UTILITY THEORY

UTILITY FUNCTION

- Utility functions are a product of Utility Theory which is one of the disciplines that helps to address the challenges of building knowledge under uncertainty.

- The agents operate under certain degree of uncertainly and need to rely on probabilities to quantify the outcome of possible states. That probabilistic function is what we call **Utility Functions.**

UTILITY THEORY AND MEU

- Utility Theory is the discipline that lays out the foundation to create and evaluate Utility Functions.
- Utility Theory uses the notion of **Expected Utility (EU)** as a value that represents the average utility of all possible outcomes of a state, weighted by the probability that the outcome occurs.
- The other key concept of Utility Theory is known as the Principle of **Maximum Utility(MEU)** which states that any rational agent should choose to maximize the agent's EU.

Utility function

- **Utility function** (denoted U)
  - Quantifies how we "value" outcomes, i.e., it reflects our preferences
- **Decision making**:
  - uses expected utilities (denoted EU)

$$EU(X) = \sum_{x \in \Omega_X} P(X = x) U(X = x)$$

$U(X = x)$ the utility of outcome x

- The principle of **maximum expected utility** **<u>(MEU)</u>** says that a rational agent should chose the action that maximizes the agent's expected utility.

<u>Important !!!</u>

- Under some conditions on preferences we can always design the utility function that fits our preferences

Utility theory

- Defines axioms on preferences that involve uncertainty and ways to manipulate them.
- Uncertainty is modeled through **lotteries**
  - **Lottery:**
- [p : A; (1 - p) : C]
    - Outcome A with probability p
    - Outcome C with probability (1-p)
- The six constraints discussed next are known as the axioms of utility theory. The axioms are the most obvious semantic constraints on preferences with lotteries.

- **Notation:**

  $\succ$ - preferable
  $\sim$ - indifferent (equally preferable)

Utility Theory Axioms

There are six fundamental axioms that setup the foundation of Utility Theory.

1. Orderability
2. Transitivity
3. Continuity
4. Substitutability
5. Monotonicity
6. Decomposability

- **Orderability:** Given any two states, the rational agent prefers one of them, else the two as equally preferable.

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

- **Transitivity:** Given any three states, if an agent prefers *A* to *B* and prefers *B* to C, agent must prefer *A* to C.

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- **Continuity:** If some state *B* is between *A* and C in preference, then there is a *p* for which the rational agent will be indifferent between state B and the lottery in which A comes with probability p, C with probability (1-p).

$$(A \succ B \succ C) \Rightarrow \exists p \, [p : A; (1-p) : C] \sim B$$

- **Substitutability:** If an agent is indifferent between two lotteries, *A* and *B*, then there is a more complex lottery in which A can be substituted with B.

$$(A \sim B) \Rightarrow [p : A; (1-p) : C] \sim [p : B; (1-p) : C]$$

- **Monotonicity:** If an agent prefers *A* to *B*, then the agent must prefer the lottery in which A occurs with a higher probability

$$(A \sim B) \Rightarrow [p : A; (1-p) : C] \sim [p : B; (1-p) : C]$$

- **Decomposability:** Compound lotteries can be reduced to simpler lotteries using the laws of probability.

$$[p : A; (1-p) : [q : B; (1-q) : C]] \Rightarrow$$
$$[p : A; (1-p)q : B; (1-p)(1-q) : C]$$

UTILITY THEORY

- **If the agent obeys the axioms of the utility theory, then**

1. there exists a real valued function U such that

$$U(A) > U(B) \Leftrightarrow A \succ B$$
$$U(A) = U(B) \Leftrightarrow A \sim B$$

2. The utility of the lottery is the expected utility, that is the sum of utilities of outcomes weighted by their probability

$$U[p : A; (1-p) : B] = pU(A) + (1-p)U(B)$$

[Maximum Expected Utility Principle]

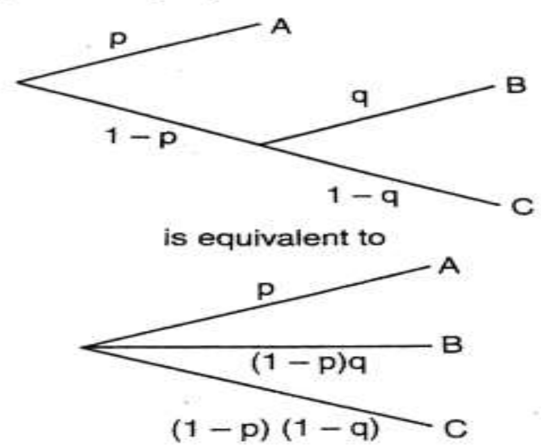


Fig. 3.12.

Decision Theoretic Expert System

Decision Network

- A **decision network** is a graphical representation of a finite sequential decision problem.
- Decision networks extend belief networks to include decision variables and utility.
- a **decision network** is a directed acyclic graph (DAG) with

  Chance Nodes(Ovals): represent attribute.

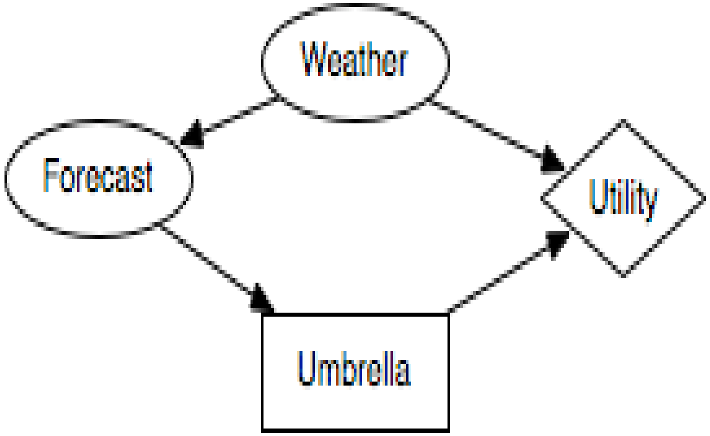
  Connection represent conditional effects.

  Additional Nodes

  Decision Nodes(Rectangles):represent possible decision

  Utility Nodes(Diamonds): calculates the utility of the decision.

Arcs coming into decision nodes represent the information that will be available when the decision is made. Arcs coming into chance nodes represents probabilistic dependence. Arcs coming into the utility node represent what the utility depends on.



: Decision network for decision of whether to take an umbrella

Simple decision network for a decision of whether the agent should take an umbrella when it goes out. The agent's utility depends on the weather and whether it takes an umbrella. However, it does not get to observe the weather. It only gets to observe the forecast. The forecast probabilistically depends on the weather.

Suppose the random variable *Weather* has domain *{norain,rain}*, the random variable *Forecast* has domain *{sunny,rainy,cloudy}*, and the decision variable *Umbrella* has domain *{takeIt,leaveIt}*. There is no domain associated with the utility node.

The designer also must specify the probability of the random variables given their parents. Suppose *P(Weather)* is defined by

$$P(Weather=rain)=0.3.$$

P(Forecast|Weather) is given by

Weather	*Forecast*	Probability
norain	*sunny*	0.7
norain	*cloudy*	0.2
norain	*rainy*	0.1
rain	*sunny*	0.15
rain	*cloudy*	0.25
rain	*rainy*	0.6

Suppose the utility function, *Utility(Weather,Umbrella)*, is

Weather	*Umbrella*	*Utility*
norain	*takeIt*	20
norain	*leaveIt*	100
rain	*takeIt*	70
rain	*leaveIt*	0

There is no table specified for the *Umbrella* decision variable. It is the task of the planner to determine which value of *Umbrella* to select, depending on the forecast.

ALGORITHM

.ne algorithm for evaluating decisions networks is the following:

- Set evidence variables for the current state
- For each possible value of the decision node
  - Set decision node to that value
  - Calculate posterior probabilities for the state nodes that are parents to the utility node
  - Compute expected utility for this action
- Return action that maximizes the expected utility.

Decision Theoretic Expert System

- The decision networks help to develop the expert systems that recommend optimal decisions, reflecting the preference of the user and available evidence.

- System can automate the process of
  - action selection and
  - drawing conclusions from evidence