1.12 Normal Forms

Conjunctive Normal Form (CNF).

A compound proposition p is said to be conjunctive normal form (CNF) of r simple propositions $p_1, p_2, \ldots p_r$ if the expression p can be expresed as $p = p_1 \wedge p_2 \wedge \ldots \wedge p_n$ where each $p_i =$ Disjunction of a finite number of the simple propositions of each of $p_1, p_2, \ldots p_n$ or their negations and $p_i \neq p_j$ for all distinct i, j

Example. (i) $p = (\sim p_1 \vee p_3 \vee p_2) \wedge (\sim p_1 \vee \sim p_2 \vee \sim p_3) \wedge (p_1 \vee p_2 \vee p_3)$ is a CNF of the simple propositions p_1, p_2, p_3

But $p = (\sim p_1 \vee p_2) \wedge (\sim p_1 \vee \sim p_2 \vee \sim p_3)$ is not a CNF since the expression $(\sim p_1 \vee p_2)$ does not contain p_3 or its negation.

(ii) $p = p \vee (\sim p \wedge q)$ is not a CNF

(iii) $(\sim p \vee q) \wedge (\sim q \vee p)$ is a CNF of the two simple propositions p and q.

Disjunctive Normal Form (DNF). A compound proposition p is said to be Disjunctive Normal Form (DNF) of r simple propositions $p_1, p_2, \ldots . p_r$ if the expression p can be expressed as

$p = p_1 \vee p_2 \vee \ldots . \vee p_n$.

where each p_i is conjunction of some of the simple propositions or their negations without any repeatations and no p_i is contained in any $p_j (j \neq i)$.

Example: (1) $P = (p \wedge \sim r) \vee (\sim q \wedge p \wedge r) \vee (\sim p \wedge \sim q)$ is a DNF.

Here $P_1 = p \wedge \sim r$ is conjunction of p and $\sim r$, $P_2 = \sim q \wedge p \wedge r$ is conjunction of $\sim q, p$ and r and $P_3 = \sim p \wedge \sim q$ etc.

(2) $(p \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (\sim r \wedge p \wedge q)$ is not DNF since $P_1 = p \wedge \sim r$ is contained in $P_3 = \sim r \wedge p \wedge q = p \wedge \sim r \wedge q = (p \wedge \sim r) \wedge q$.