Thursday
24-03-2022

ENGINEERING ECONOMICS

※※※ PRINCIPLES OF ENGINEERING ECONOMY

(i) The choice (decision) is among alternatives. The feasible alternatives need to be identified and then defined for subsequent analysis.

(ii) Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision.

(iii) The prospective outcomes of the feasible alternatives, economic and other, should be consistently developed from a defined view point (perspective).

(iv) Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of the feasible alternatives.

(v) Selection of a preferred alternative (decision making) requires the use of a criterion (or criteria)

(vi) Uncertainity is inheret in projecting (or estimating) the future outcomes of the feasible alternatives and should be recognized in their analysis and comparison.

(vii) Improved decision making results from an adaptive process; to the extent practicable, initial projected outcomes of the selected alternative and actual results achieved should be subsequently compared.

PROBLEM SOLVING & DECISION MAKING

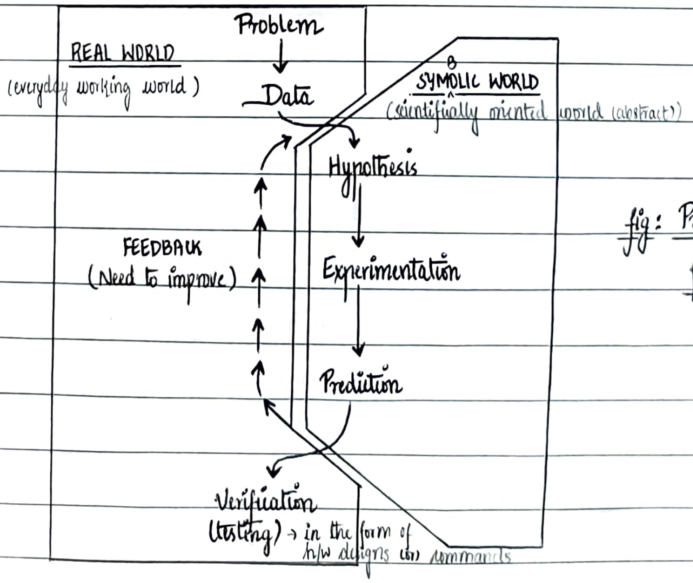

fig: Problem solving process

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* Decision made now is based on data from past performances and establishes a course of action that will result in some future outcomes.

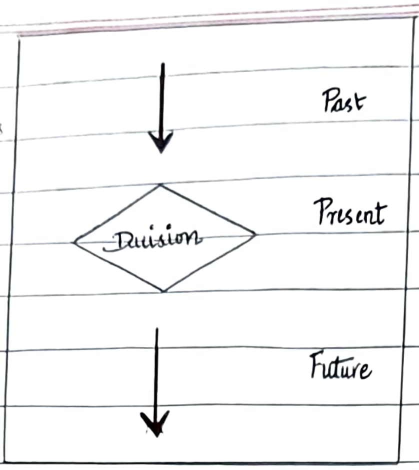

fig: Decision-making process

INTUITION AND ANALYSIS

BASIS FOR COMPARISON	INTUITION	ANALYSIS
Decision	Quick decision based on immediate perception of mind.	Decision based on rationality and reasoning.
Based	Experience and imagination based	Data based.
Dominate	Thumb rules and chances dominate.	Formulas, tables and graphs dominate.
Procedures	Undefined procedures.	Standard operating (SOP) procedures.
Used (in)	Used more in start-up companies.	Used more in long-standing companies.
Time	Time and energy saving.	Time and energy consuming.
Used (for)	Used for less significant problems.	Used for most significant problems

TACTICS AND STRATEGY

BASIS FOR COMPARISON	TACTICS (skill of dealing (or) handling difficult situations)	STRATEGY
Meaning	A carefully planned action made to achieve a specific objective is called "tactics."	A long range blue print of an organization's expected image and destination is known as "strategy."
Concept	Determining how the strategy is to be executed.	An organized set of activities that can lead the company to differentiation.

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Nature	Preventive in nature	Competitive in nature
What is it ?	Action	Action plan
Focus on	Task	Purpose
Formulated at	Middle - level (department heads)	Top - level (BOD, CEO, senior executive)
Approach	Reactive	Proactive
Flexibility	High	Comparatively less
Orientation	Towards the present conditions	Future oriented
Risk involved	Low	High

25-03-22

SENSITIVITY AND SUB - OPTIMIZATION
(minute analysis)

* A sensitivity analysis can be conducted on any problem to explore the effects of deviations from the original problem conditions.
* Sub - optimization occurs when there is a larger problem than the analyst had visualized. (large problem → sub-problems → analysed → solution)
* 3 regularly encumbered perspectives that lead to sub-optimization are :
  (i) the cross-eyed view.
  (ii) the short - sighted view.
  (iii) turn vision view point.

LAW OF SUPPLY AND DEMAND

* The demand and supply of a product are interdependent and they are sensitive w.r.t. price of product.
* A decrease in the price of a product, increases the demand for the product and decreases its supply.
* EQUILIBRIUM POINT - The point of intersection of the supply curve and the demand curve is known as the "equilibrium point."
(the quantity of supply = quantity of demand)

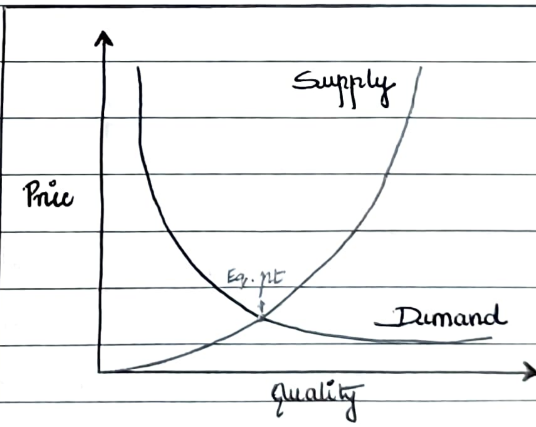

fig: Demand & Supply Curve

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(*) The shape of the demand curve is influenced by the following factors:
(i) income of the people
(ii) prices of related goods
(iii) tastes of customers (customer priorities)

(*) The shape of the supply curve is affected by the following factors:
(i) costs of the input
(ii) technology
(iii) weather
(iv) prices of related goods

INTEREST AND INTEREST FACTORS

Interest is nothing but the rent to be paid for the money borrowed. The period may be a week, monthly, quaterly, bi-annual or yearly.

Rate of Interest
- Simple Interest
- Compound Interest
  - Nominal interest rate
  - Effective interest rate
  - Continuous compounding

SIMPLE INTEREST

* Simple Interest, $\boxed{I = P \times N \times i}$

where $I \rightarrow$ interest earned for particular time periods
$P \rightarrow$ principal amount lent or borrowed
$N \rightarrow$ no. of interest periods (Ex: years)
$i \rightarrow$ interest rate per interest period

* If P is a fixed value, the annual interest charged is constant. Therefore the total amount / final amount / future sum of amount to be paid is,

$$F = P + I = P + (P \times i \times N)$$

$$\Rightarrow \boxed{F = P(1 + iN)}$$

Q: The rental cost of money is a loan of ₹ 1000 for 2 months at 10%. Use simple interest to calculate.

→ Given : $P = ₹ 1000$, $N = 2 \text{ months} = 2/12 \text{ years}$, $i = 10\% = 0.1$

$I = P \times N \times i$

$I = 1000 \times \frac{2}{12} \times 0.1$

$I = 16.667$

$F = P + I$

$F = 1000 + 16.667$

$\boxed{F = ₹.\ 1016.667}$

(*) Exact simple interest - to calculate exact simple interest, choose any 2 months and then solve the given problem (by taking days into consideration).

Ex: January & February (non-leap year) $\Rightarrow$ $F = P(1 + iN)$

$F = 1000\left[1 + 0.1\left(\frac{31 + 28}{365}\right)\right]$

$\boxed{F = ₹.\ 1016.00}$

* When F is a future sum of money to be paid when N is not a full year. There are two ways to calculate the simple interest earned during the period of the loan,

(i) When ordinary SI is used, the year is divided into twelve 30-days periods (or) a year is considered.

(ii) In exact SI, a year has precisely the calendar number of days and N is the fraction of number of days the loan is in effect that year.

Q: The rental cost of money is a loan of ₹. 2000 for 2 months at 10%.

→ (i) Ordinary SI : $F = P(1 + iN) = 2000\left(1 + 0.1\left(\frac{2}{12}\right)\right) = ₹.\ 2034$

(ii) Exact SI : $F = P(1 + iN) = 2000\left[1 + 0.1\left(\frac{31 + 28}{365}\right)\right] = ₹.\ 2032$

Q: A loan of ₹. 200 is made for a period of 13 months from January 1 to January 31 the following year at a simple interest rate of 10%, what future amount is due at the end of the loan period.

→ Given : $P = 200$, $i = 0.1$, $N = 13$ months

(i) Ordinary SI : $F = P(1 + iN)$

$$F = 200\left[1 + 0.1\left(\frac{13}{12}\right)\right] = 200 \times 1.108$$

$$\boxed{F = ₹.\ 221.6}$$

(ii) Exact SI : $F = P(1 + iN)$

$$F = 200\left[1 + 0.1\left(\frac{365 + 31}{365}\right)\right] = 200 \times 1.109$$

non-leap year

$$\boxed{F = ₹.\ 221.8}$$

Q: What sum must be loaned at 8% simple interest to earn ₹. 350 in 4 years.

→ Given : $I = ₹.\ 350$, $i = 8\% = 0.08$, $N = 4$ years , $P = ?$

$$I = P \times i \times N \Rightarrow P = \frac{I}{i \times N} = \frac{350}{0.08 \times 4}$$

$$\Rightarrow \boxed{P = ₹.\ 1093.75}$$

29-03-22

Q: How long will it take ₹. 800 to yield ₹. 72 in simple interest at 4%.

→ Given : $P = ₹.\ 800$, $I = ₹.\ 72$, $i = 0.04$, $N = ?$

$$I = P \times i \times N \Rightarrow N = \frac{I}{P \times i} = \frac{72}{800 \times 0.04}$$

$$\Rightarrow \boxed{N = 2.25 \text{ years}} = 2\tfrac{1}{4} \text{ years}$$

Q: At what rate will 65.07 yield ₹. 8.75 in simple interest in 3 years 6 months ?

→ Given : $P = ₹.\ 65.07$, $I = ₹.\ 8.75$, $N = 3 + \frac{1}{2} = 3.5$ years , $i = ?$

$$I = P \times i \times N \Rightarrow i = \frac{I}{P \times N} = \frac{8.75}{65.07 \times 3.5}$$

$$\Rightarrow i = 0.038$$

$$\Rightarrow \boxed{i = 3.8\ \%}$$

Q: How long will it take any sum to triple itself at 5% simple interest rate

→ Given : $i = 5\%$, $P = ₹.\ 100$ (assume) , $I = ₹.\ (3 \times 100) = 300$, $N = ?$

$$N = \frac{I}{P \times i} = \frac{300}{\cancel{100} \times 5/\cancel{100}} \Rightarrow \boxed{N = 60 \text{ years}}$$

(*) The effective interest rate is given by, $\boxed{i_{eff} = \left(1 + \frac{r}{N}\right)^N - 1}$

Q: Determine the effective interest rate for a nominal annual rate 6% that is compounded : (i) Semi-annually , (ii) Quaterly , (iii) Monthly , (iv) Daily

→ Given : $r = 6\% = 0.06$

(i) Semi-annually	(ii) Quaterly	(iii) Monthly	(iv) Daily
$N = 2$	$N = 4$	$N = 12$	$N = 365$
$i_{eff} = \left(1+\frac{r}{N}\right)^N - 1$	$i_{eff} = \left(1+\frac{r}{N}\right)^N - 1$	$i_{eff} = \left(1+\frac{r}{N}\right)^N - 1$	$i_{eff} = \left(1+\frac{r}{N}\right)^N - 1$
$i_{eff} = \left(1+\frac{0.06}{2}\right)^2 - 1$	$i_{eff} = \left(1+\frac{0.06}{4}\right)^4 - 1$	$i_{eff} = \left(1+\frac{0.06}{12}\right)^{12} - 1$	$i_{eff} = \left(1+\frac{0.06}{365}\right)^{365} - 1$
$i_{eff} = 0.061$	$i_{eff} = 0.061$	$i_{eff} = 0.062$	$i_{eff} = 0.062$
$\boxed{i_{eff} = 6.09\ \%}$	$\boxed{i_{eff} = 6.136\ \%}$	$\boxed{i_{eff} = 6.168\ \%}$	$\boxed{i_{eff} = 6.183\ \%}$

Q: A personal loan of ₹. 1000 is made for a period of 18 months at an interest rate of $1\frac{1}{2}\%$ per month on the unpaid balance. If the entire amount owed is repaid in a lump sum at the end of that time, determine the effective annual interest rate.

Agar effective intrest nikal rahe ho toh given rate ko in terms of annual intrest lena hoga

Tuesday
29-03-22

→ Given : $P = ₹\ 1000$, $N = 18$ months , $r = 1\frac{1}{2}\% \times 12 = 1.5\% \times 12 = 18\%$

$i_{eff} = ?$

$$i_{eff} = \left(1 + \frac{r}{N}\right)^N - 1 = \left(1 + \frac{0.18}{18}\right)^{18} - 1$$

$$i_{eff} = 0.196$$

$$\boxed{i_{eff} = 19.615\ \%}$$

* Formula for compound amount is given by , $\boxed{F = P(1+i)^N}$

Q: Find the compound amount of ₹ 100 for 4 years at 6% compounded annually.

→ Given : $P = ₹\ 100$, $N = 4$ years , $i = 6\%$, $F = ?$

$$F = P(1+i)^N = 100(1 + 0.06)^4$$

$$F = 100 \times 1.262$$

$$\boxed{F = 126.248 \text{ Rs.}}$$

01-04-22

Q: A loan of ₹ 2000 , if the interest rate is 10% per year. If interest had not been paid each year but, had been allowed to compound, how much interest would be due to the lender as a lump sum at the end of 6 years ?

→ Given : $P = ₹\ 2000$, $i = 10\%$, $N = 6$ years , $I = ?$

$$F = P(1+i)^N \qquad I = F - P$$

$$F = 2000(1.1)^6 \qquad I = 3543.122 - 2000$$

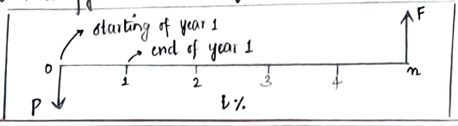
$$F = ₹\ 3543.122 \qquad \boxed{I = 1543.122 \text{ Rs.}}$$

Q: Accumulate a principle of ₹ 1000 for 5 years 9 months at a nominal rate of 12% compounded monthly. How much interest is earned ?

→ Given : $P = ₹\ 1000$, $N = (60 + 9) = 69$ months , $i = (12\%)/12 = 1\%$

$F = ?$

* Cash flow (time) diagrams is strongly recommended for situations in which the analyst needs to clarify (or) visualize what is involved when flow of money occurs at various times.



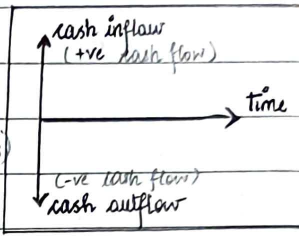
$$F = P(1+i)^N = 1000(1+0.01)^{69}$$

$$F = 1000 \times 1.987$$

$$\boxed{F = ₹\ 1986.894}$$

CASH FLOW DIAGRAMS

Cash flow is the difference between total cash inflow and cash outflow for a specified period of time. (cash inflow – receipts, cash outflow – expenditures)



NOTATIONS

Notations used for Compound Interest calculations :

i = effective interest rate per interest period
N = no. of compound periods.
P = present sum of money.
F = future sum of money.
A = end of period cash flows in a uniform series continuing for a specified number of periods, starting at the end of the first period.

INTEREST FORMULAS (for discrete compounding and cash flows)

FOR SINGLE CASH FLOWS

(i) Single Payment Compound Amount
(ii) Single Payment Present Worth

FOR UNIFORM SERIES (annuities)

(i) Uniform Series Compound Amount
(ii) Uniform Series Present Worth
(iii) Equal Payment Series Sinking Fund
(iv) Equal Payment Series Annual Equivalent Amount
(v) Arithmetic Gradient Conversion Factor (to uniform series)

TO FIND	GIVEN	FACTORS WHICH TO MULTIPLY "GIVEN"	FACTOR NAME	FACTOR FUNCTIONAL SYMBOL
For single cash flows				
F	P	$(1+i)^N$	Single Payment Compound Amount	$(F/P, i\%N)$
P	F	$\frac{1}{(1+i)^N}$	Single Payment Present Worth	$(P/F, i\%N)$
For uniform series (annuities)				
F	A	$\frac{(1+i)^N - 1}{i}$	Uniform Series Compound Amount	$(F/A, i\%N)$
P	A	$\frac{(1+i)^N - 1}{i(1+i)^N}$	Uniform Series Present Worth	$(P/A, i\%N)$
A	F	$\frac{1}{(1+i)^N - 1}$	Equal Payment Series Sinking Fund	$(A/F, i\%N)$
A	P	$\frac{i(1+i)^N}{(1+i)^N - 1}$	Annual Equivalent Amount (capital recovery)	$(A/P, i\%N)$
A	G	$\left[\frac{1}{i} - \frac{N}{(1+i)^{N-1}}\right]$	Arithmetic Gradient Conversion Factor	$(A/G, i\%N)$

Table 1 : Discrete Compounding Interest Factors and Symbols

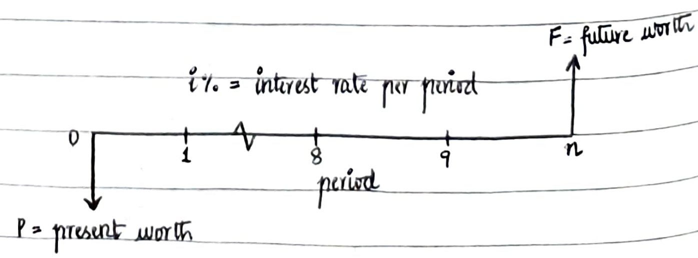
1. Problems on Compound Amount Factor (single payment)

Given : P   To find : F   Symbol : $(F/P, i\%N)$

Formula : $P(F/P, i\%N) = F$

$\boxed{P(1+i)^N = F}$

Cash flow diagram :



Friday
01-04-22

Q: A future amount F, is equivalent to ₹.1500. Now when 8 years separates the amounts and the annual interest is 12%. What is the value of F?

→ Given: $P = ₹.1500$, $N = 8$ years, $i = 12\% = 0.12$, $F = ?$

$F = P(F/P, i\%, N)$

$$F = P(1+i)^N$$

$$F = 1500(1+0.12)^8 = 1500 \times 2.476$$

$$\boxed{F = ₹.\ 3713.945}$$

Q: A person deposits a sum of ₹. 10,000/- in a bank at a nominal rate of interest of 12% for 10 years. Find the maturity amount of the deposit after 10 years, if the compounding is done quaterly.

→ Given: $P = ₹.10,000$, $i_{(r)} = 12\%$, $N = 10$ years, $F = ?$

$F = P(F/P, i\%, N)$

$$i_{eff} = \left(1 + \frac{r}{N}\right)^N - 1$$

(here, N = 4 (no. of interest periods year))

$$i_{eff} = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$i_{eff} = 12.551\%$$ //

$$F = P(1+i_{eff})^N$$

$$F = 10,000(1+0.126)^{10}$$

$$\boxed{F = ₹.\ 32,620.378}$$

Q: Suppose you borrow ₹.8,000 now, with the promise to repay the loan principal plus accumulated interest in 4 years at $i = 10\%$ per year. How much would you owe at the end of 4 years?

→ Given: $P = ₹.8000$, $i = 10\% = 0.1$, $N = 4$ years, $F = ?$

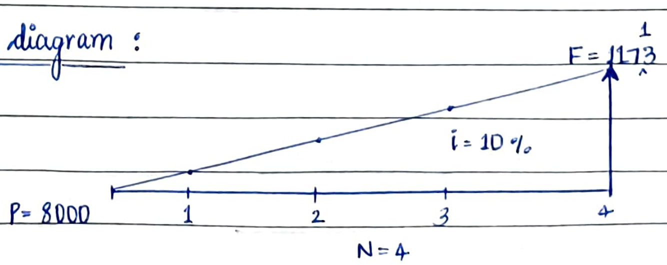
$$F = P(F/A, i\%, N)$$

$$F = P(1+i)^N$$

$$F = 8000(1+0.1)^4 = 8000 \times 1.464$$

$$\boxed{F = 11712.8 \approx ₹.\ 11713/-}$$

Cash flow diagram:



Tuesday
19-04-22

Q: Suppose that ₹ 10,000 is borrowed now at 15% interest per annum. A partial repayment of ₹ 3000 is made 4 years from now. The amount that will be remain to be paid then is most nearly ________.

→ Given : $P = ₹\ 10000$, $i = 15\% = 0.15$, $N = 4$

$F = P(1+i)^N$

$F = 10000(1+0.15)^4$

$F = 17490.063$ Rs.

The amount to be paid is,

F - partial repaid amt

$\Rightarrow 17490.063 - 3000 = \boxed{₹\ 14490}$

Problems on

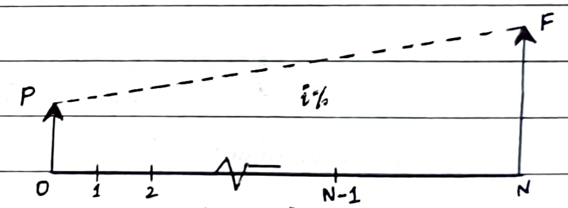
2. Present Worth Factor (single payment)

Given : F   To find : P   Symbol : $(P/F, i\%, N)$

Formula : $P = F(P/F, i\%, N)$

$$\boxed{P = F\left[\frac{1}{(1+i)^N}\right]}$$

Cash-flow diagram :



End of time period

Q: An investor has an option to purchase a tract of land that will be worth ₹ 10,000 in 6 years. If the value of the land increases at 8% each year, how much should the investor be willing to pay now for this property.

→ Given : $F = ₹\ 10,000$, $N = 6$ years , $i = 8\% = 0.08$, $P = ?$

$P = F(P/F, i\%, N)$

$P = \dfrac{F}{(1+i)^N}$

$P = \dfrac{10000}{(1+0.08)^6} = \dfrac{10000}{1.587}$

$\boxed{P = 6301.696 \text{ Rs.}}$

Tuesday
19-04-22

Q: A person wishes to have a future sum of ₹ 10 lakhs for his daughter's engineering education in 15 years from now, what is the single payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 12% rate of interest compounded annually.

→ Given : F = ₹. 10,00,000 , N = 10 years , $i = 12\% = 0.12$, P = ?

$$P = F\,(P/F\ ,\ i\%\,N)$$

$$P = \frac{F}{(1+i)^N} = \frac{1000000}{(1+0.12)^{10}} = \frac{1000000}{3.106}$$

$$\boxed{P = ₹.\ 3,21,973.237}$$

Q: If the same person wishes to have choice of investing in a private bank which pays a rate of interest of 11% but compounded quaterly, should he go for it? His desire to receive 10,00,000 , 10 years from now.

→ Given : F = ₹. 10,00,000 , N = 10 years , $i = 11\% = \frac{0.11}{4} = 0.028$

$N = 10 \times 4$

$N = 40$

$$P = \frac{F}{(1+i)^N} = \frac{10,00000}{(1+0.028)^{40}}$$

$$P = \frac{10,00,000}{2.960}$$

$$\boxed{P = ₹.\ 3,37,852.221}$$

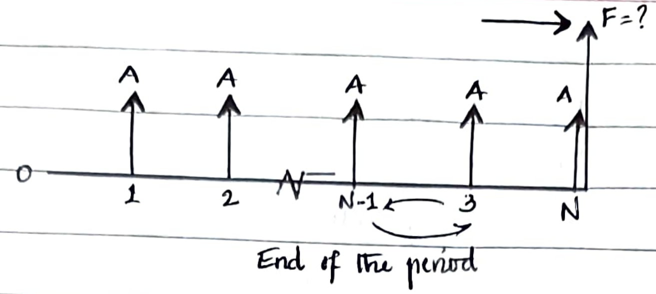
3. Problems on Series Compound - Amount Factor (uniform series)

Given : A   To find : F   Symbol : $(F/A\ ,\ i\%\,N)$

Formula : $F = A\,(F/A\ ,\ i\%\,N)$

$$\boxed{F = A\left[\frac{(1+i)^N - 1}{i}\right]}$$

Cash flow diagram :



Q: A 45 year old person is planning for his retired life. He plans to direct ₹. 30,000/- from his bonus as investment every year for the next 15 years. The banks gives 10 % interest rate compaunded annually. Find the maturity value of his account when he is 60 years old.

→ Given : A = ₹. 30,000 , N = 15 years , i = 10 % = 0.1 , F = ?

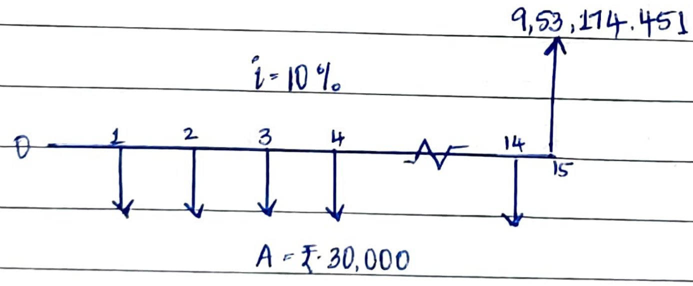
$$F = A\,(A/F,\ i\%\,N)$$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$F = 30000\left[\frac{(1.1)^{15} - 1}{0.1}\right]$$

$$\boxed{F = 9,53,174.451 \text{ Rs.}}$$

Cash-flow diagram :



20-04-22

Q: A woman desires to have ₹. 1,00,000 in her retirement savings plan after working for 25 years. She will accomplish this by depositing ₹. A each year in a savings account that earns 6% per year. How much must she save each year?

→ Given : F = ₹. 1,00,000 , N = 25 years , i = 6 % = 0.06 , A = ?

$$A = F\,(A/F,\ i\%\,N)$$

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

$$A = \frac{1{,}00{,}000 \times i}{(1+0.06)^{25} - 1} = \frac{1 \text{ lakh} \times i}{3.292}$$

$$\boxed{A = ₹.\ 1822.672}$$

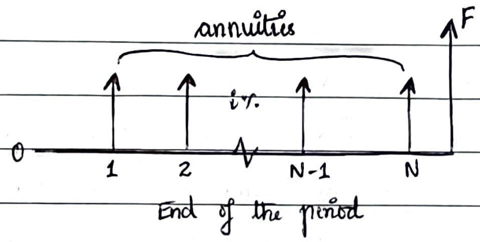
4. Problems on Sinking Fund Factor (uniform series)

Given : F   To find : A   Symbol : $(A/F, i\%, N)$

Formula : $A = F(A/F, i\%, N)$

$$\boxed{A = F\left[\frac{i}{(1+i)^N - 1}\right]}$$

Cash-flow diagram :



Q: A person estimates an expenditure of ₹ 5 lakh for his daughter's wedding about 10 years from now. He plans to deposit an equal amount at the end of every year for the next 10 years at a rate of interest of 10% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 8 years.

→ Given: F = ₹. 5 lakh, N = 10 years, i = 0.1, A = ?

$$A = F(A/F, i\%, N)$$

$$A = \frac{F \times i}{(1+i)^N - 1} = \frac{5{,}00{,}000 \times 0.1}{(1+0.1)^{10} - 1}$$

$$A = ₹.\ 31{,}372.69$$

- per year, 31,372.697 × 8 = ₹. 2,50,981.58 /-

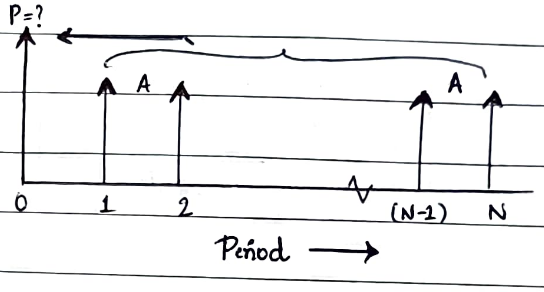
5. ## Problems on Series Present Worth Factor (uniform series)

Given : A To find : P Symbol : (P/A , i %, N)

Formula : $P = A\,(P/A,\ i\%,\ N)$

$$\boxed{P = A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]}$$

Cash flow diagram :



Q: It is estimated that a certain piece of equipment can save ₹. 6000 per year in labour and material costs. The equipment has an expected life to 5 years and no salvage value. If the company must earn a 20% rate of return on such investments, how much could be justified now for the purchase of this piece of equipment? Draw a cashflow diagram.

Given : A = ₹. 6000 , N = 5 years , i = 0.2 , P = ?

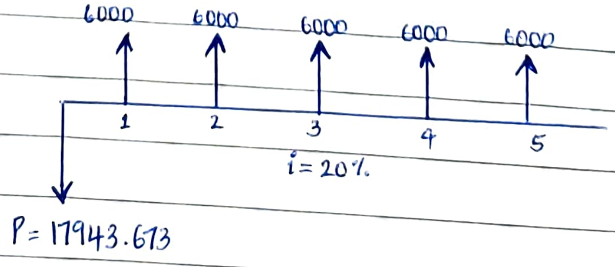
$$P = A\,(P/A,\ i\%,\ N)$$

$$P = A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

$$P = 6000\left[\frac{(1.2)^5 - 1}{0.2\,(1.2)^5}\right] = 6000 \times 2.991$$

$$\boxed{P = ₹.\ 17{,}943.673}$$

Cashflow diagram :



(↑ → saved money)

Q: Suppose that installation of low-loss thermal windows in your area is expected to save ₹ 150 a year on your home heating bill for next 18 years. If you can earn 8% a year on other investments, how much could you afford to spend now for these windows?

→ Given : ₹ 150 = A , N = 18 years , i = 0.08 , P = ?

$$P = A\,(P/A,\ i\%\ N)$$

$$P = A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

$$P = 150\left[\frac{(1+0.08)^{18} - 1}{0.08\,(1+i)^{18}}\right] = 150 \times 9.372$$

$$\boxed{P = ₹\ 1405.783}$$

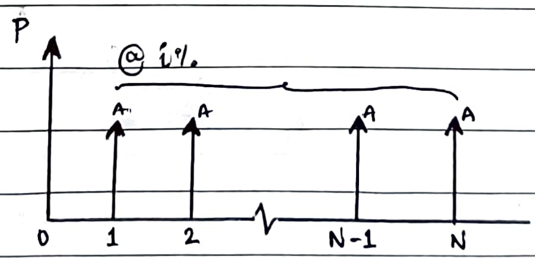
6. Problems on Capital Recovery Factor (uniform series)

Given : P   To find : A   Symbol : $(A/P,\ i\%\ N)$

Formula : $A = P\,(A/P,\ i\%\ N)$

$$\boxed{A = P\left[\frac{i\,(1+i)^N}{(1+i)^N - 1}\right]}$$

Cashflow diagram :



End of time period

Q: A proposed product modification to avoid production difficulties will require an immediate expenditure of ₹. 14,000 to modify certain dies. What annual savings must be realized to recover this expenditure in four years with interest at 10%?

→ Given : P = ₹. 14000 , N = 4 years , i = 10% = 0.1 , A = ?

$$A = P\,(A/P,\ i\%\ N)$$

$$A = P\left[\frac{i\,(i+1)^N}{(1+i)^N - 1}\right]$$

$$A = 14000\left[\frac{0.1\,(1.1)^4}{(1.1)^4 - 1}\right] = 14000 \times 0.315$$

$$\boxed{A = ₹.\ 4416.591}$$

7. Problems on Arithmetic Gradient Conversion Factor (uniform series)

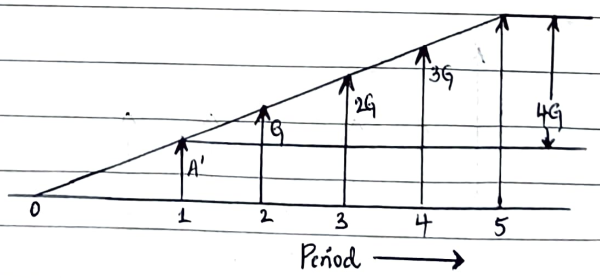
Given : G To find : A Symbol : $(A/G,\ i\%.N)$

Formula : $A = G\,(A/G,\ i\%.N)$

$$\boxed{A = A' \pm G\left[\frac{1}{i} - \frac{N}{(1+i)^N}\right]}$$

where, A' → first payment / initial payment
+ → used when there is an increase in payment
− → used when there is a decrease in payment
G → gradient increase in the cash flow receipts

Cash flow diagram :



Q: Assume that an funding was originally set up to provide a ₹ 10,000. First payment with payments decreasing by ₹.1000 each year during the 10 years funding life. What constant annual payment for 10 years would be equivalent to the original funding plan if $i = 8\%$. ?

→ Given : $A' = ₹\ 10,000$, $G = ₹.1000$, $N = 10$ years, $i = 8\% = 0.08$

$A = ?$

Wednesday
20-04-2022

$$A = G(A/G, i\%.N)$$

$$A = A' - G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = 10000 - 1000\left[\frac{1}{0.08} - \frac{10}{(1.08)^{10} - 1}\right]$$

$$A = 10000 - 1000\,(3.871)$$

$$\boxed{A = ₹.\ 6128.686}$$

21-04-22

Q: A film star is at the height of his career. He wants to invest ₹. 10 lakhs from the end of this year and follow it up with 9 lakhs, 8 lakhs and so on for the next 5 years, when his income would go on diminishing. Find the maturity amount 6 years later if a film producer agrees to pay him 15% rate of interest, compounded annually.

→ Given: $A' = 10$ lakhs, $G = 1$ lakh, $N = 6$ years, $i = 15\% = 0.15$, $A = ?$
$F = ?$

$$A = A' - G(A/G, i\%.N)$$

$$A = 1000000 - 100000\left[\frac{1}{0.15} - \frac{6}{(1.15)^6 - 1}\right]$$

$$A = 1000000 - 100000\,(2.097)$$

$$\boxed{A = ₹.\ 7{,}90{,}280.96}$$

$$F = A(F/A, i\%.N)$$

$$F = 790280.96\left[\frac{(1.15)^6 - 1}{0.15}\right]$$

$$\boxed{F = ₹.\ 69{,}17{,}912.813}$$

** Q: A company 3 years ago borrowed ₹. 40,000 to pay for a new machine tool, agreeing to repay the loan in 100 monthly payments at an annual nominal interest rate of 12% compounded monthly. The company now wants to pay off the loan. How much would this payment be, assuming no penalty costs for early payout?

→ Given: $P = ₹.\ 40{,}000$, $N = 100$, $i = 12\%/12 = 1\%$

(i) $A = P(A/P, i\%.N)$

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$A = 40000\left[\frac{0.01(1.01)^{100}}{(1.01)^{100} - 1}\right]$$

A = 634.63 Rs

(ii) $F = P(F/P, i\%.N)$

36 ⇒ ? ⇒ 3 years

$$F = P(1+i)^N$$

$$F = 40000\,(1.01)^{36}$$

$$F = ₹.\ 57{,}230.751$$

Thursday
-04-22

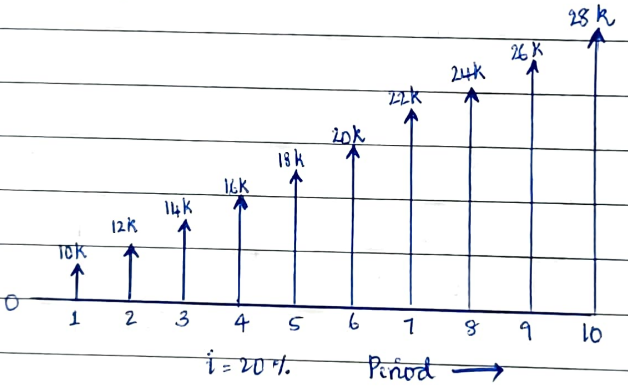
(iii) Installment payed for 3 years = $A \times 36 = 634.63 \times 36$
$= ₹\, 22,846.68$ /-

(iv) Total / Final amount = $57230.751 - 22846.68$

$\boxed{\text{Total amt.} = 34,384.071 \text{ Rs.}}$

Q: A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is ₹ 10,000 at the end of the first year and there after he wishes to deposit the same amount (Rs. 10,000) with an annual increase at ₹ 2000 for the next 9 years with an interest rate of 20%. Find the total amount at the end of the 10th year of the above series.

→ Cash flow diagram :



Given : $A' = ₹\, 10,000$, $G = ₹\, 2000$, $i = 20\% = 0.2$, $N = 10$ years

$$A = A' + G\,(A/G,\, i\%,\, N)$$

$$A = A' + G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = 10000 + 2000\left[\frac{1}{0.2} - \frac{10}{(1.2)^{10} - 1}\right]$$

$$A = 10000 + 2000\,(3.074)$$

$$\boxed{A = ₹\, 16147.724}$$

$$F = A\,(F/A,\, i\%,\, N)$$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$F = 16147.724\left[\frac{(1.2)^{10} - 1}{0.2}\right]$$

$$\boxed{F = ₹\, 4,19,173.642}$$

(salvage value - junk value)

PRESENT WORTH COMPARISON

CONDITIONS FOR PRESENT WORTH COMPARISON

(i) Cash flows are known.
(ii) Cash flows are in constant-value dollars.
(iii) The interest rate is known.
(iv) Comparisons are made with before-tax cash flows.
(v) Comparisons do not include intangible considerations.
(vi) Comparisons do not include consideration of the availability of funds to implement alternatives.

REVENUE - DOMINATED CASH FLOW DIAGRAM

(profit - dominated)

In a revenue-dominated cash flow diagram, the profit, revenue, salvage value (all inflows to an organization) will be assigned with positive sign. The costs (outflows) will be assigned with negative sign.

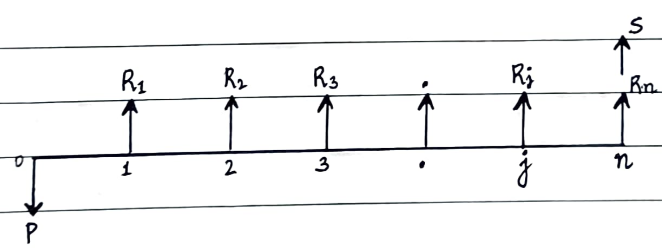


fig: Revenue dominated cash flow diagram

Here, $P \rightarrow$ initial investment
$R_j \rightarrow$ net revenue at the end of the j^{th} year.
$i \rightarrow$ interest rate, compounded annually
$S \rightarrow$ salvage value at the end of the n^{th} year.

(*) To find the present worth of the above cash flow diagram, the formula is,

$$\boxed{PW(i) = -P + R_1\left[\frac{1}{(1+i)^1}\right] + R_2\left[\frac{1}{(1+i)^2}\right] + \cdots + R_j\left[\frac{1}{(1+i)^j}\right] + R_n\left[\frac{1}{(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]}$$

In this formula, the expenditure is assigned a -ve sign and revenues are assigned a +ve sign.

COST - DOMINATED CASH FLOW DIAGRAM

In a cost dominated cash flow diagram, the costs (outflows) will be assigned with positive sign and the profit, revenue, salvage values (all inflows), etc. will be assigned with negative sign.

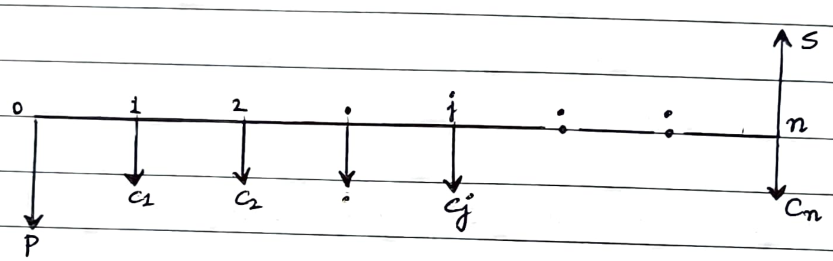


fig: Cash - dominated cash flow diagram

Here, P → an initial investment

C_j → the net cost of operation and maintenance at the end of j^{th} year.

S → the salvage value at the end of n^{th} year.

(*) To compute the present worth amount of the above cash flow diagram for a given interest rate i, the formula is,

$$\boxed{PW(i) = P + C_1\left[\frac{1}{(1+i)^1}\right] + C_2\left[\frac{1}{(1+i)^2}\right] + \dots + C_j\left[\frac{1}{(1+i)^j}\right] + C_n\left[\frac{1}{(1+i)^n}\right] \quad S\left[\frac{1}{(1+i)^n}\right]}$$

In the above formula, the expenditure is assigned a +ve value/sign and the revenue is assigned a -ve sign.

NOTE : If we have more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared. Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

Friday
22-04-22

Problems

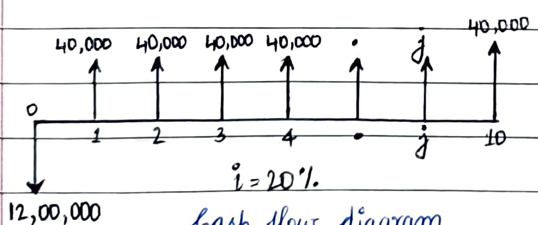
Q: Alpha Industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues w.r.t each of the technologies are given below in the table. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

(Revenue - dominated)

	INITIAL SALARY (Rs.)	ANNUAL REVENUE (Rs.)	LIFE (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

→ **TECHNOLOGY 1 :**

Given : P = ₹. 12,00,000 , A = ₹. 4 lakhs
N = 10 years , i = 20%



Cash flow diagram

$PW(20\%) = -P + A(P/A, i\%, N)$

$PW = -1200000 + 400000 \left[\frac{(i+1)^N - 1}{i(i+1)^N}\right]$

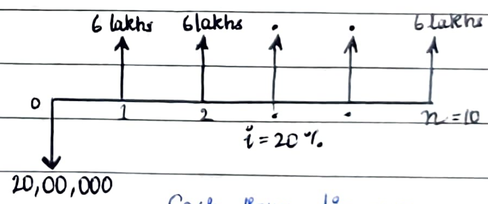
$PW = -1200000 + 400000 \left[\frac{5.192}{0.2(1.2)^{10}}\right]$

$PW = -1200000 + 400000(4.193)$

$\boxed{PW = ₹.\ 4,77,073.973}$

TECHNOLOGY 2 :

Given : P = ₹. 20 lakhs , A = ₹. 6 lakhs
N = 10 years, i = 20%



Cash flow diagram

$PW(20\%) = -P + A(P/A, i\%, N)$

$PW(20\%) = -2000000 + 600000 \left[\frac{(i+1)^N - 1}{i(i+1)^N}\right]$

$PW(20\%) = -2000000 + 600000 \left[\frac{5.192}{0.2(1.2)^{10}}\right]$

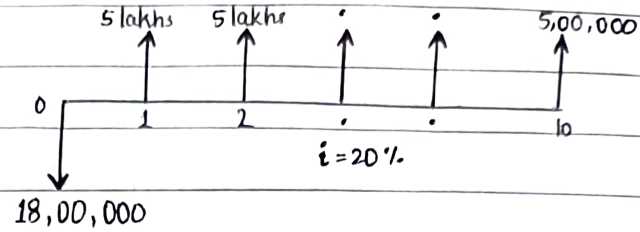
$PW(20\%) = -2000000 + 600000(4.193)$

$\boxed{PW(20\%) = ₹\ 5,15,610.959}$

TECHNOLOGY 3 :

Given : P - ₹. 18 lakhs , A - ₹. 5,00,000 , i = 20 % , N = 10 years

Cashflow diagram :



$$PW\ (20\%) = -18{,}00{,}000 + 5{,}00{,}000 \left[\frac{5.192}{0.2\ (1.2)^{10}} \right]$$

$$PW\ (20\%) = -18{,}00{,}000 + 5{,}00{,}000\ (4.193)$$

$$\boxed{PW\ (20\%) = ₹.\ 2{,}96{,}342.466}$$

∴ Technology 2 is suggested for implementation to expand the production as the present worth of technology 2 is the highest among all the technologies.

Q: An engineer has two bids for an elevator to be installed in a new building. The details of the bids for elevators are as follows :

(Cost-dominated)

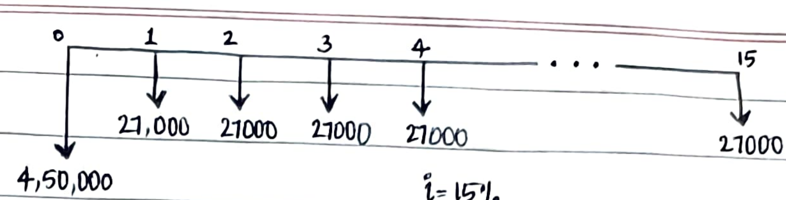
BID	ENGINEER'S ESTIMATES		
	Initial Cost (Rs.)	Annual operatn & maintenance (Rs.)	Service life (years)
Alpha Elevator Inc.	4,50,000	27, 000	15
Beta Elevator Inc.	5, 40, 000	28, 500	15

Determine which bid should be accepted based on the present worth method of comparison assuming 15 % interest rate, compounded annually.

→ BID 1 : Alpha Elevator Inc.

Given : P = ₹. 4,50,000 , A = ₹. 27,000 , N = 15 years , i = 15 %

Cashflow diagram :



$$PW\,(15\%) = P + A\,(P/A\,,\,i\%\,N)$$

$$PW\,(15\%) = P + A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

$$PW\,(15\%) = 4{,}50{,}000 + 27000\left[\frac{(1+0.15)^{15} - 1}{0.15\,(1+0.15)^{15}}\right]$$

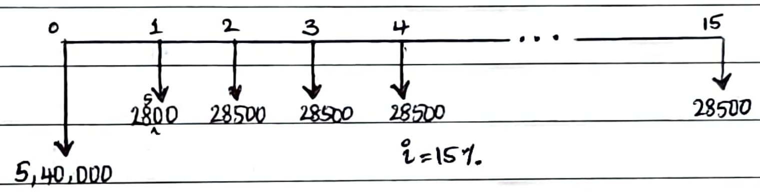
$$PW\,(15\%) = 4{,}50{,}000 + 27000(5.847)$$

$$\boxed{PW\,(15\%) = ₹\ 6{,}07{,}878.993}$$

BID 2 : Beta Elevator Inc.

Given : P = ₹ 5,40,000 , A = ₹ 28,500 , N = 15 Years , i = 15%

Cashflow diagram :



$$PW\,(15\%) = P + A\,(P/A\,,\,i\%\,N)$$

$$PW\,(15\%) = P + A\left[\frac{(1+i)^N - 1}{i\,(1+i)^N}\right]$$

$$PW\,(15\%) = 540000 + 28500\left[\frac{(1+0.15)^{15} - 1}{0.15\,(1+0.15)^{15}}\right]$$

$$PW\,(15\%) = 540000 + 28500\,(5.847)$$

$$\boxed{PW\,(15\%) = ₹\ 7{,}06{,}650.048}$$

∴ The elevator from Alpha Elevator Inc. is to be purchased and installed in the new building as the total present worth cost of bid 1 < bid 2.

26-04-22

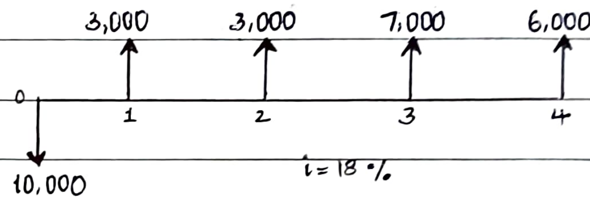
Q: The investment proposals A and B have the net cash flows as follows :

PROPOSAL	END OF YEARS				
	0	1	2	3	4
A (Rs.)	-10,000	3,000	3,000	7,000	6,000
B (Rs.)	-10,000	6,000	6,000	3,000	3,000

Compare the present worth of A with that of B at $i = 18\%$. Which proposal should be selected?

→ **A :** Given : $i = 18\%$

Cash flow diagram :



$$PW_A(18\%) = -10{,}000 + 3000(P/F, 18\%, 1) + 3000(P/F, 18\%, 2) + 7000(P/F, 18\%, 3) + 6000(P/F, 18\%, 4)$$

$$PW_A(18\%) = -10{,}000 + \frac{3000}{(1+0.18)^1} + \frac{3000}{(1+0.18)^2} + \frac{7000}{(1+0.18)^3} + \frac{6000}{(1+0.18)^4}$$

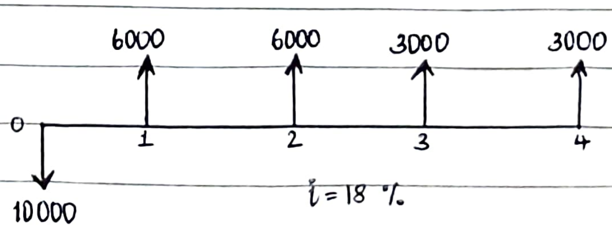
$$PW_A(18\%) = -10{,}000 + \frac{3000}{1.18} + \frac{3000}{1.392} + \frac{7000}{1.643} + \frac{6000}{1.939}$$

$$PW_A(18\%) = -10{,}000 + 2542.373 + 2154.553 + 4260.416 + 3094.733$$

$$\boxed{PW_A(18\%) = ₹.\ 2{,}052.072}$$

B : Given : $i = 18\%$

Cash flow diagram :



$$PW_B(18\%) = -10000 + 6000(P/F, 18\%, 1) + 3000(P/F, 18\%, 3) + 6000(P/F, 18\%, 2) + 3000(P/F, 18\%, 4)$$

Tuesday
26-04-22

$$PW_B(18\%) = -10,000 + \frac{6000}{(1+0.18)^1} + \frac{6000}{(1+0.18)^2} + \frac{3000}{(1+0.18)^3} + \frac{3000}{(1+0.18)^4}$$

$$PW_B(18\%) = -10,000 + \frac{6000}{1.18} + \frac{6000}{1.392} + \frac{3000}{1.643} + \frac{3000}{1.939}$$

$$PW_B(18\%) = -10,000 + 5084.746 + 4309.107 + 1825.893 + 1547.367$$

$$\boxed{PW_B(18\%) = ₹.\ 2,767.113}$$

$\therefore$ At $i = 18\%$, the present worth of proposal B is higher than that of proposal A. Hence, proposal B must be selected.

Q: