L-T

Priority Queue

elements

a_1	a_2	a_3	a_4	a_5	a_6
i_1	i_2	i_3	i_4	15	10

importance factor

Queue → Priority Queue

↓ FIFO　　↓ FIFO

↓ priority → Importance factor.

Priority Queue

Min-priority (minimum priority element)

Max priority queue (maximum priority elem...)

popped

- insert
- getMax / getMin
- removemax / removemin

→ top() → pop() {stack}

L-II

Problem →

n → insert ?

↓

get min/max ? ⟶ priority basis.

⇓

element = priority

(MIN priority queue)

14, 9, 8, ~~7~~, ~~1~~, ~~2~~ 20

1 get min → 1
2 get min → 2
3 insert → 20
4 get min → 3

	insert	get min/max	remove min/max
Arrays - (unsorted)	O(1)	O(n)	O(n)
- (sorted)	O(n)	O(1)	O(n) = shifting.
Linked List - (unsorted)	O(1)	O(n)	O(n)
- (sorted)	O(n)	O(1)	O(1)
BST	O(h) h=height	O(h)	O(h) {worst case h=n}
Best Balanced BST	$\boxed{O(\log n)}$	$\boxed{O(\log n)}$	$\boxed{O(\log n)}$
Hash-map	O(1)	O(n)	O(n)

Heap

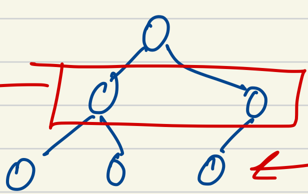
→ Issues with Balenced BST

- → Balencing
- → Complicated storing

Heaps

- — A complete Binary tree
- — Heap order property

→ Complete Binary Tree (CBT)

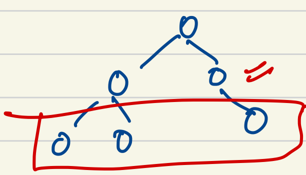


→ CBT

completly n-1 filled ←

← n^{th} not filled (L to R)

n^{th}, Right



→ Not a CBT

left to Right

(n^{th} level filling contraint)

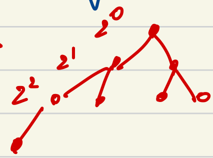
Heap property

① → CBT

↳ Height → overcoming balancing of BST

↳ minimum no. of nodes with height h CBT.

h=4 →

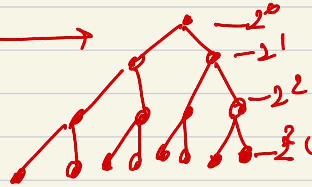


} Minimum

total = $2^0+2^1+2^2+2^{h-2}+1 = \underline{\underline{2^{h-1}}}$

↳ maximum " " " " " "

h=4 →



} h=4.

$\frac{a_1(r^{n+1}-1)}{r-1}$

$2^0+2^1+2^2+2^3+ \dots 2^{h-1}$

$1+2+2^2+2^3+ \dots 2^h$

$= \underline{\underline{2^h-1}}$

CBT → n

$$\boxed{2^{h-1} \le n \le 2^h-1}$$

$2^{h-1} < n$ (a)

$n \le 2^h-1$ (b)

for (a)

$$2^{h-1} \le n \Rightarrow \log(2^{h-1}) \le \log n$$

$$\Downarrow$$

$$(h-1)\log 2 \le \log n$$

$$h-1 \le \log_2 n$$

$$h-1 \le \log_2 n$$

$$\boxed{h \le \log_2 n + 1}$$

for (b)

$$n \le 2^h - 1$$

$$n+1 \le 2^h$$

$$\log(n+1) \le h \log 2$$

$$\boxed{\log_2(n+1) \le h}$$

$$\log_2(n+1) \le h \le \log_2 n + 1$$

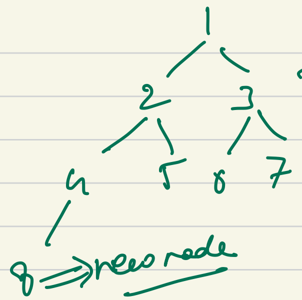
$$O(\log_2 n) \le h \le O(\log_2 n)$$

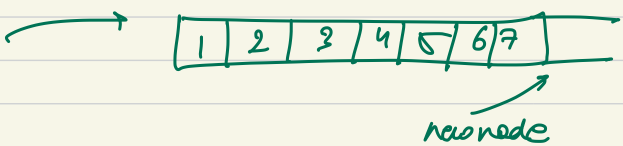
$$\Downarrow$$

$O(\log_2 n) \longrightarrow$ Best structure for priority <u>queue</u>

→ Storing of BST →

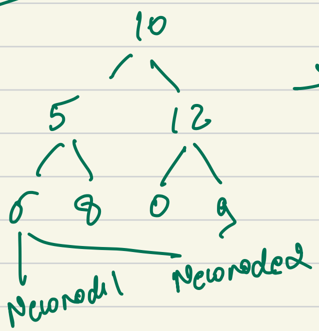
• level order traversal, first empty place is to be picked for inserting the value in BST, but it is <u>O(n)</u>, hence CBT solves this

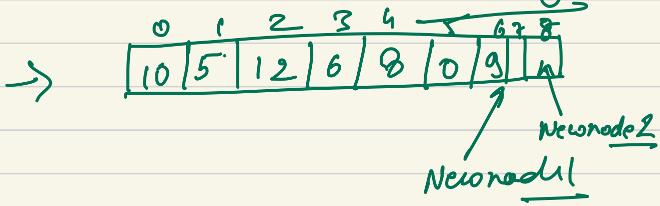




$0 \to 1, 2$
$1 \to 3, 4$
$2 \to 5, 6$

$\underbrace{2i+1, \ 2i+2}_{\text{child places in array}}$





$2i+1$
$2i+1 = \frac{9}{2}$
} child's place

parent ⟶ child

$i \longrightarrow 2i+1, \ 2i+2$

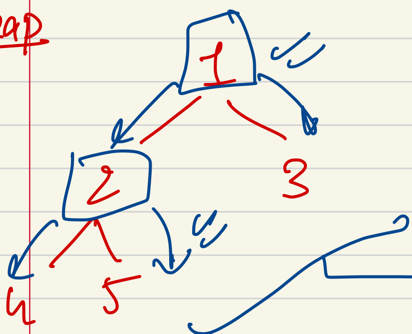
child ⟶ parent

$i \longrightarrow \frac{i-1}{2}$

Priority Queue
- Min
- Max

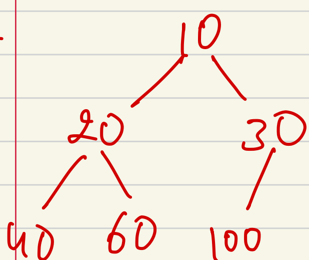
② Heap property → Heap order property →
- Min-heap
- Max-heap

Min heap



Root must have value less than both of his child

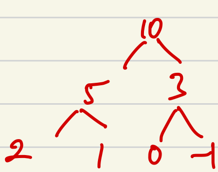
Valid heap (min) ✓



Min-max heap order property validated
- min-heap ✓

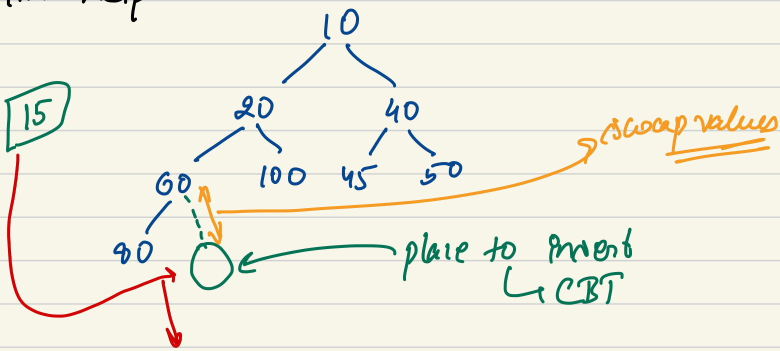
Max heap →

All parents have value greater than both childs

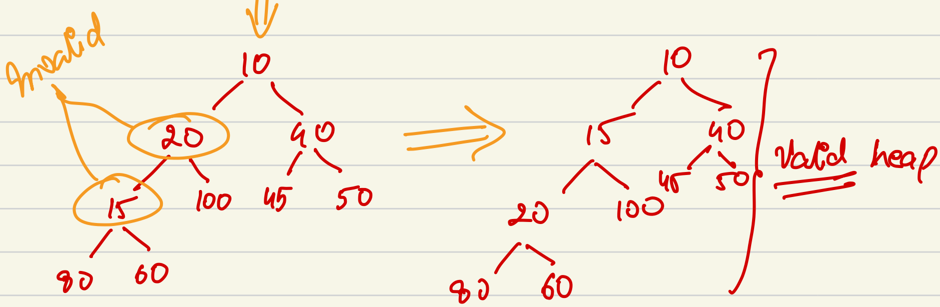


Heap Insert →

- Min heap →



Invalid heap order property



$O(h)$ → worst case insert function

$O(\log n)$ → CBT

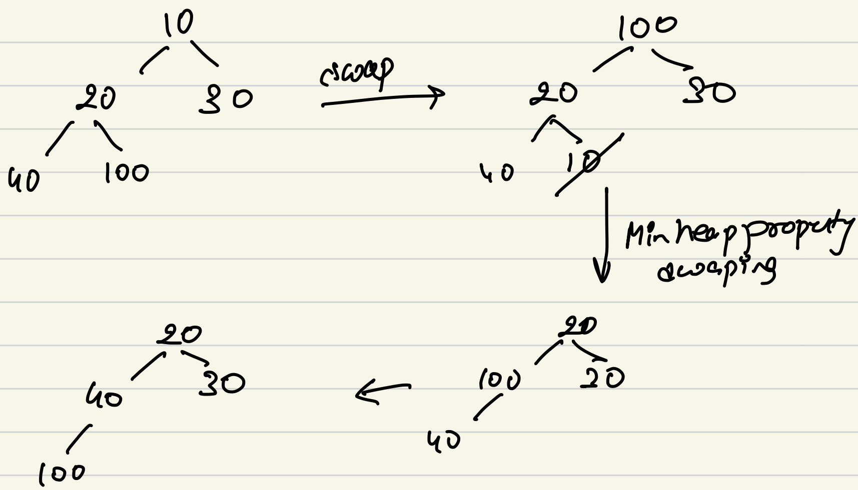
[Up-heapify]

Min heap

Heap delete →

→ swap if needed
→ Delete the desired item maintaining the properties.

Delete(10)



Min heap → minimum value at root.

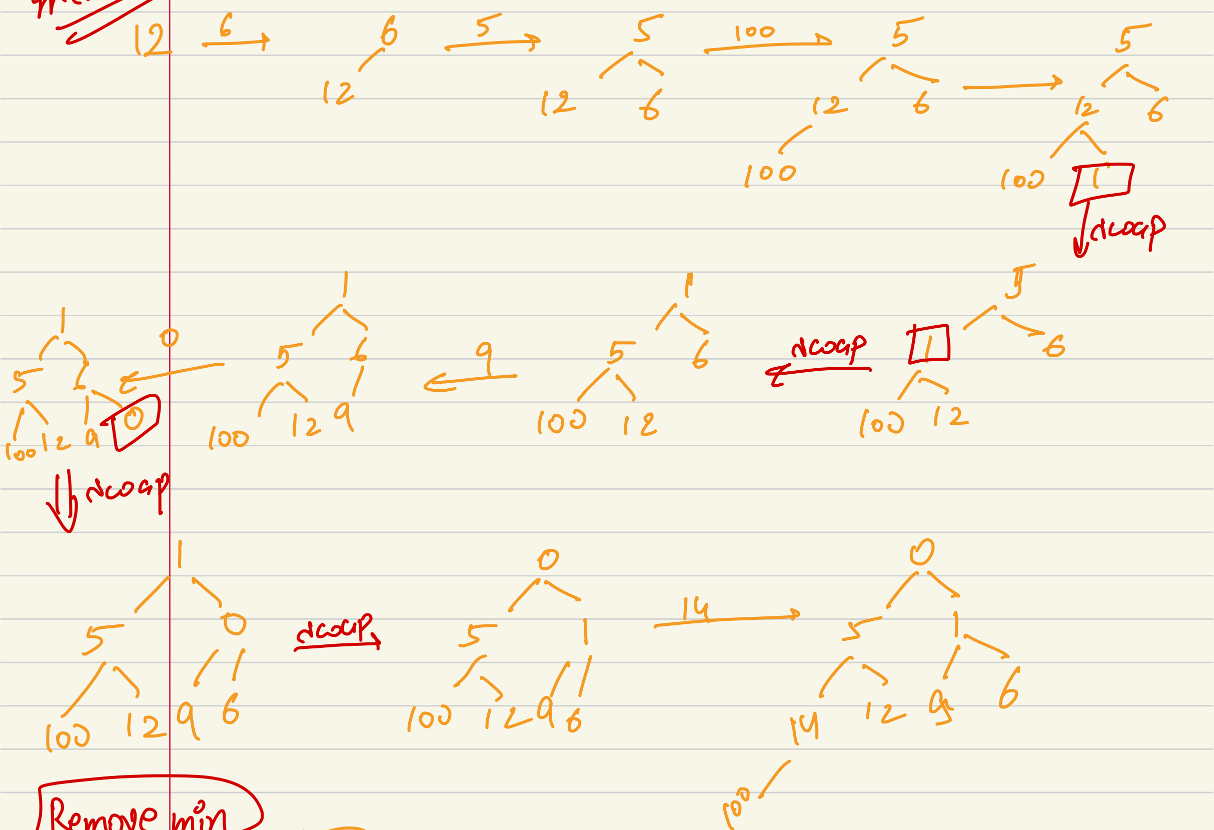
get() → return root value [O(1)]

delete() → desired properties must stand ≡ [$O(\log_2 n)$]
≡ [$O(n)$]

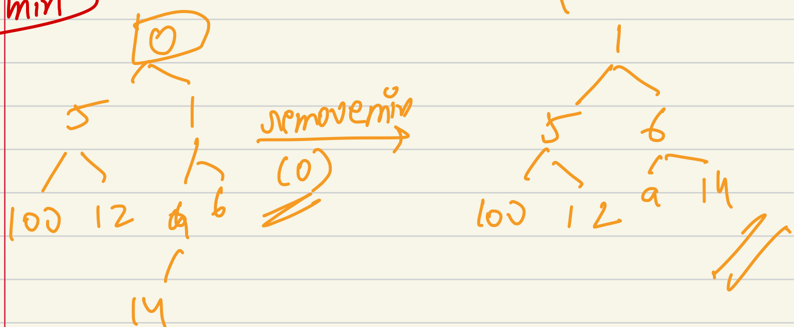
→ Draw Min heap & call remove_min() twice?

~~12~~, ~~6~~, ~~5~~, ~~100~~, ~~1~~, ~~9~~, ~~0~~, ~~14~~

Inserting



Remove min



```
class PriorityQueue {
  vector<int> pq;

 public :

 PriorityQueue() {

 }

 bool isEmpty() {
   return pq.size() == 0;
 }

 // Return the size of priorityQueue - no of elements present
 int getSize() {
   return pq.size();
 }

 int getMin() {
   if(isEmpty()) {
     return 0;   // Priority Queue is empty
   }
   return pq[0];
```

```
  return pq[0];
}

void insert(int element) {
  pq.push_back(element);

  int childIndex = pq.size() - 1;

  while(childIndex > 0) {
    int parentIndex = (childIndex - 1) / 2;

    if(pq[childIndex] < pq[parentIndex]) {
      int temp = pq[childIndex];
      pq[childIndex] = pq[parentIndex];
      pq[parentIndex] = temp;
    }
    else {
      break;
    }
    childIndex = parentIndex;
  }

}
```

```
// down-heapify

int parentIndex = 0;
int leftChildIndex = 2 * parentIndex + 1;
int rightChildIndx = 2 * parentIndex + 2;

while(leftChildIndex < pq.size()) {
    int minIndex = parentIndex;
    if(pq[minIndex] > pq[leftChildIndex]) {
        minIndex = leftChildIndex;
    }
    if(rightChildIndx < pq.size() && pq[rightChildIndx] < pq[minIndex]) {
        minIndex = rightChildIndx;
    }
    if(minIndex == parentIndex) {
        break;
    }
    int temp = pq[minIndex];
    pq[minIndex] = pq[parentIndex];
    pq[parentIndex] = temp;

    parentIndex = minIndex;
    leftChildIndex = 2 * parentIndex + 1;
    rightChildIndx = 2 * parentIndex + 2;
}

return ans;
```

Heap sorting

Time complexity → $O(\underbrace{n\log n}_{\text{inserting}} + \underbrace{\log n}_{\text{remove/swaping}})$

$O(n\log n)$

Space complexity → $O(n)$

↓

Inplace heapsort

↓

$O(1)$

→ Inplace Heap Sort → No new space to be used. $O(1)$ space complexity at average case.

Min heap

a → 5, 8, 3, 1, 6

heap

0	1	2	3	4
5	8	3	1	6

unsorted array

$i = 1$

child index $= i$

parent index $= \frac{i-1}{2}$

compare

5

8

till reach the end

```
using namespace std;


void inplaceHeapSort(int input[], int n) {
  // Build the heap in input array
  for(int i = 1; i < n; i++) {

    int childIndex = i;
    while(childIndex > 0) {
      int parentIndex = (childIndex - 1) / 2;

      if(pq[childIndex] < pq[parentIndex]) {
        int temp = pq[childIndex];
        pq[childIndex] = pq[parentIndex];
        pq[parentIndex] = temp;
      }
      else {
        break;
      }
      childIndex = parentIndex;
    }
  }

  // Remove elements
```

pq

first swap pq[0], pq[pq.size-1]

```
int size = n;

while(size > 1) {
  int temp = pq[0];
  pq[0] = pq[size - 1];
  pq[size-1] = temp;

  size--;

  int parentIndex = 0;
  int leftChildIndex = 2 * parentIndex + 1;
  int rightChildIndx = 2 * parentIndex + 2;

  while(leftChildIndex < size) {
    int minIndex = parentIndex;
    if(pq[minIndex] > pq[leftChildIndex]) {
      minIndex = leftChildIndex;
    }
    if(rightChildIndx < size && pq[rightChildIndx] < pq[minIndex]) {
      minIndex = rightChildIndx;
    }
    if(minIndex == parentIndex) {
      break;
    }
    int temp = pq[minIndex];
    pq[minIndex] = pq[parentIndex];
    pq[parentIndex] = temp;

    parentIndex = minIndex;
    leftChildIndex = 2 * parentIndex + 1;
    rightChildIndx = 2 * parentIndex + 2;
}
```

Inbuilt Priority Queue → STL → Cpp.

#include <queue>

priority_queue< template >

Max priority queue by default

- isEmpty → empty()
- getSize → size()
- void insert(e) → push(e)
- getMin() → T top() → max element
- To removeMin() → void pop() → Delete root (max element)

```cpp
#include <queue>

int main() {
  priority_queue<int> pq;

  pq.push(16);
  pq.push(1);
  pq.push(167);
  pq.push(7);
  pq.push(45);
  pq.push(32);

  cout << "Size : " << pq.size() << endl;
  cout << "Top : " << pq.top() << endl;

  while(!pq.empty()) {
    cout << pq.top() << endl;
    pq.pop();
  }
}
~
```

Nearly sorted

Medium Accuracy: **75.25%** Submissions: **25K+** Points: **4**

Given an array of **n** elements, where each element is at most **k** away from its target position, you need to sort the array optimally.

Example 1:

```
Input:
n = 7, k = 3
arr[] = {6,5,3,2,8,10,9}
Output: 2 3 5 6 8 9 10
Explanation: The sorted array will be
2 3 5 6 8 9 10
```

```
class Solution
{
    public:
    //Function to return the sorted array.
    vector <int> nearlySorted(int arr[], int num, int K){
        // Your code here
        vector<int> ans;
        priority_queue<int,vector<int>, greater<int>> pq;

        for(int i=0;i<K+1;i++)
        {
            pq.push(arr[i]);
        }
        //cout<<pq.top();
        for(int i=K+1;i<num;i++)
        {
            ans.push_back(pq.top());
            pq.pop();
            pq.push(arr[i]);

        }
        while(!pq.empty())
        {
            ans.push_back(pq.top());
            pq.pop();
        }
        //sort(ans.begin(),ans.end());
        return ans;
    }
```

Kth smallest element

Medium Accuracy: **35.17%** Submissions: **414K+** Points: **4**

Given an array **arr[]** and an integer **K** where K is smaller than size of array, the task is to find the **Kth smallest** element in the given array. It is given that all array elements are distinct.

Example 1:

```
Input:
N = 6
arr[] = 7 10 4 3 20 15
K = 3
Output : 7
```

```
class Solution{
    public:
    // arr : given array
    // l : starting index of the array i.e 0
    // r : ending index of the array i.e size-1
    // k : find kth smallest element and return using this function
    int kthSmallest(int arr[], int l, int r, int k) {
        //code here
        priority_queue<int> pq;
        for(int i=0;i<k;i++)
        {
            pq.push(arr[i]);
        }
        for(int i=k;i<r+1;i++)
        {
            if(pq.top()>arr[i])
            {
                pq.pop();
                pq.push(arr[i]);
            }

        }
        return pq.top();
```

K largest elements

Basic Accuracy: **61.15%** Submissions: **45K+** Points: **1**

Given an array of N positive integers, print k largest elements from the array.

Example 1:

```
Input:
N = 5, k = 2
arr[] = {12,5,787,1,23}
Output: 787 23
Explanation: First largest element in
the array is 787 and the second largest
is 23.
```

```cpp
class Solution
{
    public:
    //Function to return k largest elements from an array.
    vector<int> kLargest(int arr[], int n, int k)
    {
        // code here
        priority_queue<int,vector<int>,greater<int>> pq;
        for(int i=0;i<k;i++)
        pq.push(arr[i]);

        for(int i=k;i<n;i++)
        {
            if(pq.top()<arr[i])
            {
                pq.pop();
                pq.push(arr[i]);
            }
        }
        vector<int> ans;
        while(!pq.empty())
        {
            ans.push_back(pq.top());
            pq.pop();
        }
        sort(ans.begin(),ans.end(),greater<int>());
        return ans;
    }
};
```