Knowledge Representation

Inference in Propositional Logic, First order Predicate logic, Resolution, Logical Reasoning, Forward chaining, Backward chaining, Knowledge representation techniques: Semantic networks and frames.

Techniques of knowledge representation

There are mainly four ways of knowledge representation which are given as follows:

1. Logical Representation
2. Semantic Network Representation
3. Frame Representation
4. Production Rules

Logical Representation

Logical representation is a language with some **definite rules** which deal with propositions and has no ambiguity in representation.

It represents a conclusion based on various conditions and lays down some important **communication rules**.

It also consists of precisely defined **syntax and semantics** which supports the sound inference. Each sentence can be translated into logics using syntax and semantics.

Syntax:

- Syntaxes are the rules which decide how we can construct legal sentences in the logic.
- It determines which symbol we can use in knowledge representation.
- How to write those symbols.

Semantics:

- Semantics are the rules by which we can interpret the sentence in the logic.
- Semantic also involves assigning a meaning to each sentence.

Logical representation can be categorized into mainly two logics:

- **Propositional Logics**
- **Predicate logics(FOPL)**

Propositional Logics

Propositional logic is a simple form of logic which is also known as Boolean logic. A proposition has TRUTH values (0 and 1) which means it can have one of the two values i.e. True or False.

Example:

a) It is Sunday.

b) The Sun rises from West (False proposition)

c) 3+3= 7(False proposition)

d) 5 is a prime number.

Propositional logic is used in artificial intelligence for planning, problem-solving and most importantly for decision-making.

Types of Propositions

There are two types of Propositions:

1. Atomic Propositions

2. Compound propositions

Atomic Proposition: Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

Example:

a) 2+2 is 4, it is an atomic proposition as it is a **true** fact.

b) "The Sun is cold" is also a proposition as it is a **false** fact.

Compound proposition: Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

Example:

a) "It is raining today, and street is wet."

b) "Ankit is a doctor, and his clinic is in Mumbai."

Logical Connectives

We will now introduce number of connectives which will allow us to build up complex propositions.

1. Negation: A sentence such as $\neg$ P is called negation of P.

2. Conjunction: A sentence which has $\wedge$ connective such as, **P $\wedge$ Q** is called a conjunction.

Example: Rohan is intelligent and hardworking. It can be written as,

P= Rohan is intelligent,

Q= Rohan is hardworking. → P$\wedge$ Q.

3. Disjunction: A sentence which has $\vee$ connective, such as **P $\vee$ Q** is called disjunction, where P and Q are the propositions.

Example: "Ritika is a doctor or Engineer",

Here P= Ritika is Doctor. Q= Ritika is Engineer, so we can write it as **P $\vee$ Q**.

4. Implication: A sentence such as P → Q, is called an implication. Implications are also known as if-then rules. It can be represented as

If it is raining, then the street is wet.

Let P= It is raining, and Q= Street is wet, so it is represented as P → Q

5. Biconditional: A sentence such as **P⇔ Q is a Biconditional sentence,**

example If I am breathing, then I am alive

P= I am breathing, Q= I am alive, it can be represented as P ⇔ Q.

Truth Tables

For Negation:

P	¬ P
True	**False**
False	**True**

For Conjunction:

P	Q	P∧ Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**False**

For disjunction:

P	Q	P V Q.
True	True	**True**
False	True	**True**
True	False	**True**
False	False	**False**

For Implication:

P	Q	P-> Q
True	True	**True**
True	False	**False**
False	True	**True**
False	False	**True**

For Biconditional:

P	Q	P<=> Q
True	True	**True**
True	False	**False**
False	True	**False**
False	False	**True**

Logical identities & Equivalence Laws

Logical Identities and Some Equivalence Laws

1. Idempotency	$P \vee P = P$ $P \wedge P = P$
2. Associativity	$(P \vee Q) \vee R = P \vee (Q \vee R)$ $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$
3. Commutativity	$P \vee Q = Q \vee P$ $P \wedge Q = Q \wedge P$ $P \leftrightarrow Q = Q \leftrightarrow P$
4. Distributivity	$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
5. De Morgan's Laws	$\sim(P \vee Q) = \sim P \wedge \sim Q$ $\sim(P \wedge Q) = \sim P \vee \sim Q$
6. Conditional Elimination	$P \rightarrow Q = \sim P \vee Q$
7. Bi-conditional elimination	$P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$

8. Negation $\sim(\sim p) = p$

9. Contradiction
$p \wedge \sim p = F$

10. Tautology
$p \vee \sim p = T$

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11. Or-Simplification
$p \vee p = p$
$p \vee T = T$
$p \vee F = p$
$p \vee (p \wedge q) = p$

12. and Simplification
$p \wedge p = p$
$p \wedge T = p$
$p \wedge F = F$
$p \wedge (p \vee q) = p$

13. Absorption $p \wedge (\sim p \vee q) = p \wedge q$

1. Use the logical equivalences to show that **~(p v ~(p ^ q))** is a contradiction.

2. (P -> Q) ^ (R->Q) = (P V R) -> Q

Limitations of Propositional logic

- We cannot represent relations like ALL, some, or none with propositional logic. Example:
  - **All the girls are intelligent.**
  - **Some apples are sweet.**
- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

Inference

In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, **so generating the conclusions from evidence and facts is termed as Inference**.

Inference Rule: Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.

Types of Inference rules:

1. Modus Ponens
2. Modus Tollens
3. Hypothetical Syllogism
4. Disjunctive Syllogism
5. Addition
6. Simplification
7. Conjunction

Rule of Inference	Name	Rule of Inference	Name
$\dfrac{P}{\therefore P \vee Q}$	Addition	$\dfrac{\begin{array}{c}P \vee Q \\ \neg P\end{array}}{\therefore Q}$	Disjunctive Syllogism
$\dfrac{\begin{array}{c}P \\ Q\end{array}}{\therefore P \wedge Q}$	Conjunction	$\dfrac{\begin{array}{c}P \to Q \\ Q \to R\end{array}}{\therefore P \to R}$	Hypothetical Syllogism
$\dfrac{P \wedge Q}{\therefore P}$	Simplification		
$\dfrac{\begin{array}{c}P \to Q \\ P\end{array}}{\therefore Q}$	Modus Ponens		
$\dfrac{\begin{array}{c}P \to Q \\ \neg Q\end{array}}{\therefore \neg P}$	Modus Tollens		

Addition: The Addition rule is one the common inference rule, and it states that If P is true, then PVQ will be true.

Notation of Addition: $\dfrac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P

Statement-2: I have Chocolate ice-cream.

Conclusion: I have vanilla or chocolate ice-cream. ==> (PVQ)

Proof by Truth-Table:

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

Conjunction: If P and Q are two premises, we can use Conjunction rule to derive P∧Q.

$$\begin{array}{c} P \\ Q \\ \hline \therefore P \wedge Q \end{array}$$

Example

Let P – "He studies very hard"

Let Q – "He is the best boy in the class"

Therefore – "He studies very hard and he is the best boy in the class"

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

Simplification: The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \wedge Q}{Q}$ Or $\frac{P \wedge Q}{P}$

Example

"He studies very hard and he is the best boy in the class", P∧Q

Therefore − "He studies very hard"

Proof by Truth-Table:

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

Modus Ponens: The Modus Ponens rule is one of the most important rules of inference, and it states that if P and $P \rightarrow Q$ is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $\dfrac{P \rightarrow Q, \quad P}{\therefore Q}$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> $P \rightarrow Q$

Statement-2: "I am sleepy" ==> P

Conclusion: "I go to bed." ==> Q.

Hence, we can say that, if $P \rightarrow Q$ is true and P is true then Q will be true.

Proof by Truth table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

Modus tollens: The Modus Tollens rule state that if P→ Q is true and **¬ Q is true, then ¬ P** will also true. It can be represented as:

Notation for Modus Tollens: $$\frac{P \rightarrow Q, \ \sim Q}{\sim P}$$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q

Statement-2: "I do not go to the bed."==> ~Q

Statement-3: Which infers that "**I am not sleepy**" => ~P

Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

Hypothetical Syllogism: The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:

$$\frac{P \rightarrow Q, Q \rightarrow R}{\therefore P \rightarrow R}$$

Example:

Statement-1: If you have my home key then you can unlock my home. **P→Q**

Statement-2: If you can unlock my home then you can take my money. **Q→R**

Conclusion: If you have my home key then you can take my money. **P→R**

Proof by truth table:

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1

Disjunctive Syllogism: The Disjunctive syllogism rule state that if P∨Q is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\dfrac{P \vee Q,\ \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>P∨Q

Statement-2: Today is not Sunday. ==> ¬P

Conclusion: Today is Monday. ==> Q

Proof by truth-table:

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

Applying rules of inference

Assume the following statements (hypotheses):

- It is not sunny this afternoon and it is colder than yesterday.
- We will go swimming only if it is sunny.
- If we do not go swimming then we will take a canoe trip.
- If we take a canoe trip, then we will be home by sunset.

Show that all these lead to a conclusion:

- We will be home by sunset.

Applying rules of inference

Text:

(1) It is not sunny this afternoon and it is colder than yesterday.

(2) We will go swimming only if it is sunny.

(3) If we do not go swimming then we will take a canoe trip.

(4) If we take a canoe trip, then we will be home by sunset.

Propositions:

- p = It is sunny this afternoon, q = it is colder than yesterday, r = We will go swimming , s= we will take a canoe trip
- t= We will be home by sunset

Translation:

- **Assumptions:** (1) $\neg p \wedge q$, (2) ?
- **We want to show:** t

Applying rules of inference

- **Approach:**
- p = It is sunny this afternoon, q = it is colder than yesterday, r = We will go swimming , s= we will take a canoe trip
- t= We will be home by sunset

Translation: "We will go swimming only if it is sunny".

- Ambiguity: $r \to p$ or $p \to r$?
- Sunny is a must before we go swimming
- Thus, if we indeed go swimming it must be sunny, therefore $r \to p$

Applying rules of inference

Text:

(1) It is not sunny this afternoon and it is colder than yesterday.

(2) We will go swimming only if it is sunny.

(3) If we do not go swimming then we will take a canoe trip.

(4) If we take a canoe trip, then we will be home by sunset.

Propositions:

- p = It is sunny this afternoon, q = it is colder than yesterday, r = We will go swimming , s= we will take a canoe trip
- t= We will be home by sunset

Translation:

- **Assumptions:** (1) $\neg p \wedge q$, (2) $r \to p$, (3) $\neg r \to s$, (4) $s \to t$
- **We want to show:** t

Proofs using rules of inference

Translations:

- **Assumptions:** $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, $s \rightarrow t$
- **We want to show:** t

Proof:

- 1. $\neg p \wedge q$ Hypothesis
- 2. $\neg p$ Simplification
- 3. $r \rightarrow p$ Hypothesis
- 4. $\neg r$ Modus tollens (step 2 and 3)
- 5. $\neg r \rightarrow s$ Hypothesis
- 6. s Modus ponens (steps 4 and 5)
- 7. $s \rightarrow t$ Hypothesis
- 8. t Modus ponens (steps 6 and 7)
- **end of proof**