UNIT - 4

EQUIVALENT ANNUAL WORTH COMPARISONS: Equivalent Annual Worth Comparison methods, Situations for Equivalent Annual Worth Comparison, Consideration of asset life, Comparison of assets with equal and unequal lives, Use of sinking fund method, Annuity contract for guaranteed income, Exercises, Problems.

4.1 Equivalent Annual worth Comparison methods:

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost- based comparison will be selected as the best alternative.

4.2 Situations for Equivalent Annual Worth Comparison

The term annual worth suggests a positive value.If the calculations will producce a negative value it indicates that the equivalent value of negative cash flow for disburesements is greater than the corresponding positive cash flows.The situations for euivalent annual worth comparison are as follows.

1.Consolidation of cash flows

2.Recovery of invested capital

3.Net cah flow comparision

1.Consolidation of cash flows- In any project the general question will be what's it worth. It is difficult to expect from the proposal until the receipts and disburements associated with its conduct are collectively analyzed.

2. Recovery of invested capital- The investers want to know the recovery of invested capital plus the desired rate of return.It is spread over the life of the investment ,it is convinient to convert capital recovery costs to the same annual pattern.

3. Net cah flow comparision- If the worth is measured by revenues, the criterion is strictly economic, highest net worth is preffered. But, when alternatives have only costs and no income, a low EAC is prefered.

4.3 Consideration of asset life

Eeconomic life -The asset's economic life is defined as the number of years in which the asset returns more value to the owner than it costs to own, operate, and maintain. When these costs exceed returns, the acquisition is beyond its economic life.

An asset's economic life can be shortened or terminated by a number of different factors, including: Wear, degradation, or damage which can lower asset performance and raise maintenance and operation costs.Obsolescence, which can raise maintenance costs and render asset performance relatively inefficient when compared to more current alternatives. Changes in company operations, product offerings, or the company's business model, which reduce the value certain assets can deliver.

Service life or ownership life -An asset's service life is defined as the number of years the acquisition will actually be in service, and all may contribute to the owner's judgement as to what the ownership life should be.

Ownership life begins when the decision to acquire the asset begins causing costs. This may include costs that occur before the actual arrival or asset use begins, such as loan origination fees, planning costs, transportation costs, or set up costs. Ownership life ends when the asset stops causing costs and in fact has no continuing financial impact of any kind.

4.4 Comparison of assets with equal and unequal lives

Comparison of assets with equal lives

For comparing assets having equal lives, the cash flows may be converted to equivalent annual costs. If initial cost and salvage value is involved, then annual equivalent worth can be derived as

$$\text{Equivalent annual cost (EAC)} = P\,(A/P, i, n) - S\,(A/F, i, n)$$

We know that $(A/P, i, n) = i\,(1+i)^n/(1+i)^n - 1$

$$[i\,(1+i)^n/(1+i)^n - 1] - i = i/(1+i)^n - 1$$

$$(A/P, i, n) - i = (A/F, i, n)$$

Replacing the value of the above equation in

$$EAC = P\,(A/P, i, n) - S\,(A/F, i, n)$$

$$EAC = P\,(A/P, i, n) - S\,\{(A/P, i, n) - i\}$$

$$= (P-S)\,(A/P, i, n) + Si$$

Comparison of assets with unequal lives

In case of assets having unequal lives, each of the cash flows are converted in terms equivalent annual amount. If this annual amount is the cost, then assets having least annual equivalent cost are preferred.

4.5 Use of sinking fund method

The sinking fund factor is applied to compute the annuity required to accumulate a certain future amount. Organizations are sometimes obligated by legislated or contractual agreements to establish a fund, separate from internal operations to accumaulate a specified amount by a specified time.This accumulation is called a sinking fund.Provision for sinking fund requires set aside a portion of the income derived from sales, or taxes each year in order to retire a bond issue.

Problem 1:

A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of gas. The cost per litre of gas for the first year of operation is Rs. 21. He feels that the cost of gas will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of gas. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume i = 18%. If the rental car is preferred, then the company car will find some other use within the company.

Average number of km run/year = 20,000 km

Number of km/litre of gas = 9 km

Therefore,

Gas consumption/year = 20,000/9 = 2222.2 litre Cost/litre of gas for the 1st year = Rs. 21

Cost/litre of gas for the 2nd year = Rs. 21.00 + Re. 1.00 = Rs. 22.00

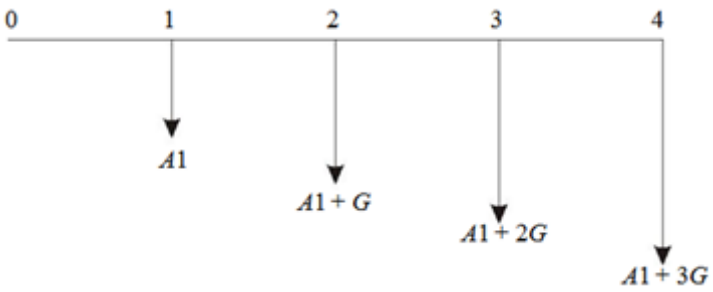
Cost/litre of gas for the 3rd year = Rs. 22.00 + Re. 1.00 = Rs. 23.00

Cost/litre of gas for the 4th year = Rs. 23.00 + Re. 1.00 = Rs. 24.00

Fuel expenditure for 1st year = 2222.2 x 21 = Rs. 46,666.20 Fuel expenditure for 2nd year = 2222.2 x 22 = Rs. 48,888.40 Fuel expenditure for 3rd year = 2222.2 ´x 23 = Rs. 51,110.60 Fuel expenditure for 4th year = 2222.2 x 24 = Rs. 53,332.80

The annual equal increment of the above expenditures is Rs. 2,222.20 (G).

The cash flow diagram for this situation is shown in Fig.4.1



$A1$ = Rs. 46,666.20 and G = Rs. 2,222.20

$$A = A1 + G(A/G, 18\%, 4)$$
$$= 46,666.20 + 2222.2(1.2947)$$
$$= \text{Rs. } 49,543.28$$

The proposal of using the company car by spending for gas by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

Problem 2:A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	Down payment	Yearly equal installment	No. of installments
	(Rs.)	(Rs.)	
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Determine the best alternative based on the annual equivalent method by assuming i = 20%, compounded annually.

Solution

Alternative 1

Down payment, P = Rs. 5, 00,000

Yearly equal installment, A = Rs. 2, 00,000 n = 15 years

i = 20%, compounded annually

The cash flow diagram for manufacturer 1 is shown in Fig

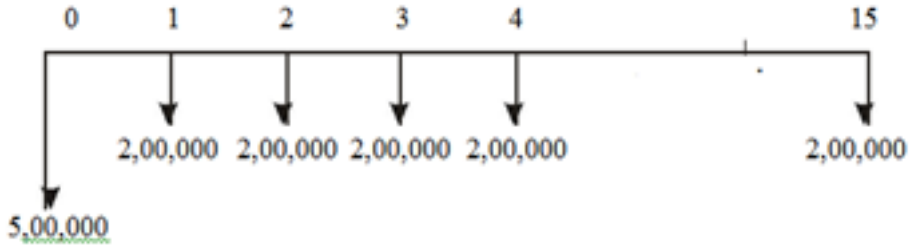


Figure 4.2 Cash flow diagram for manufacturer 1

The annual equivalent cost expression of the above cash flow diagram is

$$AE_1 (20\%) = 5, 00,000(A/P, 20\%, 15) + 2, 00,000$$

$$= 5,00,000(0.2139) + 2,00,000$$

$$= 3,06,950$$

Alternative 2

Down payment, P = Rs. 4, 00,000

Yearly equal installment, A = Rs. 3, 00,000 n = 15 years

i = 20%, compounded annually

The cash flow diagram for the manufacturer 2 is shown in Figure 4.3

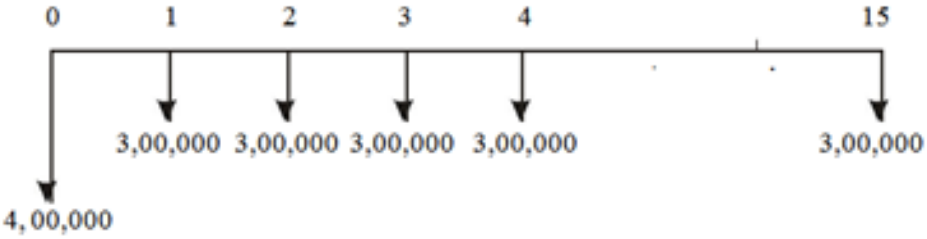


Figure 4.3 Cash flow diagram for manufacturer 2

The annual equivalent cost expression of the above cash flow diagram is

AE_2 (20%) = 4, 00,000(A/P, 20%, 15) + 3, 00,000

= 4,00,000(0.2139) + 3,00,000

= Rs. 3, 85,560.

Alternative 3

Down payment, P = Rs. 6, 00,000

Yearly equal installment, A = Rs. 1, 50,000 n = 15 years

i = 20%, compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig

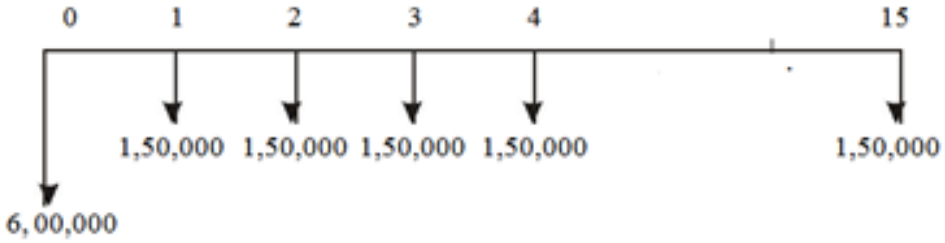


Figure 4.4 Cash flow diagram for manufacturer 3

The annual equivalent cost expression of the above cash flow diagram is

AE_3 (20%) = 6, 00,000(A/P, 20%, 15) + 1, 50,000

= 6,00,000(0.2139) + 1,50,000

= Rs. 2, 78,340.

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

Problem 3: A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B

Investment (Rs.)	– 1,50,000	– 1,75,000
Annual equal return (Rs.)	+ 60,000	+ 70,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming i = 25%.

Solution

Alternative A

Initial investment, P = Rs. 1,50,000

Annual equal return, A = Rs. 60,000

Salvage value at the end of machine life, S = Rs. 15,000

Life = 5 years

Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative A is shown in Figure 4.5

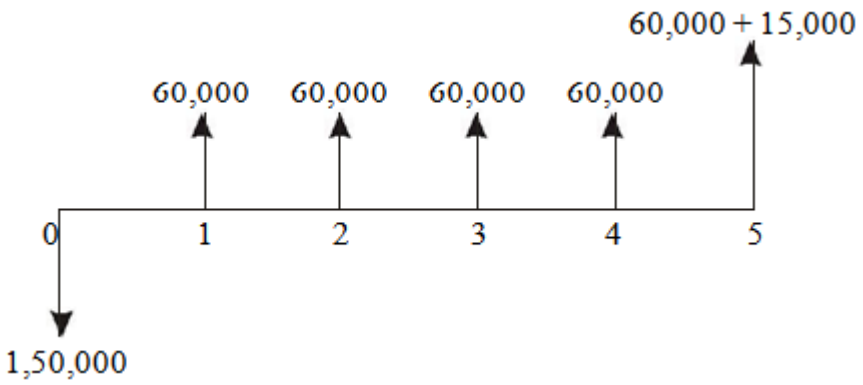


Figure 4.5 Cash flow diagrams for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:

AE_A (25%) = –1, 50,000(A/P, 25%, 5) + 60,000 + 15,000(A/F, 25%, 5)

= –1,50,000(0.3718) + 60,000 + 15,000(0.1218)

= Rs. 6,057

Alternative B

Initial investment, P = Rs. 1, 75,000

Annual equal return, A = Rs. 70,000

Salvage value at the end of machine life, S = Rs. 35,000

Life = 5 years

Interest rate, i = 25%, compounded annually

The cash flow diagram for alternative B is shown in Figure 4.6

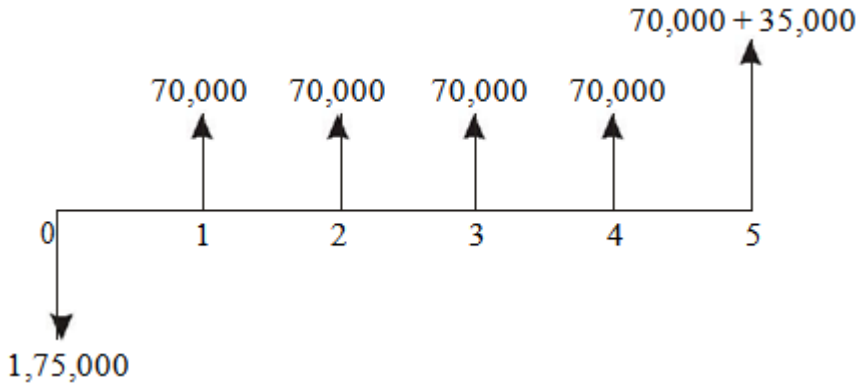


Figure 4.6 Cash flow diagrams for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is

$AE_B(25\%) = -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5)$

$= -1,75,000(0.3718) + 70,000 + 35,000(0.1218)$

= Rs. 9,198

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

Problem 4: Two possible routes for laying a power line are under study. Data on the routes are as follows:

	Around the lake	*Under the lake*
Length	15 km	5 km
First cost (Rs.)	1,50,000/km	7,50,000/km
Useful life (years)	15	15
Maintenance cost (Rs.)	6,000/km/yr	12,000/km/yr
Salvage value (Rs.)	90,000/km	1,50,000/km
Yearly power loss (Rs.)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

Solution Alternative 1— Around the lake

First cost = 1,50,000 x 15 = Rs. 22,50,000 Maintenance cost/yr = 6,000 x15 = Rs. 90,000

Power loss/yr = 15,000 x 15 = Rs. 2,25,000

Maintenance cost and power loss/yr = Rs. 90,000 + Rs. 2,25,000 = Rs. 3,15,000

Salvage value = 90,000 ´ 15 = Rs. 13,50,000

The cash flow diagram for this alternative is shown in Figure 4.7

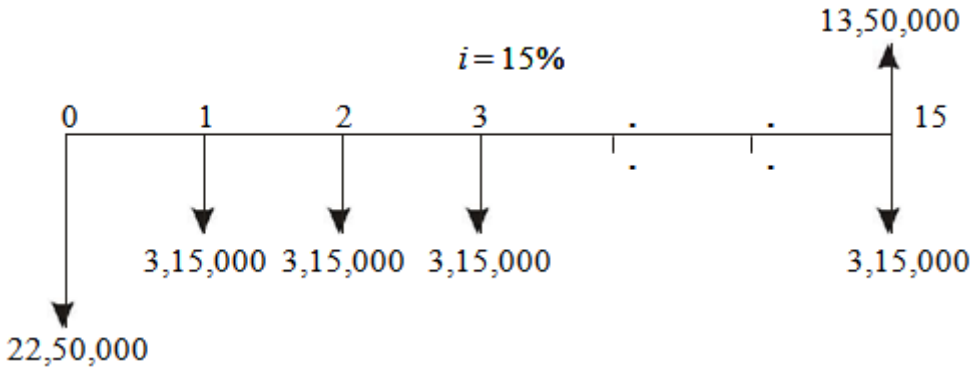


Figure 4.7 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$AE_1(15\%) = 22,50,000(A/P, 15\%, 15) + 3,15,000 – 13,50,000(A/F, 15\%, 15)$

$= 22,50,000(0.1710) + 3,15,000 – 13,50,000(0.0210)$

$= \text{Rs. } 6,71,400$

Alternative 2— Under the lake

First cost = 7,50,000 x 5 = Rs. 37,50,000

Maintenance cost/yr = 12,000 x5 = Rs. 60,000

Power loss/yr = 15,000 x 5 = Rs. 75,000

Maintenance cost and power loss/yr = Rs. 60,000 + Rs. 75,000 = Rs. 1,35,000

Salvage value = 1,50,000 x 5 = Rs. 7,50,000

The cash flow diagram for this alternative is shown in Fig 4.8

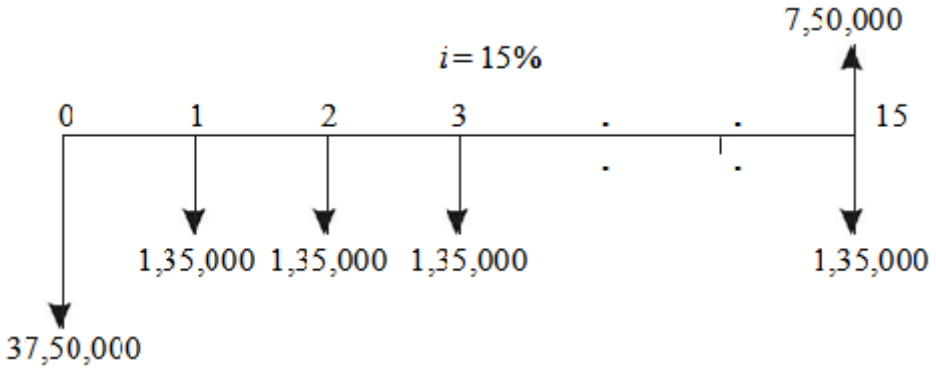


Figure 4.8 Cash flow diagram for alternative 2

The annual equivalent cost expression of the above cash flow diagram is

$AE_2(15\%) = 37,50,000(A/P, 15\%, 15) + 1,35,000 – 7,50,000(A/F, 15\%, 15)$

$= 37,50,000(0.1710) + 1,35,000 – 7,50,000(0.0210)$

$= \text{Rs. } 7,60,500$

The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

Problem 5: Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be "on the road". The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.

(a) Purchase for cash at Rs. 3,90,000.

(b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company.

Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1, 20,000 at the end of the third year, which option should he use to obtain it?

Alternative 1—Purchase car for cash

Purchase price of the car = Rs. 3,90,000

Life = 3 years = 36 months

Salvage value after 3 years = Rs. 1, 20,000

Interest rate =12% (nominal rate, compunded annually

= 1% compounded monthly

The cash flow diagram for alternative 1 is shown in figure 4.9

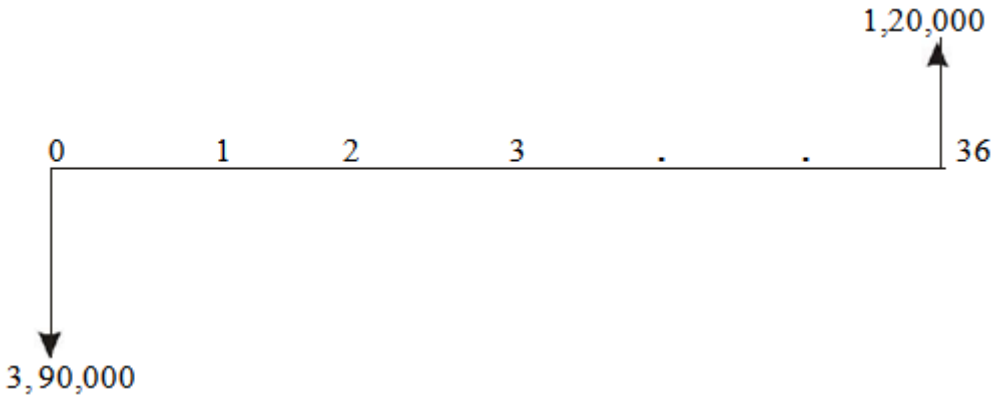


Figure 4.9 cash flow diagram for alternative 1

The monthly equivalent cost expression [*ME*(1%)] of the above cash flow diagram is

ME(1%) = 3,90,000(*A*/*P*, 1%, 36) – 1,20,000(*A*/*F*, 1%, 36)

= 3,90,000(0.0332) – 1,20,000(0.0232)

= Rs. 10,164

Alternative 2—Use of car under lease

Monthly lease amount for 36 months = Rs. 10,500

The cash flow diagram for alternative 2 is illustrated in Figure 4.10

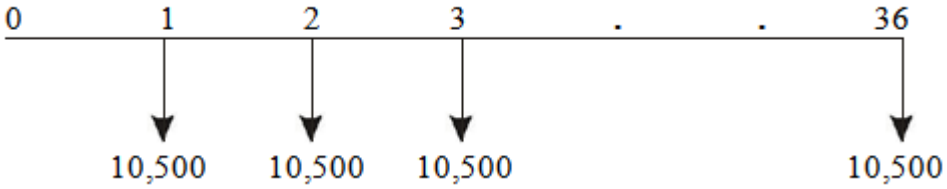


Figure 4.10 cash flow diagram for alternative 2

Monthly equivalent cost = Rs.10,500. The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

Problem 6: A company wants to buy a machine, the details are given in the following table.

	Machine A	*Machine B*
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	

At 15% interest rate, which machine should be purchased?

Solution- Machine A

Initial cost = Rs. 3,00,000 Useful life (years) = 4

Salvage value at the end of machine life = Rs. 2,00,000 Annual maintenance = Rs. 30,000

Interest rate = 15%, compounded annually

The cash flow diagram of machine A is shown in figure 4.11

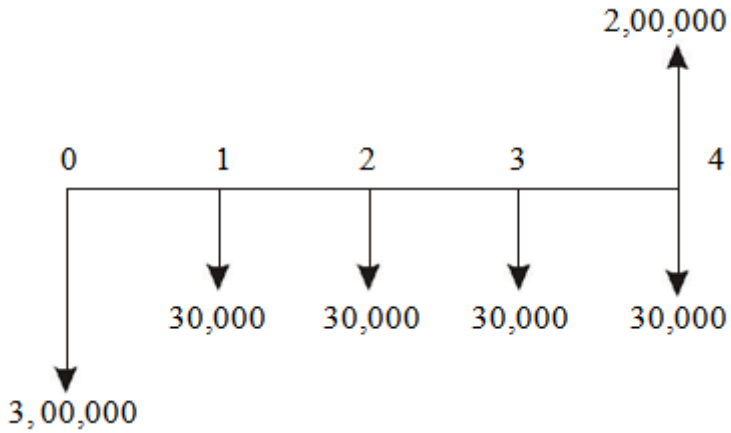


Figure 4.11 cash flow diagram of machine A

The annual equivalent cost expression of the above cash flow diagram is

$AE(15\%) = 3{,}00{,}000(A/P, 15\%, 4) + 30{,}000 - 2{,}00{,}000(A/F, 15\%, 4)$

$= 3{,}00{,}000(0.3503) + 30{,}000 - 2{,}00{,}000(0.2003)$

= Rs. 95,030

Machine B

Initial cost = Rs. 6,00,000

Useful life (years) = 4

Salvage value at the end of machine life = Rs. 3,00,000

Annual maintenance = Rs. 0.

Interest rate = 15%, compounded annually

The cash flow diagram of machine B is shown in figure 4.12

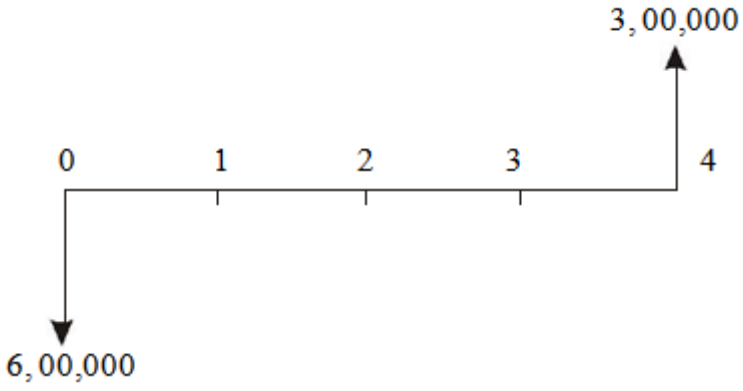


Figure 4.12 cash flow diagram of machine B

The annual equivalent cost expression of the above cash flow diagram is

$AE(15\%) = 6{,}00{,}000(A/P, 15\%, 4) - 3{,}00{,}000(A/F, 15\%, 4)$

$= 6{,}00{,}000(0.3503) - 3{,}00{,}000(0.2003)$

= Rs. 1,50,090

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

Problem 7: Jothi Lakshimi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend, has offered Jothi Lakshimi two alternative ways to pay for the equipment.

(a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.

(b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made.

If Jothi Lakshimi believes 12% is a suitable interest rate, which alternative is the best for her?

Solution:

Alternative 1

Down payment = Rs. 60,000

Payment after one year = Rs. 15,000

The cash flow diagram for alternative 1 is shown in Figure 4.13

Figure 4.13 cash flow diagram for alternative 1

The present worth equation of the above cash flow diagram is

$$PW(12\%) = 60{,}000 + 15{,}000(P/F, 12\%, 1)\ 60{,}000 + 15{,}000(0.8929)$$

$$= 73{,}393.50$$

The above present worth is represented in Figure 4.14

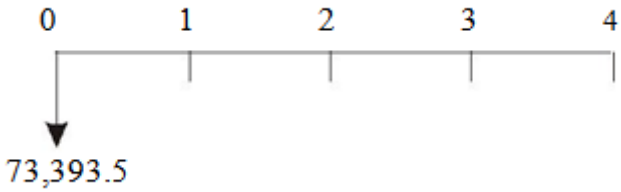


Figure 4.14 Resultant cash flow diagram for alternative 1

The annual equivalent expression of the above cash flow diagram is

$$AE(12\%) = 73{,}393.5(A/P, 12\%, 4)$$

= 73,393.5(0.3292)

= Rs. 24,161.14

Alternative 2

Payment after four years = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Figure 4.15

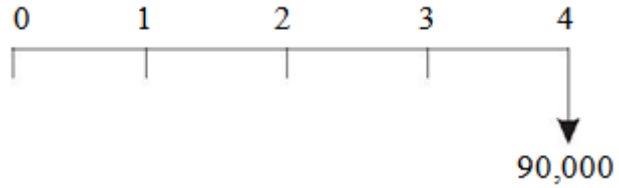


Figure 4.15 cash flow diagram for alternative 2

The annual equivalent cost expression of the above cash flow diagram is

$AE(12\%) = 90{,}000(A/F, 12\%, 4)$

= 90,000(0.2092)

= Rs. 18,828

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshimi should select alternative 2 for purchasing the home equipment.

Problem 8: A transport company has been looking for a new tyre for its truck and has located the following alternatives

Brand	Tyre warranty (months)	Price per tyre (Rs.)
A	12	1,200
B	24	1,800
C	36	2,100
D	48	2,700

If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

Solution

In all the cases, the interest rate is 12%. This is equivalent to 1% per month.

Brand A

Tyre warranty = 12 months

Price/tyre = Rs. 1,200

The cash flow diagram for brand A is shown in Figure 4.16

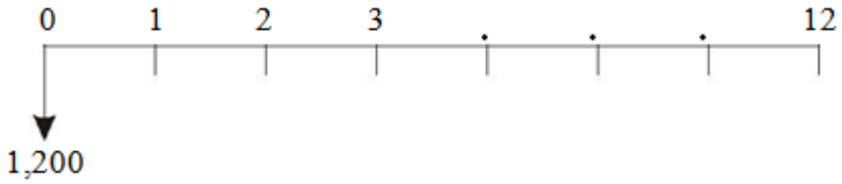


Figure 4.16 cash flow diagram for brand A

The annual equivalent cost expression of the above cash flow diagram is

$AE(1\%) = 1{,}200(A/P, 1\%, 12)$

$= 1{,}200(0.0888)$

= Rs. 106.56

Brand B

Tyre warranty = 24 months Price/tyre = Rs. 1,800

The cash flow diagram for brand B is shown in Figure 4.17

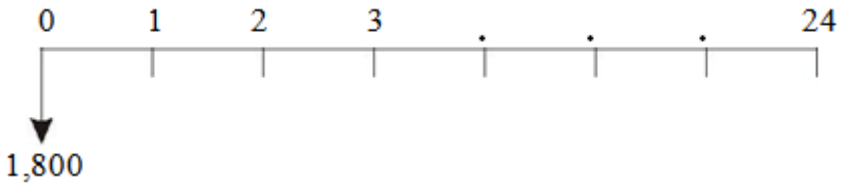


Figure 4.17 cash flow diagram for brand B

The annual equivalent cost expression of the above cash flow diagram is

$AE(1\%) = 1{,}800(A/P, 1\%, 24)$

$= 1{,}800(0.0471)$

= Rs. 84.78

Brand C

Tyre warranty = 36 months Price/tyre = Rs. 2,100

The cash flow diagram for brand C is shown in Figure 4..18

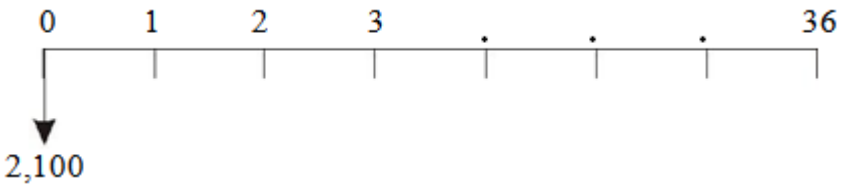


Figure 4.18 cash flow diagram for brand C

The annual equivalent expression of the above cash flow diagram is

$AE(1\%) = 2{,}100(A/P, 1\%, 36)$

$= 2{,}100(0.0332)$

= Rs. 69.72

Brand D

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Figure 4.19

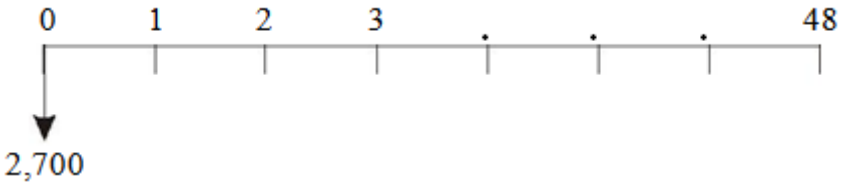


Figure 4.19 cash flow diagram for brand D

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 2{,}700(A/P, 1\%, 48)$$
$$= 2{,}700(0.0263)$$
$$= \text{Rs. } 71.01$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.