Engineering Economics

Module -3

Equivalent Annual Worth Comparisons and Rate of Return Calculations

Equivalent Annual Worth Comparisons - Equivalent Annual Worth Comparison methods, Situations for equivalent annual worth comparison, Considerations of asset life, Use of sinking fund method, Numerical Exercises.

Rate of Return Calculations – Rate of Return, Minimum acceptable rate of return, IRR , Numerical exercises on Rate of return calculations.

Annual worth(AW)

The AW method of comparison, is commonly used for comparing alternatives. AW means that all incomes and disbursements (irregular and uniform) are converted into an equivalent uniform annual (end-of-period) amount, which is the same period.

There are 5 basic methods to have results of selected alternative or to be analysed

Equivalent worth :

i) Present worth (PW)

ii) **Annual worth (AW)**

iii) Future worth (FW)

Rate of returns

i) Internal rate of return(IRR)

ii) External rate of return(ERR)

Equivalent Annual worth comparison Method

1. Structure of a Capital recovery Annuity

The main structure of annual-worth calculations is the capital recovery factor, which converts a lumpsum to an equivalent annuity.

This annuity usually represents an investment in an asset that is expected to generate a positive future cash flow, and duration of annuity is the life of asset.

The cost of asset is uniform negative payments. This negative cash flow is offset by the positive revenue produced by the asset in establishing the net equivalent annual worth of the investment.

The capital recovery factor (A/P,i,N) accounts for both the repayments of invested capital P and interest earned on the unrecovered portion of the investment.

Example

1. An asset is purchased for Rs. 40,000 . Expected life is 4 yrs and no salvage value at the end of its life. The purchaser intends to recover the 40,000 investment over 4 yrs plus interest of 40,000 would have earned if it had been invested elsewhere. Interest rate is 10%. The series of equal payments that will return the capital plus interest is computed as

Equivalent annual payment A = P(A/P,i%,N)

=40,000(A/P,10,4)

A=12,620

2. A large gasoline station is required by the city to install vapour containment equipment on its gasoline pump nozzles the immediate conversion cost will be 1,80,000 with an estimated Rs.600 per year for maintenance. Its necessary to update the equipment every 3yrs at a cost of 3500. The station pumps an average of 1 million gallons of gasoline per month. On an annual basis, what would be the price increase per

gallon necessary to pay for the conversion over a 6-year period? Include the sixth year's update cost in analysis and assume an interest rate of 14 percent.

Solution : P=1,80,000 A=600/year i=14% N=6 yrs

Once in 3yrs for update equipment = Rs.3500

Method 1:

A = P(A/P,14%,6) + 600 + [(F(P/F,14%,3) + (F(P/F,14%,6)] (A/P,14%,6)

= 1,80,000 (A/P,14%,6) + 600 + [3500(P/F,14%,3) + 3500(P/F,14%,6)] (A/P,14%,6)

=1,80,000 (0.25716) + 600 + [3500 (0.67497) + 3500 (0.45559)] (0.25716)

=46288.8 + 600 + [2362.395 + 1594.565] (0.25716)

=46888.8 + 1017.571

A=47,906.371

12 millions gallons =47,906.371

1 gallon = 47,906.371 / 12000000 = Rs. 0.00399 / gallon

= 0.4 Paise / gallon

Method 2 Alternate expression can also be used as,

A = P(A/P,14%,6) + 600 + F(A/F,14%,3) + F(A/F,14%,6)

=1,80,000 (0.25716) + 600 + 3500 (0.29073) + 3500 (0.11716)

= 46288.8 + 600 + 1017.555 + 410.06

A=48,316.415

12 millions gallons =48,316.415

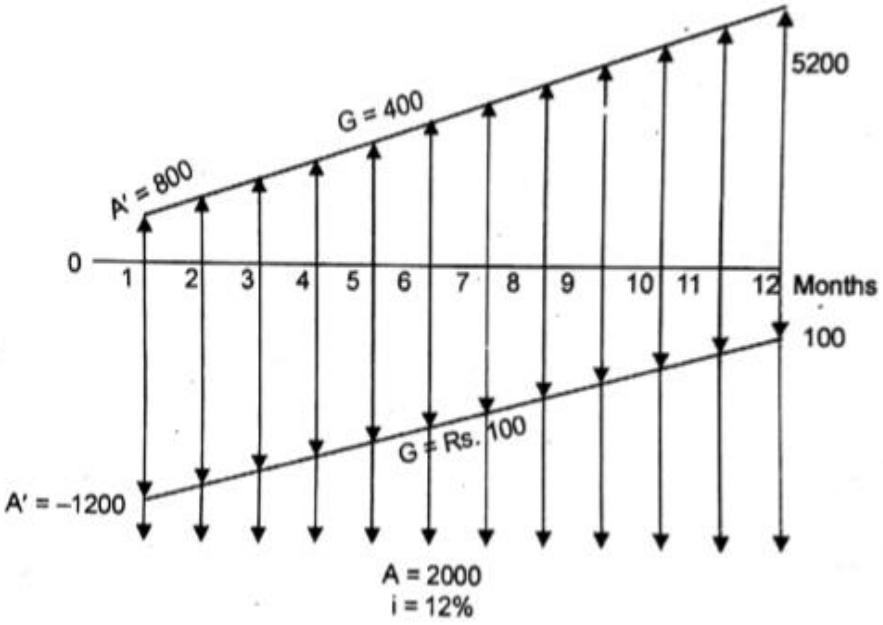
1 gallon = 48,316.415 / 12000000 = Rs. 0.0040/ gallon

= 0.4 Paise / gallon

Consolidation of Cash flows

3. A consultant proposes a training program for clerks. Program lasts 1 year , cost Rs. 2000 per month. The use of this program savings in the first months should amount to Rs. 800 and should increase by Rs.400/month for the rest of the year.Operational and work interference are expected to boost clerical costs by Rs.1200 the first month, but this amount decline in equal increments at the rate of Rs. 100 per month. If the required return on money is 12% compounded monthly and the program must pay for itself with in 1 year.

Solution : i= r / N = 0.12 / 12 = 0.01per period
N = 12 months



Equivalent monthly worth of savings = A1 + G (A/G,i%,N)
=800 + 400 (A/G,1%, 12)
=800 + 400 (5.3681)
= Rs. 2952.4

Equivalent monthly worth of costs = -A – [A1 – G (A/G,1%,12)]
= - 2000 – [1200 – 100 (5.3681)]
= -2661.9

Equivalent net monthly cash flow = 2952.4 – 2662

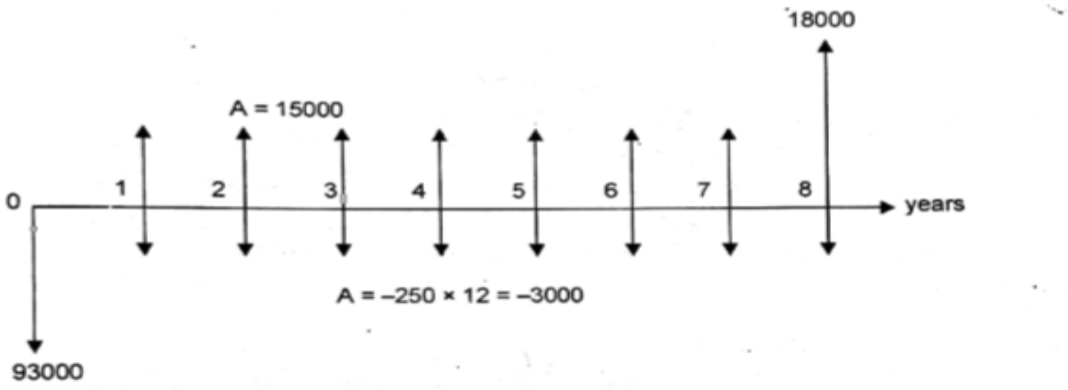
= **Rs. 290**

Recovery of Inverted capital

4. The purchase of a new bus will reduce labour cost for maintenance by Rs. 15000 per year. The price of the bus is Rs.93000 and its operating costs will exceed those of present equipment by Rs.250 per month. Salvage value is 18000 in 8 years and Should the bus be purchased if i=7% .

Solution :

P=93,000 i=7% S=18,000 N=8



EAW = - P (A/P,7%,8) + F(A/F,7%,8) + 15,000 – 3000

= -93,000 (0.1675) + 18,000 (0.0975) +12,000

EAW = -1822 loss per year for 8 years

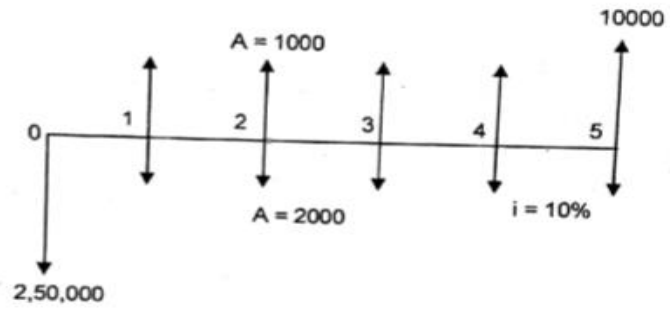
EAW calculations shows that the purchase and use of the bus will cause a loss equivalent of Rs. 1822 per year for 8 years.

Net Cash flow Comparison

Example. 1. *The machine A and B have same service life of 5 years. The other expenses are given below. If the money is worth 10% P.A. which machine is more economical?*

Cash flows	*M/CA*	*M/CB*
First cost	250000	150000
Uniform end of year maintenance	2000	4000
Over-haul at the end of 3rd year	–	3000
Salvage value	10,000	–
Yearly savings on account of less inspection.	1000	–

M/CA:



Machine A

AW = - P (A/P,i %, N) + F (A/F , i% ,N) + 1000 – 2000

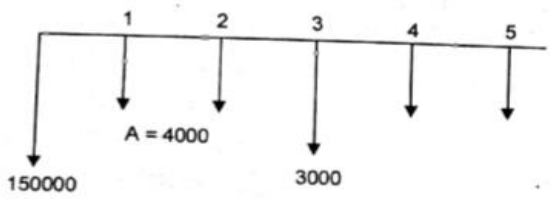
= - 250000 (A/P,10%,5) + 10000 (A/F,10%,5) -1000

= -250000 (0.26380) + 10000 (0.16380) – 1000

AW = - 65,312

Machine B

M/CB :



$$AW = -150{,}000\,(A/P, i\%N) - [(3000\,(P/F, i\%N)\,(A/P, i\%\,N)] - 4000$$

$$= -150{,}000\,(A/P, 10\%\,5) - (3000(P/F, 10\%\,3)\,(A/P, 10\%\,5) - 4000$$

$$= -39570 - 618.3 - 4000$$

$$\boxed{AW = -44188.3}$$

4.4 CONSIDERATION OF ASSET LIFE

Translating cash flows to equivalent annuities is a mechanical process that becomes almost automatic with practice. Understanding the meaning of an economic comparison and being able to explain it's significance to others are the critical skills.

4.4.1 Definitions of Assets Life

The time value analysis, N takes on a special meaning to represent the life of an asset that loses value as a function of use or time. The frequently used term to describe the life of an asset are listed as follows:

Ownership Life or Service Life is the period of time an asset is kept in service by the owner's. It implies useful service from the time of purchase until disposal. A machine can have a physical life longer than its service life the machine is still physically sound, but there is no useful function for it to perform.

Accounting Life is a life expectancy based primarily on book keeping and tax considerations. It may or may not correspond to the period of usefulness and economic desirability.

Economic Life is the period of time extending from date of installation to date of retirement (by demotion or disposal) from the primary intended service. The need for retirement is signalled in an engineering economy study. When the equivalent cost of a new asset (challenger) is less than the equivalent cost of keeping asset (defender) for an additional period of time.

4.4.2 Comparison of Assets with Equal and Unequal Lives

The lease or buy questions is raised with increasing regularity that corresponds to the rapid growth of leasing companies. It is now possible to lease almost any type of production equipment that is not custom-designed for narrowly specialized service. Important tax and inflation considerations involved in the lease or buy choice.

Example 1. *Alternative with Equal Annual Costs.*

A machine needed for 3 years can be purchased for 70,000 and sold at the end of the period for about 20,000. A comparable machine can be leased for 27737/-per year. If a firm expects a return of 20% on investments should it buy or lease the machine when end-of-year payments are expected.

Solution : P=70,000 S=20,000 i=20% n=3

Equivalent Annual Cost to buy

Equivalent annual cost (EAC) = CR and R + S*I + O &M costs

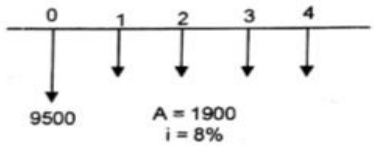
EAC = (P-S) (A/P,i%,N) + S * i
=(70,000 - 20,000) (A/P,20%,3) + 20,000*0.2
=50,000 * 0.4747 + 4000
EAC=27,735

Comparison of assets with unequal lives

Example 2: *Two machines performs the same function. First machine has a cost of Rs. 9500/- relatively high operating costs of Rs. 1900 per year more than those of the second machine and short life is 4 years. The second machine Rs. 25100 and can be kept in service economically for 8 years. The scrap value from either machine at the end of its life will barely cover it's removal cost which is preferred when the minimum alternative rate of return is 8 percent?*

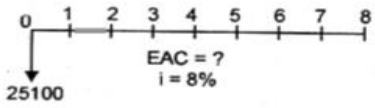
Solution.

First Machine



EAC = 1900 + 9500 (A/P, *i*% N)
= 1900 + 9500 (A/P, 8%, 4)
= 1900 + 9500 (0.30192)
= Rs. 4768

Second Machine



EAC = 25100 (A/P, 8%, 8)
= 25100 (0.17401)
EAC = 4368

The second machine has a lower annual cost for service during next 8 years and is preferred.

Refer Problems of Annual Equivalent worth method from Paneerselvam textbook (softcopy is shared in google classroom) Page No. 68 to 85

Problem on Use of a Sinking Fund

1. A company has three proposals for expanding its business operations. The details are as follows:

Alternative	*Initial cost* (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
*A*1	25,00,000	8,00,000	10
*A*2	20,00,000	6,00,000	10
*A*3	30,00,000	10,00,000	10

Each alternative has insignificant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

Solution : Alternative 1

AEM = - P (A/P, i %, n) + 800000

= -2500000 (A/P,15%,10) + 800000

= -2500000 (0.19925) + 800000

AEM= 3,01,875

Alternative 2

AEM = - P (A/P, i %, n) + 600000

= -2000000 (A/P,15%,10) + 600000

= -2000000 (0.19925) + 600000

AEM= 2,01,500

Alternative 3

AEM = - P (A/P, i %, n) + 1000000

= -3000000 (A/P,15%,10) + 1000000

= -3000000 (0.19925) + 1000000

AEM= 4,02,250

Average Annual Cost Method

i) Capital recovery (C.R) = (I – S) / n

ii) $i(Iav) = \frac{I+S}{2} * i$

iii) Operating and maintenance cost + labour cost

Average annual cost (AAC) = i + ii + iii

1. An investment of Rs. 20,000 in an off gas monitoring system will have a salvage value of Rs. 6000 after an economic life of 5 yrs. Maintenance and operating costs are 4400 / year and the firm cost of capital is 10% . What is the average annual cost of this investment.

   Solution : I=20,000 S=6000 i=10% N=5 yrs

   (i) C.R = 20,000 -6000 / 5 = **2800**

   (ii) Iav= (20,000+6000)/2 * 0.1

   =26000/2 * 0.1

   =13000 * 0.1

   Iav =1300

   (iii) O & M costs= 4400

Average Annual Cost (AAC)= i + ii + iii

=2800 + 1300 + 4400

AAC=8500

Problem 3. *The following 2 m/c's are being considered for purchase since they are multipurpose, it is impossible to allocate revenues to them, which of the 2 should be purchased.*

Description	*Machine A*	*Machine B*
Capital cost	*Rs. 13,000*	*Rs. 8,000*
Labour cost	*8,000/year*	*9,000/year*
Maintenance cost	*300/year*	*400/year*
Economic life	*7 years*	*7 years*
Salvage value	*2000*	*2000*
Cost of capital	*10%*	*10%*

Solution. *For m/c A:*

(*i*) $\text{C.R.} = \frac{I-S}{n} = \frac{13,000-2,000}{7} = \frac{11,000}{7} = 1571.428.$

(*ii*) $i(I_{av}) = \frac{I+S}{2} \times i = \frac{13,000+2,000}{2} \times 0.1 = 750$

(*iii*) **Maintenance and Operating Costs and Labour Cost = 300 + 8000 = 8300/year and labour cost.**

∴ **The average annual cost for m/c A**

= *i + ii + iii*
= 1571.428 + 750 + 8300
= Rs. 10,621.4285.

For m/c B:

(*i*) $\text{C.R.} = \frac{I-S}{n} = \frac{8,000-2000}{7} = 857.1428.$

(*ii*) $i(I_{av}) = \frac{I+S}{2} \times i = \frac{8,000+2,000}{2} \times 0.1 = 500$

(*iii*) Operating, Maintenance and Labour Cost
= 9000 + 400
= 9400/year

∴ The average annual cost for m/c B
= *i + ii + iii*
= 857.1428 + 500 + 9400
= Rs. 10,757.1428

Conclusion : The average annual cost of m/c A being less than the m/c B, So m/c A should be preferred.

Problem 4. *J.M. Wal Company is a large industrial sewing shop which manufacturers such garments as aluminised asbestus suits for the iron industry heavy duty car top carrier covers, industry approns and so on. Essentially they have several different kinds of machine through many different jobs pass.*

At present they are considering the purchase of a new m/c. The first alternative is lower in price but will incur a higher labour cost while the second is higher in price but requires less labour. In addition, the maintenance cost for their lower priced machine is estimated to be Rs. 1000 for the first year increasing by Rs. 100/year over the life of the machine. The higher priced machine has an estimated first year maintenance cost of Rs. 1500 and an increase of Rs. 200/year. The other economic data are as follows:

Description	*Machine A*	*Machine B*
Capital cost	*15,000*	*25,000*
Labour cost	*14,000*	*7,000*
Salvage value	*2,000*	*1,000*
Economic life	*10 yrs.*	*10 yrs.*

If their cost of capital is 15%, which machine should they purchase.

Solution. Step I : To calculate the average annual maintenance cost

For m/c A:

Machine A

I=15,000

Labour cost= 14,000

S=2000
N=10
i=15%
Average Annual Maintenance cost =
[1000+1100+1200+1300+1400+1500+1600+1700+1800+1900] / 10
=1450/year

C.R= I-S / n = 15,000 – 2000 / 10 =13,000/10 = 1300
Iav= [I+S]/2 * i = [15,000 + 2000]/2 * 0.15 = 1275
O & M costs + labour costs = 1450 + 14,000 =15450
AAC of machine A= 1300 + 1275 + 15450 = **18,025**

Machine B

I = 25,000 S= 1000 n=10 i=15% labour cost=7000
Average maintenance cost = [1500
+1700+1900+2100+2300+2500+2700+2900+3100+3300] /10
=2400 / year

C.R = 25,000 – 1000 / 10 = 2400
IAv= [25000+1000]/2 * 0.15 = 13000 * 0.15=1950
O & M costs + labour costs = 2400 + 7000 = 9400

AAC for machine B = 2400 + 1950 + 9500 =**13,750**
The average annual cost of m/c B being less than that of m/c A, so we will prefer m/c B only.

will prefer m/c B only.

Problem 5. *The Campbell Company is considering the purchase of one of 2 overhead cranes for their fabrication shop. For crane A the capital cost would be Rs. 25,000 while for B it would be Rs. 18,000. Both have an expected life of 10 years. The labour costs for both would be the same but maintenance cost would not be. For A, the maintenance cost would be Rs. 1000/yr while for B it would be Rs. 500 the first year and increases by Rs. 200 each year there after. The salvage value for A at the end its economic life would be Rs. 8000 while for B it would be 3000. If the cost of capital to the firm is 15% which crane should be purchased.*

Solution. Data Given:

Capital Cost of Crane A = Rs. 25,000

Capital cost of crane B = Rs. 18,000

Expected Life = 10 years

To calculate Maintenance Costs for *m/c* or Crane A and B :

For Crane A:

Average Annual Cost for Crane A = [1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000 + 1000]/10

= 1000/year.

For Crane B:

Average Annual Maintenance Cost = [500 + 700 + 900 + 1100 + 1300 + 1500 + 1700 + 1900 + 2100 + 2300]/10

= 1400/year.

Salvage value for crane A = Rs. 8000

Salvage value for crane B = Rs. 3000

Interest =15%

Crane A:

(*i*) $\text{C.R.} = \frac{I-S}{n} = \frac{25,000-8,000}{10} = \frac{17,000}{10} = 1700$

(*ii*) $i(I_{av}) = \frac{I+S}{2} \times i = \frac{25,000+8,000}{2} \times 0.15 = 2475$

(*iii*) Maintenance Cost = 1000/year

Average Annual Cost = (*i* + *ii* + *iii*) = 1700 + 2475 + 1000

A.A.C. = 5175

Crane B:

(*i*) $\text{C.R.} = \frac{I-S}{n} = \frac{18,000-3,000}{10} = 1500$

(*ii*) $i(I_{av}) = \frac{I+S}{2} \times i = \frac{18,000+3,000}{2} \times 0.15 = 1575$

(*iii*) Maintenance Cost = 1400/year

Average Annual Cost = (*i* + *ii* + *iii*)

= 1500 + 1575 + 1400

A.A.C. = 4475

Conclusion : As the average annual cost of crane B is less than that of A, so we will purchase crane B only.

Problem 6. *A public utility district, (P.U.D.) has found that it can lease a data a processing service at an annual cost of Rs. 9100/year for 5 years. The cost includes all maintenance and operating cost. Alternatively it can purchase Rs. 10,000 worth of equipment*

which will have a salvage value of Rs. 3000 in 5 years and will require Rs. 7000/years in labour cost to operate. In addition maintenance is expected to be Rs. 500 for the first year and increased by Rs. 100/year thereafter. The public utility is limited by the federal power consumption to 8% return on any investment. Using an average investment criteria, determine whether it is more economical for the P.U.D. to lease the service or purchase the equipment.

Solution. Data:

Lease Data

Cost of data processing servicing machine = Rs. 9100

(*i.e.*,) C = Rs. 9100

Purchase Data

Cost of the data processing servicing machine if purchased

I = Rs.10,000.

Salvage value = Rs.3000

Life = 5 years

Labour costs = Rs.7000.

Maintenance and operating costs $= \frac{500 + 600 + 700 + 800 + 900}{5}$

$= 700$/year.

$i = 8\% = 0.08$.

Average annual cost of m/c purchased:

(*i*) $\text{C.R.} = \frac{I - S}{n} = \frac{10,000 - 3,000}{5} = 1400$/year.

(*ii*) $i(I_{av}) = \frac{I + S}{2} \times i = \frac{10,000 + 3,000}{2} \times 0.08 = 520$/year.

(*iii*) Operating and Maintenance Cost

= Rs. 7700/year

Average Annual Cost = (*i*) + (*ii*) + (*iii*) = 1400 + 520 + 7700

Conclusion : The average annual cost for the data processing servicing unit purchased is greater than that of leased. So we will go in for the leased one only.

Advantages

(i) It is relatively simple and quick to calculate.
(ii) It is not difficult to visualize an investment costing an equal amount each year.
(iii) Alternative with different life times can be effectiveness compared.
(iv) Cost comparison can be made without knowledge of income or considering the taxes on incomes.

Disadvantages

(i) An approximation is used which ignores the timing of cash flows by computing the interest on the average investment.

(ii) Uniform or straight line depreciation of assets is usually followed.
(iii) Total costs are not explicitly considered.

4.6 EQUIVALENT ANNUAL COST METHOD

Equivalent annual cost method is a time adjusted (more accurate) method of calculatin an equal amount cost over the life of an investment. It is similar to average annual cos method. It is more exact than A.A.C. method. In this method the compounded interest i taken into account. The costs included in this equivalent annual cost method are:

(i) Capital Recovery and Return (*i.e.*,) (C.R. and R)
(ii) Interest on the Salvage ($i \times s$)
(iii) Other Annual Operating and Maintenance Costs

Equivalent Annual Cost = (C.R. and R + i(S) + O and M)

$$\text{C.R. and R} = (\text{I} - \text{S}) \times \left[\frac{1}{(PV_a)_i^n}\right]$$

$$\text{CR and R} = (\text{I-S})\ \frac{1}{(\frac{\text{P}}{\text{A}},\text{i\%,N})}$$

OR

$$\text{CR and R} = (\text{I-S})\ (\text{A/P,i\%,N})$$

1. An investment of Rs. 20,000 in an off gas monitoring system will have a salvage value of Rs. 6000 after an economic life of 5 yrs. Maintenance and operating costs are 4400 / year and the firm cost of capital is 10% . What is the average annual cost of this investment.

   Solution :

   I=20,000 S=9600 Economic life=5 yrs O&M = 4400/yr

   i= 10% = 0.1

   C.R. and R = (I-S) (A/P,10%,5) = (20,000-6000) * (0.2637) = 3693

   S * I =6000 * 0.1 = 600

   O & M costs=4400

EAC = 3693 + 600+4400 = **8693**

Conclusion: When compared with an annual average cost of 8500/yr EAC is larger because the effect of Compound interest(C.I) on the capital investment is taken into account in EAC.

method.

Problem 2 *The following 2 m/c's are being considered for purchase since they are multipurpose, it is impossible to allocate revenues to them, which of the 2 should be purchased.*

Description	*Machine A*	*Machine B*
Capital cost	*Rs. 13,000*	*Rs. 8,000*
Labour cost	*8,000/year*	*9,000/year*
Maintenance cost	*300/year*	*400/year*
Economic life	*7 years*	*7 years*
Salvage value	*2000*	*2000*
Cost of capital	*10%*	*10%*

Solution: For *m/c* A:

(*i*) $$\text{C.R. and R} = (\text{I}-\text{S})\left[\frac{1}{(\text{PV}_a)i^n}\right]$$

$$= (13{,}000-2000)\left[\frac{1}{(\text{PV}_a)^7_{0.1}}\right] \quad \text{from table}$$

$$= (13{,}000-2000)\times\frac{1}{4.868}$$

P/A, 10%, 7

$$\text{C.R. and R} = 2{,}259.46$$

$$(\text{PV}_a)^7_{0.1} = \frac{1}{(1+i)^7}$$

$$= \frac{1}{(1+.1)^1}+\frac{1}{(1+.1)^2}+\frac{1}{(1+.1)^3}+\frac{1}{(1+.1)^4}+\frac{1}{(1+.1)^5}+\frac{1}{(1+.1)^6}+\frac{1}{(1+.1)^7} = 4.868$$

(*ii*) $i \times \text{Salvage} = 2000 \times 0.1 = 200$

(*iii*) Operation, Labour and Maintenance charges
= 8000 + 300 = 8300

Now Equivalent Annual Cost for the Machine A
= 2,259.46 + 200 + 8300 = 10,759.46.

For m/c B:

(*i*) $$\text{C.R. and R} = (8000-2000)\times\frac{1}{(\text{PV}_a)^7_{0.1}}$$

$$= 6000\times\frac{1}{4.868} = 1232.53$$

(*ii*) $i \times \text{Salvage} = 2000 \times 0.1 = 200$

(*iii*) Operation, Labour and Maintenance Charges
= 9000 + 400 = 9400

Equivalent Annual Cost for the Machine B
= 1232.53 + 200 + 9400
= 10,832.53.

Conclusion : The Equivalent Annual Cost of m/c A and m/cB being greater than that of in average annual cost. It is because we consider the compound interest on capital investment in the Equivalent Annual Cost Method. Select m/c A only.

Problem 3. *J.M. Wal Company is a large industrial sewing shop which manufacturers such garments as aluminised asbestus suits for the iron industry heavy duty car top carrier covers, industry approns and so on. Essentially they have several different kinds of machine through many different jobs pass.*

At present they are considering the purchase of a new m/c. The first alternative is lower in price but will incur a higher labour cost while the second is higher in price but requires less labour. In addition, the maintenance cost for their lower priced machine is estimated to be Rs. 1000 for the first year increasing by Rs. 100/year over the life of the machine. The higher priced machine has an estimated first

less labour. In addition, the maintenance cost for their lower priced machine is estimated to be Rs. 1000 for the first year increasing by Rs. 100/year over the life of the machine. The higher priced machine has an estimated first year maintenance cost of Rs. 1500 and an increase of Rs. 200/year. The other economic data are as follows:

Description	*Machine A*	*Machine B*
Capital cost	*15,000*	*25,000*
Labour cost	*14,000*	*7,000*
Salvage value	*2,000*	*1,000*
Economic life	*10 yrs.*	*10 yrs.*

If their cost of capital is 15%, which machine should they purchase.

Solution.

(*i*) For m/c A: C.R. and R: $= (I - S) \times \left[\frac{1}{(PV_a) i^n}\right]$

$= (15000 - 2000) \times \left[\frac{1}{(PV_a)^{10}_{0.15}}\right]$

$= 13,000 \times \frac{1}{5.0154}$

$(PV_a)^{10}_{0.15}$ = 0.8695 + 0.7561 + 0.6575 + 0.571 + 0.497 + 0.432 + 0.3759 + 0.326 + 0.284 + 0.2471 = 5.0154.

C.R and R = 2,592.01

(*ii*) $i \times$ salvage = 2000 × 0.15 = 300.

(*iii*) Operation, Labour and Maintenance Cost

= 14,000 + 1450 = 15,450.

Equivalent Annual Cost for m/c A = 2592.01 + 300 + 15,450 = 18,342.01

For m/c B:

(*i*) C.R and Return $= (I - S) \times \left[\frac{1}{(PV_a) i^n}\right]$

$= (25,000 - 1000) \times \frac{1}{5.0154} = 4785.26.$

(*ii*) $i \times$ salvage = 1000 × 0.15 = 150.

(*iii*) Operation, Labour and Maintenance Costs

= 7000 + 2400 = 9400.

Equivalent Annual Cost for M/C B = 4785.26 + 150 + 9400

= 14,335.26

Result: The E.A.C. being greater than that of A.A.C. of both the m/c, it is because we consider the C.I. on the capital investment in the Equivalent Annual Cost Method.

Alternative

Equivalent Annual Cost method can be done by another *formula also.*

(*i.e.*) *Equivalent Annual Cost* $= (I - S)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] + iS + O$ and M costs

Problem 4.

Machine A	*Machine B*
Capital cost = Rs. 10,000	Rs. 7000
Estimated life = 10 yrs.	10 yrs.
Salvage value = 4000	2000
Annual disbursement = 1000 (Maintenance)	1500
Cost of Capital = 6%	6%

(*a*) *Which of the following 2 alternatives is preferred?*

(a) Which of the following 2 alternatives is preferable?

Solution. Considering Machine A:

(i) C.R. and R $= (I - S)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

$= (10,000 - 4,000)\left[\frac{0.06(1.06)^{10}}{(1.06)^{10} - 1}\right]$

$= 6,000 \times 0.1358 = 815.2077$

(ii) $i \times$ salvage $= 4,000 \times 0.06 = 240.$

(iii) Operating and Maintenance Cost = 1000.

Equivalent Annual Cost for m/c A $= i + ii + iii = 2055.2077$

Considering Machine B:

(i) C.R. and R. $= (I - S)\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

$= (7,000 - 2,000)\left[\frac{0.06(1.06)^{10}}{(1.06)^{10} - 1}\right]$

$= 5000 \times 0.1358 = 679.$

(ii) i S $= 2000 \times 0.06 = 120.$

(iii) Operation and Maintenance Cost = Rs. 1500.

Now the Equivalent Annual Cost for m/c B:

$= i + ii + iii = 679 + 120 + 1500$

E.A.C. $= 2299.$

Conclusion : It is clear from the calculations that the Equivalent Annual Cost for the m/c A is less than that of B, so we will prefer only m/c A.

Rate of Return Calculations

Refer Problems of Rate of return calculations from Paneerselvam textbook (softcopy is shared in google classroom) Page No. 88 to 97

IRR CALCULATIONS

1. A piece of land adjacent to a highway road is likely to increase in value. The present value now is Rs. 60,000 and is expected to be worth Rs.1,30,000 within in 5yrs. During that period it can be rented for pasture at Rs.1300 per year. Annual taxes presently are Rs.650 and will likely remain constant. What rate of return will be earned on the investment if the estimates are accurate?

Solution : P=60,000 F=1,30,000 N=5 yrs A=1300/year A(expenses)=650

$$PW = -P + A(P/A,i\%,5) - A(P/A,i\%,5) + F(P/F,i\%,5)$$

= -60,000 + 1300(P/A,i%,5) – 650 (P/A,i%,5) + 1,30,000(P/F,i%,5)

PW= -60,000 + 650(P/A,i%,5) + 1,30,000(P/F,i%,5)

PW(i=10%) = -60,000 + 650(P/A,10%,5) + 1,30,000 (P/F,10%,5)

=23,181.15

When i=15%

PW = -60,000 + 650(P/A,15%,5) + 1,30,000 (P/F,15%,5)

= - 60,000 + 650(3.35216) + 1,30,000(0.49718)

PW= -60,000 + 2178.9 + 64,633.4 = 6,812.3

When i=18% , PW = -1,144

The approximate value of i is determined by linear interpolation method,

$$i = 15\% + \frac{[(15-18)\%\ (6812.3)]}{6812.3-(-1144)}$$

i=17.569%

2. The subassemblies for a product are purchased for Rs. 81 a piece.The annual demand is 450 units and is expected to be continued for 3yrs. At this time the new product under development will be ready for manufacturing. The initial investment is Rs. 22,000 , the production costs of subassemblies will be Rs.19,500 for the first-year and Rs. 13,250 in each of the last 2 years. The equipment will have no salvage value.
   Solution : P=22,000 N=3yrs
   Present annual cost = 450 * 81 = 36,450
   Net savings (1st year) = 36,450 – 19,500 =16,950
   Net savings (2nd & 3rd year) = 36,450 – 13,250 = 23,200

   PW = - P + F(P/F,i%,1) + F(P/F,i%,2) + F(P/F,i%,3)
   = -22,000 + 16950 (P/F,i%,1) + 23200(P/F,i%,2) + 23200 (P/F,i%,3)

   When i=10% , PW = 30011.88

i=50% , PW = 6485.21
i=60% , PW = 3320.408
i=70% , PW = 720.46
i=80% , PW = -1444.706

The approximate value of i is determined by linear interpolation method,

$$i = 70\% + \frac{[(80-70)\%\ (720.46)]}{720.46-(-1444.706)}$$

i=73.3275%

3. A construction firm can lease a crane required on a project for 3 years for Rs. 1,80,000 payable now, with the maintenance cost included. The alternative is to buy a crane for Rs. 2,40,000 and sell it at the end of 3 years for Rs.1,00,000. Annual maintenance costs are expected to be Rs. 5000 the first 2 yrs. and 10,000 the third year. At what interest rate would the two alternatives be equivalent?

Solution : Lease = Rs. 1,80,000 for 3 years

P or I=2,40,000 N= 3yrs F=1,00,000 A1=5000 A2=5000 A3=10,000 i=?

PW = -P – 5000(P/F,i%,1) – 5000(P/F,i%,2) – 10,000(P/F,i%,3) + 1,00,000(P/F,i%,3)

When , i=5%

PW = -2,40,000 – 5000 (P/F,5%,1) – 5000(P/F,5%,2) – 10,000(P/F,5%,3) + 1,00,000 (P/F,5%,3)

PW = - 1,71,555

When i=10% , PW = -1,81,060.5

The interest rates of the two alternatives are equal @ 10%.

4. An automobile company is planning to buy a robot for its forging unit. It has identified two different companies for the supply of the robot. The details of cost and incremental revenue of using robots are summarized in the following table.

	Brand	
	Speedex	Giant
Initial cost Rs.	5,00,000	9,00,000
Annual incremental revenue Rs.	80,000	2,50,000
Life yrs.	8	8
Salvage value Rs.	40,000	60,000

MARR = 12%. Suggest the best brand of robot to the company based on ROR method.

Solution:

SpeedEx

P=5,00,000 A=80,000 N=8 F=40,000

PW = -P + A(P/A,i%,8) + F(P/F,i%,8)

= - 5,00,000 + 80,000 (P/A,i%,8) + 40,000(P/F,i%,8)

When i=10%,

PW = -5,00,000 + 80,000(P/A,10%,8) + 40,000 (P/F,10%,8)

= -5,00,000 + 80,000 (5.335) + 40,000(0.4665)

PW = -54540

When i=5%

PW = 44,112

$i = 5\% + \frac{[(10-5)\%\ (44112)]}{44112-(-54540)} = 7.235\%$

Giant

P=9,00,000 A=2,50,000 N= 8 F=60,000

PW = - 9,00,000 + 2,50,000(P/A,i%,8) + 60,000(P/F,i%,8)

When i=20% , PW= 73,206

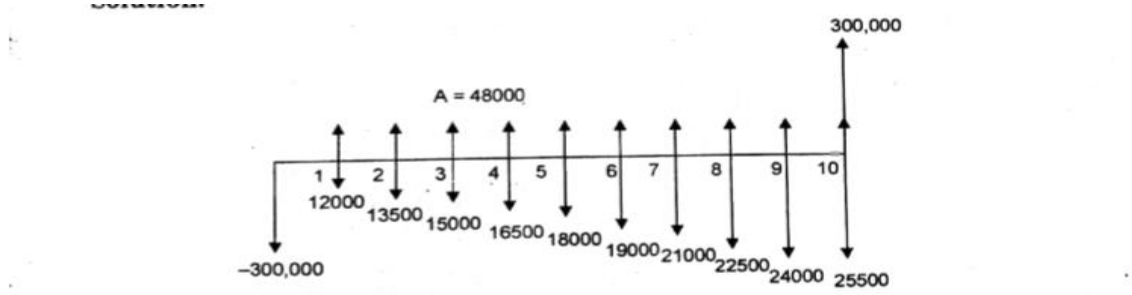
When i=25% , PW = -57682

i = $20\% + \frac{[(25-20)\%\ (73206)]}{73206-(-57682)}$ = **22.79%** is the best one since its more than MARR.

5. An investor has an opportunity to purchase a commercial rental property for Rs. 3,00,000. The current occupants have signed a 10 year lease at a constant annual rent of Rs. 48,000 and maintenance costs and taxes on the structure are currently 12,000 and are expected to increase at a rate of Rs.1500 per year over the 10-year period. Assuming that the property can be sold for at least the purchase price when the current lease expires, determine the investor's minimum expected rate of return.

Solution : P= 3,00,000 A=48,000 (revenue) F=3,00,000

A1=12,000 G=1500



PW = -P + A (P/A,i%,10) + F (P/F,i%,10) – [A1 + G(A/G,i%,10)](P/A,i%,10)

= - 3,00,000 + 48,000 (P/A,i%,10) + 3,00,000(P/F,i%,10) – [12000 + 1500 (A/G,i%,10)] (P/A,i%,10)

When i=10%

PW = -3,00,000 + 48,000(P/A,10%,10) + 3,00,000(P/F,10%,10) – [12000 + 1500 (A/G,10%,10)] (P/A,10%,10)

= -3,00,000 + 48,000 (6.1445) + 3,00,000 (0.3855) – [12000 + 1500 (3.7254)] (6.1445)

= -3,00,000 + 294936 + 115650 – [12,000 + 5588.1] (6.1445)

= -3,00,000 + 294936 + 115650 – [17588.1 * 6.1445]

PW(10%) = 2515.92

Similarly calculating ,

When i=15%, PW= -70624.916

$i = 10\ \% + \frac{[(15-10)\%\ (2515.92)]}{2515.92-(-70624.916)}$ = **10.206%**

5.8 RATE OF RETURN (R.O.R.) CAN ALSO BE CALCULATED IN THE FOLLOWING MANNER

Rate of Return expresses the percentage of profit to investment.

R.O.R. = Profit to Investment.

R.O.R. = $\frac{P}{I}$ and is expressed as an interest rate (i)

Assumption: Is that, the time factors are not considered or alternatively that the investment and profits all occur within the first time period. Profits consist of the net operating advantage, less the depreciation and taxes if applicable.

The unadjusted R. O. R. has not been so adjusted (uses an approximation much like that used in the A.A.C. method).

Whereas in the unadjusted R.O.R., does take compound interest into account.

(*a*) R.O.R (Unadjusted before taxes).

$$\underset{\text{(before taxes)}}{\text{R.R.U.}_{BT}} = \frac{\text{O.A / yr. – Dep / yr.}}{I_{average}}$$

$$\underset{\text{(aftertaxes)}}{\text{R.R.U.}_{BT}} = \frac{\text{O.A / yr. – Dep / yr. – Tax / yr.}}{I_{average}}$$

Problem 1. *A Nuclear utility in Chio has an opportunity to install an irradiation tube in its reactor during a forthcoming shut down. The tube will cost Rs. 10,000 initially and will be of no value (highly contaminated) after use. During the 4 year life of the project, the tube will be used to collect radioactive isotopes which can be sold to yield an operating advantage of Rs. 3000/yr.*

(a) *Find before taxes rate of return.*

(b) *If the utility claims at 8% interest on its debt equity capital structure. Is this worthwhile project from an economic standpoint?*

Solution.

Given data :

Capital Investment = Rs. 10,000

n = 4 year

O.A. = Rs. 3000/year

Depreciation = Rs. 2500/year

$$I_{av} = \frac{I+S}{2} = \frac{10,000+0}{2} = 5000.$$

(a) Rate of Return before taxes $= \frac{3000-2500}{5000} = 0.1$/year

(b) *Result:* The project is worthwhile as the required is less than what we are getting.

Problem 2. *A metal brokerage firm has an opportunity to invest 35,00,000 in UO_2 and make an immediate sale of a new England Electric Utility 43,50,000. What is the simple rate of return?*

Solution. Simple rate of return $= \frac{\text{Profit}}{\text{Investment}}$

Profit = Utility − Investment

= 43,50,000 − 35,00,000

= 8,50,000.

R.O.R simple $= \frac{8,50,000}{3,500,000} = 0.24$ (*i.e.*) 24%.

Problem 3. *A Rs. 20,000 m/c is expected to last 10 yrs and have a salvage of Rs. 2000. It will generate increased income (before depreciation) of Rs. 4000/year. but neccessitate that Rs. 1100 be paid in taxes. What is the unadjusted R.O.R assuming straightline depreciation?*

Solution. Given data:

Cost of Machine (I) = Rs. 20,000

Life of the machine (n) = 10 year

Operating Advantage = Rs. 4000/year

Taxes = Rs. 1100

Salvage value 'S' = Rs. 2,000.

$$R.O.R_{\substack{(AT)\\(Unadjusted)}} = \frac{OA/year - Dep/year - Taxes}{I_{av}}$$

$$I_{av} = \frac{20,000-2000}{20} = 11,000$$

$$\text{Depreciation} = \frac{I-S}{n} = \frac{20,000-2000}{10} = 1800.$$

$$R.O.R._{AT} = \frac{4000-1800-1100}{11,000} = 0.1$$

$R.O.R._{AT}$ = 10%

Conclusion. $R.O.R._{AT}$ is 10%.

5.8.1 Advantages

1. It defines a rate of return at which profits are earned & can thus be compared with similar rates from other projects.
2. It is a useful measure to compare against the cost of capital.
3. It can be interpreted either on the basis of the total project (*i.e.*) R.O.R. simple or on annual basis (*i.e.*) Rate of Return unadjusted.

5.8.2 Disadvantages

1. It does not adjust for the compounding value of income.
2. It does not consider the total magnitude of the project.
3. It requires the estimates of return as well as cost.

Rate of Return (Adjusted) or Internal Rate of Return)

Defination : The Internal rate of return (I.R.R.) for an investment is the discount rate that equates the present value of expected cash outflows with the present value of the expected cash inflows.

Mathematically it is represented by rate '*r*'

i.e.

$$r = \left[\sum_{t=0}^{n} \frac{A_t}{(1+r)^t}\right] = 0$$

where A is the cash flow for a period t whether it be a net cash outflow or inflow. n is the number of years.

$$A_O = \frac{A_1}{(1+r)^1} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + \frac{A_4}{(1+r)^4} + \ldots.$$

where r is the rate of return.

Problems to be solved(IRR)

1. A firm has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternative		
	*A*1	*A*2	*A*3
Investment	Rs. 1,50,000	Rs. 2,10,000	Rs. 2,55,000
Annual net income	Rs. 45,570	Rs. 58,260	Rs. 69,000

Find the best alternative based on the rate of return method of comparison.

Solution: A1 = 15.81% , A2=12.03% , A3=11 %

A1 is selected.

2. Consider the following cash flow of a project:

Year	0	1	2	3	4	5
Cash flow	–10,000	4,000	4,500	5,000	5,500	6,000

Find the rate of return of the project.

Solution: 37.447%

3. A company is in the process of selecting the best alternative among the following three mutually exclusive alternatives:

Alternative	*Initial investment*	*Annual revenue* (Rs.)	*Life* (years)
*A*1	Rs. 5,00,000	1,00,000	10
*A*2	Rs. 8,00,000	1,40,000	10
*A*3	Rs. 3,00,000	70,000	10

Find the best alternative based on the rate of return method of comparison.
Solution : A1 = 15.1% , A2=11.72% , A3 = 19.5%
A3 will be selected since it has highest rate of return value.

4. A Proposal has an initial cost of Rs. 1500 and a positive cash flow that returns Rs. 200 the first year and increases by Rs. 200 each of the following years until the end of the 5 year study period. What is the rate on the proposal?
   Solution : **i=21.98%**
5. Two mutually exclusive projects are being considered. Project X requires Rs.500 now and results in a return amounting to a one-time only profit of Rs. 1000 in 5yrs from now. Project Y also requires 500 now but will return Rs. 170 per year for each of the next 5 yrs. Given MARR of 14% which project should be adopted?
   Solution : Project X , i= 14.88% Project Y , i=20.82%
   Project Y will be adopted.
6. A company is considering the purchase of a new piece of testing equipment that is expected to produce Rs. 8000 additional profit during the first year of operation. This amount will probably decrease by Rs.500 per year for each additional year of ownership. The equipment costs Rs.20,000 and will have an estimated salvage value of Rs. 3000 after 8 years of use. How does the proposal match up against a MARR of 18%?

PW = -20,000 + [8000 – 500 (A/G,i%,8)](P/A,i%,8) + 3000(P/F,i%,8)

Solution : i=31.227%

Problems to be solved (AW)

1. A company has three proposals for expanding its business operations. The details are as follows:

Alternative	*Initial cost* (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
1	25,00,000	8,00,000	10
2	20,00,000	6,00,000	10
3	30,00,000	10,00,000	10

Each alternative has in significant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

2. A company must decide whether to buy machine A or machine B.

	Machine A	*Machine B*
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	0

At 15% interest rate, which machine should be purchased?

3. Find the best alternative using the annual equivalent method of comparison. Assume an interest rate of 15% compounded annually.

Alternative	*A*	*B*	*C*
Initial cost (Rs.)	5,00,000	8,00,000	6,00,000
Annual receipt (Rs.)	2,00,000	1,50,000	1,20,000
Life (years)	10	10	10
Salvage value (Rs.)	1,00,000	50,000	30,000