

## Problems on Pigeonhole Principle

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I.E.M.

1. Prove that in a set containing  $n$  positive integers there must be a subset such that the sum of all numbers in it is divisible by  $n$ .

[10 marks]

2. In a bag there are some balls of the same size that are coloured by 7 colours and for each colour the number of balls is 77. At least how many balls are needed to be picked out at random to ensure that one can obtain 7 groups of 7 balls each such that in each group the balls are homochromatic?

[10 marks]

3. There are 5 points in a square of side length 2. Prove that there exist 2 of them having a distance not more than  $\sqrt{2}$ .

[5 marks]

4. There is a sequence of 100 integers. Prove that there is a sequence of consecutive terms such that the sum of these terms is divisible by 99.

[5 marks]

5. 76 points are aligned so that each row has 19 points and each column has 4 points. (A column is perpendicular to a row.) Each point is painted in red, blue or yellow. Prove that there exists a monochromatic rectangle. (i.e. 4 vertices are of same colour) with its sides parallel to the rows and columns.

[10 marks]