**MODULE 5**

REPLACEMENT ANALYSIS: Introduction, reasons for replacement, Individual Replacement of machinery or equipment with/without value of money, Group Replacement Policies, Problems.

### 7.1 Introduction

Replacement analysis is concerned with the question, when is it time to replace existing equipment with a new one? When the old one wears out." It is possible, after all, to keep a 1957 Chevy running up to the present day, if you're prepared to spend enough time and money on it. Conversely, it may be worth replacing an IBM XT with a Pentium well before the former breaks down. The figure 7.1 clearly explains about replacement of an equipment.

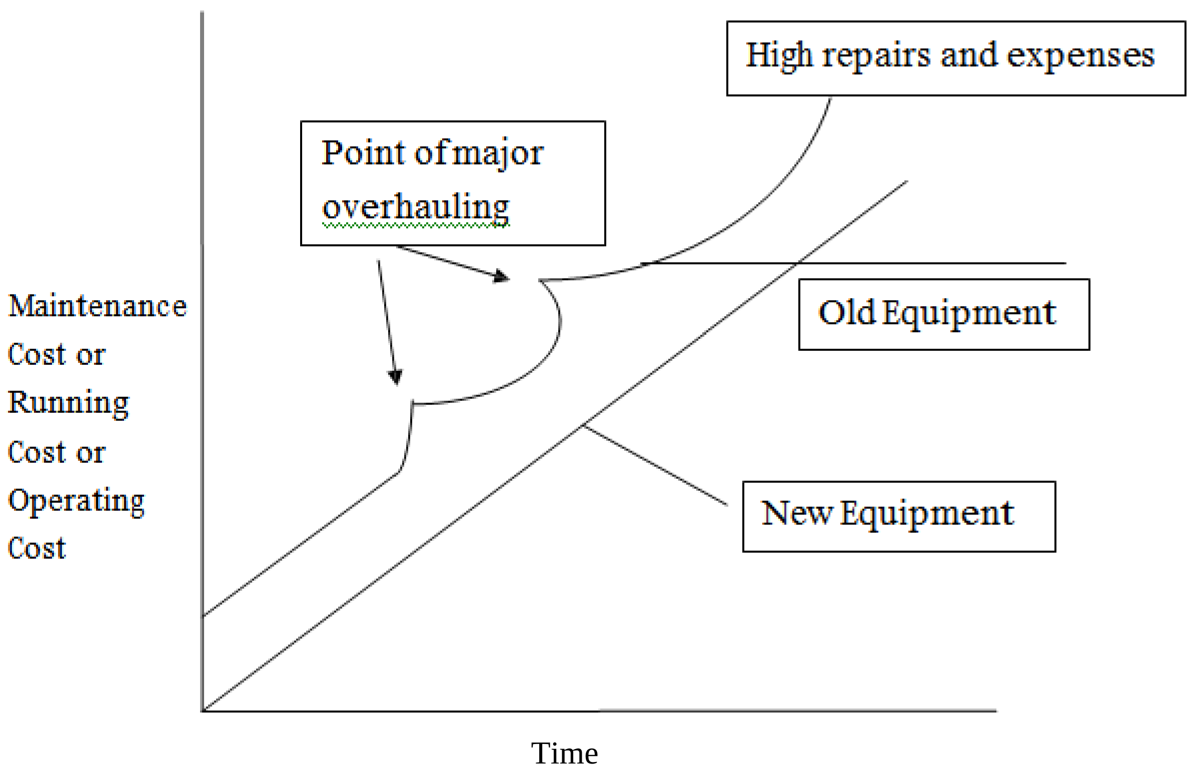


Figure 7.1 Replacement of equipment

### 7.2 Reasons for Replacement

1. Technological development

2. Poor performance over years
3. Unable to meet the required demands

Capital equipment that deteriorates with time: It is concerned with the equipment an machinery that deteriorates with time.Many people feel that equipment should not be replaced until it is physically worn off. But, it is not correct, preferable equipment must be constantly renewed and updated otherwise it will be in the risk of failure or it may become obsolete.

**Reasons for replacement**

1. Deterioration
2. Obsolescence
3. Technological development
4. Inadequacy

Deterioration is the decline in the performance of the equipment as compared to the new equipment. It may occur due to wear and tear. Due to this

a) Increase in the maintenance cost.

b) Reduces the product quality

c) Decreases the rate of production

d) Increases the labor cost

e) Reduces the efficiency of the equipment

**7 .3Models:**

**Model 1**:

"Replacement of items whose maintenance Cost increases with time and the value of the money remains constant during the period"

**Model 2:** "replacement of items whose maintenance cost increases with time and value of money also changes with time".

**Model 3**: "Group Replacement policy"

**Model1**: Notation and symbols

C- Purchase cost of the machinery or equipment

S- Salvage value or resale value or scrap value of the machinery or equipment

Tc- total cost increased on the item or equipment during the period y

$T_C = C + m(Y) - S$

Where M(Y) is the cumulative maintenance cost in that period.

G(Y) $\rightarrow$ Average cost incurred on the equipment or item during the period. G(Y) $= T_C / y$

**Problem 1:**
The cost of the machine is Rs 6100/- and its scrap value is Rs 100 at the end of every year. The maintnance cost found from experience are as follows:

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| M.C | 100 | 250 | 400 | 600 | 900 | 1200 | 1600 | 2000 |

When should the machine be replaced?

**Solution:**

Given: C=Rs 6100

S=Rs 100

| Year (Y) | Maintenance | Cumulative Maintenance cost m(y) | Total Cost C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 100 | 100 | 6100 | 6100 |
| 2 | 250 | 350 | 6350 | 3175 |
| 3 | 400 | 750 | 6750 | 2250 |
| 4 | 600 | 1350 | 7350 | 1837.5 |
| 5 | 900 | 2250 | 8250 | 1650 |
| 6 | 1200 | 3450 | 9450 | 1575 |
| 7 | 1600 | 5050 | 11050 | 1578.5 |
| 8 | 2000 | 7050 | 13050 | 1631.25 |

It's clear from the above analysis that the machine has to replaced at the end of $6^{th}$ year or at the beginning of $7^{th}$ year because the maintenance cost of $7^{th}$ year is more than the average cost of the machine i.e 1578.5 > 1575.5

**Problem 2:**

Machine A costs Rs 9000/- annual operating cost is Rs 200/- for the first year and then increases by Rs 2000/- every year. Determine the best age at which the machine should be replaced and what would be the average cost of owning is operating cost of the machine? Machine B costs Rs 10000/- annual operating cost is Rs 400/- for the first year and then increased by Rs. 800/- every year you now have a machine A which is one year old. Should you replace it with B? If so when? Assume machines have no resale value and that the future costs are not discounted.

**Solution:**

Machine A:

Cost price C = Rs 9000 S = Rs 0

| Year (Y) | Maintenance Cost | Cumulative Maintenance cost m(y) | Total cost ($T_C$ ) C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 200 | 200 | 9200 | 9200 |
| 2 | 2200 | 2400 | 11400 | 5700 |
| 3 | 4200 | 6600 | 15600 | 5200 |
| 4 | 6200 | 12800 | 21800 | 5450 |
| 5 | 8200 | 21000 | 30000 | 6000 |

**Machine B:**

Cost price C =Rs 10,000 S = 0

| Year (Y) | Maintenance Cost | Cumulative Maintenance cost m(y) | Total cost ($T_C$ ) C-S + m(y) | Average Cost g(y) |
|---|---|---|---|---|
| 1 | 400 | 400 | 10400 | 10400 |
| 2 | 1200 | 1600 | 11600 | 5800 |
| 3 | 2000 | 3600 | 13600 | 4533.33 |
| 4 | 2800 | 6400 | 16400 | 4100 |
| 5 | 3600 | 10000 | 20000 | 4000 |
| 6 | 4400 | 14400 | 24400 | 4066.67 |

Machine A should be replaced at the end of the 3rd year and the average cost is 5200/- . machine B should be replaced at the end of 5th year and the average cost is Rs 4000/-. The I year maintenance cost of machine A which is a year old is Rs2200/- < which is less than the average cost of machine B i.e. 4000. The II year maintenance cost of machine A ( 1 year old) is 4200 > avg cost of machine B (i.e. 4000/-) The III year maintenance cost of machine A is 6200/- which is greater than the average cost of machine B. i.e. 4000/-. It is clear from above analysis that machine A that is one year old can be used one more year from now and then replaced with machine B.

**Model 2**

"Replacement of items whose maintenance cost increases with time and the value of money also changes with time" The maintenance cost varies with time and we want to find out the optimum time period at which the items will be replaced value of money decreases with a constant rate which is known as depreciation ratio or discounted factor which is given by V= 1/ (1+i)^n-1

for the value of 1 rupee

where i rate of interest,

n- no. of years

**Problem 5:**

A company buys a machine for Rs 6000/-. The maintenance cost are expected to be Rs 300/- in each year for the first 2 years and go up annually as follows 700, 1000, 1500, 2000, and 2500. Assume the money is worth of 20% per year. When the machine should be replaced.

**Solution**

C = 6000/-

I = 20%

Assumption: - in this to solve the problem we assume that the maintenance cost is

spent on the machine at the beginning of each year as 1.

| Year | Maintenance cost | Present value of 1 Re P/F, 20% | Present value of Maintenance cost | Cumulative Present value Maintenance cost m (y) | Total cost $T_C$ = C-S + m(y) | Cumulative Present value of 1 Re | Weighted Average cost (TC/Cum present Value) |
|---|---|---|---|---|---|---|---|
| 1 | 300 | 1 | 300 | 300 | 6300 | 1 | 6300 |
| 2 | 300 | 0.833 | 249.9 | 549.9 | 6549.9 | 1.833 | 3573.32 |
| 3 | 700 | 0.694 | 486.1 | 1036 | 7036 | 2.527 | 2784.32 |
| 4 | 1000 | 0.578 | 578 | 1614 | 7614 | 3.105 | 2452.17 |
| 5 | 1500 | 0.482 | 723 | 2337 | 8337 | 3.587 | 2324.22 |
| 6 | 2000 | 0.401 | 802 | 3139 | 9139 | 3.988 | 2291.62 |
| 7 | 2500 | 0.334 | 835 | 3974 | 9974 | 4.322 | 2307.72 |

Specimen calculations for 3$^{rd}$ year by

$V = 1/(1+i)^{n-1}$

$1/(1.2)^2$

=0.694

=0.694*700 = 486.1

The machine should be replaced at the end of 6$^{th}$ year or at the beginning of 7$^{th}$ year because the maintenance cost in the 7$^{th}$ year is more than the average cost of the machine 2500 > 2291.62

**Problem 6:**

A machine cost Rs 10000/- the operating cost is Rs 500/- for the first 5 years and then increased by Rs 100/- every year subsequently from the 6$^{th}$ year onwards.Assuming the money is worth of 10% per year. Find the optimal length of time to hold the machine before replacement.

**Solution**:

C=10,000, i = 10%

S = 0

Assumption:

To solve the problem, we assume that maintenance cost spent on the equipment is at the beginning of each year.

Specimen Calculations for 3rd year

V= 1/ (1+i)^n-1

$= 1/ (1.1)^3$

=0.826

= 0.826 x 500

= 413

| Year | Maintenance cost | Present value of 1 Re | Present value of Maintenance cost | Cumulative Present value Maintenance cost m (y) | Total cost $T_C$ =C-S + m(y) | Cumulative Present value of 1 Re | Weighted Average cost |
|---|---|---|---|---|---|---|---|
| 1 | 500 | 1 | 500 | 500 | 10500 | 1 | 10500 |
| 2 | 500 | 0.909 | 454.5 | 954.5 | 10954.5 | 1.909 | 5738.34 |
| 3 | 500 | 0.826 | 413 | 1367.5 | 11367.5 | 2.735 | 4156.31 |
| 4 | 500 | 0.751 | 375.5 | 1743 | 11743 | 3.486 | 3368.62 |
| 5 | 500 | 0.683 | 341.5 | 2084.5 | 12084.5 | 4.169 | 2898.66 |
| 6 | 600 | 0.621 | 372.6 | 2457.1 | 12457.1 | 4.79 | 2600.65 |
| 7 | 700 | 0.564 | 394.8 | 2851.9 | 12851.9 | 5.354 | 2400.43 |
| 8 | 800 | 0.513 | 410.4 | 3262.3 | 13262.3 | 5.867 | 2260.49 |
| 9 | 900 | 0.466 | 419.4 | 3681.7 | 13681.7 | 6.333 | 2160.38 |
| 10 | 1000 | 0.424 | 424 | 4105.7 | 14105.7 | 6.757 | 2087.57 |
| 11 | 1100 | 0.385 | 423.5 | 4529.2 | 14529.2 | 7.142 | 2034.33 |
| 12 | 1200 | 0.350 | 420 | 4949.2 | 14949.2 | 7.492 | 1995.36 |
| 13 | 1300 | 0.318 | 413.4 | 5362.6 | 15362.6 | 7.81 | 1967.04 |
| 14 | 1400 | 0.289 | 404.6 | 5767.2 | 15767.2 | 8.099 | 1946.81 |
| 15 | 1500 | 0.263 | 394.5 | 6161.7 | 16161.7 | 8.362 | 1932.76 |
| 16 | 1600 | 0.239 | 382.4 | 6544.1 | 16544.1 | 8.601 | 1923.51 |
| 17 | 1700 | 0.217 | 368.9 | 6913 | 16913 | 8.818 | 1918.01 |
| 18 | 1800 | 0.198 | 356.4 | 7269.4 | 17269.4 | 9.016 | 1915.42 |
| 19 | 1900 | 0.180 | 342 | 7611.4 | 17611.4 | 9.196 | 1915.11 |
| 20 | 2000 | 0.163 | 326 | 7937.4 | 17937.4 | 9.359 | 1916.59 |

Conclusion: The machine should be replaced at the end of $19^{th}$ year or at the beginning of 20 year because the maintenance cost in the $20^{th}$ year is more than the average cost of $19^{th}$ year i.e. 1916.59 > 1915.11

It is better to replace at the end of $10^{th}$ year Rs 18000.40/-

***EXAMPLE*** A firm is considering replacement of an equipment, whose first cost is Rs. 4,000 and the scrap value is negligible at the end of any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 200 every year thereafter.

(a) When should the equipment be replaced if $i$ = 0%?

(b) When should the equipment be replaced if $i$ = 12%?

(a) When $i$ = 0%. In this problem,

(i) First cost = Rs. 4,000

(ii) Maintenance cost is Rs. 0 during the first year and it increases by Rs. 200 every year thereafter.

This is summarized in column B of Table 8.1.

**Table 8.1** Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 0%)

| *End of year* (*n*) | *Maintenance cost at end of year* | *Summation of maintenance costs* | *Average cost of maintenance through year given* | *Average first cost if replaced at year end given* | *Average total cost through year given* |
|---|---|---|---|---|---|
| | | ΣB | C/A | 4,000/A | D + E |
| A | B (Rs.) | C (Rs.) | D (Rs.) | E (Rs.) | F (Rs.) |
| 1 | 0 | 0 | 0 | 4,000.00 | 4,000.00 |
| 2 | 200 | 200 | 100 | 2,000.00 | 2,100.00 |
| 3 | 400 | 600 | 200 | 1,333.33 | 1,533.33 |
| 4 | 600 | 1,200 | 300 | 1,000.00 | 1,300.00 |
| 5 | 800 | 2,000 | 400 | 800.00 | 1,200.00 |
| 6 | 1,000 | 3,000 | 500 | 666.67 | 1,166.67* |
| 7 | 1,200 | 4,200 | 600 | 571.43 | 1,171.43 |

*Economic life of the machine = 6 years

Column C summarizes the summation of maintenance costs for each replacement period. The value corresponding to any end of year in this column represents the total maintenance cost of using the equipment till the end of that particular year.

$$\text{Average total cost} = \frac{\text{First cost (FC)} + \text{Summation of maintenance cost}}{\text{Replacement period}}$$

$$= \frac{\text{FC}}{n} + \frac{\text{Column C}}{n}$$

$$= \begin{matrix}\text{Average first cost for} \\ \text{the given period}\end{matrix} + \begin{matrix}\text{Average maintenance cost} \\ \text{for the given period}\end{matrix}$$

$$\text{Column F} = \text{Column E} + \text{Column D}$$

The value corresponding to any end of year ($n$) in Column F represents the average total cost of using the equipment till the end of that particular year.

For this problem, the average total cost decreases till the end of year 6 and then it increases. Therefore, the optimal replacement period is six years, i.e. economic life of the equipment is six years.

(b) When interest rate, $i$ = 12%. When the interest rate is more than 0%, the steps to be taken for getting the economic life are summarized with reference to Table 8.2.

**Table 8.2** Calculations to Determine Economic Life (First cost = Rs. 4,000, Interest = 12%)

| *End of year* (*n*) | *Maintenance cost at end of year* | *P/F*, 12%, *n* | *Present worth as of beginning of year 1 of maintenance costs* | *Summation of present worth of maintenance costs through year given* | *Present worth of cumulative maintenance cost & first cost* | *A/P*, 12%, *n* | *Annual equivalent total cost through year given* |
|---|---|---|---|---|---|---|---|
| | | | (B × C) | Σ D | E + Rs. 4,000 | | F × G |
| A | B (Rs) | C | D (Rs.) | E (Rs.) | F (Rs.) | G | H (Rs.) |
| 1 | 0 | 0.8929 | 0.00 | 0.00 | 4,000.00 | 1.1200 | 4,480.00 |
| 2 | 200 | 0.7972 | 159.44 | 159.44 | 4,159.44 | 0.5917 | 2,461.14 |
| 3 | 400 | 0.7118 | 284.72 | 444.16 | 4,444.16 | 0.4163 | 1,850.10 |
| 4 | 600 | 0.6355 | 381.30 | 825.46 | 4,825.46 | 0.3292 | 1,588.54 |
| 5 | 800 | 0.5674 | 453.92 | 1,279.38 | 5,279.38 | 0.2774 | 1,464.50 |
| 6 | 1,000 | 0.5066 | 506.60 | 1,785.98 | 5,785.98 | 0.2432 | 1,407.15 |
| 7 | 1,200 | 0.4524 | 542.88 | 2,328.86 | 6,328.86 | 0.2191 | 1,386.65* |
| 8 | 1,400 | 0.4039 | 565.46 | 2,894.32 | 6,894.32 | 0.2013 | 1,387.83 |
| 9 | 1,600 | 0.3606 | 576.96 | 3,471.28 | 7,471.28 | 0.1877 | 1,402.36 |
| 10 | 1,800 | 0.3220 | 579.60 | 4,050.88 | 8,050.88 | 0.1770 | 1,425.00 |

*Economic life of the machine = 7 years

The steps are summarized now:

1. Discount the maintenance costs to the beginning of year 1.

$$\text{Column D} = \text{Column B} \times \frac{1}{(1+i)^n}$$

$$= \text{Column B} \times (P/F,\, i,\, n) = \text{Column B} \times \text{Column C}.$$

2. Find the summation of present worth of maintenance costs through the year given (Column E = Σ Column D).

3. Find Column F by adding the first cost of Rs. 4,000 to Column E.
4. Find the annual equivalent total cost through the years given.

$$\text{Column H} = \text{Column F} \times \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$= \text{Column F} \times (A/P,\ 12\%,\ n) = \text{Column F} \times \text{Column G}$$

5. Identify the end of year for which the annual equivalent total cost is minimum.

For this problem, the annual equivalent total cost is minimum at the end of year 7. Therefore, the economic life of the equipment is seven years.

***EXAMPLE*** The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.

Find the economic life of the machine assuming interest rate, $i$ = 15%.

| *End of year* ($n$) | *Operation cost at the end of year (Rs.)* | *Maintenance cost at the end of year (Rs.)* | *Salvage value at the end of year (Rs.)* |
|---|---|---|---|
| 1 | 3,000 | 300 | 9,000 |
| 2 | 4,000 | 400 | 8,000 |
| 3 | 5,000 | 500 | 7,000 |
| 4 | 6,000 | 600 | 6,000 |
| 5 | 7,000 | 700 | 5,000 |
| 6 | 8,000 | 800 | 4,000 |
| 7 | 9,000 | 900 | 3,000 |
| 8 | 10,000 | 1,000 | 2,000 |
| 9 | 11,000 | 1,100 | 1,000 |
| 10 | 12,000 | 1,200 | 0 |

***Solution***

First cost = Rs. 20,000

Interest rate = 15%

The other details are summarized in Table 8.3 along with regular calculations for determining the economic life.

**Table 8.3** Calculations to Determine Economic Life

(First Cost = Rs. 20,000, Interest Rate = 15%)

| *End of year* (*n*) | *Operation cost at the end of year* | *Maintenance cost at the end of year* | *Sum of operation and maintenance costs at the end of year* | *P/F, 15%, n* | *Present worth as of beginning of year 1 of sum of operation & maintenance cost* | *Cumulative sum of column F through year designated* | *Salvage value at the end of year* | *Present worth as of beginning of year 1 of salvage value* | *Total present worth* | *A/P, 15%, n* | *Annual equlant. total cost through year given* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | B + C | | D × E | Σ F | | H × E | G + 20,000 – I | | J × K |
| A | B (Rs.) | C (Rs.) | D (Rs.) | E | F (Rs.) | G (Rs.) | H (Rs.) | I (Rs.) | J (Rs.) | K | L (Rs.) |
| 1 | 3,000 | 300 | 3,300 | 0.8696 | 2,869.68 | 2,869.68 | 9,000 | 7,826.40 | 15,043.28 | 1.1500 | 17,299.77 |
| 2 | 4,000 | 400 | 4,400 | 0.7562 | 3,326.84 | 6,196.52 | 8,000 | 6,048.80 | 20,147.72 | 0.6151 | 12,392.86 |
| 3 | 5,000 | 500 | 5,500 | 0.6575 | 3,616.25 | 9,812.77 | 7,000 | 4,602.50 | 25,210.27 | 0.4380 | 11,042.01 |
| 4 | 6,000 | 600 | 6,600 | 0.5718 | 3,773.88 | 13,586.65 | 6,000 | 3,430.80 | 30,155.85 | 0.3503 | 10,563.59 |
| 5 | 7,000 | 700 | 7,700 | 0.4972 | 3,828.44 | 17,415.09 | 5,000 | 2,486.00 | 34,929.09 | 0.2983 | 10,419.35 * |
| 6 | 8,000 | 800 | 8,800 | 0.4323 | 3,804.24 | 21,219.33 | 4,000 | 1,729.20 | 39,490.13 | 0.2642 | 10,433.29 |
| 7 | 9,000 | 900 | 9,900 | 0.3759 | 3,721.41 | 24,940.74 | 3,000 | 1,127.70 | 43,813.04 | 0.2404 | 10,532.66 |

*Economic Life = 5 years

Total annual equivalent cost

$$= \left[ \begin{array}{c}\text{Cumulative sum of} \\ \text{present worth as of} \\ \text{beginning of year 1} \\ \text{of operation and} \\ \text{maintenance costs}\end{array} + \begin{array}{c}\text{First} \\ \text{cost}\end{array} - \begin{array}{c}\text{Present worth} \\ \text{as of beginning} \\ \text{of year 1 of} \\ \text{salvage value}\end{array} \right] \times (A/P,\ 15\%,\ n)$$

i.e. Column L = (Column G + 20,000 – Column I) × Column K
= Column J × Column K

In Column L, the annual equivalent total cost is minimum for $n = 5$. Therefore, the economic life of the machine is five years.

***EXAMPLE 8.3*** A company has already identified machine A and determined the economic life as four years by assuming 15% interest rate. The annual equivalent total cost corresponding to the economic life is Rs. 2,780.

Now, the manufacturer of machine B has approached the company. Machine B, which has the same capacity as that of machine A, is priced at Rs. 6,000. The maintenance cost of machine B is estimated at Rs. 1,500 for the first year and an equal yearly increment of Rs. 300 thereafter.

If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

*Determination of economic life and corresponding annual equivalent total cost of machine B.* The details of machine B are summarized in Table 8.4 along with the usual calculations to determine the economic life.

**Table 8.4** Calculations to Determine Economic Life (First Cost = Rs. 6,000, Interest = 15%)

| *End of year* ($n$) | *Maintenance cost for end of year* | *P/F, 15%, n* | *Present worth as of beginning of year 1 of maintenance costs* | *Summation of present worth of maintenance costs through year given* | *Column E + Rs. 6000* | *A/P, 15%, n* | *Annual equivalent total cost through year given* |
|---|---|---|---|---|---|---|---|
| | | | B × C | Σ D | | | F × G |
| A | B (Rs) | C | D (Rs.) | E (Rs.) | F (Rs.) | G | H (Rs.) |
| 1 | 1,500 | 0.8696 | 1,304.40 | 1,304.40 | 7,304.40 | 1.1500 | 8,400.06 |
| 2 | 1,800 | 0.7561 | 1,360.98 | 2,665.38 | 8,665.38 | 0.6151 | 5,330.08 |
| 3 | 2,100 | 0.6575 | 1,380.75 | 4,046.13 | 10,046.13 | 0.4380 | 4,400.21 |
| 4 | 2,400 | 0.5718 | 1,372.32 | 5,418.45 | 11,418.45 | 0.3503 | 3,999.88 |
| 5 | 2,700 | 0.4972 | 1,342.44 | 6,760.89 | 12,760.89 | 0.2983 | 3,806.57 |
| 6 | 3,000 | 0.4323 | 1,296.90 | 8,057.79 | 14,057.79 | 0.2642 | 3,714.07 |
| 7 | 3,300 | 0.3759 | 1,240.47 | 9,298.26 | 15,298.26 | 0.2404 | 3,677.70 |
| 8 | 3,600 | 0.3269 | 1,176.84 | 10,475.10 | 16,475.10 | 0.2229 | 3,672.30 * |
| 9 | 3,900 | 0.2843 | 1,108.77 | 11,583.87 | 17,583.87 | 0.2096 | 3,685.58 |
| 10 | 4,200 | 0.2472 | 1,038.24 | 12,622.11 | 18,622.11 | 0.1993 | 3,711.39 |

*Economic life of the machine = 8 years

Column B of Table 8.4 summarizes the yearly maintenance costs of machine B. The first cost of machine B is equal to Rs. 6,000.

Annual equivalent total cost

$$= \left[\begin{array}{l}\text{Summation of present worth} \\ \text{of maintenance cost through} \\ \text{year}\end{array} + \begin{array}{c}\text{First} \\ \text{cost}\end{array}\right] \times (A/P,\ 15\%,\ n)$$

$$= (\text{Column E} + \text{Rs. } 6{,}000) \times \text{Column G}$$

$$= \text{Column F} \times \text{Column G}$$

In Column H, the minimum annual equivalent total cost occurs when $n$ is equal to 8. Hence the economic life of machine B is 8 years and the corresponding annual equivalent total cost is Rs. 3,672.30.

RESULT

Minimum annual equivalent total cost for machine A = Rs. 2,780
Minimum annual equivalent total cost for machine B = Rs. 3,672.30

Since the minimum annual equivalent total cost of machine A is less than that of machine B, machine A is selected as the best machine which has the economic life of four years. (*Note:* Selection of the best machine is based on the minimum annual equivalent total cost. The comparison is made over the minimum common multiple of the lives of machine A and machine B, i.e. $4 \times 2 = 8$ years).

## 8.5 REPLACEMENT OF EXISTING ASSET WITH A NEW ASSET

In this section, the concept of comparison of replacement of an existing asset with a new asset is presented. In this analysis, the annual equivalent cost of each alternative should be computed first. Then the alternative which has the least cost should be selected as the best alternative. Before discussing details, some preliminary concepts which are essential for this type of replacement analysis are presented.

### 8.5.1 Capital Recovery with Return

Consider the following data of a machine. Let

$P$ = purchase price of the machine,

$F$ = salvage value of the machine at the end of machine life,

$n$ = life of the machine in years, and

$i$ = interest rate, compounded annually

The corresponding cash flow diagram is shown in Fig. 8.3.

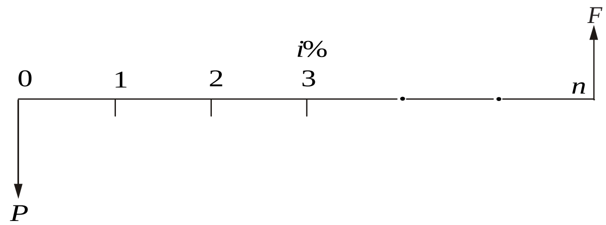


**Fig. 8.3** Cash flow diagram of machine.

The equation for the annual equivalent amount for the above cash flow diagram is

$$AE(i) = (P - F) \times (A/P, i, n) + F \times i + A$$

This equation represents the *capital recovery with return*.

### 8.5.2 Concept of Challenger and Defender

If an existing equipment is considered for replacement with a new equipment, then the existing equipment is known as the *defender* and the new equipment is known as *challenger*.

Assume that an equipment has been purchased about three years back for Rs. 5,00,000 and it is considered for replacement with a new equipment. The supplier of the new equipment will take the old one for some money, say, Rs. 3,00,000. This should be treated as the present value of the existing equipment and it should be considered for all further economic analysis. The purchase value of the existing equipment before three years is now known as *sunk cost*, and it should not be considered for further analysis.

***EXAMPLE 8.4*** Two years ago, a machine was purchased at a cost of Rs. 2,00,000 to be useful for eight years. Its salvage value at the end of its life is Rs. 25,000. The annual maintenance cost is Rs. 25,000. The market value of the present machine is Rs. 1,20,000. Now, a new machine to cater to the need of the present machine is available at Rs. 1,50,000 to be useful for six years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 12%, find whether it is worth replacing the present machine with the new machine.

***Solution*** ***Alternative 1—Present machine***

Purchase price = Rs. 2,00,000

Present value ($P$) = Rs. 1,20,000

Salvage value ($F$) = Rs. 25,000

Annual maintenance cost ($A$) = Rs. 25,000

Remaining life = 6 years

Interest rate = 12%

The cash flow diagram of the present machine is illustrated in Fig. 8.4. The

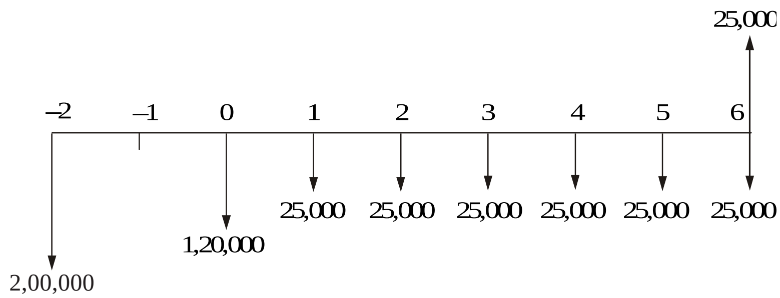


**Fig. 8.4** Cash flow diagram for alternative 1.

annual maintenance cost for the preceding periods are not shown in this figure. The annual equivalent cost is computed as

$$
\begin{aligned}
AE(12\%) &= (P - F)(A/P, 12\%, 6) + F \times i + A \\
&= (1{,}20{,}000 - 25{,}000)(0.2432) + 25{,}000 \times 0.12 + 25{,}000 \\
&= \text{Rs. } 51{,}104
\end{aligned}
$$

***Alternative 2—New machine***

Purchase price ($P$) = Rs. 1,50,000
Salvage value ($F$) = Rs. 20,000
Annual maintenance cost ($A$) = Rs. 14,000
Life = 6 years
Interest rate = 12%

The cash flow diagram of the new machine is depicted in Fig. 8.5.

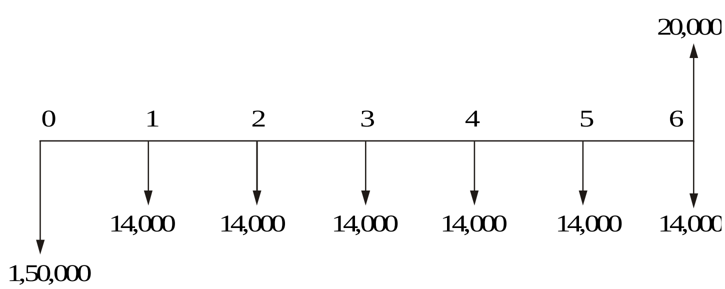


**Fig. 8.5** Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$
\begin{aligned}
AE(12\%) &= (P - F)(A/P, 12\%, 6) + F \times i + A \\
&= (1{,}50{,}000 - 20{,}000)(0.2432) + 20{,}000 \times 0.12 + 14{,}000 \\
&= \text{Rs. } 48{,}016
\end{aligned}
$$

Since the annual equivalent cost of the new machine is less than that of the present machine, it is suggested that the present machine be replaced with the new machine.

***EXAMPLE 8.5*** A diesel engine was installed 10 years ago at a cost of Rs. 50,000. It has a present realizable market value of Rs. 15,000. If kept, it can be expected to last five years more, with operating and maintenance cost of Rs. 14,000 per year and to have a salvage value of Rs. 8,000 at the end of the fifth year. This engine can be replaced with an improved version costing Rs. 65,000 which has an expected life of 20 years. This improved version will have an estimated annual operating and maintenance cost of Rs. 9,000 and ultimate salvage value of Rs. 13,000. Using an interest rate of 15%, make an annual equivalent cost analysis to determine whether to keep or replace the old engine.

***Solution   Alternative 1—Old diesel engine***

Purchase price = Rs. 50,000
Present value ($P$) = Rs. 15,000
Salvage value ($F$) = Rs. 8,000
Annual operating and maintenance cost ($A$) = Rs. 14,000
Remaining life ($n$) = 5 years
Interest rate = 15%

The cash flow diagram of the old diesel engine is shown in Fig. 8.6.

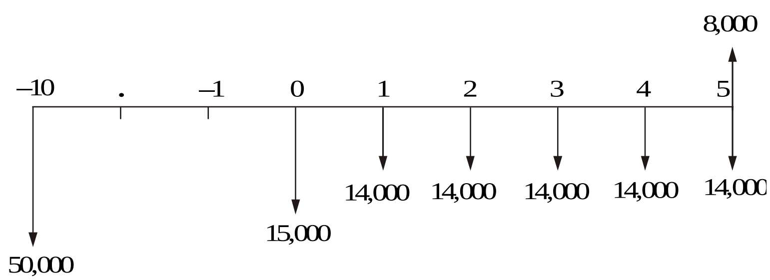


**Fig. 8.6** Cash flow diagram for alternative 1.

The formula for the annual equivalent cost is

$$
\begin{aligned}
AE(15\%) &= (P - F)(A/P, 15\%, 5) + F \times i + A \\
&= (15{,}000 - 8{,}000)(0.2983) + 8{,}000 \times 0.15 + 14{,}000 \\
&= \text{Rs. } 17{,}288.10
\end{aligned}
$$

***Alternative 2—New diesel engine***

Present value ($P$) = Rs. 65,000
Salvage value ($F$) = Rs. 13,000
Annual operating and maintenance cost ($A$) = Rs. 9,000
Life ($n$) = 20 years
Interest rate = 15%

The cash flow diagram of the new diesel engine is shown in Fig. 8.7.

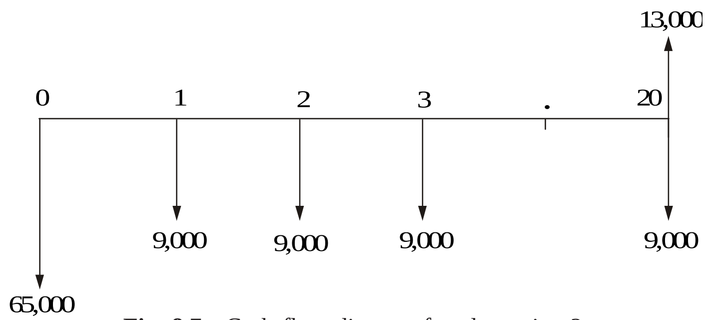


**Fig. 8.7** Cash flow diagram for alternative 2.

The formula for the annual equivalent cost is

$$\begin{aligned} AE(15\%) &= (P - F)(A/P,\ 15\%,\ 20) + F \times i + A \\ &= (65{,}000 - 13{,}000)(0.1598) + 13{,}000 \times 0.15 + 9{,}000 \\ &= \text{Rs. } 19{,}259.60 \end{aligned}$$

For comparing the engines based on equal lives (20 years), the annual equivalent figures are given in Fig. 8.8. Equal lives are nothing but the least common multiple of the lives of the alternatives.

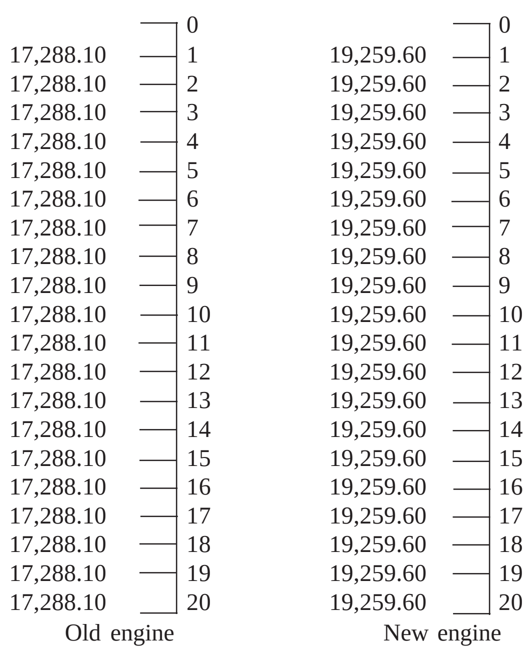


**Fig. 8.8** Cash flow diagram of alternatives based on common lives.

Since the annual equivalent cost of the old diesel engine is less than that of the new diesel engine, it is suggested to keep the old diesel engine. Here, an important assumption is that the old engine will be replaced four times during the 20 years period of comparison.

***EXAMPLE 8.6*** A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 6,60,000 and would make the bridge fit for an additional five years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 4,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 15,00,000 and would meet the foreseeable requirements of the next 40 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 96,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 4,20,000. Assume that the money costs the state 10%. What would you recommend?

***Solution*** There are two alternatives:

1. Reinforce the existing bridge.
2. Replace the existing bridge by a new prestressed concrete bridge.

***Alternative 1—Reinforce the existing bridge***

Cost of reinforcement ($P$) = Rs. 6,60,000
Salvage value after 5 years ($F$) = Rs. 4,00,000
The excess annual maintenance cost over prestressed concrete bridge ($A$) = Rs. 96,000
Life ($n$) = 5 years
Interest rate ($i$) = 10%

The cash flow diagram of alternative 1 is illustrated in Fig. 8.9.

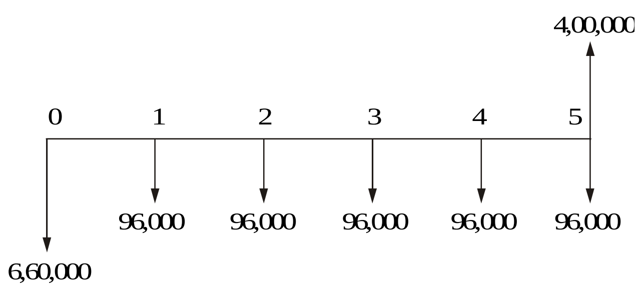


**Fig. 8.9** Cash flow diagram for alternative 1.

The annual equivalent cost of the alternative 1 is computed as

$$\begin{aligned} AE(10\%) &= (P - F)(A/P, 10\%, 5) + F \times i + A \\ &= (6{,}60{,}000 - 4{,}00{,}000)(0.2638) + 4{,}00{,}000 \times 0.10 + 96{,}000 \\ &= \text{Rs. } 2{,}04{,}588 \end{aligned}$$

***Alternative 2—Replace the existing bridge by a new prestressed concrete bridge***

Cost of prestressed concrete bridge ($P$) = Rs. 15,00,000
Excess scrap value of steel over the demolition cost of the current bridge ($X$) = Rs. 4,20,000

Life ($n$) = 40 years
Interest rate ($i$) = 10%

Note that the excess maintenance cost of the reinforced bridge over the prestressed concrete bridge is included in alternative 1.

The cash flow diagram for alternative 2 is shown in Fig. 8.10.

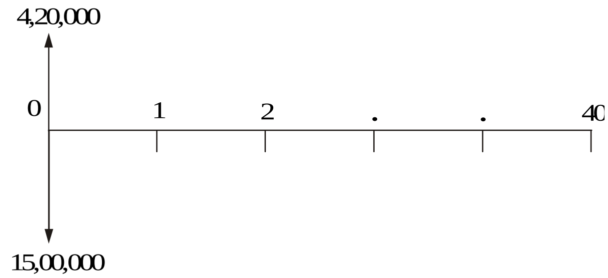


**Fig. 8.10** Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is calculated as

$$\begin{aligned} AE(10\%) &= (P - X)\,(A/P,\ 10\%,\ 40) \\ &= (15{,}00{,}000 - 4{,}20{,}000) \times 0.1023 \\ &= \text{Rs. } 1{,}10{,}484 \end{aligned}$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Based on equal lives comparison over 40 years, alternative 2 is selected as the best alternative.

Thus, it is suggested to go in for prestressed concrete bridge.

***EXAMPLE 8.7*** Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Due to the fast development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are now tabulated.

| | *Old 10 hp motor* | *New 5 hp motor* | *New 15 hp motor* |
|---|---|---|---|
| Purchase cost ($P$) Rs. | 25,000 | 10,000 | 35,000 |
| Life in years ($n$) | 10 | 7 | 7 |
| Salvage value at the end of machine life (Rs.) | 1,500 | 800 | 4,000 |
| Annual operating & maintenance cost (Rs.) | 1,600 | 1,000 | 500 |

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 15%, find the best alternative.

***Solution*** There are two alternatives to cope with the situation:

1. Augmenting the present 10 hp motor with an additional 5 hp motor.
2. Replacing the present 10 hp motor with a new 15 hp motor.

***Alternative 1—Augmenting the present 10 hp motor with an additional 5 hp motor***

Total annual equivalent cost = Annual equivalent cost of 10 hp motor + Annual equivalent cost of 5 hp motor

**Calculation of annual equivalent cost of 10 hp Motor**

Present market value of the 10 hp motor ($P$) = Rs. 10,000
Remaining life ($n$) = 7 years
Salvage value at the end of motor life ($F$) = Rs. 1,500
Annual operation and maintenance cost ($A$) = Rs. 1,600
Interest rate, $i$ = 15%

The cash flow diagram of this alternative is shown in Fig. 8.11.

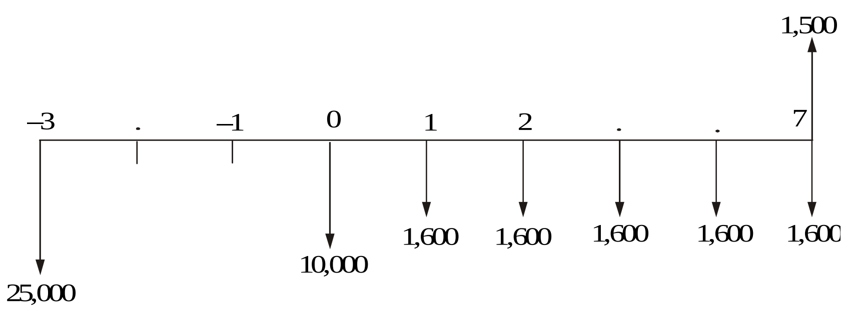


**Fig. 8.11** Cash flow diagram for 10 hp motor.

The annual equivalent cost of the 10 hp motor is calculated as

$$
\begin{aligned}
AE(15\%) &= (P - F)(A/P, 15\%, 7) + F \times i + A \\
&= (10{,}000 - 1{,}500)(0.2404) + 1{,}500 \times 0.15 + 1{,}600 \\
&= \text{Rs. } 3{,}868.40
\end{aligned}
$$

**Calculation of annual equivalent cost of 5 hp motor**

Purchase value of the 5 hp motor ($P$) = Rs. 10,000
Life ($n$) = 7 years
Salvage value at the end of motor life ($F$) = Rs. 800
Annual operation and maintenance cost ($A$) = Rs. 1,000
Interest rate, $i$ = 15%

The cash flow diagram of the 5 hp motor is illustrated in Fig. 8.12.

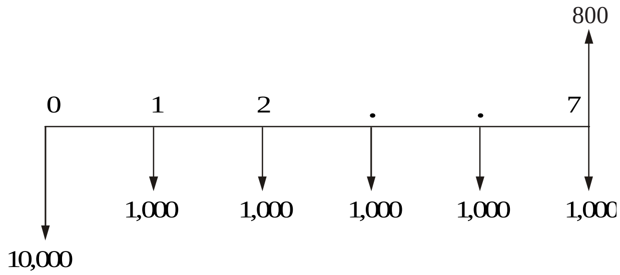

**Fig. 8.12** Cash flow diagram for 5 hp motor.

The annual equivalent cost of the 5 hp motor is computed as

$$AE(15\%) = (P - F)(A/P, 15\%, 7) + F \times i + A$$
$$= (10{,}000 - 800)(0.2404) + 800 \times 0.15 + 1{,}000$$
$$= \text{Rs. } 3{,}331.68$$

Total annual equivalent cost of the alternative 1 = Rs. 3,868.40
+ Rs. 3,331.68
= Rs. 7,200.08

***Alternative 2—Replacing the present 10 hp motor with a new 15 hp motor***

Purchase value of the 15 hp motor ($P$) = Rs. 35,000
Life ($n$) = 7 years
Salvage value at the end of motor life ($F$) = Rs. 4,000
Annual operation and maintenance cost ($A$) = Rs. 500
Interest rate, $i$ = 15%

The cash flow diagram of this alternative is shown in Fig. 8.13.

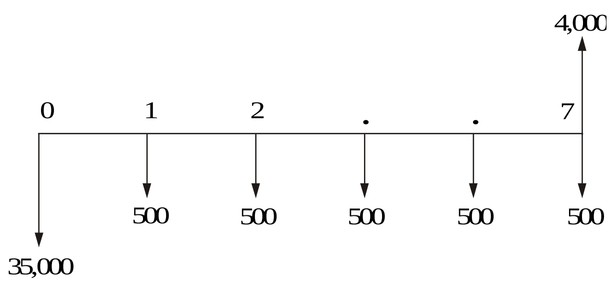

**Fig. 8.13** Cash flow diagram for alternative 2.

The annual equivalent cost of alternative 2 is

$$AE(15\%) = (P - F)\ (A/P, 15\%, 7) + F \times i + A$$
$$= (35{,}000 - 4{,}000)(0.2404) + 4{,}000 \times 0.15 + 500$$
$$= \text{Rs. } 8{,}552.40$$

The total annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, it is suggested that the present 10 hp motor be augmented with a new 5 hp motor.

***EXAMPLE 8.8*** A machine was purchased two years ago for Rs. 10,000. Its annual maintenance cost is Rs. 750. Its life is six years and its salvage value at the end of its life is Rs. 1,000. Now, a company is offering a new machine at a cost of Rs. 10,000. Its life is four years and its salvage value at the end of its life is Rs. 4,000. The annual maintenance cost of the new machine is Rs. 500. The company which is supplying the new machine is willing to take the old machine for Rs. 8,000 if it is replaced by the new machine. Assume an interest rate of 12%, compounded annually.

(a) Find the comparative use value of the old machine.
(b) Is it advisable to replace the old machine?

***Solution Old machine*** Let the comparative use value of the old machine be $X$.

Remaining life ($n$) = 4 years.
Salvage value of the old machine ($F$) = Rs. 1,000
Annual maintenance cost ($A$) = Rs. 750
Interest rate, $i$ = 12%

The cash flow diagram of the old machine is depicted in Fig. 8.14.

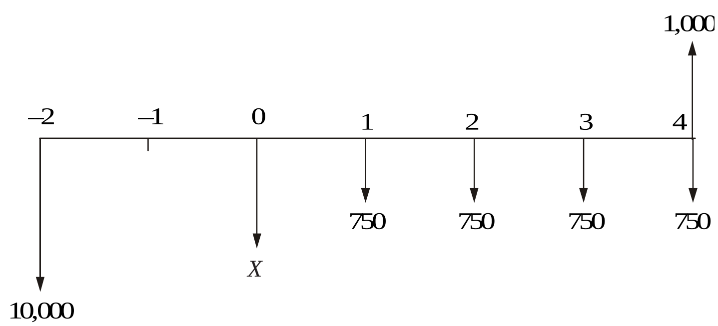


**Fig. 8.14** Cash flow diagram for old machine.

The annual equivalent cost of the old machine is computed as

$$AE(12\%) = (X - F)(A/P, 12\%, 4) + F \times i + A$$
$$= (X - 1{,}000)(0.3292) + 1{,}000 \times 0.12 + 750$$

***New machine***

Cost of the new Machine ($P$) = Rs. 10,000
Life ($n$) = 4 years.
Salvage value of the new machine ($F$) = Rs. 4,000
Annual Maintenance cost ($A$) = Rs. 500
Interest rate, $i$ = 12%

The cash flow diagram of the new machine is illustrated in Fig. 8.15.

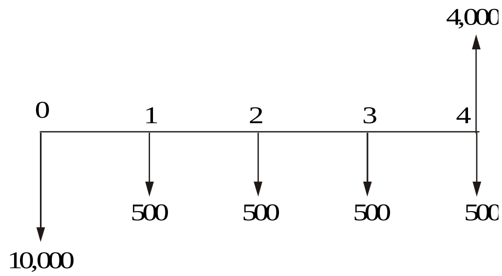


**Fig. 8.15** Cash flow diagram for new machine.

The annual equivalent cost of the new machine is illustrated as

$$\begin{aligned} AE(12\%) &= (P - F)\,(A/P,\ 12\%,\ 4) + F \times i + A \\ &= (10{,}000 - 4{,}000)(0.3292) + 4{,}000 \times 0.12 + 500 \\ &= \text{Rs. } 2{,}955.20 \end{aligned}$$

Now, equate the annual equivalent costs of the two alternatives and solve for $X$.

$$(X - 1{,}000)(0.3292) + 1{,}000 \times 0.12 + 750 = 2{,}955.20$$

$$X = \text{Rs. } 7{,}334.14$$

The comparative use value of the old machine is Rs. 7,334.14, which is less than the price (Rs. 8,000) offered by the company which is supplying the new machine in the event of replacing the old machine by the new machine.

Therefore, it is advisable to replace the old machine with the new one.

## QUESTIONS

**1.** List and explain the different types of maintenance.

**2.** Discuss the reasons for replacement.

**3.** Define ‘economic life’ of an equipment.

**4.** Distinguish between breakdown maintenance and preventive maintenance.

**5.** A firm is considering replacement of an equipment, whose first cost is Rs. 1,750 and the scrap value is negligible at any year. Based on experience, it was found that the maintenance cost is zero during the first year and it increases by Rs. 100 every year thereafter.

(a) When should the equipment be replaced if $i$ = 0%?

(b) When should the equipment be replaced if $i$ = 12%?

**6.** The following table gives the operation cost, maintenance cost and salvage value at the end of every year of a machine whose purchase value is Rs. 20,000.

(a) Find the economic life of the machine assuming interest rate ($i$) of 0%

(b) Find the economic life of the machine assuming interest rate of 15%.

| *End of year* ($n$) | *Operation cost* | *Maintenance cost* | *Salvage value* |
|---|---|---|---|
| 1 | 2,000 | 200 | 10,000 |
| 2 | 3,000 | 300 | 9,000 |
| 3 | 4,000 | 400 | 8,000 |
| 4 | 5,000 | 500 | 7,000 |
| 5 | 6,000 | 600 | 6,000 |
| 6 | 7,000 | 700 | 5,000 |
| 7 | 8,000 | 800 | 4,000 |
| 8 | 9,000 | 900 | 3,000 |
| 9 | 10,000 | 1,000 | 2,000 |
| 10 | 11,000 | 1,100 | 1,000 |

7. A manufacturer is offered two machines A and B. A is priced at Rs. 8,000 and maintenance costs are estimated at Rs. 500 for the first year and an equal increment of Rs. 100 from year 2 to year 5, and Rs. 1,500 for the sixth year and an equal increment of Rs. 500 from year 7 onwards.

   Machine B which has the same capacity is priced at Rs. 6,000. The maintenance costs of the machine B are estimated at Rs. 1,000 for the first year and an equal yearly increment of Rs. 200 thereafter.

   If the money is worth 15% per year, which machine should be purchased? (Assume that the scrap value of each of the machines is negligible at any year.)

8. Three years back, a machine was purchased at a cost of Rs. 3,00,000 to be useful for 10 years. Its salvage value at the end of its estimated life is Rs. 50,000. Its annual maintenance cost is Rs. 40,000. The market value of the present machine is Rs. 2,00,000. A new machine to cater to the need of the present machine is available at Rs. 2,50,000 to be useful for 7 years. Its annual maintenance cost is Rs. 14,000. The salvage value of the new machine is Rs. 20,000. Using an interest rate of 15%, find whether it is worth replacing the present machine with the new one.

9. A steel highway bridge must either be reinforced or replaced. Reinforcement would cost Rs. 8,60,000 and would make the bridge adequate for an additional seven years of service. If it is reinforced, it is estimated that its net salvage value would be Rs. 5,00,000 at the time it is retired from service. The new prestressed concrete bridge would cost Rs. 18,00,000 and would meet the foreseeable requirements of the next 35 years. Such a bridge would have no salvage value. It is estimated that the annual maintenance cost of the reinforced bridge would exceed that of the concrete bridge by Rs. 1,00,000. If the bridge is replaced by a new prestressed concrete bridge, the scrap value of the steel would exceed the demolition cost by Rs. 5,20,000. Assume that the money costs the state 12%. What would you recommend?

10. Three years back, a municipality purchased a 10 hp motor for pumping drinking water. Its useful life was estimated to be 10 years. Its annual operation and maintenance cost is Rs. 1,500. Due to rapid development of that locality, the municipality is unable to meet the current demand for water with the existing motor. The municipality can cope with the situation either by augmenting an additional 5 hp motor or replacing the existing 10 hp motor with a new 15 hp motor. The details of these motors are given in the following table.

| | *Old 10 hp motor* | *New 5 hp motor* | *New 15 hp motor* |
|---|---|---|---|
| Purchase cost ($P$) Rs. | 20,000 | 8,000 | 30,000 |
| Life in years ($n$) | 10 | 7 | 7 |
| Salvage value at the end of machine life (Rs.) | 1,200 | 800 | 3,500 |
| Annual operating & maintenance cost (Rs.) | 1,500 | 900 | 450 |

The current market value of the 10 hp motor is Rs. 10,000. Using an interest rate of 18% find the best alternative.