Uncertainty- Probability, Bayes Rule, Belief Networks

UNCERTAINTY

- We have learned knowledge representation using first-order logic and propositional logic with certainty, which means we were sure about the predicates.

- With this knowledge representation, we might write $A \rightarrow B$, which means if A is true then B is true,

- but consider a situation where we are not sure about whether A is true or not then we cannot express this statement, this situation is called **uncertainty.**

CAUSES OF UNCERTAINTY

Following are some causes of uncertainty to occur in the real world.

1. Information occurred from unreliable sources.
2. Experimental Errors
3. Equipment fault
4. Temperature variation
5. Climate change.

PROBABILISTIC REASONING

- Probabilistic reasoning is a way of knowledge representation where we apply the concept of probability to indicate the uncertainty in knowledge.

- We use probability in probabilistic reasoning because it provides a way to handle the uncertainty that is the result of someone's laziness and ignorance.

- In the real world, there are lots of scenarios, where the certainty of something is not confirmed, such as **"It will rain today," "A match between two teams or two players."** These are probable sentences for which we can assume that it will happen but not sure about it, so here we use **probabilistic reasoning.**

NEED OF PROBABILISTIC REASONING IN AI

- When there are unpredictable outcomes.
- When specifications or possibilities of predicates becomes too large to handle.
- When an unknown error occurs during an experiment.

In probabilistic reasoning, there are two ways to solve problems with uncertain knowledge:

- **Bayes' rule.**
- **Bayesian Statistics.**

PROBABILITY

- Probability can be defined as a chance that an uncertain event will occur.
- It is the numerical measure of the likelihood that an event will occur.
- The value of probability always remains between 0 and 1 that represent ideal uncertainties.

$0 \leq P(A) \leq 1$, where $P(A)$ is the probability of an event A.

$P(A) = 0$, indicates total uncertainty in an event A.

$P(A) = 1$, indicates total certainty in an event A.

We can find the probability of an uncertain event by using the below formula.

$$\textbf{Probability of occurrence} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

- $P(\neg A)$ = probability of a not happening event.
- $P(\neg A) + P(A) = 1$.

- **Event:** Each possible outcome of a variable is called an event.
- **Sample space:** The collection of all possible events is called sample space.
- **Random variables:** Random variables are used to represent the events and objects in the real world.
- **Prior probability:** The prior probability of an event is probability computed before observing new information.
- **Posterior Probability:** The probability that is calculated after all evidence or information has taken into account. It is a combination of prior probability and new information.

Conditional probability:

- Conditional probability is a probability of occurring an event when another event has already happened.
- Let's suppose, we want to calculate the event A when event B has already occurred, "the probability of A under the conditions of B", it can be written as:

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

Where **$P(A \wedge B)$= Joint probability of A and B**

P(B)= Marginal probability of B.

- If the probability of A is given and we need to find the probability of B, then it will be given as:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}$$

EXAMPLE

In a class, there are 70% of the students who like English and 40% of the students who likes English and mathematics, and then what is the percent of students those who like English also like mathematics?

Let, A is an event that a student likes Mathematics

B is an event that a student likes English.

$$P(A|B) = \frac{P(A \wedge B)}{P(B)} = \frac{0.4}{0.7} = 57\%$$

Hence, 57% are the students who like English also like Mathematics.

BAYES' THEOREM

- Bayes' theorem is also known as **Bayes' rule, Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.

- In probability theory, it relates the conditional probability and marginal probabilities of two random events.

- It is a way to calculate the value of P(B|A) with the knowledge of P(A|B).

- Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

As from product rule we can write:

$$P(A \wedge B) = P(A|B)\, P(B) \text{ or}$$

Similarly, the probability of event B with known event A:

$$P(A \wedge B) = P(B|A)\, P(A)$$

Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)} \quad \text{....(a)}$$

The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

P(A|B) is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

P(B|A) is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.

P(A) is called the **prior probability**, probability of hypothesis before considering the evidence

P(B) is called **marginal probability**, pure probability of an evidence.

In the equation (a), in general, we can write P (B) = P(A)*P(B|Ai), hence the Bayes' rule can be written as:

$$P(A_i|B) = \frac{P(A_i)*P(B|A_i)}{\sum_{i=1}^{k} P(A_i)*P(B|A_i)}$$

Example: What is the probability that a patient has diseases meningitis with a stiff neck?

Given Data:

A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:

The Known probability that a patient has meningitis disease is 1/30,000.The Known probability that a patient has a stiff neck is 2%

Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis, so we can calculate the following as:

P(a|b) = 0.8

P(b) = 1/30000

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} = \frac{0.8*(\frac{1}{30000})}{0.02} = 0.001333333.$$

P(a)= .02

Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

APPLICATION OF BAYES' THEOREM IN ARTIFICIAL INTELLIGENCE

Following are some applications of Bayes' theorem:

1. It is used to calculate the next step of the robot when the already executed step is given.

2. Bayes' theorem is helpful in weather forecasting.

Bayesian Belief Network

- Bayesian belief network is key computer technology for dealing with probabilistic events and to solve a problem which has uncertainty.

We can define a Bayesian network as:

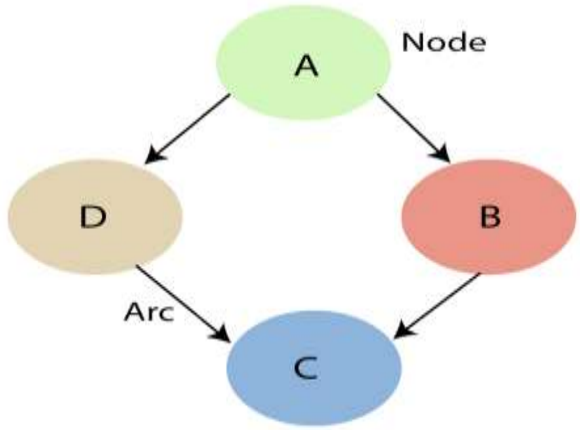
- "A Bayesian network is a probabilistic graphical model which represents a set of variables and their conditional dependencies using a directed acyclic graph."
- It is also called a **Bayes network, belief network, decision network**, or **Bayesian model**.

- Bayesian Network can be used for building models from data and experts opinions, and it consists of two parts:

  1. Directed Acyclic Graph

  2. Table of conditional probabilities.

A Bayesian network graph is made up of nodes and Arcs (directed links), where:



- Each **node** corresponds to the random variables, and a variable can be **continuous** or **discrete**.
- **Arc or directed arrows** represent the causal relationship or conditional probabilities between random variables.

- In the diagram, A, B, C, and D are random variables represented by the nodes of the network graph.
- If we are considering node B, which is connected with node A by a directed arrow, then node A is called the parent of Node B.
- Node C is independent of node A.

- Each node in the Bayesian network has condition probability distribution $\mathbf{P(X_i \mid Parent(X_i))}$, which determines the effect of the parent on that node.

- **Bayesian network is based on Joint probability distribution and conditional probability.**

JOINT PROBABILITY DISTRIBUTION

If we have variables x1, x2, x3,....., xn, then the probabilities of a different combination of x1, x2, x3.. xn, are known as Joint probability distribution.

- **$P[x_1, x_2, x_3, \ldots\ldots, x_n]$**, it can be written as the following way in terms of the joint probability distribution.

$$= P[x_1 | x_2, x_3, \ldots\ldots, x_n] P[x_2, x_3, \ldots\ldots, x_n]$$

$$= P[x_1 | x_2, x_3, \ldots\ldots, x_n] P[x_2 | x_3, \ldots\ldots, x_n] \ldots\ldots P[x_{n-1} | x_n] P[x_n].$$

In general for each variable Xi, we can write the equation as:

$$P(X_i | X_{i-1}, \ldots\ldots\ldots, X_1) = P(X_i | \mathrm{Parents}(X_i))$$

Explanation of Bayesian network:

Let's understand the Bayesian network through an example by creating a directed acyclic graph:

Example: Harry installed a new burglar alarm at his home to detect burglary. The alarm reliably responds at detecting a burglary but also responds for minor earthquakes. Harry has two neighbors David and Sophia, who have taken a responsibility to inform Harry at work when they hear the alarm. David always calls Harry when he hears the alarm, but sometimes he got confused with the phone ringing and calls at that time too. On the other hand, Sophia likes to listen to high music, so sometimes she misses to hear the alarm. Here we would like to compute the probability of Burglary Alarm.

Problem:

Calculate the probability that alarm has sounded, but there is neither a burglary, nor an earthquake occurred, and David and Sophia both called the Harry.

Solution:

- The Bayesian network for the above problem is given below. The network structure is showing that burglary and earthquake is the parent node of the alarm and directly affecting the probability of alarm's going off, but David and Sophia's calls depend on alarm probability.
- The network is representing that our assumptions do not directly perceive the burglary and also do not notice the minor earthquake, and they also not confer before calling.
- The conditional distributions for each node are given as conditional probabilities table or CPT.
- Each row in the CPT must be sum to 1 because all the entries in the table represent an exhaustive set of cases for the variable.
- In CPT, a boolean variable with k boolean parents contains 2^K probabilities. Hence, if there are two parents, then CPT will contain 4 probability values

List of all events occurring in this network:

- Burglary (B)
- Earthquake(E)
- Alarm(A)
- David Calls(D)
- Sophia calls(S)

We can write the events of problem statement in the form of probability: P[D, S, A, B, E], can rewrite the above probability statement using joint probability distribution:

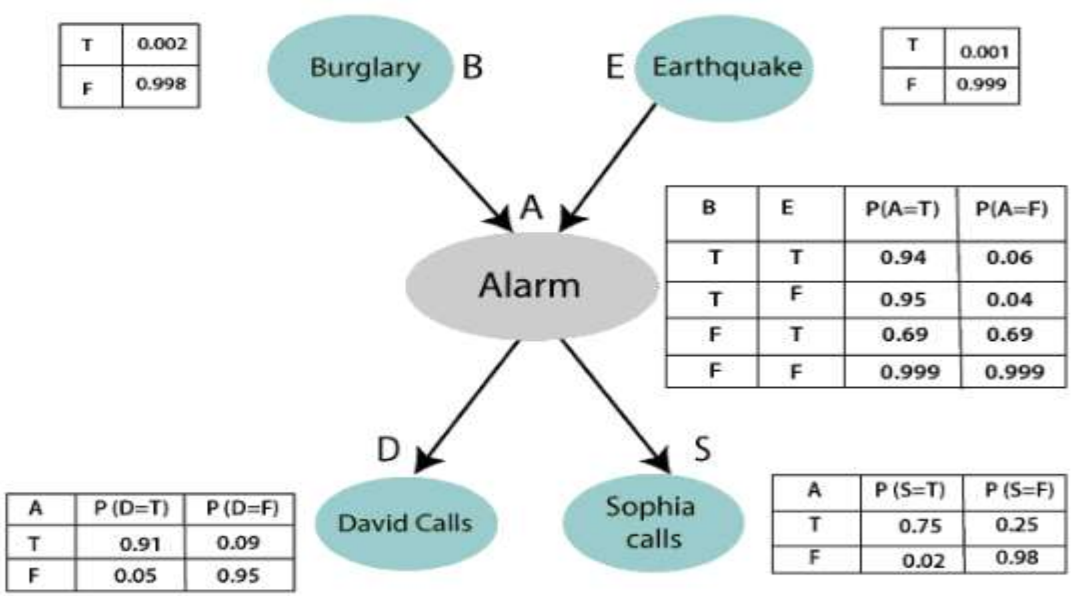
P[D, S, A, B, E]= P[D | S, A, B, E]. P[S, A, B, E]

=P[D | S, A, B, E]. P[S | A, B, E]. P[A, B, E]

= P [D| A]. P [S| A, B, E]. P[A, B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B, E]

= P[D | A]. P[S | A]. P[A| B, E]. P[B |E]. P[E]



T	0.002
F	0.998

T	0.001
F	0.999

B	E	P(A=T)	P(A=F)
T	T	0.94	0.06
T	F	0.95	0.04
F	T	0.69	0.69
F	F	0.999	0.999

A	P (D=T)	P (D=F)
T	0.91	0.09
F	0.05	0.95

A	P (S=T)	P (S=F)
T	0.75	0.25
F	0.02	0.98

The Conditional probability of Alarm A depends on Burglar and earthquake:

B	E	P(A= True)	P(A= False)
True	True	0.94	0.06
True	False	0.95	0.04
False	True	0.31	0.69
False	False	0.001	0.999

Conditional probability table for David Calls:

The Conditional probability of David that he will call depends on the probability of Alarm.

A	P(D= True)	P(D= False)
True	0.91	0.09
False	0.05	0.95

Conditional probability table for Sophia Calls:

The Conditional probability of Sophia that she calls is depending on its Parent Node "Alarm."

A	P(S= True)	P(S= False)
True	0.75	0.25
False	0.02	0.98

From the formula of joint distribution, we can write the problem statement in the form of probability distribution:

$P(S, D, A, \neg B, \neg E) = P(S|A) * P(D|A) * P(A|\neg B \wedge \neg E) * P(\neg B) * P(\neg E)$.

DEFAULT REASONING

- This is a very common form of **non-monotonic reasoning.** We want to draw conclusions based on what is most likely to be true.
- A **logic** is **non-monotonic** if some conclusions can be invalidated by adding more knowledge. The **logic** of definite clauses with negation as failure is **non-monotonic**.
- **Non-monotonic reasoning** is useful for representing defaults.

  - **Example:** let the KB contain:

    Typically birds fly.
    Penguins do not fly.
    Tweety is a bird.

  - It is plausible to conclude that Tweety flies.
  - However if the following information is added to KB

    Tweety is a penguin

    the previous conclusion must be retracted and, instead, the new conclusion that Tweety does not fly will hold.

We will discuss two approaches to do this:

- Non-Monotonic logic.
- Default logic.