Factor Analysis Example

May 6, 2022

[23]:
```
from sklearn.decomposition import FactorAnalysis
from sklearn.datasets import load_digits
import matplotlib.pyplot as plt
import pandas as pd
```

0.1 Loading the mnist dataset from sklearn

[24]:
```
mnist = load_digits()
mnist.data.shape
```

[24]:
```
(1797, 64)
```

[25]:
```
pd.DataFrame(mnist.data)
```

[25]:

	0	1	2	3	4	5	6	7	8	9	…	54	55	\
0	0.0	0.0	5.0	13.0	9.0	1.0	0.0	0.0	0.0	0.0	…	0.0	0.0	
1	0.0	0.0	0.0	12.0	13.0	5.0	0.0	0.0	0.0	0.0	…	0.0	0.0	
2	0.0	0.0	0.0	4.0	15.0	12.0	0.0	0.0	0.0	0.0	…	5.0	0.0	
3	0.0	0.0	7.0	15.0	13.0	1.0	0.0	0.0	0.0	8.0	…	9.0	0.0	
4	0.0	0.0	0.0	1.0	11.0	0.0	0.0	0.0	0.0	0.0	…	0.0	0.0	
…	…	…	…	…	…	…	…	…	…	…	…	…	…	
1792	0.0	0.0	4.0	10.0	13.0	6.0	0.0	0.0	0.0	1.0	…	4.0	0.0	
1793	0.0	0.0	6.0	16.0	13.0	11.0	1.0	0.0	0.0	0.0	…	1.0	0.0	
1794	0.0	0.0	1.0	11.0	15.0	1.0	0.0	0.0	0.0	0.0	…	0.0	0.0	
1795	0.0	0.0	2.0	10.0	7.0	0.0	0.0	0.0	0.0	0.0	…	2.0	0.0	
1796	0.0	0.0	10.0	14.0	8.0	1.0	0.0	0.0	0.0	2.0	…	8.0	0.0	

	56	57	58	59	60	61	62	63
0	0.0	0.0	6.0	13.0	10.0	0.0	0.0	0.0
1	0.0	0.0	0.0	11.0	16.0	10.0	0.0	0.0
2	0.0	0.0	0.0	3.0	11.0	16.0	9.0	0.0
3	0.0	0.0	7.0	13.0	13.0	9.0	0.0	0.0
4	0.0	0.0	0.0	2.0	16.0	4.0	0.0	0.0
…	…	…	…	…	…	…	…	…
1792	0.0	0.0	2.0	14.0	15.0	9.0	0.0	0.0
1793	0.0	0.0	6.0	16.0	14.0	6.0	0.0	0.0
1794	0.0	0.0	2.0	9.0	13.0	6.0	0.0	0.0

```
1795  0.0  0.0  5.0  12.0  16.0  12.0  0.0  0.0
1796  0.0  1.0  8.0  12.0  14.0  12.0  1.0  0.0

[1797 rows x 64 columns]
```

[26]:
```
mnist.target.shape
```

[26]:
```
(1797,)
```

[27]:
```
len(mnist.data[0])
```

[27]:
```
64
```

0.2 Factor Analysis

Factor Analysis (FA).

A simple linear generative model with Gaussian latent variables.

The observations are assumed to be caused by a linear transformation of lower dimensional latent factors and added Gaussian noise. Without loss of generality the factors are distributed according to a Gaussian with zero mean and unit covariance. The noise is also zero mean and has an arbitrary diagonal covariance matrix.

0.3 Performing factor analysis

Here, we reduce the dimensionality of the features to 10

[28]:
```
fa = FactorAnalysis(n_components=10, random_state=123)
z = fa.fit_transform(mnist.data)
z.shape
```

[28]:
```
(1797, 10)
```

[29]:
```
pd.DataFrame(z)
```

[29]:
```
             0         1         2         3         4         5         6  \
0    -0.137682 -0.345500  0.299633 -1.964890  0.131388 -0.543294  0.854779
1    -0.876863 -0.212599 -0.289595  1.597254 -0.686167  0.263516 -1.761568
2    -1.079785  0.634491 -0.079147  1.519223 -1.585191 -0.357723  1.440194
3     0.643335  0.736808  0.653374  0.087544 -0.467310  0.866322 -0.744105
4    -1.567630 -0.107409 -0.666044  0.147479 -0.726444  2.162796  0.622288
…          …         …         …         …         …         …         …
1792 -0.203052 -0.018559  1.886885 -0.685014 -0.258193 -0.236861 -0.760494
1793 -0.123894 -0.128567 -0.239166 -2.097233 -0.104741 -0.887154  0.705658
1794 -0.703092 -0.072442 -0.646984  0.425525 -1.183108 -0.665684 -0.145152
1795 -0.331626 -0.025683  1.309561 -0.548834 -0.007503  0.085643 -0.897940
1796  0.606129  0.839921 -0.420771 -0.575240 -1.150315  0.289251  1.012708
```

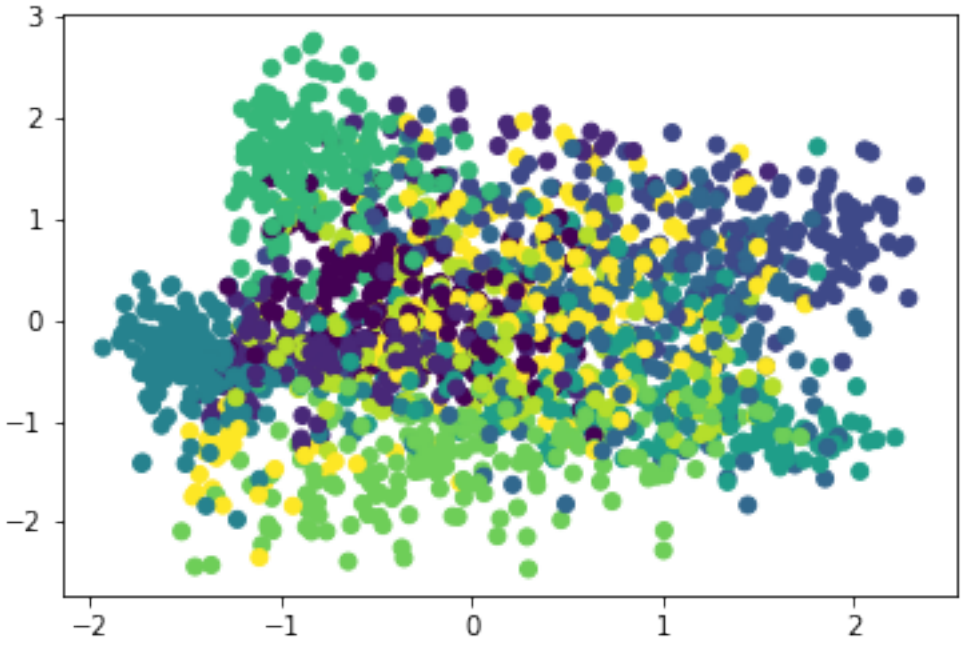
```
              7         8         9
0     -0.882733  0.105022  0.346685
1      0.089412 -0.288956  0.767700
2     -1.429385 -0.904890 -1.681560
3      1.349558  0.013035  1.048830
4      0.083669  0.675323  0.074913
…           …         …         …
1792  -0.301666 -0.065745  0.642226
1793  -1.447609 -0.172683  0.272573
1794  -0.648046 -1.308386  1.197597
1795  -0.811411 -0.082541  0.580797
1796   0.066969 -0.923151  0.771381

[1797 rows x 10 columns]
```

0.4 Data visualisation

Each colour represents a digit

[30]:
```
plt.scatter(z[:, 0], z[:, 1], c=mnist.target)
```

[30]:
```
<matplotlib.collections.PathCollection at 0x7f2067de7430>
```



0.5 Observe for the data where the digit is 2

[31]:
```
digit = 2
df_digit = [mnist.data[i] for i in range(len(mnist.data)) if mnist.target[i] ==␣
 ↪digit]
len(df_digit)
```

[31]:
```
177
```

0.6 We perform factor analysis

We fit the reduced feature dimensionality and get a FactorAnalysis object

[32]:
```
fa = FactorAnalysis(n_components=10, random_state=123)
z = fa.fit(df_digit)
z
```

[32]:
```
FactorAnalysis(n_components=10, random_state=123)
```

0.7 We have the old and the new features

[33]:
```
z.components_.shape
```

[33]:
```
(10, 64)
```

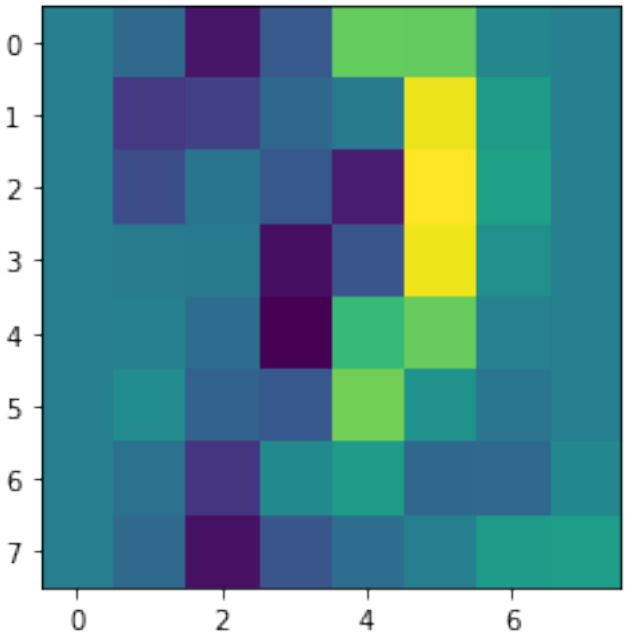
[34]:
```
df_digit[0]
```

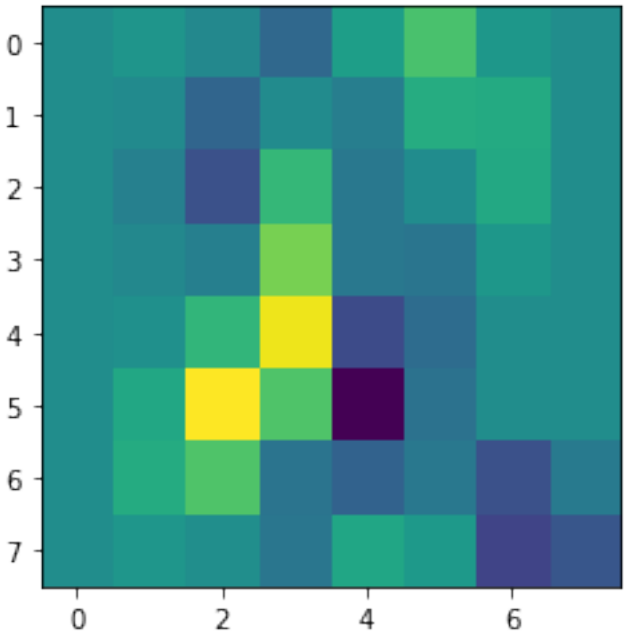
[34]:
```
array([ 0.,  0.,  0.,  4., 15., 12.,  0.,  0.,  0.,  0.,  3., 16., 15.,
       14.,  0.,  0.,  0.,  0.,  8., 13.,  8., 16.,  0.,  0.,  0.,  0.,
        1.,  6., 15., 11.,  0.,  0.,  0.,  1.,  8., 13., 15.,  1.,  0.,
        0.,  0.,  9., 16., 16.,  5.,  0.,  0.,  0.,  0.,  3., 13., 16.,
       16., 11.,  5.,  0.,  0.,  0.,  0.,  3., 11., 16.,  9.,  0.])
```

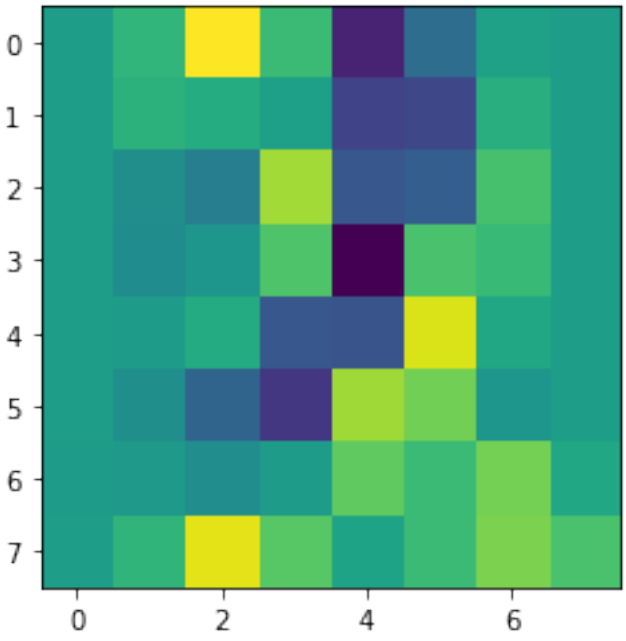
0.8 We observe the new features after dimensionality reduction

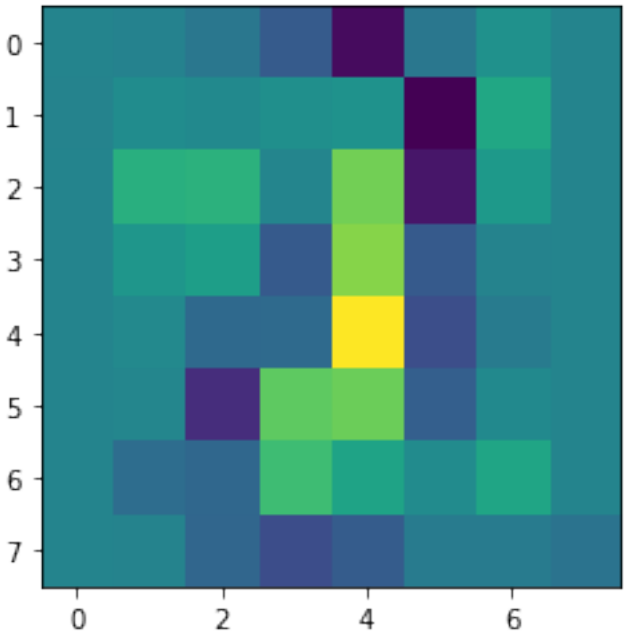
We see a faint image of 2

[35]:
```
for i in range(10):
    plt.imshow(z.components_[i].reshape(8,8), )
    plt.show()
```


0
1
2
3
4
5
6
7
0
2
4
6


0
1
2
3
4
5
6
7
0
2
4
6


0
1
2
3
4
5
6
7
0
2
4
6


0
1
2
3
4
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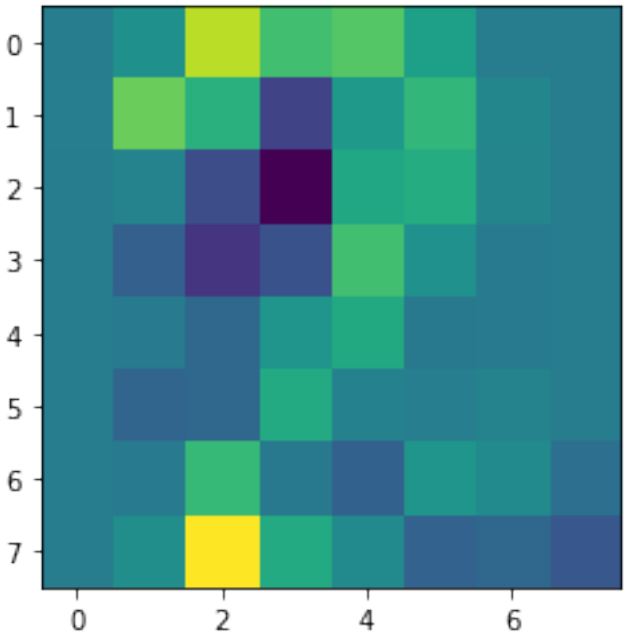
6
7
0
2
4
6


0
1
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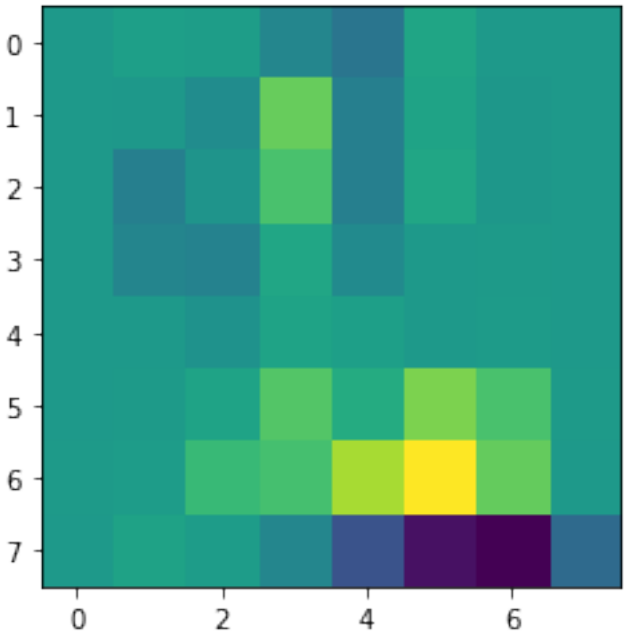
6


0
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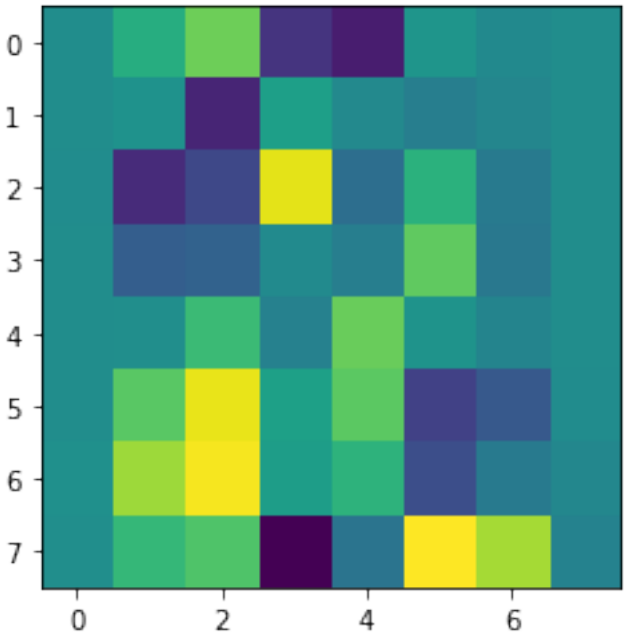
0
1
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6
7
0
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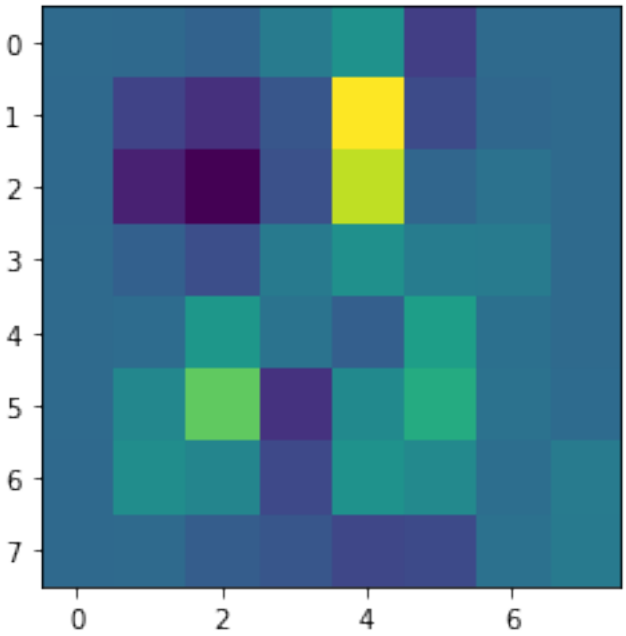
6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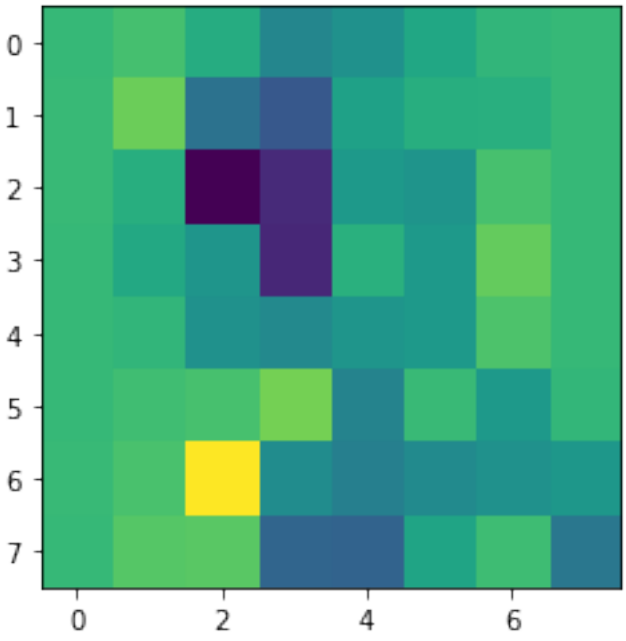
0
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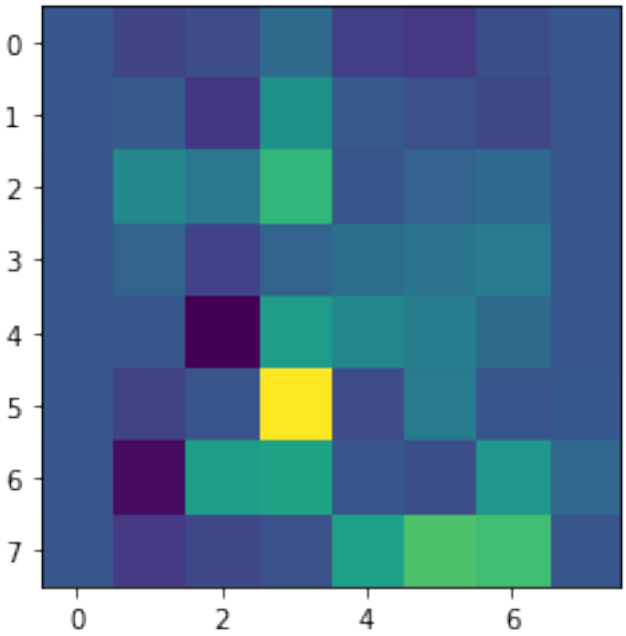
0
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