+Y+Z)** = (ZY + ZZ) (X+Y+Z) = (ZY + Z) (X+Y+Z) = Z(X+Y+Z) [Theorem 16 X+XY=Z] = ZX + ZY + ZZ = ZX + ZY + Z = Z(X+Y+1) = Z(1) = Z	4) $\mathbf{X + \bar{X}\,Y + \bar{Y} + (X + \bar{Y})\bar{X}\,Y}$ $= X + \bar{X}\,Y + \bar{Y} + X\,\bar{X}\,Y + \bar{Y}\bar{X}\,Y$ $= X + \bar{X}\,Y + \bar{Y} + 0 + 0$ $= (X + \bar{X})\,(X + Y) + \bar{Y}$ $= 1\,(X + Y) + \bar{Y}$ $= X + Y + \bar{Y}$ $= X + 1$ $= 1$

➢ Exercise Problems: Simplify using Algebraic Manipulation

1) $(\bar{A} + B).(A + B)$
2) $AB + A\bar{B} + \bar{A}\,B$
3) $B(A+C) + A\bar{B} + B\bar{C} + C$

➢ Exercise Problems: Solving using DeMorgan's Theorem

1) $(\bar{A} + C).(B + D)$
2) $A\bar{B} + C)$

➢ Minterm:

- Minterm is a product of all the literal (with or without bar) within the logic system.
- **(OR)** A single variable or the logical product of several variables. The variables may or may not be complemented.
- A variable may appear either in its normal form (X) or in its complement form ($\bar{X}$)
- If a variable **value is 0 then its complemented** otherwise it is in its normal form.
- For example, if you have two variables X & Y, there are four possible combination can be formed with AND operation. Each of these four AND operations represents one of the Boolean expressions terms and is called a Minterm or a standard product.

X	Y	Minterm	Designation
0	0	$\bar{X}\,\bar{Y}$	m_0
0	1	$\bar{X}\,Y$	m_1
1	0	$X\,\bar{Y}$	m_2
1	1	$X\,Y$	m_3

- A symbol for each Minterm is also shown in the table and is of the form mj where j denotes the decimal equivalent of the binary number of the Minterm designated.
- For example, the Minterm $\mathbf{X\,Y\,\bar{Z}}$ whose combination is **1 1 0** can be written as $\mathbf{m_6}$ as decimal equivalent of **1 1 0 is 6.**
- A Boolean expression may be represented from a given truth table by forming a Minterm for each combination of the variables **which produces as 1 in the function, and then taking the OR (Logical Addition) of all those terms**.
- Assume the truth table

X	Y	Z	Output	Minterm	Designation
0	0	0	0	$\bar{X}\,\bar{Y}\,\bar{Z}$	m_0
0	0	1	0	$\bar{X}\,\bar{Y}\,Z$	m_1
0	1	0	**1**	$\bar{X}\,Y\,\bar{Z}$	m_2
0	1	1	**1**	$\bar{X}\,Y\,Z$	m_3
1	0	0	0	$X\,\bar{Y}\,\bar{Z}$	m_4
1	0	1	0	$X\,\bar{Y}\,Z$	m_5
1	1	0	**1**	$X\,Y\,\bar{Z}$	m_6
1	1	1	0	$X\,Y\,Z$	m_7

Minterm Results 1

- The Boolean function of truth table is obtained by OR ing (Add) three Minterm i.e. 010 ($\bar{X}\,Y\,\bar{Z}$), 011 ($\bar{X}\,Y\,Z$), 110 ($X\,Y\,\bar{Z}$). Since each of these Minterm results is 1 (output).

$$\mathbf{f\,(X, Y, Z) = \overline{X}\,Y\,\overline{Z} + \overline{X}\,Y\,Z + X\,Y\,\overline{Z} = m_2 + m_3 + m_6}$$

- The above Boolean function is the sum of three product terms. This type of expression is known as Sum of Product (SOP) expression.

$$\mathbf{f\,(X, Y, Z) = \sum (2, 3, 6)}$$

- Where f is a Boolean function with three variables (X, Y, Z) and it can be read as function f is sum of 2^{nd}, 3^{rd}, and 6^{th} Minterm.
- ***Sum of Product (SOP): A Sum of product expression is a product term or several product terms logically added.***

➢ **Find the Minterm designation for the following:**

1) $X\,\overline{Y}\,\overline{Z}$

Binary Equivalent = 1 0 0

Decimal Equivalent $= 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$

$= 4 + 0 + 0$

$= 4$

So, $\mathbf{X\,\overline{Y}\,\overline{Z} = m_4}$

2) $A\,\overline{B}\,C\,\overline{D}$

Binary Equivalent = 1 0 1 0

Decimal Equivalent $= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 8 + 0 + 2 + 0$

$= 10$

So, $\mathbf{A\,\overline{B}\,C\,\overline{D} = m_{10}}$

➢ **What are the fundamental products for each of the input words; ABCD = 0010, ABCD =110, ABCD=1110. Write SOP expression.**

Solution: The SOP expression is: $\mathbf{\overline{A}\,\overline{B}\,C\,\overline{D} + A\,B\,\overline{C} + A\,B\,C\,\overline{D}}$

➢ Maxterm:

- Maxterm is a sum of all the literal (with or without bar) within the logic system.
- **(OR)** A single variable or the logical sum of several variables. The variables may or may not be complemented.
- A variable may appear either in its normal form (X) or in its complement form ($\overline{X}$)
- If a variable **<u>value is 1 then its complemented</u>** otherwise it is in its normal form.
- For example, if you have two variables X & Y, there are four possible combination can be formed with OR operation. Each of these four OR operations represents one of the Boolean expressions

terms and is called a Minterm or a standard product.

X	Y	Minterm	Designation
0	0	$X + Y$	M_0
0	1	$X + \overline{Y}$	M_1
1	0	$\overline{X} + Y$	M_2
1	1	$\overline{X} + \overline{Y}$	M_3

- A symbol for each Maxterm is also shown in the table and is of the form Mj where j denotes the decimal equivalent of the binary number of the Maxterm designated.
- For example, the Maxterm $\mathbf{X + Y + \overline{Z}}$ whose combination is **0 0 1** can be written as $\mathbf{M_1}$ as decimal equivalent of **0 0 1 is 1.**
- A Boolean expression may be represented from a given truth table by forming a Maxterm for each combination of the variables **which produces as 0 in the function, and then taking the AND (Logical Multiplication) of all those terms**.
- Assume the truth table

X	Y	Z	Output	Minterm	Designation
0	0	0	**0**	$\mathbf{X + Y + Z}$	$\mathbf{M_0}$
0	0	1	**0**	$\mathbf{X + Y + \overline{Z}}$	$\mathbf{M_1}$
0	1	0	1	$X + \overline{Y} + Z$	M_2
0	1	1	1	$X + \overline{Y} + \overline{Z}$	M_3
1	0	0	**0**	$\mathbf{\overline{X} + Y + Z}$	$\mathbf{M_4}$
1	0	1	**0**	$\mathbf{\overline{X} + Y + \overline{Z}}$	$\mathbf{M_5}$
1	1	0	1	$\overline{X} + \overline{Y} + Z$	M_6
1	1	1	**0**	$\mathbf{\overline{X} + \overline{Y} + \overline{Z}}$	$\mathbf{M_7}$

Maxterm Results 0

- The Boolean function of truth table is obtained by AND ing (Multiply) five Maxterm i.e. 000, 001, 100, 101, 111. Since each of these Maxterm results is 0 (output).

$$\mathbf{f(X, Y, Z) = (X + Y + Z)(X + Y + \overline{Z})(\overline{X} + Y + Z)(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z}) = M_0.M_1.M_4.M_5.M_7}$$

- The above Boolean function is the product of three sum terms. This type of expression is known as Product of Sum (POS) expression.

$$\mathbf{f(X, Y, Z) = \pi\ (0, 1, 4, 5, 7)}$$

- Where f is a Boolean function with three variables (X, Y, Z) and it can be read as function f is product of 0^{th}, 1^{st}, 4^{th}, 5^{th} and 7^{th} Maxterm.
- ***Product of Sum (POS): A product of sum expression is a sum term or several sum terms logically multiplied.***

➢ Canonical Form:

- **Boolean expression expressed as sum of Minterms or product of Maxterms are called canonical forms.**
- For example, the following expressions are the Minterm canonical form and Maxterm canonical form of two variables X and Y.
    - Minterm Canonical = $f(X, Y) = X + \overline{X}\,\overline{Y} + \overline{X}\,Y + X\,\overline{Y} + X\,Y$
    - Maxterm Canonical = $f(X, Y) = (X + \overline{Y})(\overline{X} + Y)(X + Y)$
- The Minterm canonical expression is the sum of all Minterms. Each Minterm contain all the variables.
- The maxterm canonical expression is the product of all Maxterms. Each Maxterm contain all the variables.

➢ Conversion of SOP into Canonical form:

1) Convert the Boolean function $f(X, Y) = X + X\,\overline{Y}$ into canonical form.
    - **Solution**: The given Boolean function $f(X, Y) = X + X\,\overline{Y}$ ------→ (i)
    - It has two variables and sum of two Minterms. The first term X is missing one variable. So to make it of two variables it can be multiplied by $(Y + \overline{Y})$, as $(Y + \overline{Y}) = 1$.

$$\text{Therefore, } X = X\,(Y + \overline{Y}) = XY + X\overline{Y}$$

    - Substitute the value of X in (i) we get

$$f(X, Y) = X\,Y + X\overline{Y} + X\,\overline{Y}$$

    - Here, the term $X\,\overline{Y}$ appear twice, according to theorem X+X =X, it is possible to remove one o them.

$$f(X, Y) = X\,Y + X\overline{Y}$$

    - Rearrange the Minterm in ascending order

$$f(X, Y) = X\overline{Y} + X\,Y$$

$$= m_2 + m_3$$

$$f(X, Y) = \sum (2, 3)$$

2) Convert the Boolean function $f(X, Y) = X + \overline{Y}\,Z$ into canonical form.

➢ Conversion of POS into Canonical form:

1) Convert the Boolean function $F(X, Y, Z) = (X + Y)(Y + Z)$ into canonical form.
    - **Solution**: The given Boolean function

$$F(X,Y,Z) = (X + Y)(Y + Z) \text{ ------→ (i)}$$

    - It has three variables and product of two Maxterms. Each Maxterm is missing one variable.

- The first term can be written as

$$X+Y = (X+Y+Z.\overline{Z}) \quad \text{Since } Z.\overline{Z} = 0$$

- Using distributive law (X + YZ) = (X + Y) (X + Z), we can write

$$X+Y = (X+Y+Z)\,(X+Y+\overline{Z}) \text{ ------} \rightarrow \text{(ii)}$$

- The Second term can be written as

$$Y + Z = (Y+Z+X.\overline{X})$$

$$Y+Z = (Y+Z+X)\,(Y+Z+\overline{X}) \text{ ------} \rightarrow \text{(iii)}$$

- Substitute (ii) and (iii) in (i) we get

$$F(X, Y, Z) = (X+Y+Z)\,(X+Y+\overline{Z})\,(Y+Z+X)\,(Y+Z+\overline{X})$$

- The term (X+Y+Z) appear twice and according to theorem X . X = X

$$F(X, Y, Z) = (X+Y+Z)\,(X+Y+\overline{Z})\,(Y+Z+\overline{X})$$

- Rearrange the Maxterm in ascending order.

$$F(X, Y, Z) = (X+Y+Z)\,(X+Y+\overline{Z})\,(\overline{X}+Y+Z)$$

$$= M_0 . M_1 . M_4$$

$$F(X, Y, Z) = \pi\,(0, 1, 4)$$

2) Convert the Boolean function F (A, B, C) = $AB + \overline{A}C$ into product of sum (POS) form.

Important
5 Marks

➢ Karnaugh Map:

- A graphical display of the fundamental products in a truth table.
- Fundamental Product: The logical product of variables and complements that produces a high output for a given input condition.
- The map method provides simple procedure for minimizing the Boolean function.
- The map method was first proposed by E.W. Veitch in 1952 known as "**Veitch Diagram**".
- In 1953, Maurice Karnaugh proposed "**Karnaugh Map**" also known as "**K-Map**".

➢ Construction of K-Map:

- The K-Map is a pictorial representation of a truth table made up of squares.
- Each square represents a Minterm or Maxterm.
- A K-Map for **n variables** is made up of **2^n squares**.

➢ Single Variable K-Map:

- A one variable K-Map is shown in the following figure.
- The one variable Boolean expression is of the form f(A).
- There are two Minterms (A and $\overline{A}$) for one variable.

- Hence the map consists of 2 squares (i.e. **2^n square, 2^1 = 2 square)**

A	$\bar{A}$	A
	$\bar{A}$	A

Minterm

A	0	1
	0	1

Basic Labelling

A		
	m_0	m_1

- In one variable K-map:
  - One square represents one Minterm.
  - Two adjacent squares represents a function which is always true i.e. f (A) = 1.

➢ Two Variable K-Map:

- The two variable Boolean expressions are of the form f (A, B).
- There are four Minterms ($\bar{A}\,\bar{B}$, $\bar{A}B$, $A\bar{B}$ and AB for two variable.
- Hence the map consists of 4 squares (i.e. **2^n square, 2^2 = 4 square)**

	$\bar{B}$	B
$\bar{A}$	$\bar{A}\,\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

Minterm

	0	1
0	00	01
1	10	11

Basic Labelling

$\bar{A}$	m_0	m_1
A	m_2	m_3

➢ Three Variable K-Map:

- The three variable Boolean expressions are of the form f (A, B, C).
- There are eight Minterms ($\bar{A}\bar{B}\bar{C}$, $\bar{A}\bar{B}C$, $\bar{A}B\bar{C}$, $\bar{A}BC$, $A\bar{B}\bar{C}$, $A\bar{B}C$, $\bar{A}B\bar{C}$, and ABC) for three variable.
- Hence the map consists of 8 squares (i.e. **2^n square, 2^3 = 8 square)**

		BC			
		$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
A	$\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
	A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$\bar{A}B\bar{C}$

Minterms

		BC			
		00	01	11	10
A	0	000	001	011	010
	1	100	101	111	110

Basic Labelling

- **Note**: The ordering of variable i.e. 00, 01, 11 & 10 is in gray (reflected binary code) one should not use straight binary code i.e. 00, 01, 10, 11. The straight binary code was used in Veitch diagram but Mr. Karnaugh modified the veitch diagram and use reflected binary code.

➢ Four Variable K-Map:

Important
5 Marks

- The four variable Boolean expressions are of the form f (A, B, C, D).
- There are sixteen Minterms for four variables.

- Hence the map consists of 8 squares (i.e. **2^n square, 2^4 = 16 square).**
- The rows and columns are numbered in a reflected code system.

CD

		$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
	$\overline{A}\overline{B}$	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}\overline{B}CD$	$\overline{A}\overline{B}C\overline{D}$
AB	$\overline{A}B$	$\overline{A}B\overline{C}\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
	AB	$AB\overline{C}\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
	$A\overline{B}$	$A\overline{B}\overline{C}\overline{D}$	$A\overline{B}\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Minterms

CD

		00	01	11	10
	00	0000	0001	0011	0010
AB	01	0100	0101	0111	0110
	11	1100	1101	1111	1110
	10	1000	1001	1011	1010

Basic Labelling

CHAPTER 2 – BOOLEAN ALGEBRA BLUE PRINT				
VSA (1 marks)	SA (2 marks)	LA (3 Marks)	Essay (5 Marks)	Total
-	02 Question	-	01 Question	03 Question
-	Question no 11, 12	-	Question no 27	09 Marks

Important Questions

➢ 2 Marks Question:

1. Prove that X + XY = X [March 2015, March 2017]
2. Define Minterm and Maxterm [March 2015, March 2016]
3. State and prove Involution law. [June 2015]
4. State and prove Commutative law using truth table. [June 2016]
5. What is principle of duality? Give Example [June 2015, March 2017]
6. Prove algebraically that $(X+Y)\ (X+Z)=X+YZ$ [March 2016]
7. Prove: (X+Y) (X+($\overline{Y}$) = X [June 2016]
8. Prove algebraically that $X+\overline{X}Y=X+Y$
9. Draw a general K-map for four variables A, B, C and D.

➢ 5 Marks Question:

1. Give the Boolean function F (W, X, Y, Z) = ∑ (0, 4, 8, 9, 10, 11, 12, 13, 15). Reduce it by using K-Map. [March 2015]
2. Reduce F (A, B, C, D) = ∑ (1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15) using K-Map [June 2015]
3. Using K-Map, Simplify the following expression in four variables
   F (A, B, C, D) = $m_1 + m_2 + m_4 + m_5 + m_9 + m_{11} + m_{12} + m_{13}$. [March 2016]