

# Social and Information Network Analysis

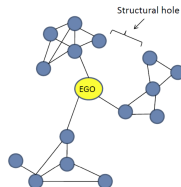
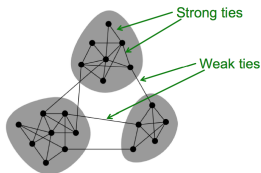
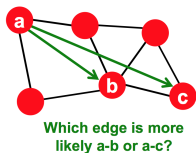
## Positive and Negative Relationships

Cornelia Caragea

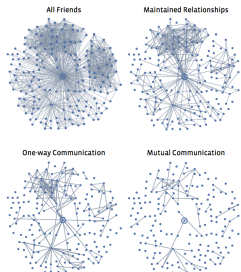
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# To Recap



- Triadic Closure
- The Strength of Weak Ties
- Tie Strength in Social Media
- Closure and Structural Holes
- Community Detection



# Positive and Negative Relationships

- Relationships in the networks thus far have **positive connotations**:
  - friendship, collaboration, sharing of information
- However, there exist also **negative (or antagonistic) relationships**:
  - controversy, disagreement, and sometimes outright conflict

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How should we reason about the mix of positive and negative relationships that take place within a network?

# Positive and Negative Relationships

- Given a network and annotated links as positive and negative
  - Goal: understand the tension between these two forces
- Underlying framework for such an analysis is the notion of structural balance

# Positive and Negative Relationships

- We focus on basic models of positive and negative relationships where the graph is complete:
  - A social network on a set of people:
    - Everyone knows everyone else
    - Edges are labeled with either  $+$  or  $-$ , where  $+$  indicates friends, and  $-$  indicates enemies
    - Examples?
    - Generally, small groups of people who all know each other, e.g., a classroom, a small company, a sports team, a fraternity or sorority, international relationships

# Positive and Negative Relationships

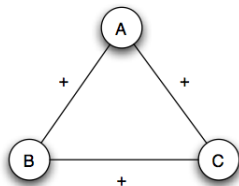
- The models can be extended to settings where
  - The graph is not complete
  - The graph structure is only “approximately balanced”
  - The dynamic aspects of the structural balance theory are captured by modeling how the set of friendships and antagonisms in a complete graph might evolve over time, as the social network implicitly seeks out structural balance.

# Structural Balance for Triangles

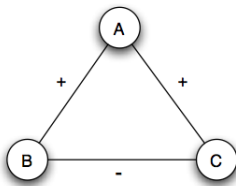
- Key idea - based on theories in social psychology:
  - Any two people in the group seen in isolation are either friends or enemies
  - For any three people in the group, certain configurations of +’s and -’s are socially and psychologically more plausible than others
  - Four distinct ways to label the three edges among three people with +’s and -’s



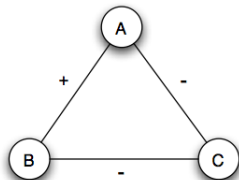
# Balanced vs. Not Balanced Triangles



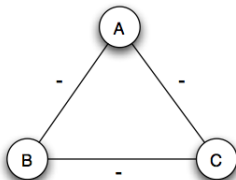
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*



(c) *A and B are friends with C as a mutual enemy: balanced.*

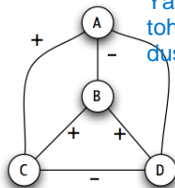
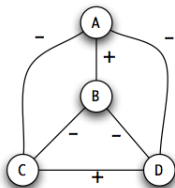


(d) *A, B, and C are mutual enemies: not balanced.*

# Structural Balance for Arbitrary Networks

- How to extend structural balanced to networks of arbitrary number of nodes (i.e.,  $N > 3$ )?
  - A labeled complete graph is balanced if every triangle in the graph is balanced

**Structural Balance Property:** For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +



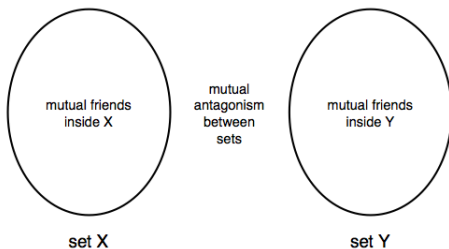
Ya toh saare dost, ya  
toh fir dushman ka  
dushman dost (-, -, +)

# Characterizing the Structure of Balanced Networks

- What can we say about the structure of a balanced network?
  - More precisely, how does a balanced network look like?
  - Note that the balanced labeled network is complete
- When is a network balanced?
  - Everyone likes each other  $\rightarrow$  all triangles have three + labels.
  - There are two groups of friends with negative relations between people in different groups

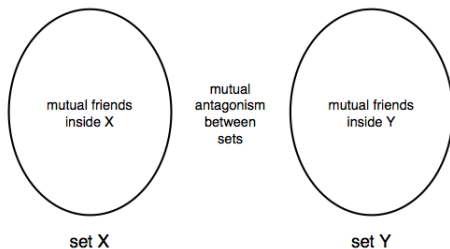
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- These are the *only* ways to have a balanced network

# The Balance Theorem

- *If a labeled complete graph is balanced, then either all pairs of nodes are friends, or else the nodes can be divided into two groups,  $X$  and  $Y$ , such that every pair of nodes in  $X$  like each other, every pair of nodes in  $Y$  like each other, and everyone in  $X$  is the enemy of everyone in  $Y$ .*

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  - **Implication:** Structural Balanced Property (a purely *local* property) implies a strong *global* property (either everyone gets along, or the world is divided into two battling factions)

# Proving the Balance Theorem

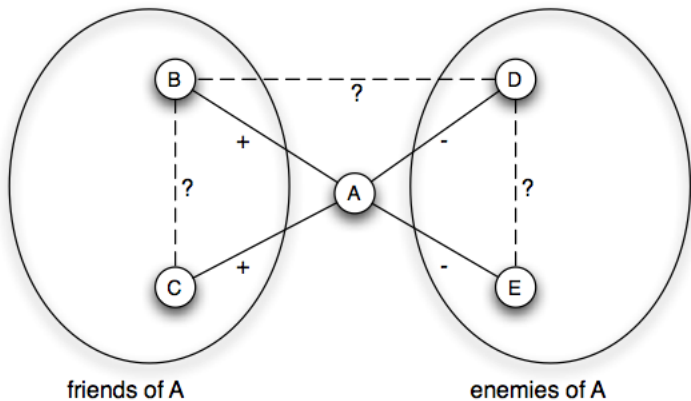
- Given a labeled balanced complete graph
- Conclude that either everyone is friends, or that there are sets  $X$  and  $Y$  as described in the claim

**Proof:**

- If no negative edges, done
- Assume there is at least one negative edge
  - Want to show that:
    - Every two nodes in  $X$  are friends
    - Every two nodes in  $Y$  are friends
    - Every node in  $X$  is an enemy of every node in  $Y$

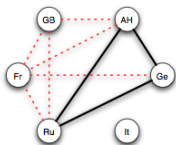


# Proving the Balance Theorem

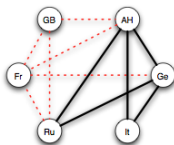


# Applications of Structural Balance - International Relations

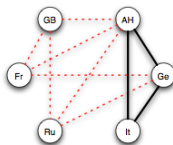
- Nodes are nations, and + and - are edge labels indicating alliances or animosity.
- Structural balance can sometimes provide an effective explanation for the behavior of nations during various international crises.



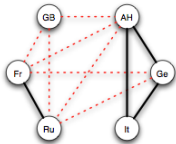
(a) *Three Emperors' League 1872-81*



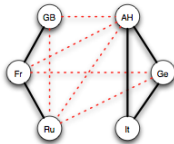
(b) *Triple Alliance 1882*



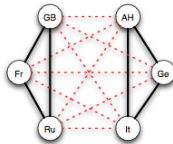
(c) *German-Russian Lapse 1890*



(d) *French-Russian Alliance 1891-94*



(e) *Entente Cordiale 1904*



(f) *British Russian Alliance 1907*

# International Relations

## Conclusion on International Relations:

- **Structural balance is not necessarily a good thing:**
  - Since its global outcome is often two implacably opposed alliances, the search for balance in a system can sometimes be seen as a slide into a hard-to-resolve opposition between two sides

# Trust, Distrust, and On-Line Ratings

- User communities on the Web where people can express positive or negative sentiments about each other:
  - The technology news site Slashdot, where users can designate each other as a "friend" or a "foe";
  - On-line product-rating sites such as Epinions, where a user can express evaluations of different products, and also express trust or distrust of other users.
    - Epinions seen as a directed graph;
    - Some patterns - easy to reason about:
    - For example, if user A trusts user B, and user B trusts user C, then it is natural to expect that A will trust C.
    - But what if A distrusts B and B distrusts C: should we expect A to trust or to distrust C?  
Appealing arguments in both directions!

# A Weaker Form of Structural Balance

To recap:

- There are two kinds of structures on a group of three people that are inherently unbalanced:
  - A triangle with 2 + edges and 1 - edge.
  - A triangle with 3 - edges.
  - Source of stress that the network might try to resolve.
  - First case stronger than the second.

Question:

- What structural properties arise when we rule out only triangles with exactly two positive edges, while allowing triangles with three negative edges to be present in the network.

# Characterizing Weakly Balanced Networks

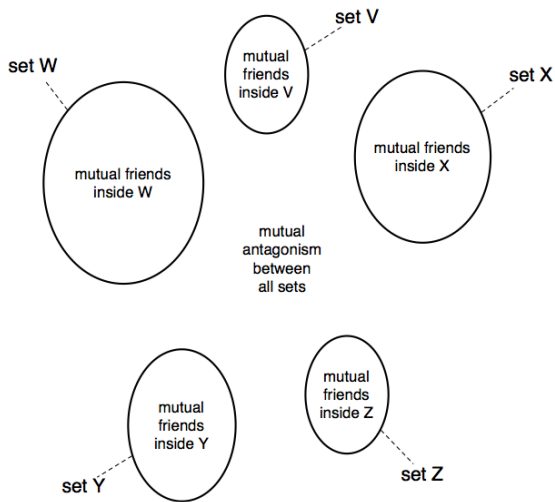
- A complete labeled graph is *weakly balanced* if the following property holds:
  - *Weak Structural Balance Property: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge.*
- How do they look like?

# Characterization of Weakly Balanced Networks

*If a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.*

- In other words,
  - Given a weakly balanced complete graph
  - Produce a division of its nodes into groups of mutual friends, such that all relations between nodes in different groups are negative.

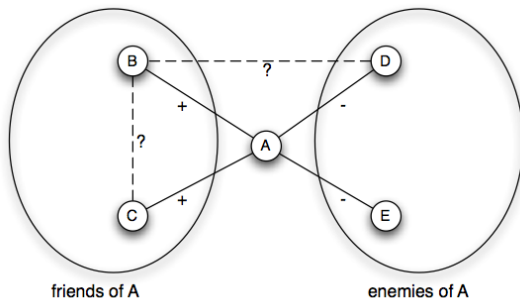
# Weakly Balanced Networks





# Proving the Characterization

- Pick any node A at random
- Show:
  - All of A's friends are friends with each other.
  - A and all his friends are enemies with everyone else in the graph.



- Repeat until all nodes are assigned to a set.

# In Summary

- We looked at the structure of a network with positive and negative relationships
  - Implication: a purely local property (such as the structural balanced property) implies a strong global property (either everyone gets along, or the world is divided into groups of battling factions)