Engineering Economics

- 7 principles of EE

1. Among alternatives

2. Differences in future outcomes

3.Outcomes of feasible alternatives

4.Common unit of measurement

5.Selection preferred alternative

6.Uncertainity

7.Improvement/revisit the decision

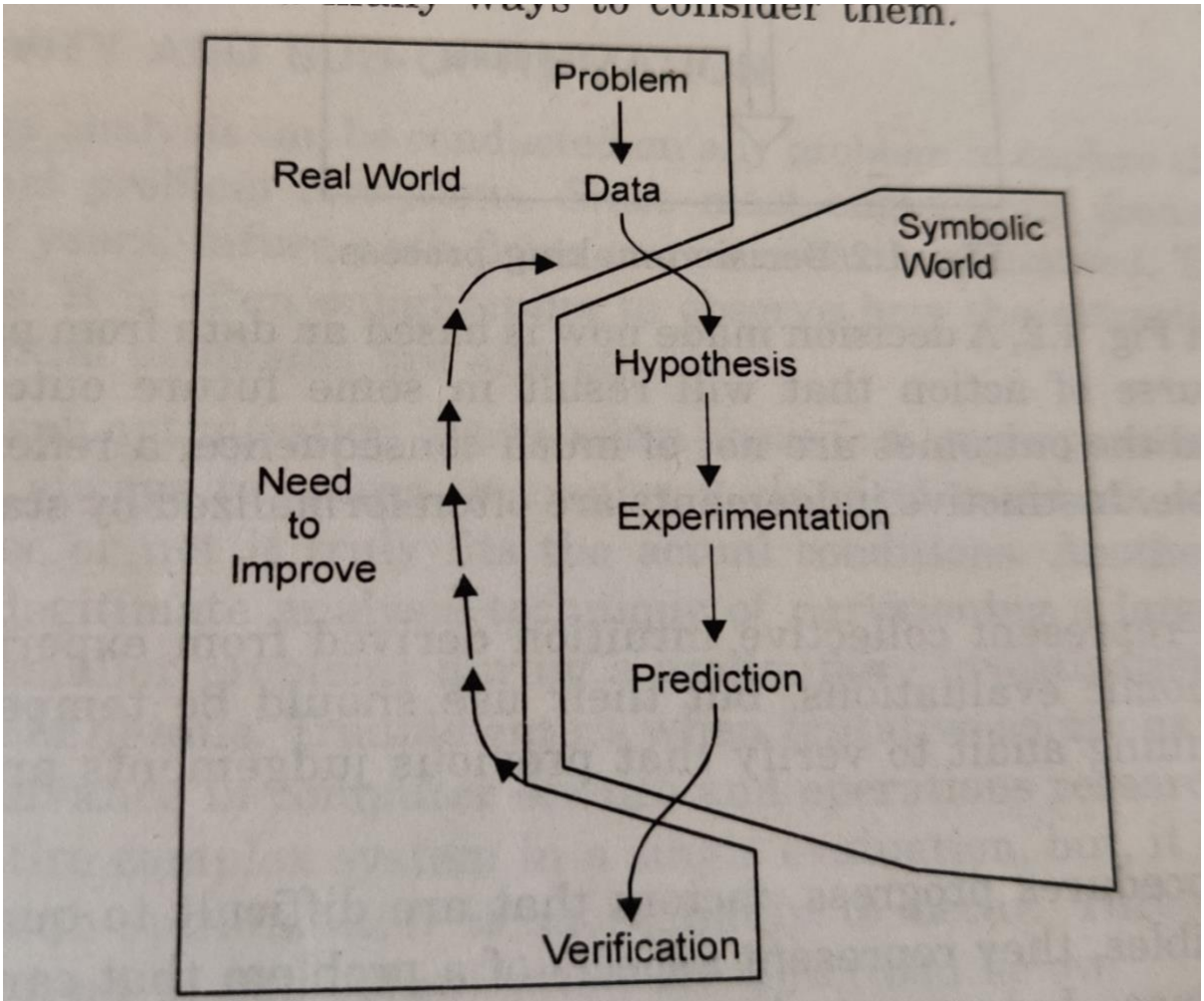


Fig. 1.1 Problem solving process.

- Interest and interest factors

Interest rate, simple / compound interest factors, Cash flow diagrams, Numerical excercises.

Interest – is a rent paid for money borrowed.

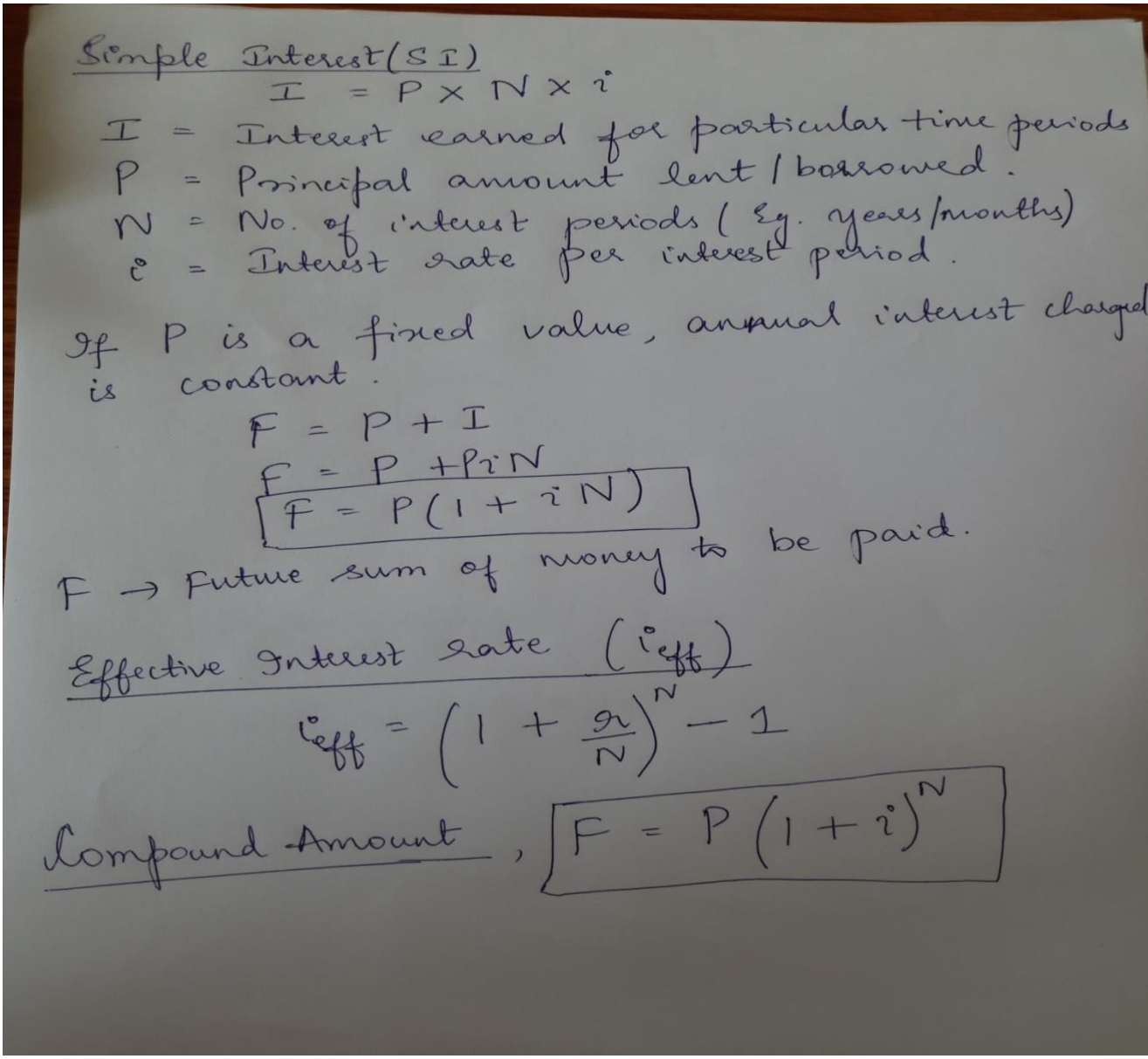
Two types

1. Simple interest
2. Compound interest ---

a)Nominal interest rates

b)Effective interest rates

c)Continuous compounding



Problems

1. The rental cost of money is a loan of Rs. 1000 for 2months @ 10%. Use simple interest.

Solution : P=1000

i=10%

N=2/12 years

F=P(1+i.N)

=1000(1+0.1*0.167)

=1016.67

F=1000(1+0.1(31+28/365))=1016.16

2.What sum must be loaned at 8% simple interest to earn rs.350 in 4years.

When N is a full year

$$I = P*i*N$$

$$350=P*0.08*4$$

$$P=350/(0.08*4)$$

$$P= 1093.75$$

i=8%

N=4

I=350

P=?

3.How long will it take rs.800 to yield rs.72 in simple interest at 4%.

N=?

P=800

I=72

i=4%

I=P*i*N

72=800*0.04*N

N=2.25years

4.At what rate will 65.07 yield Rs. 8.75 in simple interest in 3 years 6 months?

i=?

P=65.07

I=8.75

N=3.6years

I=P*i*N

8.75=65.07*i*3.5

i=3.8%

5.How long will it take any sum to triple itself at 5% simple interest rate? Asssume P=100

Answer : i=5%, I=300,N=60yrs

6. Determine the effective interest rate for a nominal annual rate of 6% that is compounded

i)Semi-annually – n=2

ii)Quarterly n=4

iii)Monthly n=12

iv)Daily n=365

Ans: r=6%=0.06

1)i=(1+0.06/2)^2 -1

i=6.09%

ii) i=6.13%

iii)i=6.16%

iv)6.18%

7. A personal loan of Rs.1000 is made for a period of 18 months at an interest rate of 1 ½ percent per month on the unpaid balance. If the entire amount owed is repaid in a lump sum at the end of that time, determine the effective annual interest rate.

Ans: P=1000

N=18 months

i= 1 ½ * 12=18%

i=(1+0.18/18)^18 - 1

i=19.61%

8. Find the compound amount of Rs.100 for 4yrs at 6% compounded annually.

Ans: P=100

N=4 , i=6%

F=P(1+i)^n

=100(1+0.06)^4

F=126.24

9. A loan of Rs.2000 , interest rate is 10% per year. If interest had not been paid each year but, had been allowed to compound, how much interest would be due to the lender as a lump sum at the end of 6yrs?

Ans: P=2000

i=10% , N=6

F=2000(1+0.1)^6

F=3543.12

Interest = F – P =3543.12-2000=1543.12

10. Accumulate a principle of Rs.1000 for

5yrs 9months at a nominal rate of 12% compounded monthly. How much interest is earned?

Ans: P=1000 , N=69months , i=12%=1% per month

F=1000(1+0.01)^69

=1986.89

Notation and Cash flow Diagrams

A cash flow is the difference between

total cash inflows(receipts)and

cash outflow(expenditures) for a specified period of time.

Interest formulas for Discrete Compounding and Cash flows

There are 6 most common discrete compound interest factors,

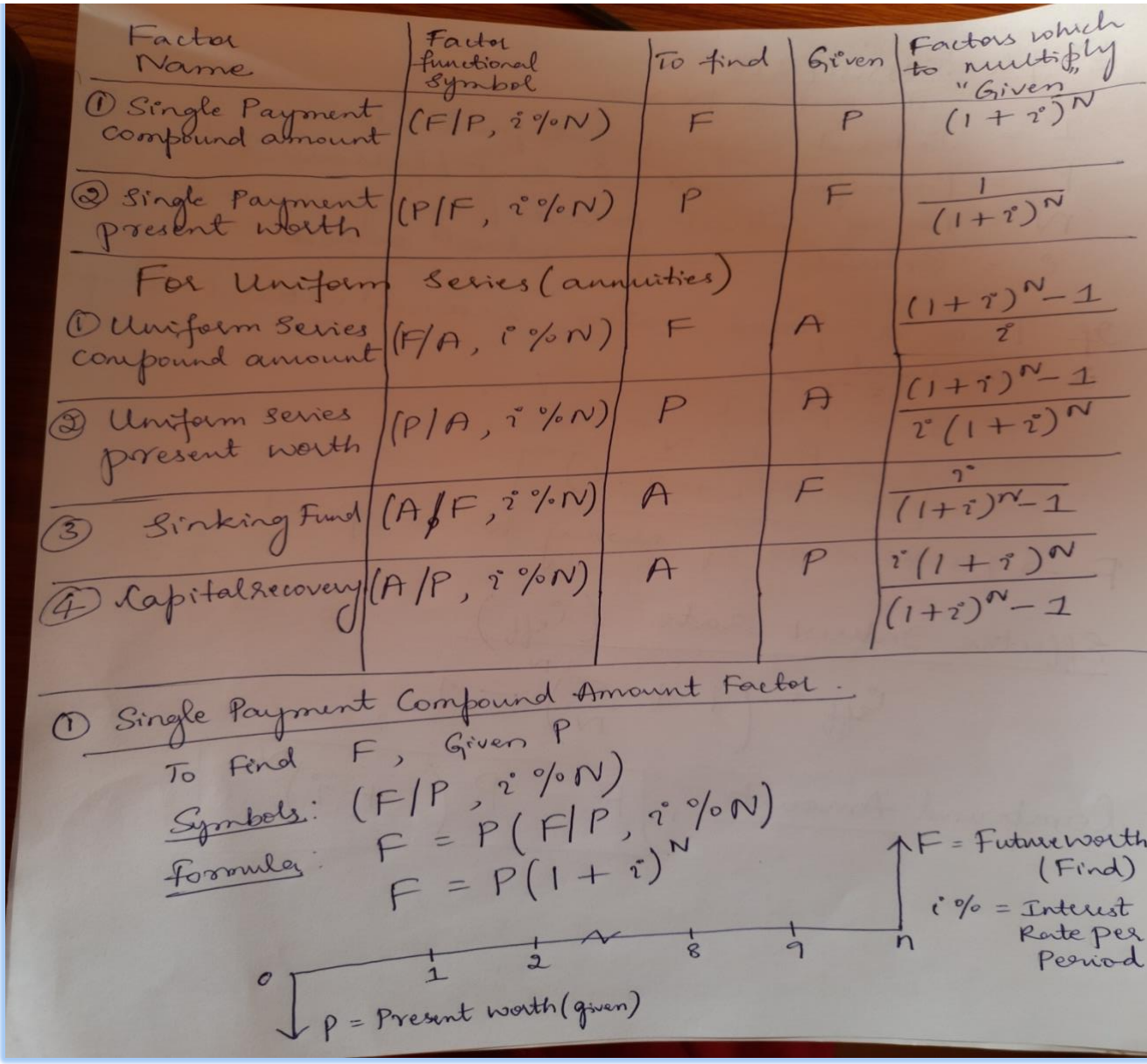
For Single Cash flows

1. Single payment compound amount P, F?

2. Single payment present worth F , P?

For Uniform series

1. Uniform series compound amount
2. Uniform series present worth
3. Equal payment series sinking fund
4. Equal payment series annual equivalent amount
5. Arithmetic gradient conversion factor (to uniform series).



Factor Name	Factor functional Symbol	To find	Given	Factors which to multiply "Given"
① Single Payment compound amount	(F/P, i%N)	F	P	$(1+i)^N$
② Single Payment present worth	(P/F, i%N)	P	F	$\frac{1}{(1+i)^N}$
For Uniform Series (annuities)				
① Uniform Series compound amount	(F/A, i%N)	F	A	$\frac{(1+i)^N-1}{i}$
② Uniform series present worth	(P/A, i%N)	P	A	$\frac{(1+i)^N-1}{i(1+i)^N}$
③ Sinking Fund	(A/F, i%N)	A	F	$\frac{i}{(1+i)^N-1}$
④ Capital Recovery	(A/P, i%N)	A	P	$\frac{i(1+i)^N}{(1+i)^N-1}$

① Single Payment Compound Amount Factor.

To Find F, Given P

Symbols: (F/P, i%N)

formula: $F = P(F/P, i\%N)$

$F = P(1+i)^N$

1. single payment compound amount

To find F , given P

1. A person deposits a sum of rs.20000 at the interest rate of 18% compounded annually for 10yrs. Find the maturity value after 10yrs?

P=20000

i=18%

F=P(F/P,i%N)

F=P(1+i)^n

=20000(1+0.18)^10

F=1,04,680

2. A person deposits a sum of rs.10,000 in a bank at a nominal rate of interest of 12% for 10yrs.Find the maturity amount of the deposit after 10yrs. if the compounding is done quarterly.

P=10,000

i=12%(compounded quarterly)

N=10

F=?

Method-1

No. of interest periods per year=4

No. of interest periods for 10yrs=10*4=40

Revised n=40

Rate of interest =12/4=3% =0.03

F=10,000(1+0.03)^40

F=32620.37

Method-2

R=(1+i/N)^n – 1

=(1+0.12/4) ^4 – 1

R=12.55%

Hence R replaces ‘i’ in the formula,

F=P(1+i)^n

=10000(1+0.1255)^10

F=32,620

3.Suppose that 10,000 is borrowed now at 15% interest per annum. A partial repayment of 3000 is made four yrs from now. The amount that will be remain to be paid then is most nearly (a) 7000 (b)8050 (c)8500 (d)13000 (e)14490.

P=10,000

i=15%

N=4

F =P(1+i)^n

=10,000(1+0.15)^4

=17490

The amount to be paid is

=17490-3000=14490

2.Single Payment present worth

To find P given F

1. An investor has an option to purchase a land that will be worth Rs.10,000 in 6yrs. If the value of land increases at 8% each year,how much should the investor be willing to pay now for this property.

F=10,000

N=6yrs

i=8%

P=?

P=F(P/F,i%N)

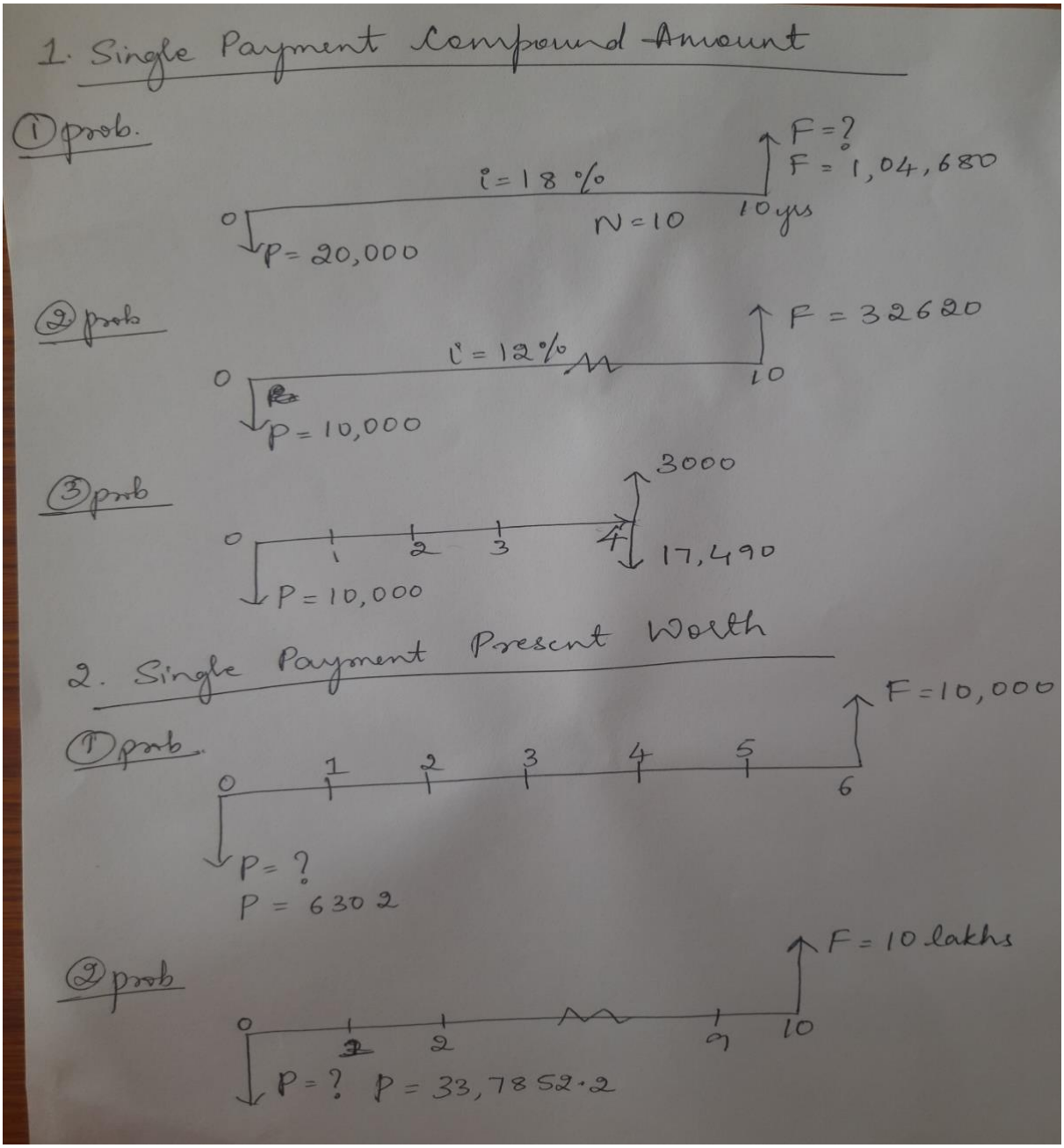
P=10,000(P/F,8%,6)

P=10,000/(1+0.08)^6

P=6302

2. If a person wishes to invest in a private bank that pays rate of interest 11% but compounded quarterly. So that he gets 10,00,000 for 10yrs from now.
   F=10 lakhs
   i=11%
   No. of interest period per year =4
   Total no. of interest periods for 10yrs=4*10=40
   Interest quarterly=0.11/4
   P=F(P/F,0.11/4,40)
   P=10,00,000/(1+0.11/4)^40
   P=3,37,852.22

Cash flow diagrams for the problems solved on single payment compound amount and present worth.



Problems to be solved

1. A future amount F , is equivalent to Rs. 1500. Now when eight years separates the amounts and the annual interest is 12%. What is the value of F? **Ans** : **3713.9**
2. Suppose you borrow Rs.8000 now, with the promise to repay the loan principal plus accumulated interest in 4years at i=10% per year. How much would you owe at the end of 4years? **Ans: 11,713**
3. A person wishes to have a future sum of Rs.10 lakhs for his daughter's engineering education in 15 years from now, what is the single payment

that he should deposit now. So, that he gets the desired amount after 10years? The banks gives 12% rate of interest compounded annually. **Ans: 3,21,973.3**

4. A person wishes to have a future sum of Rs.1,00,000 for his son's marriage after 10years from now. What is the single payment that he should deposit now. So, that he gets the desired amount after 10years? The banks gives 15% interest rate compounded annually. **Ans: 24,720**

Probem 1 .

A person invests a sum of Rs.50,000 in a bank at a nominal interest rate of 18% for 15yrs. The compounding is monthly. Find the maturity amount of the deposit after 15yrs.

P=50,000

i=18% = 18/12=1.5% (compounding monthly)

N=15 * 12=180

F=?

F=P(1+i)^n

F=50000(1+1.5/100) ^180

F=7,29,218.4

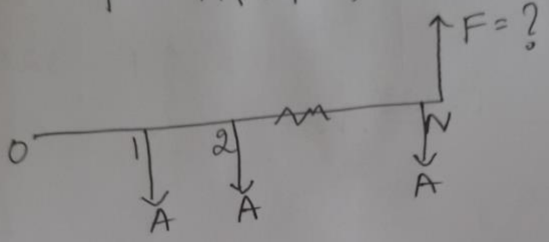
(3) Series Compound-Amount Factor (Uniform Series)

To find F, given A

Symbols: $(F/A, i\%, N)$

Formula: $F = A\dfrac{(1+i)^N - 1}{i}$

$F = A(F/A, i\%, N)$



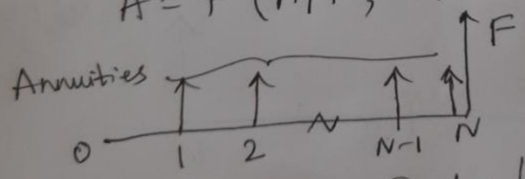
(4) Sinking Fund Factor (Uniform Series)

To find A, given F.

Symbols: $(A/F, i\%, N)$

formula: $A = F\left(\dfrac{i}{(1+i)^N - 1}\right)$

$A = F(A/F, i\%, N)$



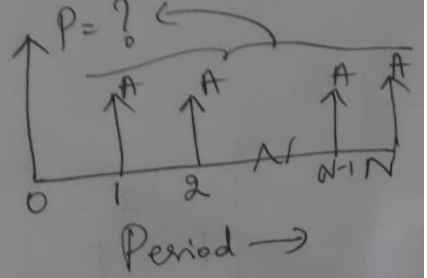
(5) Series Present Worth Factor (Uniform Series)

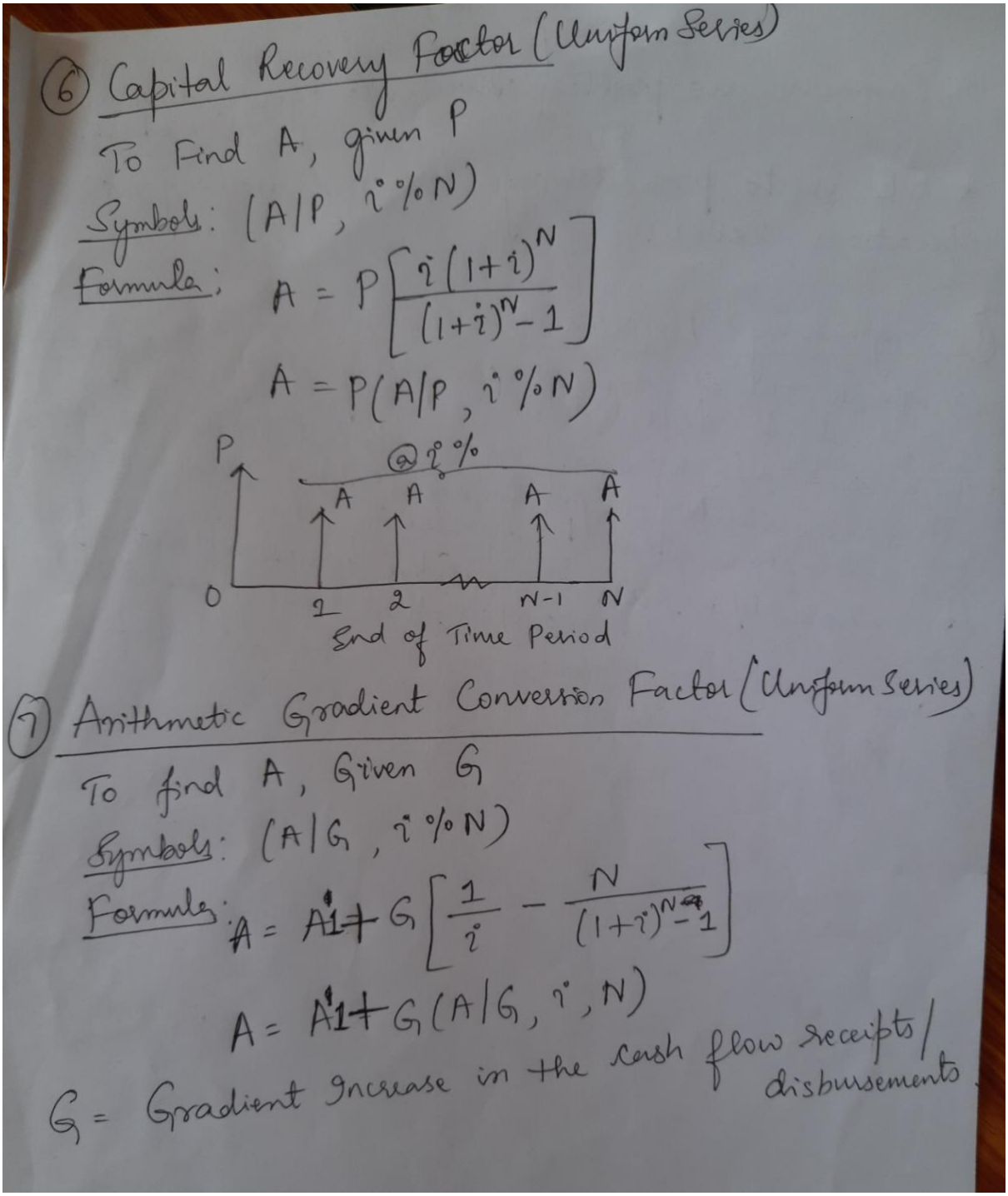
To Find P, given A

Symbols: $(P/A, i\%, N)$

formula: $P = A\left[\dfrac{(1+i)^N - 1}{i(1+i)^N}\right]$

$P = A(P/A, i\%, N)$





3. Series Compound amount factor (uniform series)

To find F, given A

1. A 45 year old person is planning for his retired life. He plans to divert Rs. 30,000 from his bonus as investment every year for the next 15yrs. The banks gives 10%

interest rate compounded annually. Find the maturity value of his account when he is 60yrs old.
A=30,000 , N=15 , i=10% , F=?
F=A(F/A,i%N)
F=30,000[(1+0.1)^15 – 1] / 0.1
F=9,53,174

2. An automobile company recently advertised its car for a down payment of Rs.1,50,000. Alternatively, the car can be taken home by customers without making any payment,but they have to pay an equal yearly amount of Rs.25,000 for 15yrs at an interest rate of 18% compounded annually.
Ans: Car value=P=1,50,000
A=25,000 , N=15 , i=18%
F=A(F/A,i%,N)
F=25000[(1+0.18)^15 – 1]/0.18
F=15,24,131.6

4. Sinking fund factor (Uniform series)

1. A person estimates an expenditure of Rs. 5 lakh for his daughter's wedding about 10yrs from now. He plans to deposit an equal amount at the end of every year for the next 10yrs at the rate of interest of 10% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 10yrs.
F=5,00,000
N=10

A=? i=10%
A=500000(A/F,i%,N)
A=500000 * 0.1 / [(1+0.1)^10 – 1]
A=31,372.7 is the annual amount to be paid.

2.A company has to replace a present facility after 15yrs at an outlay of Rs.5 L . If he plans to deposit an equal amount at the end of every year for the next 15yrs at the rate of interest of 18% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 15yrs.
F=5,00,000 , N=15 , i=18% , A=?
A=F(i/(1+i)^n – 1)
A=500000 (A/F,18%,15)
A=500000 * 0.0164
A=8200

5. Series Present Worth factor (Uniform series)

1. A company wants to set up an reserve which will help the company to have an annual equivalent amount of Rs.10,00,000 for the next 20yrs towards its employees welfare measures.The reserve is assumed to grow at the rate of 15% annually. Find the single payment that must be made now as the reserve amount.
   A=10,00,000 , N=20 , i=15% , P=?
   P=A(P/A,i%,N)
   P=[1000000(1+0.15)^20 – 1] / 0.15(1+0.15)^20

P=62,59,300

2. Suppose that installation of low-loss thermal windows in your area is expected to save Rs.150 a year on your home heating bill for next 18yrs. If you can earn 8% a year on other investments, how much could you afford to spend now for these windows?
   A=150 , N=18 , i=8% , P=?
   P=A(P/A,8%,18)
   P=150 * (1+0.08)^18 – 1 / 0.08(1+0.08)^18
   P=1405.8

6. Capital Recovery Factor (Uniform series)
   1. A proposed product modification to avoid production difficulties will require an immediate expenditure of Rs.14,000 to modify certain dies. What annual savings must be realised to recover this expenditure in 4yrs with interest at 10% ?
      i=10% , N=4 , P=14000

$$A = P * \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A=14000 * \frac{0.1(1+0.1)^4}{(1+0.1)^4 - 1}$$

A=4416.6

2. ICICI bank is offering Rs. 30 lakhs home loan to a person to buy a new apartment at a interest rate of 7.5% compounded annually. This amount should be repaid in 15yrs equal instalments. Find the annual instalment amount the person has to pay to the bank.
   P=30,00,000 N=15 i=7.5% A=?

$$A = P * \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A = 3000000 * \frac{0.075(1+0.075)^{15}}{(1+0.075)^{15} - 1}$$

A=3,39,861 per annum

If the bank decides to compound the rate of interest **monthly rather than annually**,how much less money the person has to pay annually?

No. of interest periods per year = N=12

Total no. of interest periods n=12*15=180

Annual rate of interest=7.5%

Rate of interest per month = 0.075/12=0.00625

$$A = 3000000 * \frac{0.00625(1+0.00625)^{180}}{(1+0.00625)^{180} - 1}$$

A=27,810

He has to pay 3,39,861 – (27810*12)

=6141 less/year.

3. A company 3yrs ago borrowed 40,000 to pay for a new machine tool, agreeing to repay the loan in 100 monthly payments at an annual nominal interest rate of 12% compounded monthly. The company now wants to pay off the loan. How much would this payment be, assuming no penalty costs for early payout?
   P=40,000 i=12% =12/12=1% N=100months A=?

$$A = P * \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$A = 40000 * \frac{0.01(1+0.01)^{100}}{(1+0.01)^{100} - 1}$$

A=634.6

F=P(F/P,1%,36)

=P(1+i)^n

=40000 (1+0.01)^36

F=57230.75

The worth of 40000 for 3yrs is 57,230.75 and installement payed for 3yrs = 634.6*36=22845.6

The pay off amount = 57320.75 – 22845.6 = 34385.2

7.Arithmetic Gradient Conversion Factor (Uniform series)

To find annual equivalent amount of a series with an amount A1 @ end of first year and with an equal increment (G) at the end of (n-1) yrs with i% interest.

1. Assume that an endowment was originally set up to provide a Rs. 10000 as first payment with payments decreasing by 1000 each year during the 10 year endowment life. What constant annual payment for 10yrs would be equivalent to the original endowment plan if i=8% ?
   A1=10,000 , i=8% , G=1000 , N=10

$$A = A1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = 10000 - 1000\left[\frac{1}{0.08} - \frac{10}{(1+0.08)^{10} - 1}\right]$$

A=10000 – 3871.31
A=6128.69

2. A person is planning for his retired life. He has 10 more yrs of service. He would like to deposit 20% of his salary, which is Rs.4000 at the end of 1st year and thereafter he wishes to deposit the amount with an annual increase of Rs.500 for the next 9yrs with an interest rate of 15%. Find the total amount at the end of the 10th year of above series.
   A1=4000 , G=500 , i=15% , N=10 , A=? , F=?

$$A = A1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$A = 4000 + 500\left[\frac{1}{0.15} - \frac{10}{(1+0.15)^{10} - 1}\right]$$

A=4000+500(3.3832)
A=5,691.60
F=A(F/A,15%,10)

F=5691.60 * 20.304
F=1,15,562.25

3. A film star is at the height of his carrier. He wants to invest Rs. 10 lakhs from the end of this year and follow it up with 9 lakhs, 8 lakhs and so on for the next five yrs, when his income would go on diminishing. Find the maturity amount 6 yrs later if a film producer agrees to pay him 15% rate of interest,compounded annually.
   A1=10,00,000 i=15% N=6 G=100000 F=? A=?
   A=A1 – G (A/G,15%,6)
   A=1000000 – 100000(2.09719)
   A=7,90,281

   F=A(F/A,15%,6)
   F=7,90,281 (8.75374) from discrete series table
   F=69,17,914

Problems to be solved

1. A person deposits a sum of Rs.1,00,000 in a bank for his son's education who will be admitted to a professional course after 6yrs. The bank pays 15% interest rate, compounded annually.Find the future amount of the deposited money at the time of admitting his son in the professional course.
   Ans: F=2,31,306
2. A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15yrs from now. Find the amount of money that he should deposit now in a bank if the bank gives 18% interest,compounded annually.**Ans: P=16703.2**
3. A person who is now 35yrs old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25yrs. The banks gives 20% interest rate compounded annually. Find the maturity value of his account when he is 60yrs old. **Ans: F=47,19,810**
4. A woman wishes to have Rs. 1,00,000 in her retirement savings plan after working for 25yrs. She will accomplish this by depositing Rs. A each year in a savings acoount that earns 6% per year. How much must she save each year. **Ans : A=1822**
5. A financial institution introduces a plan to pay a sum of Rs.15,00,000 after 10yrs at the rate of 18% compounded annually. Find the annual equivalent amount that a person should invest at the end of every year for the next 10yrs to receive Rs.15,00,000 after 10yrs from the institution.**Ans:A=63771.96**
6. A person wants to give scholarship to poor students of Rs.25,000 every year. He wants to deposit a lumpsum in the bank which makes him receive the required amount every year for next 20yrs. The reserve is assumed to grow annually at the rate of 9%. Find the single payment that must be made now as the reserve amount?
   Ans: P=2,28,213

7. It is estimated that a certain piece of equipment car save Rs.6000 per year in labour and materials costs. The equipment has an expected life to 5yrs. If the company must earn a 20% rate of return on such investments,how much could be justified now for the purchase of this piece of equipment? Draw a cash flow diagram. **Ans:P=17,943.7**
8. If Rs.25000 is deposited now into a savings account that earns 12% per year, what uniform annual amount could be withdrawn at the end of each year for 10yrs so that nothing would be left in the account after the tenth withdraw? **Ans : A=4422.5**
9. A bank gives a loan to a company to purchase an equivalent worth Rs.10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15yearly equal instalments. Find the instalment amount that the company has to pay to the bank.
   Ans: A=1,96,400
10. A person is planning for his retired life. He has 10 more yrs of service. He would like to deposit 20% of his salary, which is Rs.10,000 at the end of 1^{st} year and thereafter he wishes to deposit the amount with an annual increase of Rs.2000 for the next 9yrs with an interest rate of 20%. Find the total amount at the end of the 10^{th} year of above series.
    Ans : A=16147.7 and F=419173.01