Name - Kartikay Tandon
Roll no - 102117121 , CS-5

<u>Q1</u> $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2 \; -- \; x_n$ is sample size of n

$L(x_1, x_2 --- x_n) = f(x_1) . f(x_2) --- f(x_n)$

$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \right) . \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \right) ----$

taking $\ln$ both sides

$\ln(2) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2} \right) \quad -1$

take partial derivate wrt to μ in 1

$\frac{\partial \ln(2)}{\partial \mu} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i-\mu)}{2\sigma^2} \right) = 0$

Date: __.__.__

$$\Rightarrow \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\Rightarrow n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

hence $\theta_1 = \bar{x}$ is therefore sample mean.

Taking derivate wrt to σ^2 in ①

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i - \mu)^2}{2\sigma^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \times \left(\sum_{i=1}^{n} (x_i - \mu)^2 \right)$$

hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$

Q2 Binomial distri $\rightarrow$ ${}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Take log in both sides

$$\log L = \sum_{i=1}^{m} \left(\log ({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{m} \log ({}^{n}C_{x_i}) + \log \theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n - x_i)$$

apsara

Date: __.__.__

differentiate wrt ~~to~~ θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{1}{1-\theta} \Sigma(n - x_i) = 0$$

$$\frac{1}{\theta(1-\theta)} \Sigma(x_i) = \frac{n^2}{1-\theta}$$

$$\Rightarrow \quad \theta = \frac{\Sigma x_i}{n^2}$$