

## **UNIT - 4**

**EQUIVALENT ANNUAL WORTH COMPARISONS:** Equivalent Annual Worth Comparison methods, Situations for Equivalent Annual Worth Comparison, Consideration of asset life, Comparison of assets with equal and unequal lives, Use of sinking fund method, Annuity contract for guaranteed income, Exercises, Problems.

### **4.1 Equivalent Annual worth Comparison methods:**

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost-based comparison will be selected as the best alternative.

### **4.2 Situations for Equivalent Annual Worth Comparison**

The term annual worth suggests a positive value. If the calculations will produce a negative value it indicates that the equivalent value of negative cash flow for disbursements is greater than the corresponding positive cash flows. The situations for equivalent annual worth comparison are as follows.

1. Consolidation of cash flows
2. Recovery of invested capital
3. Net cash flow comparison

1. Consolidation of cash flows- In any project the general question will be what's it worth. It is difficult to expect from the proposal until the receipts and disbursements associated with its conduct are collectively analyzed.

2. Recovery of invested capital- The investors want to know the recovery of invested capital plus the desired rate of return. It is spread over the life of the investment, it is convenient to convert capital recovery costs to the same annual pattern.

3. Net cash flow comparison- If the worth is measured by revenues, the criterion is strictly economic, highest net worth is preferred. But, when alternatives have only costs and no income, a low EAC is preferred.

### **4.3 Consideration of asset life**

**Economic life** -The asset's economic life is defined as the number of years in which the asset returns more value to the owner than it costs to own, operate, and maintain. When these costs exceed returns, the acquisition is beyond its economic life.

An asset's economic life can be shortened or terminated by a number of different factors, including: Wear, degradation, or damage which can lower asset performance and raise maintenance and operation costs. Obsolescence, which can raise maintenance costs and render asset performance relatively inefficient when compared to more current alternatives. Changes in company operations, product offerings, or the company's business model, which reduce the value certain assets can deliver.

**Service life or ownership life** -An asset's service life is defined as the number of years the acquisition will actually be in service, and all may contribute to the owner's judgement as to what the ownership life should be.

Ownership life begins when the decision to acquire the asset begins causing costs. This may include costs that occur before the actual arrival or asset use begins, such as loan origination fees, planning costs, transportation costs, or set up costs. Ownership life ends when the asset stops causing costs and in fact has no continuing financial impact of any kind.

#### 4.4 Comparison of assets with equal and unequal lives

##### Comparison of assets with equal lives

For comparing assets having equal lives, the cash flows may be converted to equivalent annual costs. If initial cost and salvage value is involved, then annual equivalent worth can be derived as

$$\text{Equivalent annual cost (EAC)} = P (A/P, i, n) - S (A/F, i, n)$$

$$\text{We know that } (A/P, i, n) = i (1+i)^n / (1+i)^n - 1$$

$$[i (1+i)^n / (1+i)^n - 1] - i = i / (1+i)^n - 1$$

$$(A/P, i, n) - i = (A/F, i, n)$$

Replacing the value of the above equation in

$$\text{EAC} = P (A/P, i, n) - S (A/F, i, n)$$

$$\text{EAC} = P (A/P, i, n) - S \{ (A/P, i, n) - i \}$$

$$= (P-S) (A/P, i, n) + Si$$

##### Comparison of assets with unequal lives

In case of assets having unequal lives, each of the cash flows are converted in terms equivalent annual amount. If this annual amount is the cost, then assets having least annual equivalent cost are preferred.

#### 4.5 Use of sinking fund method

The sinking fund factor is applied to compute the annuity required to accumulate a certain future amount. Organizations are sometimes obligated by legislated or contractual agreements to establish a fund, separate from internal operations to accumulate a specified amount by a specified time. This accumulation is called a sinking fund. Provision for sinking fund requires set aside a portion of the income derived from sales, or taxes each year in order to retire a bond issue.

##### Problem 1:

A company provides a car to its chief executive. The owner of the company is concerned about the increasing cost of gas. The cost per litre of gas for the first year of operation is Rs. 21. He feels that the cost of gas will be increasing by Re.1 every year. His experience with his company car indicates that it averages 9 km per litre of gas. The executive expects to drive an average of 20,000 km each year for the next four years. What is the annual equivalent cost of fuel over this period of time?. If he is offered similar service with the same quality on rental basis at Rs. 60,000 per year, should the owner continue to provide company car for his executive or alternatively provide a rental car to his executive? Assume  $i = 18\%$ . If the rental car is preferred, then the company car will find some other use within the company.

Average number of km run/year = 20,000 km

Number of km/litre of gas = 9 km

Therefore,

Gas consumption/year =  $20,000/9 = 2222.2$  litre Cost/litre of gas for the 1st year = Rs. 21

Cost/litre of gas for the 2nd year = Rs. 21.00 + Re. 1.00 = Rs. 22.00

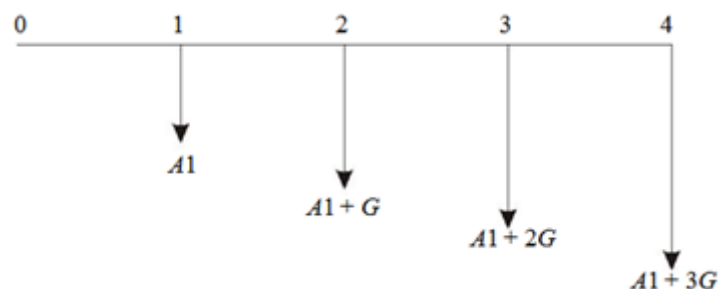
Cost/litre of gas for the 3rd year = Rs. 22.00 + Re. 1.00 = Rs. 23.00

Cost/litre of gas for the 4th year = Rs. 23.00 + Re. 1.00 = Rs. 24.00

Fuel expenditure for 1st year =  $2222.2 \times 21 = \text{Rs. } 46,666.20$  Fuel expenditure for 2nd year =  $2222.2 \times 22 = \text{Rs. } 48,888.40$  Fuel expenditure for 3rd year =  $2222.2 \times 23 = \text{Rs. } 51,110.60$  Fuel expenditure for 4th year =  $2222.2 \times 24 = \text{Rs. } 53,332.80$

The annual equal increment of the above expenditures is Rs. 2,222.20 ( $G$ ).

The cash flow diagram for this situation is shown in Fig.4.1



$A1 = \text{Rs. } 46,666.20$  and  $G = \text{Rs. } 2,222.20$

$$\begin{aligned} A &= A1 + G(A/G, 18\%, 4) \\ &= 46,666.20 + 2222.2(1.2947) \\ &= \text{Rs. } 49,543.28 \end{aligned}$$

The proposal of using the company car by spending for gas by the company will cost an annual equivalent amount of Rs. 49,543.28 for four years. This amount is less than the annual rental value of Rs. 60,000. Therefore, the company should continue to provide its own car to its executive.

**Problem 2:** A company is planning to purchase an advanced machine centre. Three original manufacturers have responded to its tender whose particulars are tabulated as follows:

Manufacturer	Down payment	Yearly equal installment	No. of installments
	(Rs.)	(Rs.)	
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Determine the best alternative based on the annual equivalent method by assuming  $i = 20\%$ , compounded annually.

### Solution

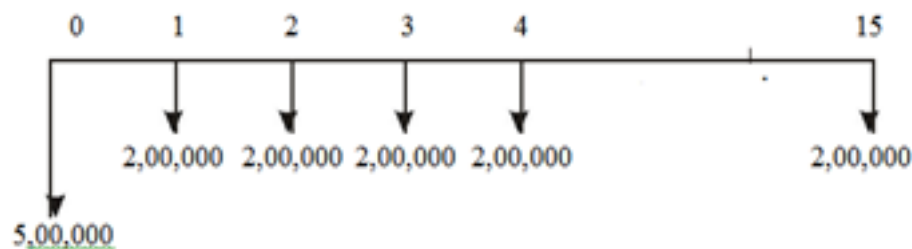
#### Alternative 1

Down payment,  $P = \text{Rs. } 5,00,000$

Yearly equal installment,  $A = \text{Rs. } 2,00,000$   $n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for manufacturer 1 is shown in Fig



**Figure 4.2** Cash flow diagram for manufacturer 1

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE_1 (20\%) &= 5,00,000(A/P, 20\%, 15) + 2,00,000 \\
 &= 5,00,000(0.2139) + 2,00,000 \\
 &= 3,06,950
 \end{aligned}$$

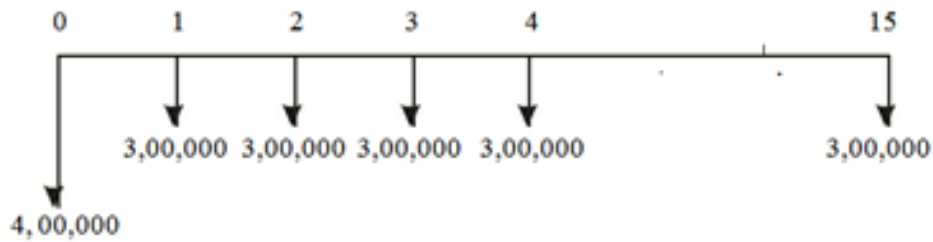
#### Alternative 2

Down payment,  $P = \text{Rs. } 4,00,000$

Yearly equal installment,  $A = \text{Rs. } 3,00,000$   $n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for the manufacturer 2 is shown in Figure 4.3



**Figure 4.3** Cash flow diagram for manufacturer 2

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_2 (20\%) &= 4,00,000(A/P, 20\%, 15) + 3,00,000 \\ &= 4,00,000(0.2139) + 3,00,000 \\ &= \text{Rs. } 3,85,560. \end{aligned}$$

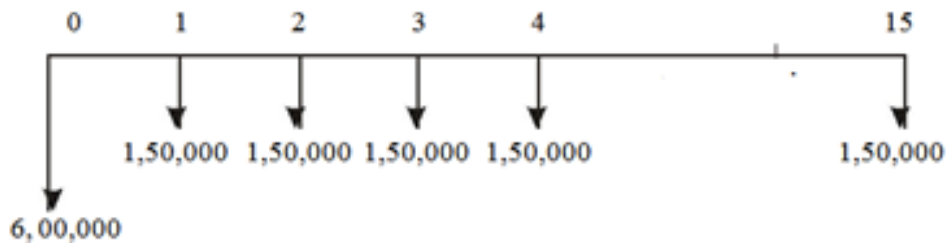
### Alternative 3

Down payment,  $P = \text{Rs. } 6,00,000$

Yearly equal installment,  $A = \text{Rs. } 1,50,000$       $n = 15$  years

$i = 20\%$ , compounded annually

The cash flow diagram for manufacturer 3 is shown in Fig



**Figure 4.4** Cash flow diagram for manufacturer 3

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE_3 (20\%) &= 6,00,000(A/P, 20\%, 15) + 1,50,000 \\ &= 6,00,000(0.2139) + 1,50,000 \\ &= \text{Rs. } 2,78,340. \end{aligned}$$

The annual equivalent cost of manufacturer 3 is less than that of manufacturer 1 and manufacturer 2. Therefore, the company should buy the advanced machine centre from manufacturer 3.

**Problem 3:** A company invests in one of the two mutually exclusive alternatives. The life of both alternatives is estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	A	B

Investment (Rs.)	- 1,50,000	- 1,75,000
Annual equal return (Rs.)	+ 60,000	+ 70,000
Salvage value (Rs.)	+ 15,000	+ 35,000

Determine the best alternative based on the annual equivalent method by assuming  $i = 25\%$ .

### Solution

#### Alternative A

Initial investment,  $P = \text{Rs. } 1,50,000$

Annual equal return,  $A = \text{Rs. } 60,000$

Salvage value at the end of machine life,  $S = \text{Rs. } 15,000$

Life = 5 years

Interest rate,  $i = 25\%$ , compounded annually

The cash flow diagram for alternative A is shown in Figure 4.5

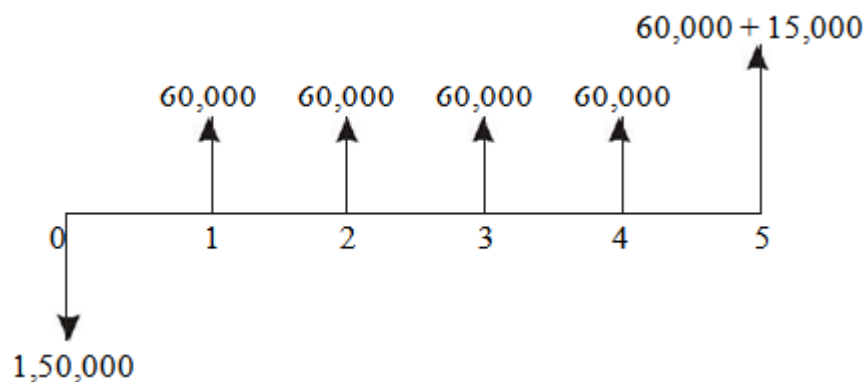


Figure 4.5 Cash flow diagrams for alternative A.

The annual equivalent revenue expression of the above cash flow diagram is as follows:

$$\begin{aligned}
 AE_A (25\%) &= -1,50,000(A/P, 25\%, 5) + 60,000 + 15,000(A/F, 25\%, 5) \\
 &= -1,50,000(0.3718) + 60,000 + 15,000(0.1218) \\
 &= \text{Rs. } 6,057
 \end{aligned}$$

#### Alternative B

Initial investment,  $P = \text{Rs. } 1,75,000$

Annual equal return,  $A = \text{Rs. } 70,000$

Salvage value at the end of machine life,  $S = \text{Rs. } 35,000$

Life = 5 years

Interest rate,  $i = 25\%$ , compounded annually

The cash flow diagram for alternative B is shown in Figure 4.6

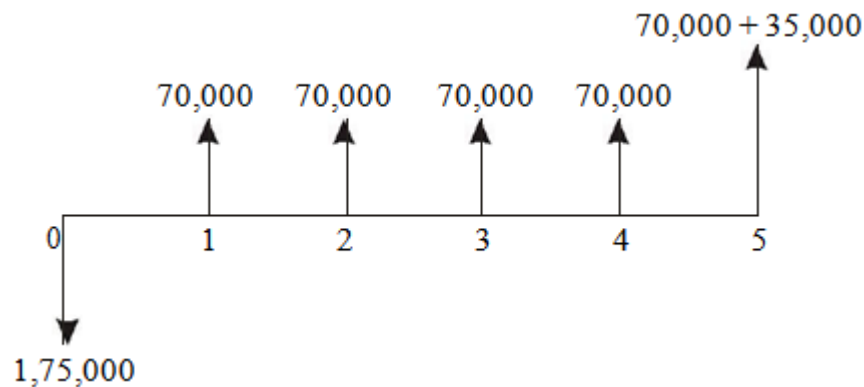


Figure 4.6 Cash flow diagrams for alternative B.

The annual equivalent revenue expression of the above cash flow diagram is

$$\begin{aligned}
 AE_B(25\%) &= -1,75,000(A/P, 25\%, 5) + 70,000 + 35,000(A/F, 25\%, 5) \\
 &= -1,75,000(0.3718) + 70,000 + 35,000(0.1218) \\
 &= \text{Rs. } 9,198
 \end{aligned}$$

The annual equivalent net return of alternative B is more than that of alternative A. Thus, the company should select alternative B.

**Problem 4:** Two possible routes for laying a power line are under study. Data on the routes are as follows:

	<i>Around the lake</i>	<i>Under the lake</i>
Length	15 km	5 km
First cost (Rs.)	1,50,000/km	7,50,000/km
Useful life (years)	15	15
Maintenance cost (Rs.)	6,000/km/yr	12,000/km/yr
Salvage value (Rs.)	90,000/km	1,50,000/km
Yearly power loss (Rs.)	15,000/km	15,000/km

If 15% interest is used, should the power line be routed around the lake or under the lake?

**Solution Alternative 1— Around the lake**

First cost =  $1,50,000 \times 15 = \text{Rs. } 22,50,000$  Maintenance cost/yr =  $6,000 \times 15 = \text{Rs. } 90,000$

Power loss/yr =  $15,000 \times 15 = \text{Rs. } 2,25,000$

Maintenance cost and power loss/yr =  $\text{Rs. } 90,000 + \text{Rs. } 2,25,000 = \text{Rs. } 3,15,000$

Salvage value =  $90,000 \times 15 = \text{Rs. } 13,50,000$

The cash flow diagram for this alternative is shown in Figure 4.7

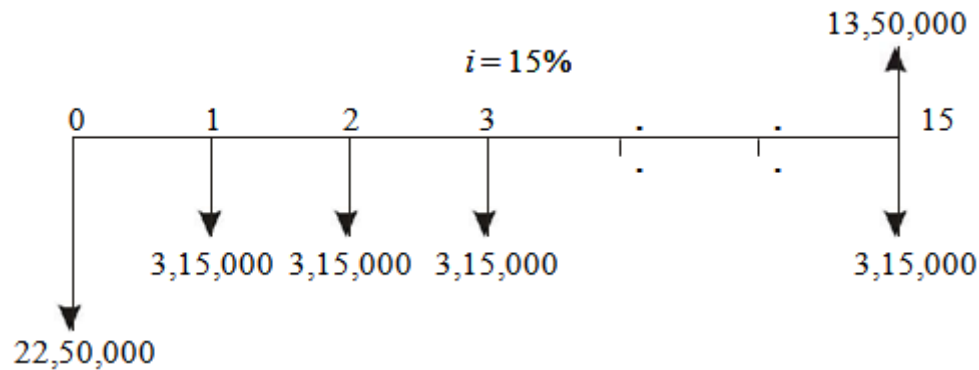


Figure 4.7 Cash flow diagram for alternative 1.

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE_1(15\%) &= 22,50,000(A/P, 15\%, 15) + 3,15,000 - 13,50,000(A/F, 15\%, 15) \\
 &= 22,50,000(0.1710) + 3,15,000 - 13,50,000(0.0210) \\
 &= \text{Rs. } 6,71,400
 \end{aligned}$$

**Alternative 2— Under the lake**

First cost = 7,50,000 x 5 = Rs. 37,50,000

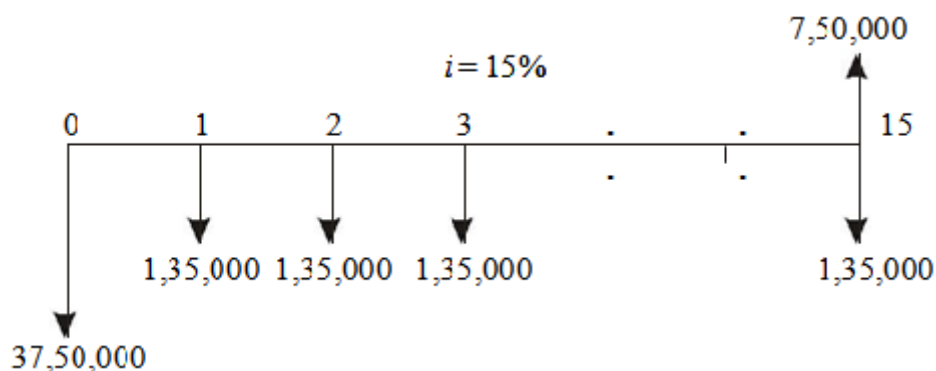
Maintenance cost/yr = 12,000 x 5 = Rs. 60,000

Power loss/yr = 15,000 x 5 = Rs. 75,000

Maintenance cost and power loss/yr = Rs. 60,000 + Rs. 75,000 = Rs. 1,35,000

Salvage value = 1,50,000 x 5 = Rs. 7,50,000

The cash flow diagram for this alternative is shown in Fig 4.8



**Figure 4.8** Cash flow diagram for alternative 2

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE_2(15\%) &= 37,50,000(A/P, 15\%, 15) + 1,35,000 - 7,50,000(A/F, 15\%, 15) \\
 &= 37,50,000(0.1710) + 1,35,000 - 7,50,000(0.0210) \\
 &= \text{Rs. } 7,60,500
 \end{aligned}$$



The annual equivalent cost of alternative 1 is less than that of alternative 2. Therefore, select the route around the lake for laying the power line.

**Problem 5:** Ramu, a salesman, needs a new car for use in his business. He expects that he will be promoted to a supervisory job at the end of third year and so his concern now is to have a car for the three years he expects to be “on the road”. The company will reimburse their salesman each month the fuel cost and maintenance cost. Ramu has decided to drive a low-priced automobile. He finds, however, that there are two different ways of obtaining the automobile. In either case, the fuel cost and maintenance cost are borne by the company.

- (a) Purchase for cash at Rs. 3,90,000.
- (b) Lease a car. The monthly charge is Rs. 10,500 on a 36-month lease payable at the end of each month. At the end of the three-year period, the car is returned to the leasing company.

Ramu believes that he should use a 12% interest rate compounded monthly in determining which alternative to select. If the car could be sold for Rs. 1, 20,000 at the end of the third year, which option should he use to obtain it?

***Alternative 1—Purchase car for cash***

Purchase price of the car = Rs. 3,90,000

Life = 3 years = 36 months

Salvage value after 3 years = Rs. 1, 20,000

Interest rate = 12% (nominal rate, compounded annually)  
= 1% compounded monthly

The cash flow diagram for alternative 1 is shown in figure 4.9

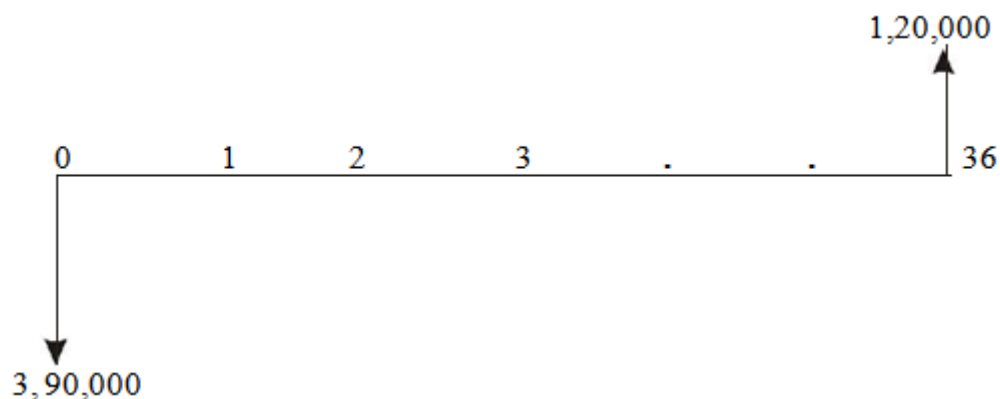


Figure 4.9 cash flow diagram for alternative 1

The monthly equivalent cost expression [ $ME(1\%)$ ] of the above cash flow diagram is

$$ME(1\%) = 3,90,000(A/P, 1\%, 36) - 1,20,000(A/F, 1\%, 36)$$

$$= 3,90,000(0.0332) - 1,20,000(0.0232)$$

$$= \text{Rs. } 10,164$$

**Alternative 2—Use of car under lease**

Monthly lease amount for 36 months = Rs. 10,500

The cash flow diagram for alternative 2 is illustrated in Figure 4.10

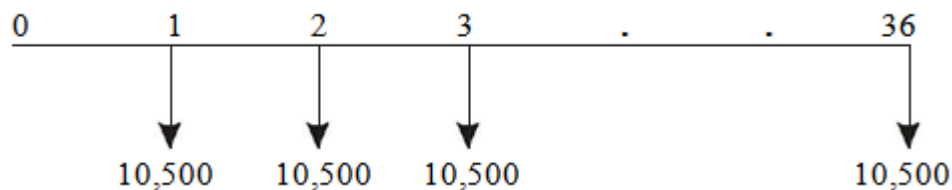


Figure 4.10 cash flow diagram for alternative 2

Monthly equivalent cost = Rs.10,500. The monthly equivalent cost of alternative 1 is less than that of alternative 2. Hence, the salesman should purchase the car for cash.

**Problem 6:** A company wants to buy a machine, the details are given in the following table.

	Machine A	Machine B
Initial cost (Rs.)	3,00,000	6,00,000
Useful life (years)	4	4
Salvage value at the end of machine life (Rs.)	2,00,000	3,00,000
Annual maintenance (Rs.)	30,000	

At 15% interest rate, which machine should be purchased?

**Solution- Machine A**

Initial cost = Rs. 3,00,000 Useful life (years) = 4

Salvage value at the end of machine life = Rs. 2,00,000 Annual maintenance = Rs. 30,000

Interest rate = 15%, compounded annually

The cash flow diagram of machine A is shown in figure 4.11

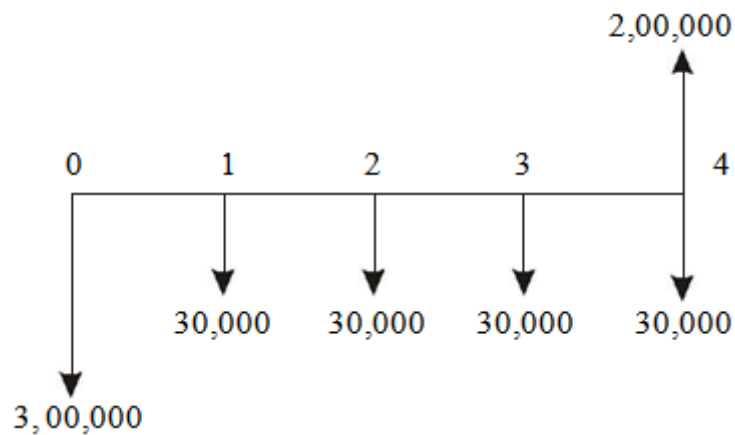


Figure 4.11 cash flow diagram of machine A

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE(15\%) &= 3,00,000(A/P, 15\%, 4) + 30,000 - 2,00,000(A/F, 15\%, 4) \\
 &= 3,00,000(0.3503) + 30,000 - 2,00,000(0.2003) \\
 &= \text{Rs. } 95,030
 \end{aligned}$$

### **Machine B**

Initial cost = Rs. 6,00,000

Useful life (years) = 4

Salvage value at the end of machine life = Rs. 3,00,000

Annual maintenance = Rs. 0.

Interest rate = 15%, compounded annually

The cash flow diagram of machine B is shown in figure 4.12

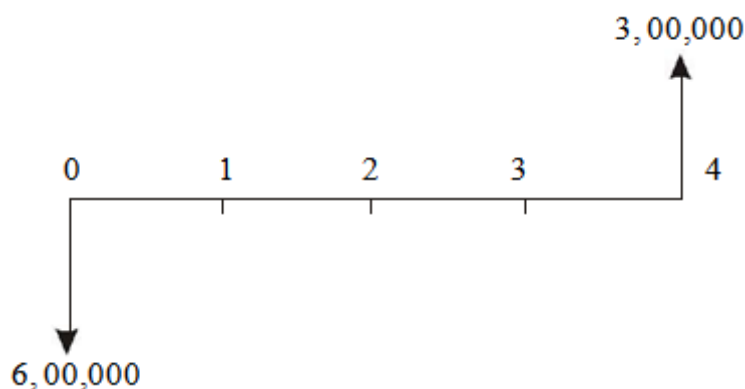


Figure 4.12 cash flow diagram of machine B

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned}
 AE(15\%) &= 6,00,000(A/P, 15\%, 4) - 3,00,000(A/F, 15\%, 4) \\
 &= 6,00,000(0.3503) - 3,00,000(0.2003) \\
 &= \text{Rs. } 1,50,090
 \end{aligned}$$

Since the annual equivalent cost of machine A is less than that of machine B, it is advisable to buy machine A.

**Problem 7:** Jothi Lakshimi has arranged to buy some home recording equipment. She estimates that it will have a five year useful life and no salvage value at the end of equipment life. The dealer, who is a friend, has offered Jothi Lakshimi two alternative ways to pay for the equipment.

- (a) Pay Rs. 60,000 immediately and Rs. 15,000 at the end of one year.
- (b) Pay nothing until the end of fourth year when a single payment of Rs. 90,000 must be made.

If Jothi Lakshimi believes 12% is a suitable interest rate, which alternative is the best for her?

**Solution:**

**Alternative 1**

Down payment = Rs. 60,000

Payment after one year = Rs. 15,000

The cash flow diagram for alternative 1 is shown in Figure 4.13



Figure 4.13 cash flow diagram for alternative 1

The present worth equation of the above cash flow diagram is

$$\begin{aligned} \text{PW}(12\%) &= 60,000 + 15,000(P/F, 12\%, 1) \\ &= 60,000 + 15,000(0.8929) \\ &= 73,393.50 \end{aligned}$$

The above present worth is represented in Figure 4.14

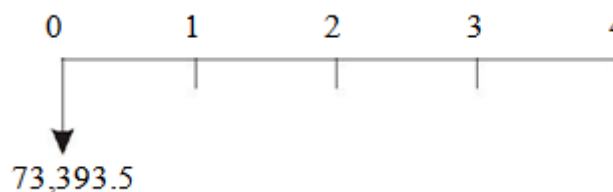


Figure 4.14 Resultant cash flow diagram for alternative 1

The annual equivalent expression of the above cash flow diagram is

$$\text{AE}(12\%) = 73,393.5(A/P, 12\%, 4)$$

$$= 73,393.5(0.3292)$$

$$= \text{Rs. } 24,161.14$$

**Alternative 2**

Payment after four years = Rs. 90,000

The cash flow diagram for alternative 2 is shown in Figure 4.15

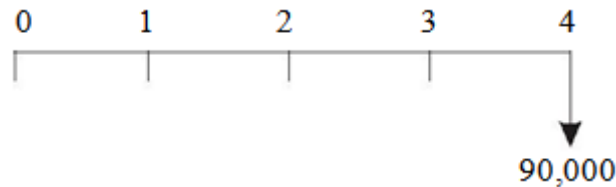


Figure 4.15 cash flow diagram for alternative 2

The annual equivalent cost expression of the above cash flow diagram is

$$AE(12\%) = 90,000(A/F, 12\%, 4)$$

$$= 90,000(0.2092)$$

$$= \text{Rs. } 18,828$$

The annual equivalent cost of alternative 2 is less than that of alternative 1. Hence, Jothi Lakshimi should select alternative 2 for purchasing the home equipment.

**Problem 8:** A transport company has been looking for a new tyre for its truck and has located the following alternatives

Brand	Tyre warranty (months)	Price per tyre (Rs.)
A	12	1,200
B	24	1,800
C	36	2,100
D	48	2,700

If the company feels that the warranty period is a good estimate of the tyre life and that a nominal interest rate (compounded annually) of 12% is appropriate, which tyre should it buy?

**Solution**

In all the cases, the interest rate is 12%. This is equivalent to 1% per month.

**Brand A**

Tyre warranty = 12 months

Price/tyre = Rs. 1,200

The cash flow diagram for brand A is shown in Figure 4.16

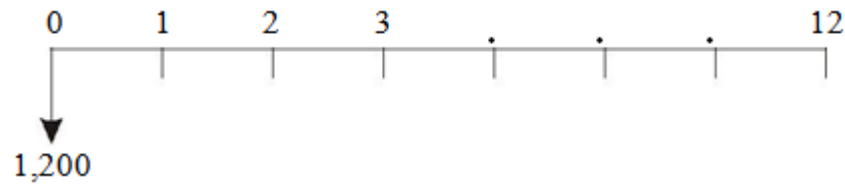


Figure 4.16 cash flow diagram for brand A

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 1,200(A/P, 1\%, 12) \\ &= 1,200(0.0888) \\ &= \text{Rs. } 106.56 \end{aligned}$$

### Brand B

Tyre warranty = 24 months Price/tyre = Rs. 1,800

The cash flow diagram for brand B is shown in Figure 4.17

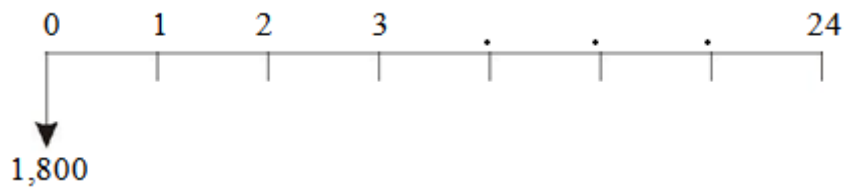


Figure 4.17 cash flow diagram for brand B

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 1,800(A/P, 1\%, 24) \\ &= 1,800(0.0471) \\ &= \text{Rs. } 84.78 \end{aligned}$$

### Brand C

Tyre warranty = 36 months Price/tyre = Rs. 2,100

The cash flow diagram for brand C is shown in Figure 4..18

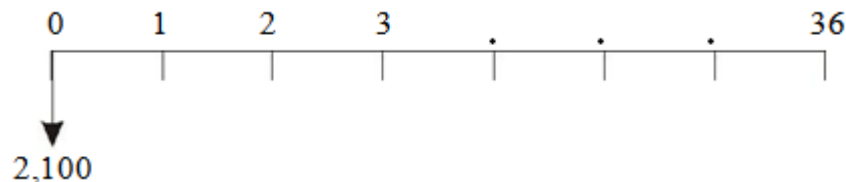


Figure 4.18 cash flow diagram for brand C

The annual equivalent expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 2,100(A/P, 1\%, 36) \\ &= 2,100(0.0332) \\ &= \text{Rs. } 69.72 \end{aligned}$$

**Brand D**

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Figure 4.19

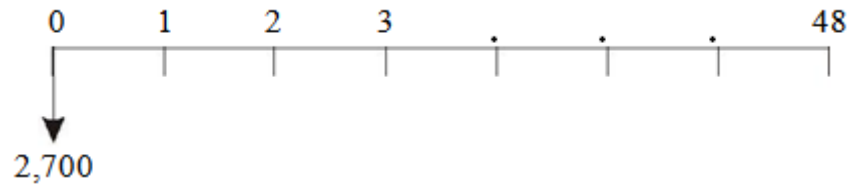


Figure 4.19 cash flow diagram for brand D

The annual equivalent cost expression of the above cash flow diagram is

$$\begin{aligned} AE(1\%) &= 2,700(A/P, 1\%, 48) \\ &= 2,700(0.0263) \\ &= \text{Rs. } 71.01 \end{aligned}$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.

## UNIT - 5

RATE OF RETURN CALCULATIONS: Rate of return, Minimum acceptable rate of return, IRR, IRR misconceptions, Cost of capital concepts, replacement models.

### 5.1 Rate of return

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative is computed. Then the alternative which has the highest rate of return is selected as the best alternative.

In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign.

A generalized cash flow diagram to demonstrate the rate of return method of comparison is shown in Figure 5.1

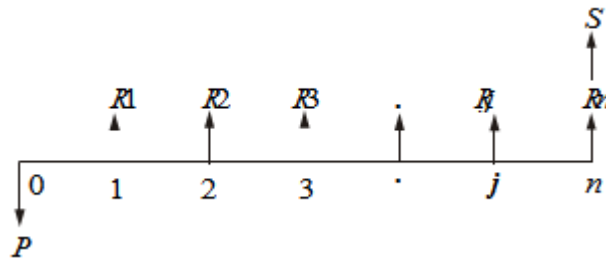


Figure 5.1 Generalized cash flow diagram

In the above cash flow diagram,  $P$  represents an initial investment,  $R_j$  the net revenue at the end of the  $j$ th year, and  $S$  the salvage value at the end of the  $n$ th year.

The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate,  $i$ .

$$\begin{aligned} PW(i) = & -P + R_1/(1+i)^1 + R_2/(1+i)^2 + \dots + R_j/(1+i)^j + \dots \\ & + R_n/(1+i)^n + S/(1+i)^n \end{aligned}$$



As shown in the figure 5.2, the present worth goes on decreasing when the interest rate is increased. The value of  $i$  at which the present worth curve cuts the X-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of  $i$  at which the present worth function reduces to zero.

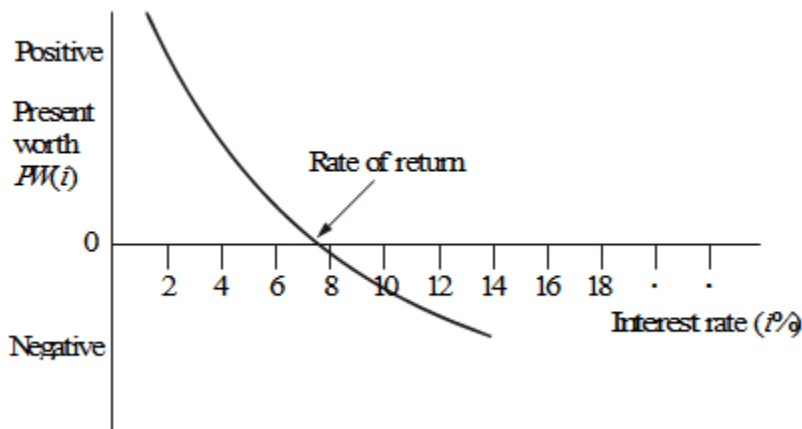


Figure 5.1 Present worth function

So, the analyst has to start with an intuitive value of  $i$  and check whether the present worth function is positive. If so, increase the value of  $i$  until  $PW(i)$  becomes negative. Then, the rate of return is determined by interpolation method in the range of values of  $i$  for which the sign of the present worth function changes from positive to negative.

## 5.2 Minimum acceptable (attractive) rate of return (MARR)

If, as is often the case, the interest rate at which a project should be evaluated is not known, a target rate, cut-off rate, or valuation rate will be used. This rate is also called the minimum attractive rate of return (abbreviated MARR). While dependent on general company policy, the MARR may also be project specific, and will normally increase with the risk attending the project.

## 5.3 IRR, IRR misconceptions

The consistency of AW and PW comparisons always agree with IRR evaluation when done correctly and there are some misconceptions

- Ranking alternatives by individual IRR values-The incremental analysis was used that ranking individual alternatives IRR values
- More than one possible rate of return-when the cash flow or cumulative cash flow of a project switches from negative to positive more than once the project may have more than one roots of the PW equation  $PW(i)=0$
- Explicit investment rate-an explicit reinvestment rate is a designated interest percentage appropriate for a specific application.
- Project balance method-this involves calculation of  $i^*$  if there are multiple I values
- Reinvestment question

**Problem 1:** A person is planning a new business. The initial outlay and cash flow pattern for the new business are as listed below. The expected life of the business is five years. Find the rate of return for the new business.

Period	0	1	2	3	4	5
Cash flow	-1,00,000	30,000	30,000	30,000	30,000	30,000 (Rs.)

### ***Solution***

Initial investment = Rs. 1,00,000

Annual equal revenue = Rs. 30,000

Life = 5 years

The cash flow diagram for this situation is shown in figure 5.3

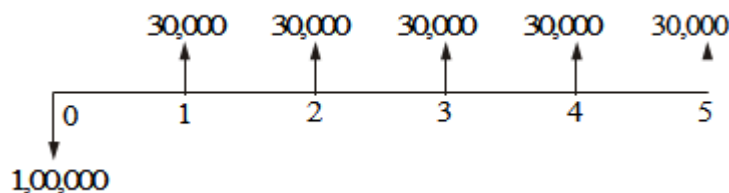


Figure 5.3 Cash flow diagram

The present worth function for the business is

$$PW(i) = -1,00,000 + 30,000(P/A, i, 5) \text{ When } i = 10\%,$$

$$PW(10\%) = -1,00,000 + 30,000(P/A, 10\%, 5)$$

$$= -1,00,000 + 30,000(3.7908)$$

$$= \text{Rs. } 13,724.$$

When  $i = 15\%$ ,

$$PW(15\%) = -1,00,000 + 30,000(P/A, 15\%, 5)$$

$$= -1,00,000 + 30,000(3.3522)$$

$$= \text{Rs. } 566.$$

When  $i = 18\%$ ,

$$PW(18\%) = -1,00,000 + 30,000(P/A, 18\%, 5)$$

$$= -1,00,000 + 30,000(3.1272)$$

$$= \text{Rs. } -6,184$$

$$i = 15\% + \frac{566 - 0}{566 - (-6184)} 3\%$$

$$i = 15\% + 0.252\%$$

$$i = 15.252\%$$

**Problem 2:** A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs. 20,00,000. The annual net profit is Rs. 3,50,000. Find the rate of return for the new business.

**Solution**

Life of the product line ( $n$ ) = 10 years

Initial outlay = Rs. 20,00,000

Annual net profit = Rs. 3,50,000

Scrap value after 10 years = 0

The cash flow diagram for this situation is shown in Figure 5.4

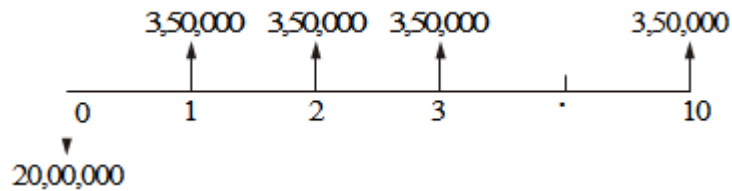


Figure 5.4 Cash flow diagram

The formula for the net present worth function of the situation is

$$PW(i) = -20,00,000 + 3,50,000(P/A, i, 10) \text{ When } i = 10\%,$$

$$PW(10\%) = -20,00,000 + 3,50,000(P/A, 10\%, 10)$$

$$= -20,00,000 + 3,50,000(6.1446)$$

$$= \text{Rs. } 1,50,610. \text{ When } i = 12\%,$$

$$PW(12\%) = -20,00,000 + 3,50,000(P/A, 12\%, 10)$$

$$= -20,00,000 + 3,50,000(5.6502)$$

$$= \text{Rs. } -22,430.$$

$$i = 10\% \frac{150610 - 0}{150610 - (-22430)} (2\%)$$

$$i = 11.74\%$$

Therefore, the rate of return of the new product line is 11.74%

**Problem 3:**

Primier plastics has identified three mutually exclusive investment proposals whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	A1	A2	A3
Investment	Rs. 1,50,000	Rs. 2,10,000	Rs. 2,55,000
Annual net	Rs. 45,570	Rs. 58,260	Rs. 69,000

Find the best alternative based on the rate of return

**Solution Calculation of rate of return for alternative A1**

Initial outlay = Rs. 1, 50,000

Annual profit = Rs. 45,570

Life = 5 years

The cash flow diagram for alternative A1 is shown in Figure5.5

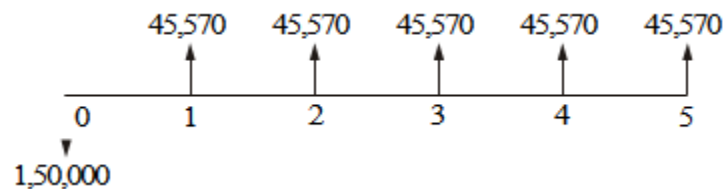


Figure 5.5 Cash flow diagram for alternative A1

The formula for the net present worth of alternative A1 is given as

$$PW(i) = -1,50,000 + 45,570(P/A, i, 5)$$

When  $i = 10\%$ ,

$$PW(10\%) = -1, 50,000 + 45,570(P/A, 10\%, 5)$$

$$= -1, 50,000 + 45,570(3.7908)$$

$$= \text{Rs. } 22,746.76$$

When  $i = 12\%$ ,

$$PW(12\%) = -1,50,000 + 45,570(P/A, 12\%, 5)$$

$$= -1,50,000 + 45,570(3.6048)$$

$$= \text{Rs. } 14,270.74$$

When  $i = 15\%$ ,

$$PW(15\%) = -1,50,000 + 45,570(P/A, 15\%, 5)$$

$$= -1,50,000 + 45,570(3.3522)$$

$$= \text{Rs. } 2,759.75$$

When  $i = 18\%$ ,

$$PW(18\%) = -1,50,000 + 45,570(P/A, 18\%, 5)$$

$$= -1,50,000 + 45,570(3.1272)$$

$$= \text{Rs. } -7,493.50$$

$$i = 15\% + \frac{2759.75 - 0}{2759.75 - (-7493.50)} 3\%$$

$$= 15\% + 0.81\%$$

$$= 15.81\%$$

Therefore, the rate of return of the alternative A1 is

***Calculation of rate of return for alternative A2***

Initial outlay = Rs. 2, 10,000

Annual profit = Rs. 58,260

Life of alternative A2 = 5 years

The cash flow diagram for alternative A2 is shown in figure 5.6

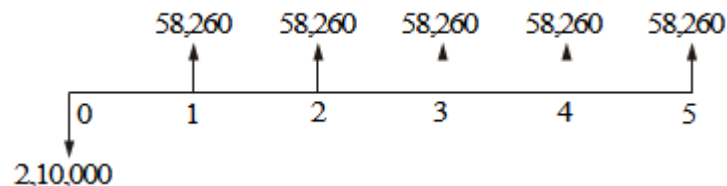


Figure 5.6 Cash flow diagrams for A2

The formula for the net present worth of this alternative is

$$PW(i) = -2,10,000 + 58,260(P/A, i, 5)$$

When  $i = 12\%$ ,

$$\begin{aligned}PW(12\%) &= -2,10,000 + 58,260(P/A, 12\%, 5) \\&= -2,10,000 + 58,260(3.6048) \\&= \text{Rs. } 15.65\end{aligned}$$

When  $i = 13\%$ ,

$$\begin{aligned}PW(13\%) &= -2,10,000 + 58,260(P/A, 13\%, 5) \\&= -2,10,000 + 58,260(3.5172) \\&= \text{Rs. } -5,087.93\end{aligned}$$

Therefore, the rate of return of alternative A2 is

$$\begin{aligned}i &= 12\% + \frac{15.65 - 0}{15.65 - (-5087.93)} 1\% \\&= 12\% + 0\% \\&= 12\%\end{aligned}$$

### ***Calculation of rate of return for alternative A3***

Initial outlay = Rs. 2, 55,000

Annual profit = Rs. 69,000

Life of alternative A3 = 5 years

The cash flow diagram for alternative A3 is depicted in Figure 5.7



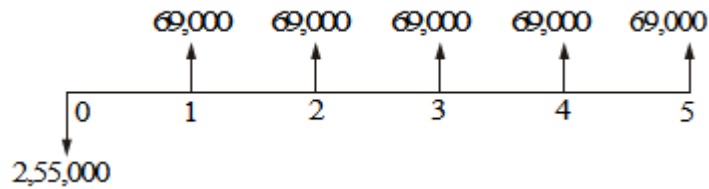


Figure 5.7 Cash flow diagram

The formula for the net present worth of this alternative A3 is

$$PW(i) = -2,55,000 + 69,000(P/A, i, 5).$$

When  $i = 11\%$ ,

$$\begin{aligned} PW(11\%) &= -2,55,000 + 69,000(P/A, 11\%, 5) \\ &= -2,55,000 + 69,000(3.6959) \\ &= \text{Rs. } 17.1 \end{aligned}$$

When  $i = 12\%$ ,

$$\begin{aligned} PW(12\%) &= -2,55,000 + 69,000(P/A, 12\%, 5) \\ &= -2,55,000 + 69,000(3.6048) \\ &= \text{Rs. } -6,268.80 \end{aligned}$$

By linier interpolation technique  $i=11\%$

The rates of return for the three alternatives are now tabulated.

Alternative	A1	A2	A3
Rate of return	15.81%	12%	11%

From the above data, it is clear that the rate of return for alternative A3 is less than the minimum attractive rate of return of 12%. So, it should not be considered for comparison. The remaining two alternatives are qualified for consideration. Among the alternatives A1 and A2, the rate of return of alternative A1 is greater than that of alternative A2. Hence, alternative A1 should be selected.

**Problem 8:** For the cash flow diagram shown in Fig. 5.8, compute the rate of return. The amounts are in rupees

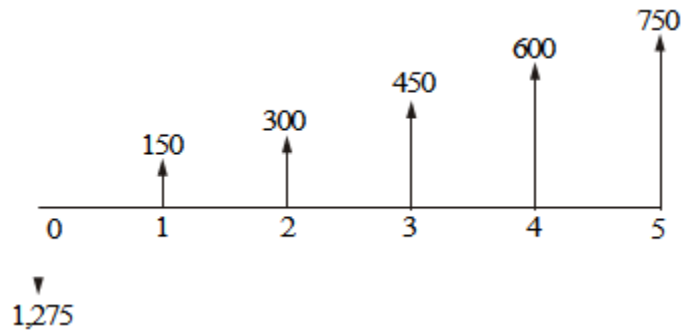


Figure 5.8 cashflow diagram

**Solution** For the positive cash flows of the problem,

$$A_1 = \text{Rs. } 150, \quad G = \text{Rs. } 150$$

The annual equivalent of the positive cash flows of the uniform gradient series is given by

$$A = A_1 + G(A/G, i, n)$$

$= 150 + 150(A/G, i, 5)$  The formula for the present worth of the whole diagram

$$= -1,275 + [150 + 150(A/G, i, 5)] \quad (P/A, i, 5)$$

$$PW(10\%) = -1,275 + [150 + 150(A/G, 10\%, 5)] \quad (P/A, 10\%, 5)$$

$$= -1,275 + [150 + 150(1.8101)] \quad (3.7908)$$

$$= \text{Rs. } 322.88$$

$$PW(12\%) = -1,275 + [150 + 150(A/G, 12\%, 5)] \quad (P/A, 12\%, 5)$$

$$= -1,275 + [150 + 150(1.7746)] \quad (3.6048)$$

$$= \text{Rs. } 225.28$$

$$PW(15\%) = -1,275 + [150 + 150(A/G, 15\%, 5)] \quad (P/A, 15\%, 5)$$

$$= -1,275 + [150 + 150(1.7228)] \quad (3.3522)$$

$$= \text{Rs. } 94.11$$

$$PW(18\%) = -1,275 + [150 + 150(A/G, 18\%, 5)] \quad (P/A, 18\%, 5)$$

$$= -1,275 + [150 + 150(1.6728)] \quad (3.1272)$$

$$= \text{Rs. } -21.24$$

Therefore, the rate of return for the cash flow diagram is

$$i = 15\% + \frac{94.11 - 0}{94.11 - (-21.24)}$$

3%

$$= 15\% + 2.45\% = 17.45\%$$

**Problem 9:**

A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	<i>Initial investment</i> (Rs.)	<i>Yearly revenue</i> (Rs.)
Alternative 1	5,00,000	1,70,000
Alternative 2	8,00,000	2,70,000

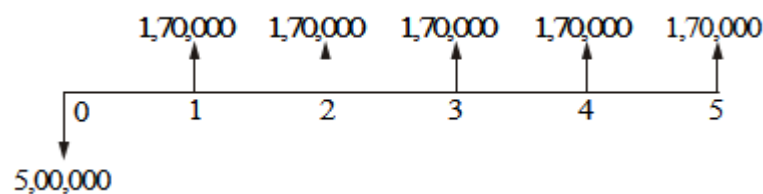
**Solution Alternative 1**

Initial outlay = Rs. 5,00,000

Annual revenue = Rs. 1,70,000

Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is shown in Fig. 5.9.



**Fig. 5.9** Cash flow diagram for alternative 1.

$$PW_1(i) = -5,00,000 + 1,70,000(P/A, i, 5)$$

$$PW_1(15\%) = -5,00,000 + 1,70,000(P/A, 15\%, 5)$$

$$= -5,00,000 + 1,70,000(3.3522)$$

$$= \text{Rs. } 69,874$$

$$PW_1(17\%) = -5,00,000 + 1,70,000(P/A, 17\%, 5)$$

$$= -5,00,000 + 1,70,000(3.1993)$$

$$= \text{Rs. } 43,881$$

$$PW_1(20\%) = -5,00,000 + 1,70,000(P/A, 20\%, 5)$$

$$= -5,00,000 + 1,70,000(2.9906)$$

$$= \text{Rs. } 8,402$$

$$PW_1(22\%) = -5,00,000 + 1,70,000(P/A, 22\%, 5)$$

$$= -5,00,000 + 1,70,000(2.8636)$$

$$= \text{Rs. } -13,188$$

By linear interpolation

$$I = 20.78\%$$

### **Alternative 2**

Initial outlay = Rs. 8,00,000

Annual revenue = Rs. 2,70,000

Life = 5 years

The cash flow diagram for alternative 2 is shown in figure 5.10

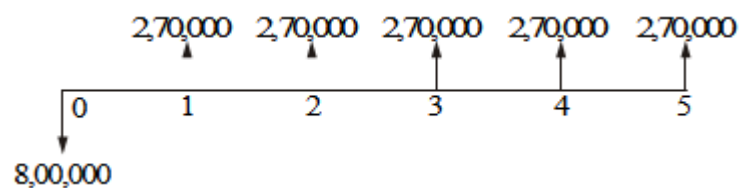


Figure 5.10 Cash flow diagram for Alternative A2

The formula for the net present worth of alternative 2 is:

$$PW_2(i) = -8,00,000 + 2,70,000(P/A, i, 5)$$

$$PW_2(20\%) = -8,00,000 + 2,70,000(P/A, 20\%, 5)$$

$$= -8,00,000 + 2,70,000(2.9906)$$

$$= \text{Rs. } 7,462$$

$$\begin{aligned}
PW_2(22\%) &= -8,00,000 + 2,70,000 (P/A, 22\%, 5) \\
&= -8,00,000 + 2,70,000 (2.8636) \\
&= \text{Rs. } -26,828
\end{aligned}$$

Thus, the rate of return of alternative 2 is = 20.435%

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

#### **5.4 Cost of capital concepts**

To compute the value of an asset, you need to know the rate of return,  $r$ , that is required to invest in the asset. To this point, The purpose of this cost of capital to show how the required rate of return for a firm (or for an individual for that matter) is determined and what the value for the required rate of return means. As you read this section, keep in mind that the reason a firm has to earn a particular rate of return on its assets is because investors who provide funds to the firm demand to receive a return on their funds. As a result, the firm must earn enough on its investments to provide the return investors demand that is,  $r$ , or the required rate of return. The concept is the same as if you borrow money for the purposes of investing in the stock market. If investments don't earn a return that covers the interest the investors are paying on the loan, which is the required rate of return in this case, then investor will lose money that is, the net present value (NPV) of the investments is negative, which means you reduce your wealth.

#### **The Logic of the Weighted Average Cost of Capital**

A firm generally uses more than one type of funds to finance its assets, and the costs of, or the returns associated with, those funds usually are not the same. For example, the existing assets of firm might be financed with some debt, which has a market return (cost) equal to 8 percent, and with some stock, or equity, which has a market return (cost) equal to 15 percent. If 50 percent of the firm's financing is debt, then the other 50 percent is equity. Thus, 50 percent of the funds the firm is using costs 8 percent while the other 50 percent costs 15 percent, and the average rate that the firm is paying is  $11\frac{1}{2}$  percent, which is the weighted average of the two costs ( $11\frac{1}{2}\% = 0.50 \times 8\% + 0.50 \times 15\%$ )

#### **5.5 Replacement Models**

It is concerned with the equipment and machinery that deteriorates with time. Many people feel that equipment should not be replaced until it is physically worn off. But, it is not correct,

preferable equipment must be constantly renewed and updated otherwise it will be in the risk of failure or it may become obsolete.

### **Reasons for replacement**

1. Deterioration
2. Obsolescence
3. Technological development
4. Inadequacy

### **Models:**

There are three replacement models

#### **Model 1:**

“Replacement of items whose maintenance Cost increases with time and the value of the money remains constant during the period”

#### **Model 2: “**

replacement of items whose maintenance cost increases with time and value of money also changes with time”.

#### **Model 3: “**

Group Replacement policy”

#### **Model 1: Notation and symbols**

C- Purchase cost of the machinery or equipment

S- Salvage value or resale value or scrap value of the machinery or equipment

Tc- total cost increased on the item or equipment during the period y

$$Tc = C + m(Y) - S$$

Where M(Y) is the cumulative maintenance cost in that period.

G(Y) Average cost incurred on the equipment or item during the period.

$$G(Y) = Tc / y$$

#### **Model II**

“Replacement of items whose maintenance cost increases with time and the value of money also changes with time”

The maintenance cost varies with time and we want to find out the optimum time period at which the items will be replaced value of money decreases with a constant rate which is known as depreciation ratio or discounted factor which is given by

$$V = 1 / (1+i)^n$$

for the value of 1 rupee

where i rate of interest , n- no. of years

**Group Replacement Policy:**

Replacement of items that fail completely We always come across situation where the probability of failure in any system increases with time. The nature of the same may be such that if any item fails then it may result in complete breakdown of the s/m. this breakdown implies loss of production, work-in-progress.

Individual replacement policy-

Whenever an item fails it should be replaced immediately.

Group replacement policy:- 2 steps

In the first step, it consists of individual replacement at the time of failure of any unit In the second step, there is a group replacement of some live units at some suitable time. In the group replacement we decide that all items in a system should be replaced after a certain interval of time irrespective of the fact that the items have failed or not with a provision that if any item fails before this time it should be replaced immediately. It requires

- a) Rate of individual replacement during the period
- b) Total cost incurred for individual and group replacement during the chosen period. The period for which the total cost incurred is minimum will be the optimum period for replacement.

**STRUCTURAL ANALYSIS OF ALTERNATIVES: Identifying and Defining alternatives, IRR analysis of mutually exclusive alternatives, Capital Budget view point, ranking criteria.**

Many people besides engineers develop and evaluate investment alternatives. However, few other professions are so intimately and regularly involved with selecting the optimum device or best way to do a particular operation. Further, the operation usually has long term consequences. That is why it is important for engineers, and others engaged in similar decision situations, to be competent economic analysts.

Analysis starts with the identification of alternatives. A need to do something originates from asking. What must be done? What can be done? What should be done? A general idea for an undertaking evolves into a family of alternatives through further questioning of the feasibility of alternative solutions and of how to do them and when. Answers to these questions may suggest several ways to accomplish the same mission or other missions that deserve attention. Perhaps the most bothersome question in the search for alternatives is, How many is enough? Decision making can be paralyzed by a continuing search for a still-better option. At the other



extreme, acting on the first option that comes to mind may ensure a fast decision but seldom a wise one. The governing objective is to develop a set of alternatives large enough to include the best possible solution. Earlier, a balance was suggested that where the cost of a less-than-ideal solution was weighed against the cost of making a more optimal decision. We should not forget this possibility when we are looking at our optimal possibilities.

### **5.6 Defining alternatives**

The basic data required for an economic analysis are the timing and amount of cash flows. These come from an understanding of the present situation and future situations. Besides the current minimum acceptable rate of return (MARR) and today's cost commitments, knowledge is needed about future objectives, resources, and operational constraints .

### **5.7 Classification of alternatives:**

The role of the engineering economist includes the identification of alternatives and analysis of their worth. Since then, alternative courses of action have been evaluated for many different situations. Alternatives that share certain features can be grouped to facilitate analysis. As indicated earlier, we can classify alternatives into dependent and independent categories.

An independent alternative is not affected by the selection of another alternative. Each proposal is evaluated on its merit and is approved if it meets the criteria of acceptability. Comparisons of independent investment proposals are designed to determine which proposals satisfy a minimum level of economic value. All those that surpass the minimum level may be implemented as long as sufficient capital is available.

### **5.8 IRR analysis of mutually exclusive alternatives**

The aim of a cash flow analysis is to put all competing alternatives into a comparable investment perspective. It acts as a screen. The alternative with the most promising future from one evaluation is then pitted against winners of other evaluations where intangible and financial considerations become critical. No organization has sufficient capital to fund all conceivable worthwhile proposals within its province. Care must be taken in all evaluations to ensure correct and consistent selections. These are two logical-sounding selection criteria that sometimes lead to inaccurate conclusions.

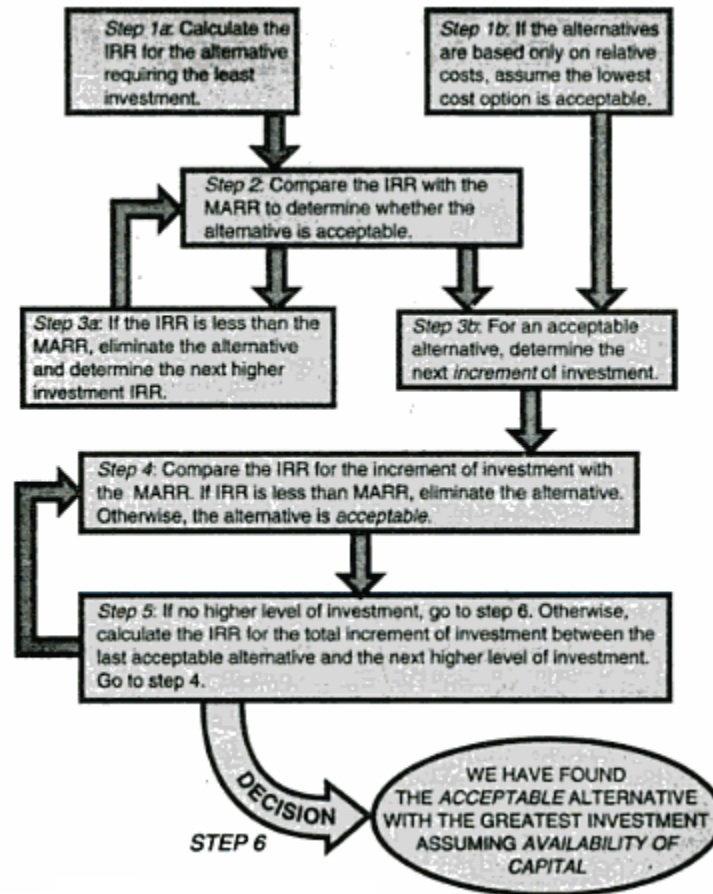


Figure 5.11: Process of evaluating alternatives

- Select the alternative that offers the highest rate of return on total investment.
- Select the alternative with the largest investment that meets the minimum required rate of return.

The Process of evaluating alternatives is shown in figure 5.11

### 5.9 Capital budget view point:

Determining a subset of eligible independent proposals for funding may be accomplished by using a capital budgeting approach.' Suppose we have four independent proposals that have been requested for funding and a limit on available capital of Rs 2300 lakhs. Data on the proposals are given in Table 5.1. It shows that independent proposals competing for funding should be picked according to their IRR values—monotonically from highest to lowest. If we rank by IRR. We get the following results.

<b>Proposal</b>	<b>IRR%</b>	<b>Proposal first cost (Lakhs Rs)</b>	<b>Cumulative first cost</b>
1	14.4	300	300
2	12.5	500	800
4	12.3	1500	2300
3	12.0	600	2900

Table 5.1 Competing proposals

### 5. 10 Ranking criteria

Given a specified MARR value, Newnan' suggests that proposals be ranked on the basis of

$$\text{Ranking Ratio} = \frac{\text{Proposal PW ( MARR)}}{\text{Proposal First cost}}$$

For the four independent proposals, we would have the following rankings by ratios (using the MARR of 10%)

<b>Proposal</b>	<b>First cost</b>	<b>PW(10%)</b>	<b>PW (10%)/First cost</b>
1	300	23.29	0.078
2	500	22.24	0.045
4	1500	61.74	0.041
3	600	21.71	0.036

The three proposals selected are the same as were chosen by the other IRR ranking method, although this is somewhat by chance. The IRR method used an effective MARR of 12.3 percent since that was the lowest IRR value for a proposal's selected approaches. The ratio method uses the company-specified MARR value.

