UNIT - 3

PRESENT WORTH COMPARISON: Conditions for present worth comparisons, Basic Present worth comparisons, Present worth equivalence, Net Present worth, Assets with unequal lives, infinite lives, Future worth comparison, Pay – back comparison, Exercises, Discussions and problems.

3.1 Conditions for present worth comparisons

Present worth: In a present worth comparison of alternatives, the costs associated with each alternative investment are all converted to a present sum of money, and the least of these values represents the best alternative. Annual costs, future payments, and gradients must be brought to the present. Converting all cash flows to present worth is referred to as discounting.

In present worth analysis, the *P* value, now called *PW*, is calculated at the MARR for each alternative. This converts all future cash flows into present dollar equivalents.This makes it easy to determine the economic advantage of one alternative over another. The PW comparison of alternatives with equal lives is straightforward. If both alternatives are used in identical capacities for the same time period, they are termed *equal-service* alternatives. For mutually exclusive alternatives the following guidelines are applied:

One alternative- Calculate PW at the MARR. If PW _ 0, the alternative is financially viable.

Two or more alternatives- Calculate the PW of each alternative at the MARR. Select the alternative with the PW value that is numerically largest, that is, less negative or more positive. The second guideline uses the criterion of numerically largest to indicate a lower PW of costs only or larger PW of net cash flows. Numerically largest is not the absolute value because the sign matters here. The selections below correctly apply this guideline.

Conditions for using the present worth comparisons:While evaluating alternatives ,the following assumptions are cosidered for better results and smooth evaluation process.

1. Cash flows are known (no risks taken)
2. Cash flows are in constant value dollars (no inflation)
3. Interest rate is known
4. Taxes are not included in calculations
5. Intangibles are not considered
6. Funds are considered to exist at all time

3.2 Basic Present worth comparisons:

The present worth of a cash flow overtime is its value today is represented as time zero (0) in a cahs flow diagram.Two general pattrens are apparent in present worth calculations:present worth equivallience and net present worth.

3.3 Present worth equivalence:

One pattren detremines the present worth equivalence of a series of future transactions. The purpose is to secure one figure that represent all the transactions for instance,a series of expenses that will occur in the future can be discounted to obtain its PW and decision can be made about whether an investment of the PW amount should be made now to avoid the expenses**.**

Assets with unequal lives:

Unequal lives among feasible alternatives somewhat complecate their analysis and comparison.To make engineering economy studies in such cases thw assumptions are emplyed

1.The repeatability assumptions

2. The co-terminated assumptions

In repeatability assumption or common multiple method , least common multiple is taken as life of the asste,based on that LCM the assent as to be repalced many times.

Problem -1

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.3,00,000, Annual operating and maintenance cost = Rs.20,000, Expected salvage value = Rs.1,25,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.2,00,000, Annual operating and maintenance cost = Rs.35,000, Expected salvage value = Rs.70,000, Useful life = 5 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

Since both alternatives have the same life span i.e. 5years, the present worth of the alternatives will be compared over a period of 5 years. The cash flow diagram of Alternative-1 is shown in Fig. 3.1.

The cash outflows i.e. costs or expenditures are represented by vertically downward arrows

whereas the cash inflows i.e. revenue or income are represented by vertically upward arrows. The same convention is adopted here.

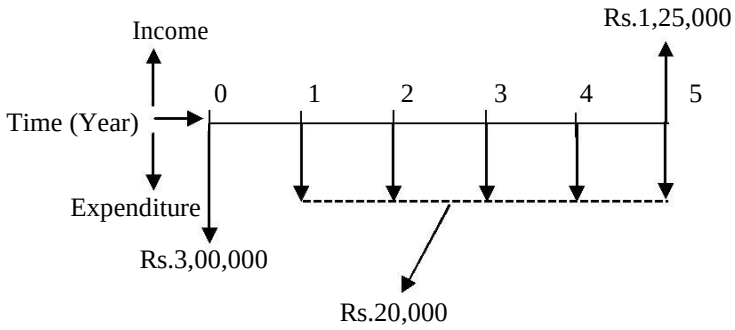


Fig. 3.1 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows; The initial cost, P = Rs.3,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.20,000 (cash outflow), Salvage value, F = Rs.1,25,000 (cash inflow).

PW_1 = - 3,00,000 – 20,000($P/A, i, n$) + 1,25,000($P/F, i, n$)

PW_1 = - 3,00,000 – 20,000($P/A, 10\%, 5$) + 1,25,000($P/F, 10\%, 5$)

Now putting the mathematical expressions of different compound interest factors in the above expression for PW_1 (in Rs.) results in the following;

$$PW_1 = -3,00,000 - 20,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 1,25,000 \times \frac{1}{(1+i)^n}$$

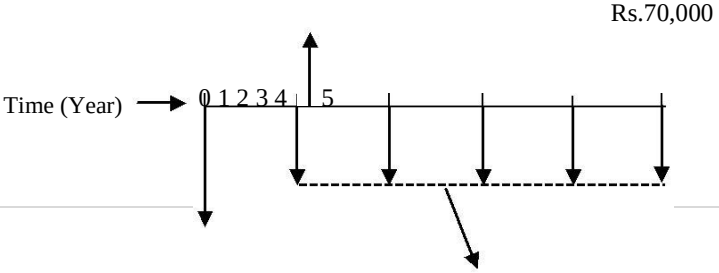
$$PW_1 = -3,00,000 - 20,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 1,25,000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$$

PW_1 = - 3,00,000 -75,816 + 77,613

***PW₁* = - Rs.2,98,203**

The cash flow diagram of Alternative-2 is shown in Fig. 3.2.



Rs.2,00,000

Rs.35,000

Fig.3.2 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

The initial cost, P = Rs.2,00,000 (cash outflow),

Annual operating and maintenance cost, A = Rs.35,000 (cash outflow), Salvage value, F = Rs.70,000 (cash inflow).

PW_2 = - 2,00,000 – 35,000(*P/A, i, n*) + 70,000(*P/F, i, n*)

PW_2 = - 2,00,000 – 35,000(*P/A, 10%, 5*) + 70,000(*P/F, 10%, 5*)

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70{,}000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70{,}000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -2{,}00{,}000 - 35{,}000 \times 3.7908 + 70{,}000 \times 0.6209$$

PW_2 = - 2,00,000 – 1,32,678 + 43,463

***PW2* = - Rs.2,89,215**

Comparing the equivalent present worth of both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent present worth compared to Alternative-1 at the interest rate of 10% per year.

The equivalent present worth of both the alternatives can also be calculated by using the values of compound interest factors from interest tables. The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

PW_1 = - 3,00,000 – 20,000(*P/A, i, n*) + 1,25,000(*P/F, i, n*)

PW_1 = - 3,00,000 – 20,000(*P/A, 10%, 5*) + 1,25,000(*P/F, 10%, 5*)

The values of compound interest factors i.e. *(P/A, i, n)* and *(P/F, i, n)* can be obtained from the interest tables . By referring to the interest table for 10% interest rate, the values

of compound interest factors i.e. *(P/A, 10%, 5)* and *(P/F, 10%, 5)* at interest rate („*i'*) of 10% and for interest period („*n'*) of 5 years are obtained at the intersection of these factors and interest period „*n*‟ equal to 5 i.e. the values are obtained from *P/A* column and*P/F* column at „*n*‟ equal to 5 from the interest table (discrete compounding) corresponding to 10% interest rate. The obtained values of *(P/A, 10%, 5)* and *(P/F, 10%, 5)* are 3.7908 and 0.6209 respectively (same as those obtained using mathematical expressions of these factors).

Now putting the values of compound interest factors in the above expression, the equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

$PW_1 = -3,00,000 - 20,000 \times 3.7908 + 1,25,000 \times 0.6209$

$PW_1 = -\ 3,00,000 - 75,816 + 77,613$

***PW1* = - Rs.2,98,203**

Now the calculation of equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is presented below.

$PW_2 = -\ 2,00,000 - 35,000(P/A, i, n) + 70,000(P/F, i, n)$

$PW_2 = -\ 2,00,000 - 35,000(P/A, 10\%, 5) + 70,000(P/F, 10\%, 5)$

Now putting the values of compound interest factors in the above expression (same as above) the equivalent present worth of Alternative-2 i.e. PW_2 is calculated as follows;

$PW_2 = -2,00,000 - 35,000 \times 3.7908 + 70,000 \times 0.6209$

$PW_2 = -\ 2,00,000 - 1,32,678 + 43,463$

***PW2* = - Rs.2,89,215**

It may be noted that in the above example only cost components and the salvage value of the alternatives were considered for comparison.

Problem 2:

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative-2 are Rs.50000 and Rs.45000 respectively. Compute the equivalent present worth of the alternatives at the same rate of interest 10% per year and find out the economical alternative.

Solution:

The cash flow diagram of Alternative-1 is shown in Fig. 3.3.

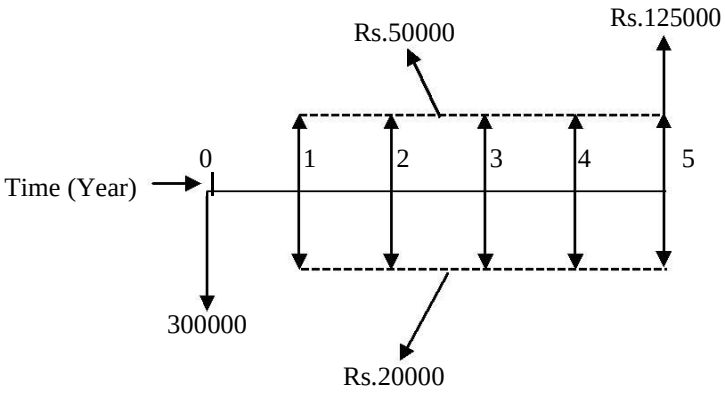


Fig. 3.3 Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 is calculated as follows;

PW_1 = - 300000 - 20000(*P/A, i, n*) + 50000*(P/A, i, n)* + 125000(*P/F, i, n*)

PW_1 = - 300000 - 20000(*P/A, 10%, 5*) + 50000(*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

PW_1 = - 300000 + (50000 – 20000) (*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

PW_1 = - 300000 + 30000(*P/A, 10%, 5*) + 125000(*P/F, 10%, 5*)

$$PW_1 = -300000 + 30000 \times \frac{(1+i)_n - 1}{i(1+i)^n} + 125000 \times \frac{1}{(1+i)^n}$$

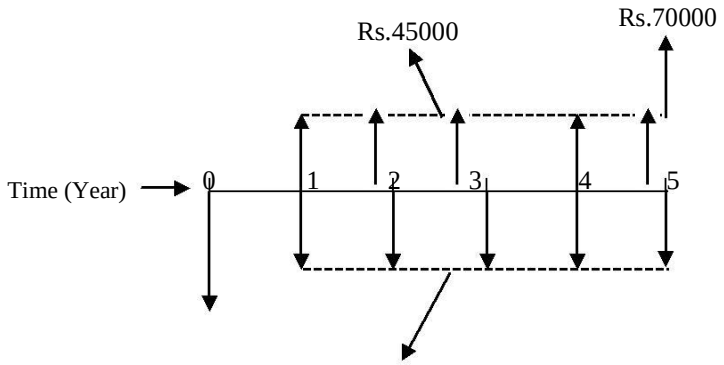
$$PW_1 = -300000 + 30000 \times \frac{(1+0.1)_5 - 1}{0.1(1+0.1)^5} + 125000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -300000 + 30000 \times 3.7908 + 125000 \times 0.6209$$

PW_1 = - 300000 + 113724 + 77613

PW_1 = - Rs.108663

The cash flow diagram of Alternative-2 is shown in Fig. 2.4.



200000

Rs.35000

Fig. 2.4 Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 i.e. PW_2 (in Rs.) is calculated as follows;

PW_2 = - 200000 - 35000(*P/A, i, n*) + 45000*(P/A, i, n)* + 70000(*P/F, i, n*)

PW_2 = - 200000 - 35000(*P/A, 10%, 5*) + 45000*(P/A, 10%, 5)* + 70000(*P/F, 10%, 5*)

PW_2 = - 200000 + (45000 – 35000) (*P/A, 10%, 5*) + 70000(*P/F, 10%, 5*)

PW_2 = - 200000 + 10000(*P/A, 10%, 5*) + 70000(*P/F, 10%, 5*)

$$PW_2 = -200000 + 10000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 70000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -200000 + 10000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 70000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -200000 + 10000 \times 3.7908 + 70000 \times 0.6209$$

PW_2 = - 200000 + 37908 + 43463

***PW2* = - Rs.118629**

Comparing the equivalent present worth of the both the alternatives, it is observed that Alternative-1 will be selected as it shows lower cost compared to Alternative-2.

Problem 3:

A construction contractor has three options to purchase a dump truck for transportation and dumping of soil at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are provided as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1st year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using present worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in Fig. 3.4

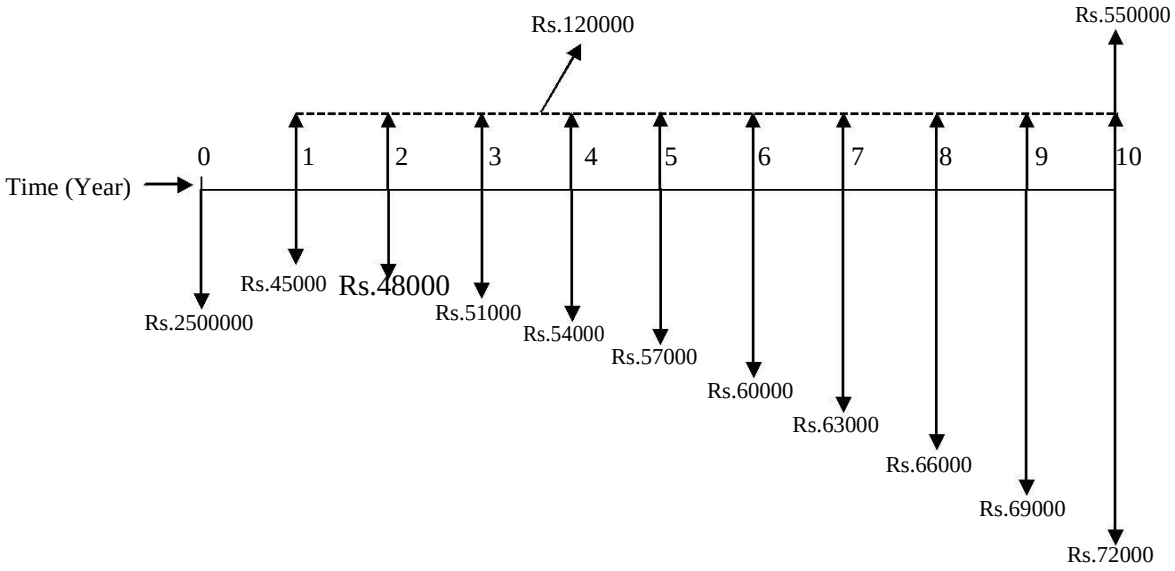
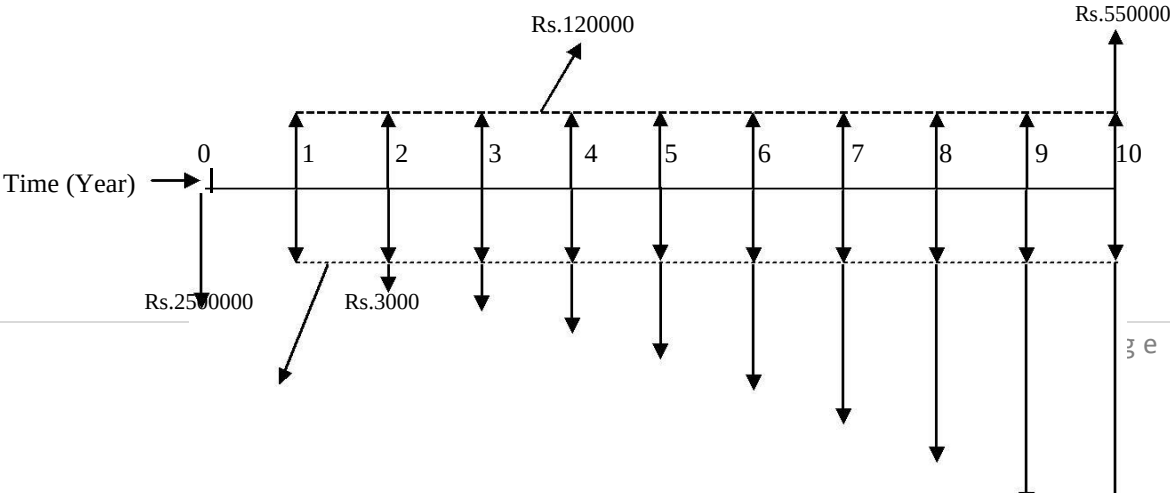


Fig. 3.4 Cash flow diagram of Option-1

For Option-1, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „2". The operating cost at the end of different years can be split into the uniform base amount of Rs.45000 and the gradient amount in multiples of Rs.3000 as shown in Fig. 3.5.



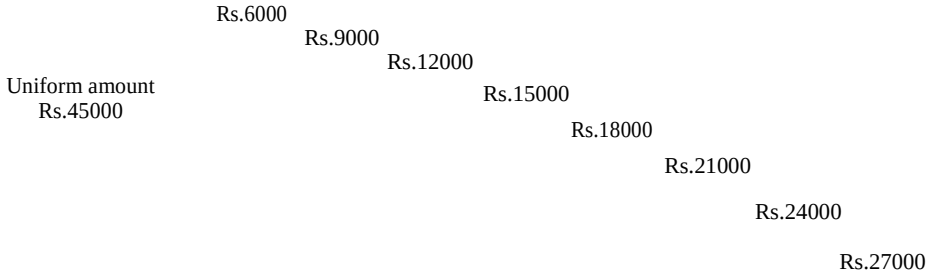


Fig. 3.5 Cash flow diagram of Option-1
with annual operating cost split into uniform base amount and gradient amount

The present worth of the uniform gradient series will be located at the beginning i.e. in year „0" i.e. 2 years before the commencement of the uniform gradient.

Now the equivalent present worth (in Rs.) of Option-1 is calculated as follows;

PW_1 = - 2500000 - 45000(*P/A, i, n*) - 3000*(P/G, i, n)* + 120000*(P/A, i, n)* + 550000(*P/F, i, n*)

PW_1 = - 2500000 - 45000*(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 120000*(P/A, 8%, 10)* + 550000*(P/F, 8%, 10)*

PW_1 = - 2500000 + (120000 - 45000) *(P/A, 8%, 10)* - 3000*(P/G, 8%, 10)* + 550000*(P/F, 8%, 10)*

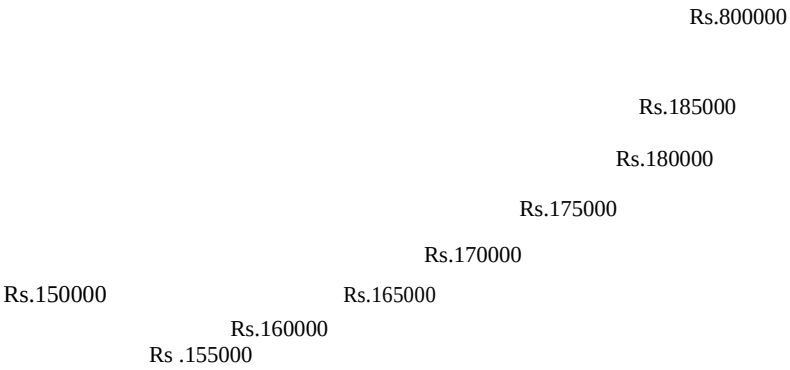
Now putting the values of different compound interest factors (the expressions in terms of *'i'* and *'n'* in the above expression for PW_1 results in the following;

$$PW_1 = -2500000 + 75000 \times 6.7101 - 3000 \times 25.9768 + 550000 \times 0.4632$$

PW_1 = - 2500000 + 503258 - 77930 + 254760

PW_1 = - Rs.1819912

The cash flow diagram of Option-2 is shown in Fig. 3.6



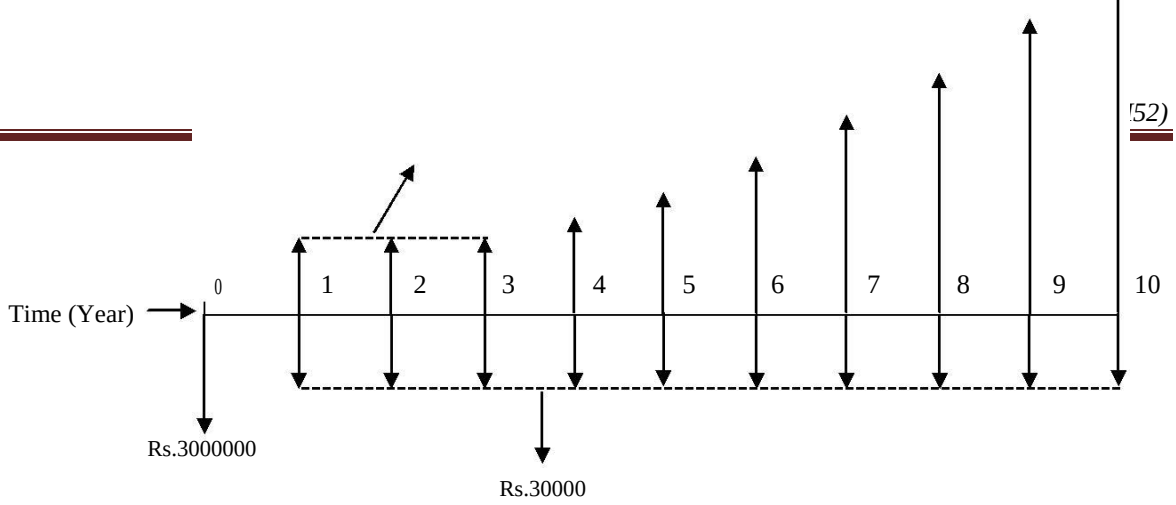


Fig. 3.6 Cash flow diagram of Option-2

For Option-2, the annual income is in the form of a positive uniform gradient series with gradient starting from end of year „4". The annual income can be split into the uniform base amount of Rs.150000 and the gradient amount in multiples of Rs.5000 starting from end of year „4" and is shown in Fig3.7

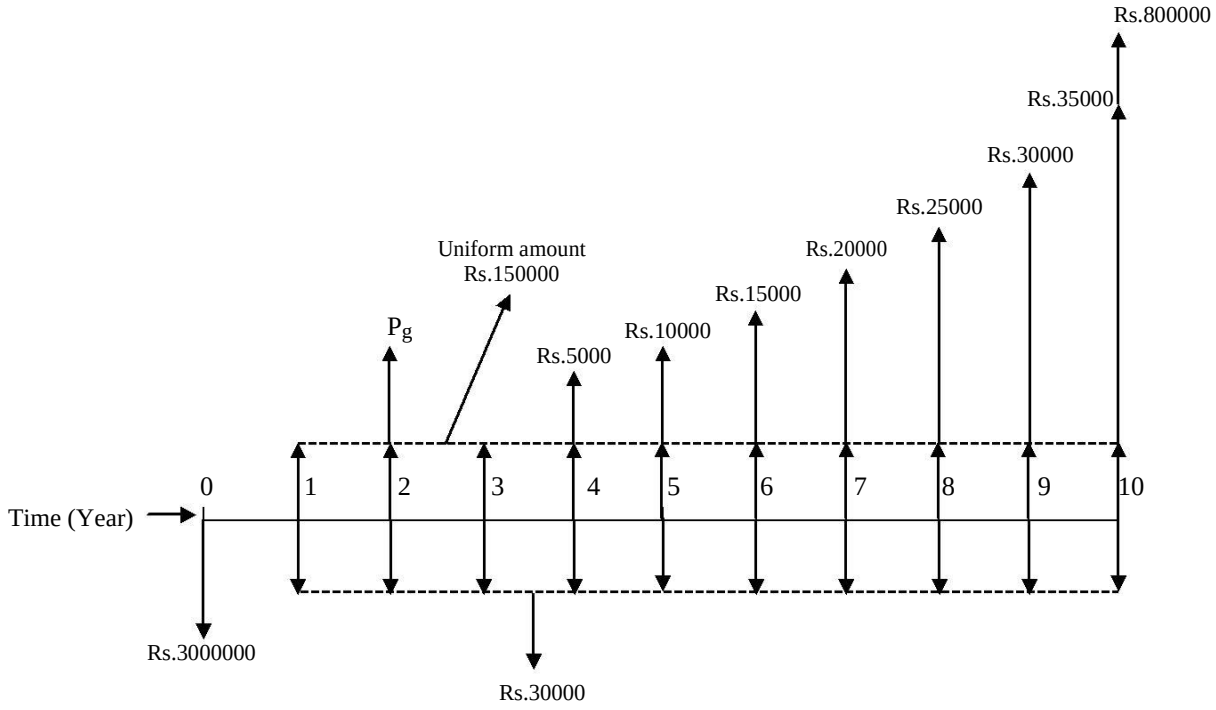


Fig.3.7 Cash flow diagram of Option-2 with
annual income split into uniform base amount and gradient amount

The equivalent present worth of the gradient series (of the annual income) starting from end of year „4" will be located at the end of year „2" i.e. 2 years before the start of the gradient. Further the present worth of this amount at beginning i.e. at time „0" will be obtained by multiplying the equivalent present worth „P_g' (shown in Fig. 3.7) at the end of year „2" (which is a future amount) with the single payment present worth factor *(P/F, i, n)*.

Now the equivalent present worth (in Rs.) of Option-2 is determined as follows;

PW_2 = - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + P_g *(P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 5000(*P/G, 8%, 8*).

PW_2 = - 3000000 - 30000(*P/A, 8%, 10*) + 150000*(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

PW_2 = - 3000000 + (150000 - 30000) *(P/A, 8%, 10)* + 5000*(P/G, 8%, 8) (P/F, 8%, 2)* + 800000(*P/F, 8%, 10*)

Now putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$PW_2 = -3000000 + 120000 \times 6.7101 + 5000 \times 17.8061 \times 0.8573 + 800000 \times 0.4632$

PW_2 = - 3000000 + 805212 + 76326 + 370560

PW_2 = - Rs.1747902

The cash flow diagram of Option-3 is shown in Fig. 3.8

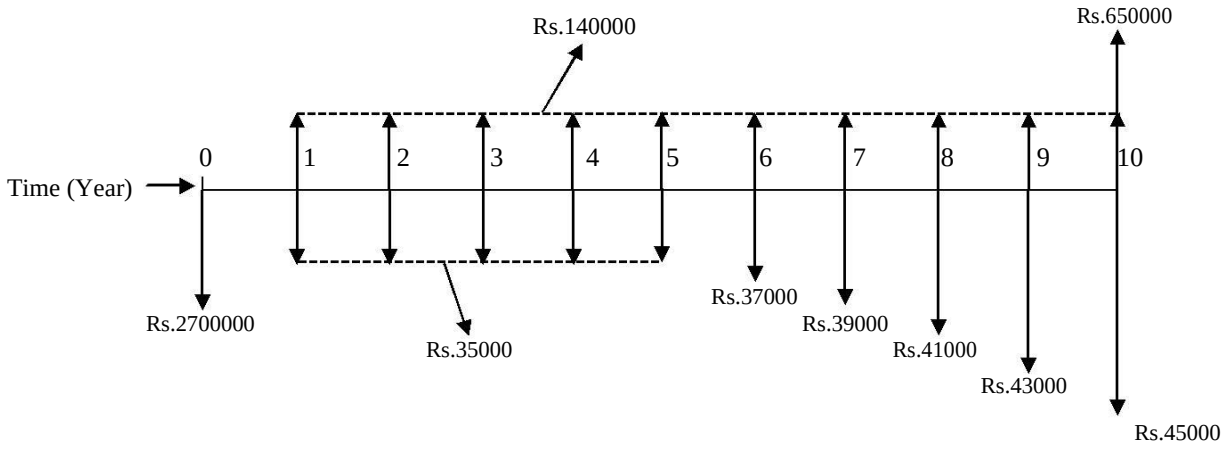


Fig. 3.8 Cash flow diagram of Option-3

For Option-3, the annual operating cost is in the form of a positive uniform gradient series with gradient starting from end of year „6". The annual operating cost can thus be split into the uniform base amount of Rs.35000 and the gradient amount in multiples of Rs.2000 starting from end of year „6" .

The equivalent present worth of the gradient series for the annual operating cost starting from

end of year „6" will be located at the end of year „4". Further the present worth of this amount at time „0" will be determined by multiplying the equivalent present worth „P_g' (shown in Fig. 2.10) at the end of year „4" with the single payment present worth factor *(P/F, i, n)*.

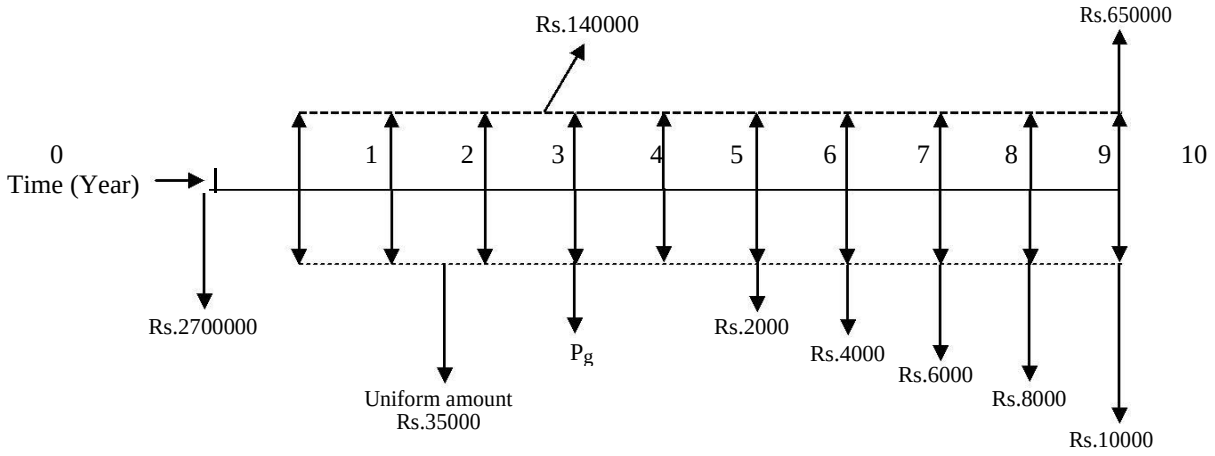


Fig. 3.9 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount

The equivalent present worth (in Rs.) of Option-3 is obtained as follows;

PW_3 = - 2700000 - 35000(*P/A, 8%, 10*) - P_g *(P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

Now in the above expression, P_g will be replaced by *G (P/G, i, n)* i.e. 2000(*P/G, 8%, 6*).

PW_3 = - 2700000 - 35000(*P/A, 8%, 10*) - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 140000*(P/A, 8%, 10)* + 650000(*P/F, 8%, 10*)

PW_3 = - 2700000 + (140000 - 35000) *(P/A, 8%, 10)* - 2000*(P/G, 8%, 6) (P/F, 8%, 4)* + 650000(*P/F, 8%, 10*)

Now putting the values of different compound interest factors in the above expression, the value of PW_3 is given by;

$$PW_3 = -2700000 + 105000 \times 6.7101 - 2000 \times 10.5233 \times 0.7350 + 650000 \times 0.4632$$

PW_3 = - 2700000 + 704561 - 15469 + 301080

PW_3 = - Rs.1709828

From the comparison of equivalent present worth of all the three mutually exclusive alternatives, it is observed that Option-3 shows lowest negative equivalent present worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck.

Problem 4:A material testing laboratory has two alternatives for purchasing a compression testing machine which will be used for determining the compressive strength of different construction materials. The alternatives are from two different manufacturing companies. The cash flow details of the alternatives are as follows;

Alternative-1: Initial purchase price = Rs.1000000, Annual operating cost = Rs.10000, Expected annual income to be generated from testing of different construction materials = Rs.175000, Expected salvage value = Rs.200000, Useful life = 10 years.

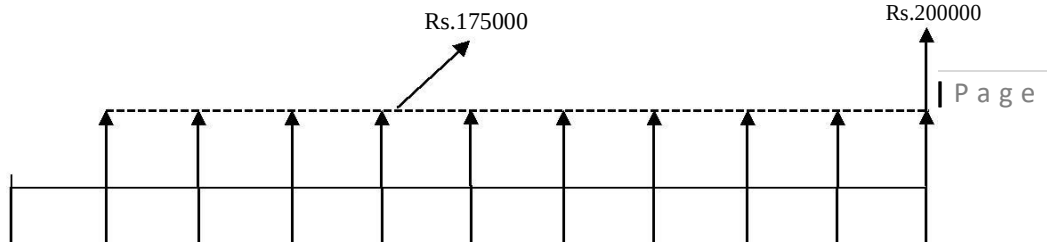
Alternative-2: Initial purchase price = Rs.700000, Annual operating cost = Rs.15000, Expected annual income to be generated from testing of different construction materials = Rs.165000, Expected salvage value = Rs.250000, Useful life = 5 years.

Using present worth method, find out the most economical alternative at the interest rate of 10% per year.

Solution:

The alternatives have different life spans i.e. 10 years and 5 years. Thus the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives. In this case the least common multiple of the life spans is 10 years. Thus the cash flow of Alternative-1 will be analyzed for one cycle (duration of 10 years) whereas the cash flow of Alternative-2 will be analyzed for two cycles (duration of 5 years for each cycle). The cash flow of the Alternative-2 for the second cycle will be exactly same as that in the first cycle.

The cash flow diagram of Alternative-1 is shown in Fig. 3.10



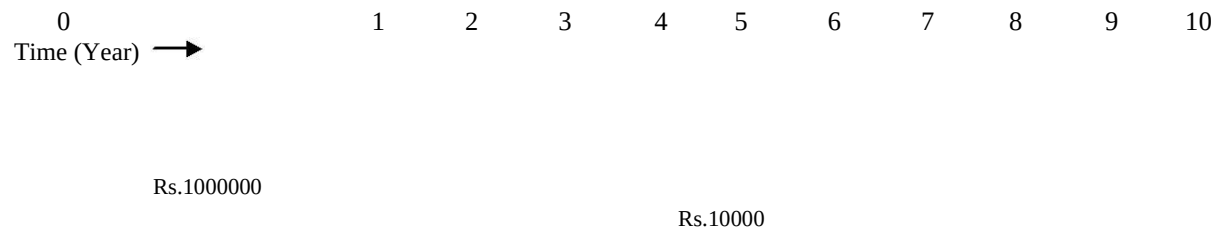


Fig. 3.10 Cash flow diagram of Alternative-1

The equivalent present worth PW_1 (in Rs.) of Alternative-1 is calculated as follows;

PW_1 = - 1000000 - 10000(*P/A, i, n*) + 175000*(P/A, i, n)* + 200000(*P/F, i, n*)

PW_1 = - 1000000 - 10000(*P/A, 10%, 10*) + 175000*(P/A, 10%, 10)* + 200000(*P/F, 10%, 10*)

PW_1 = - 1000000 + (175000 - 10000) (*P/A, 10%, 10*) + 200000(*P/F, 10%, 10*)

Putting the values of different compound interest factors in the above expression for PW_1;

$$PW_1 = -1000000 + 165000 \times 6.1446 + 200000 \times 0.3855$$

PW_1 = - 1000000 + 1013859 + 77100

PW_1 = Rs.90959

The cash flow diagram of Alternative-2 is shown in Fig. 3.11. As the least common multiple of the life spans of the alternatives is 10 years, the cash flow of Alternative-2 is shown for two cycles with each cycle of duration 5 years.

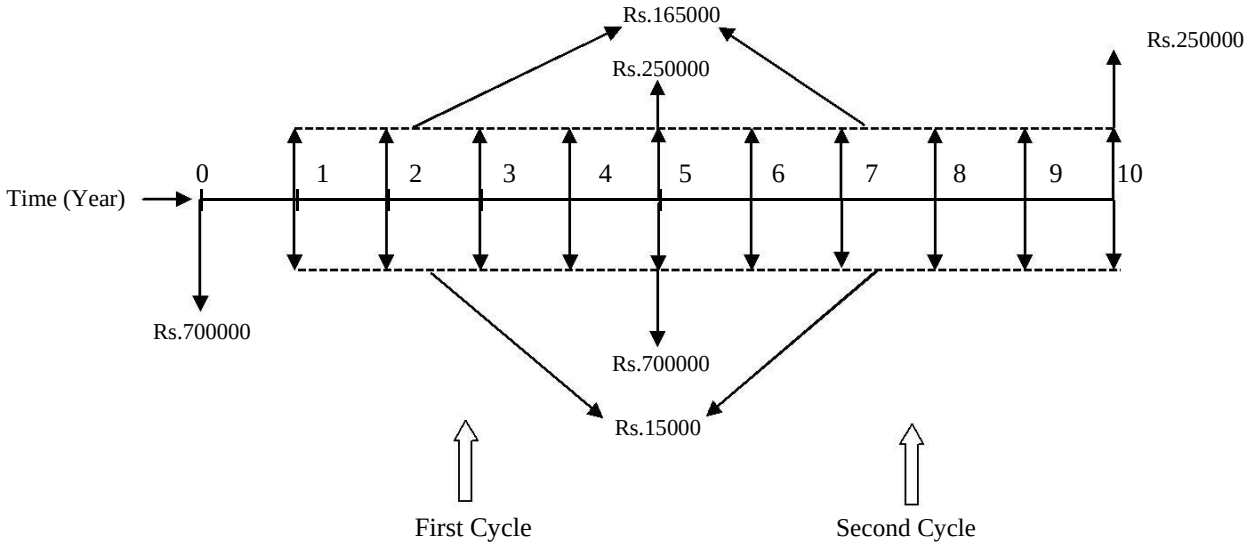


Fig. 3.11 Cash flow diagram of Alternative-2 for two cycles

In the cash flow diagram of Alternative-2, the initial purchase price of Rs.700000 is again

located at the end of year „5" i.e. at the end of first cycle or the beginning of the second cycle. In addition the annual operating cost and the annual income are also repeated in the second cycle from end of year „6" till end of year „10". Further the salvage value of Rs.250000 is also located at end of year „10" i.e. at the end of second cycle.

The equivalent present worth PW_2 (in Rs.) of Alternative-2 is determined as follows;

PW_2 = - 700000 - 15000(*P/A, 10%, 10*) + 165000*(P/A, 10%, 10)* + 250000(*P/F, 10%, 5*) - 700000*(P/F, 10%, 5)* + 250000*(P/F, 10%, 10)*

PW_2 = - 700000 + (165000 - 15000) (*P/A, 10%, 10*) - (700000 – 250000) (*P/F, 10%, 5*) + 250000*(P/F, 10%, 10)*

Putting the values of different compound interest factors in the above expression for PW_2 results in the following;

$PW_2 = -700000 + 150000 \times 6.1446 - 450000 \times 0.6209 + 250000 \times 0.3855$

PW_2 = - 700000 + 921690 - 279405 + 96375

PW_2 = Rs.38660

Thus from the comparison of equivalent present worth of the alternatives, it is evident that Alternative-1 will be selected for purchase of the compression testing machine as it shows the higher positive equivalent present worth.

Problem 5:

A construction firm has decided to purchase a dozer to be employed at a construction site. Two different companies manufacture the dozer that will fulfill the functional requirement of the construction firm. The construction firm will purchase the most economical one from one of these companies. The alternatives have different useful lives. The cash flow details of both alternatives are presented as follows;

Company-A Dozer: Initial purchase cost = Rs.3050000, Annual operating cost Rs.40000 at end of 1st year and increasing by Rs.2000 in the subsequent years till the end of useful life, Annual income = Rs.560000, Expected salvage value = Rs.1050000, Useful life = 6 years.

Company-B Dozer: Initial purchase cost = Rs.4000000, Annual operating cost = Rs.55000, Annual revenue to be generated Rs.600000 at the end of 1st year and increasing by Rs.5000 in the subsequent years till the end of useful life, Expected salvage value = Rs.1000000, Useful life = 12 years.

Using present worth method, find out the most economical alternative at the interest rate of 7% per year.

Solution:

Since the alternatives have different life spans i.e. 6 and 12 years, the comparison will be made over a time period equal to the least common multiple of the life spans of the alternatives i.e. 12 years. The cash flow of Company-A Dozer will be analyzed for two cycles i.e. duration of 6 years for each cycle. The cash flow of Company-B Dozer will be analyzed for one cycle i.e. duration of 12 years.

The cash flow diagram of Company-A Dozer is shown in Fig. 3.12. Since the least common multiple of the life spans of the alternatives is 12 years, the cash flow is shown for two cycles.

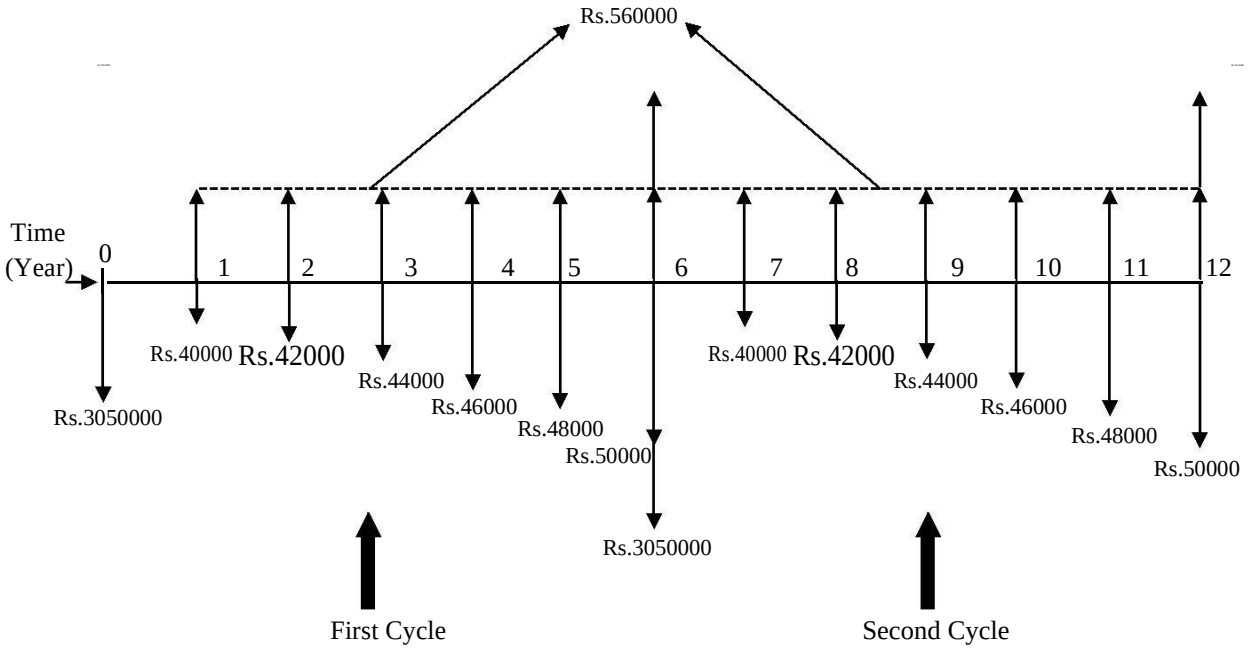


Fig. 3.12 Cash flow diagram of Company-A Dozer for two cycles

For Company-A Dozer, the annual operating cost is in the form of a positive uniform gradient series which can be split into the uniform base amount of Rs.40000 and the gradient amount in multiples of Rs.2000 starting from end of year „2" for first cycle as shown in Fig. 3.12. The equivalent present worth of this gradient for cycle one will be

located at the beginning i.e. in year „0". However for second cycle, the equivalent present worth of the gradient for the annual operating cost starting from end of year „8" (shown in Fig. 3.12) will be located at the end of year „6". Further the present worth of this amount at time „0" will be determined by multiplying the equivalent present worth of the gradient at the end of year „6" with the single payment present worth factor *(P/F, i, n)*.

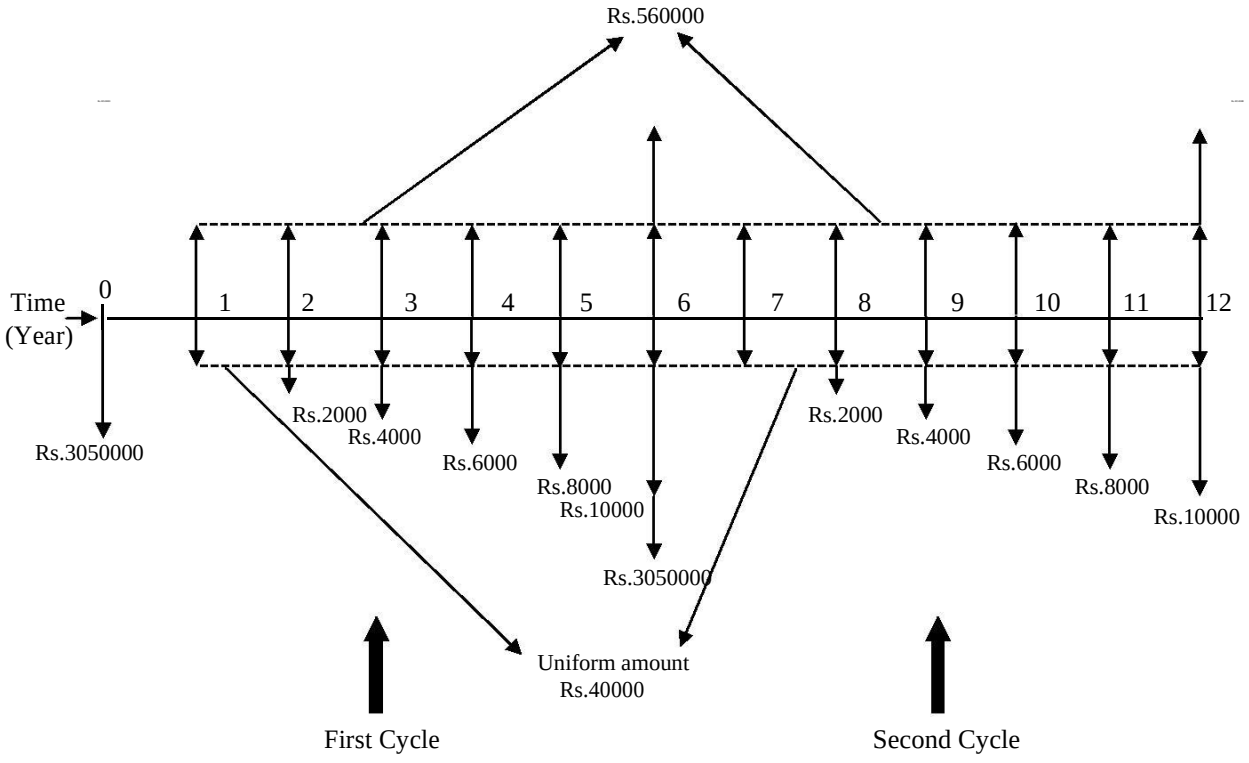


Fig. 3.13 Cash flow diagram of Company-A Dozer for two cycles
with annual operating cost split into uniform base amount and gradient amount

The equivalent present worth PW_A (in Rs.) of Company-A Dozer is calculated as follows;

PW_A = - 3050000 - 40000*(P/A, 7%, 12)* - 2000*(P/G, 7%, 6)* + 560000*(P/A, 7%, 12)* + 1050000*(P/F, 7%, 6)* - 3050000*(P/F, 7%, 6)* - 2000*(P/G, 7%, 6) (P/F, 7%, 6)* + 1050000*(P/F, 7%, 12)*

PW_A = - 3050000 + (560000 - 40000) *(P/A, 7%, 12)* - 2000*(P/G, 7%, 6)* - (3050000 - 1050000) *(P/F, 7%, 6)* - 2000*(P/G, 7%, 6) (P/F, 7%, 6)* + 1050000*(P/F, 7%, 12)*

Putting the values of different compound interest factors in the above expression;

$$PW_A = -3050000 + 520000 \times 7.9427 - 2000 \times 10.9784 - 2000000 \times 0.6663 - 2000 \times 10.9784 \times 0.6663 + 1050000 \times 0.4440$$

PW_A = - 3050000 + 4130204 - 21957 - 1332600 - 14630 + 466200

PW_A = Rs.177217

The cash flow diagram of Company-B Dozer is shown in Fig. 3.14.

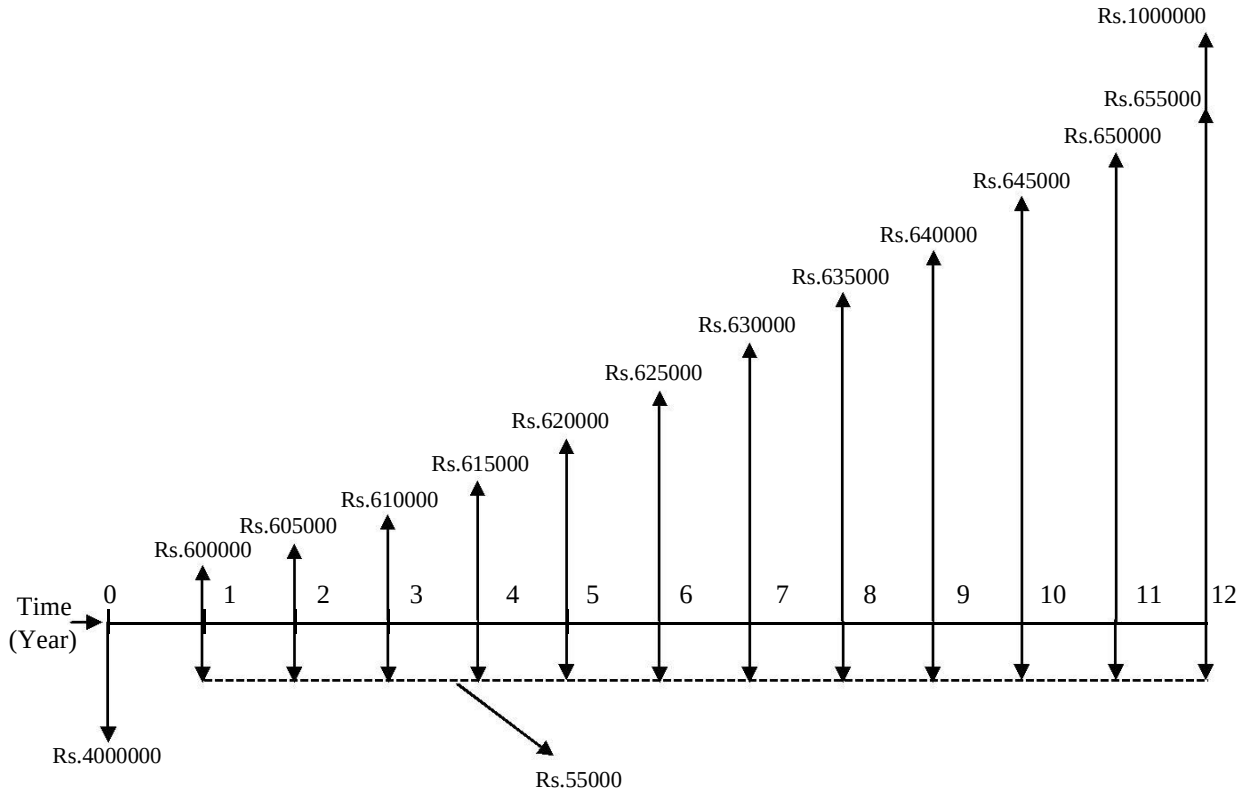


Fig. 3.14 Cash flow diagram of Company-B Dozer

For Company-B Dozer, the annual revenue is in the form of a positive uniform gradient series that can be split into the uniform base amount of Rs.600000 and gradient amount in multiples of Rs.5000 as shown in Fig. 3.14. The equivalent present worth of this gradient amount will be located at the beginning i.e. in year „0".

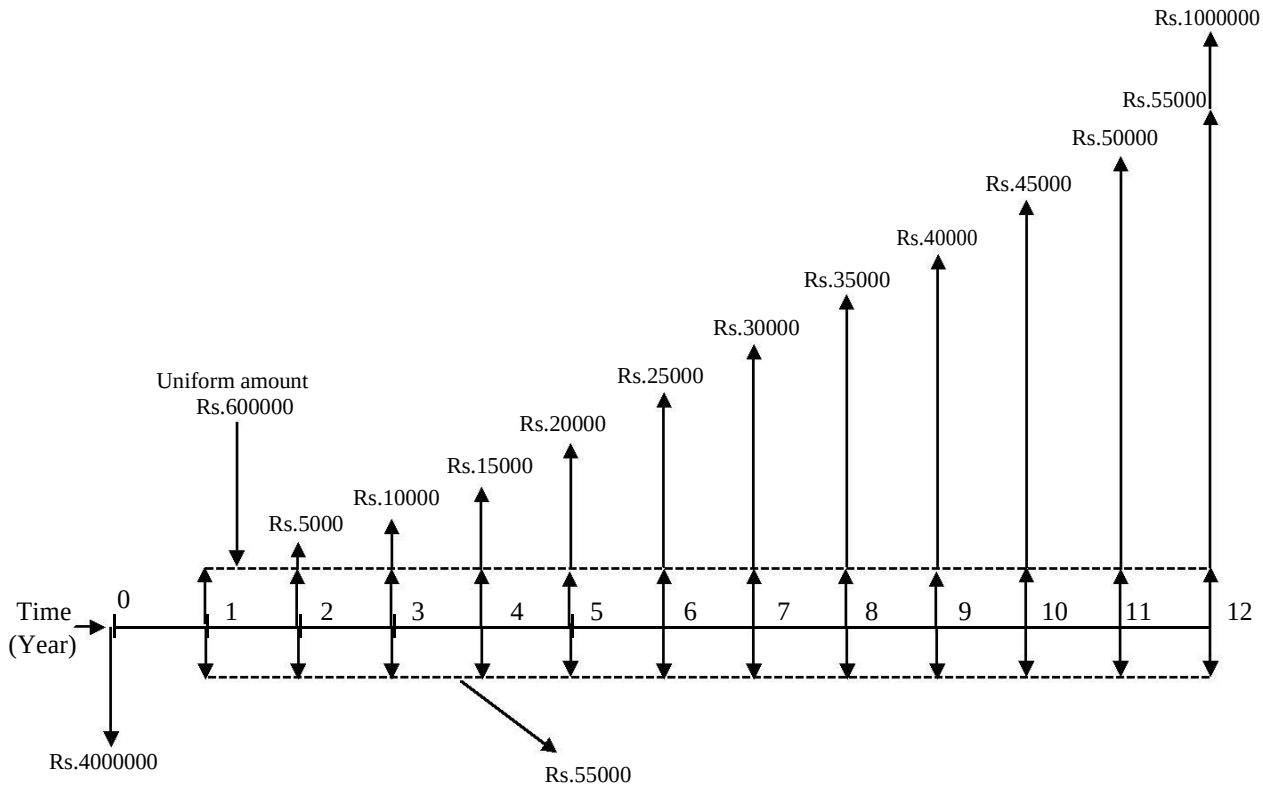


Fig. 3.15 Cash flow diagram of Company-B Dozer
with annual revenue split into uniform base amount and gradient amount

The equivalent present worth PW_B (in Rs.) of Company-B Dozer is determined as follows;

PW_B = - 4000000 - 55000*(P/A, 7%, 12)* + 600000*(P/A, 7%, 12)* + 5000*(P/G, 7%, 12)* + 1000000*(P/F, 7%, 12)*

PW_B = - 4000000 + (600000 - 55000) *(P/A, 7%, 12)* + 5000*(P/G, 7%, 12)* + 1000000*(P/F, 7%, 12)*

Now putting the values of different compound interest factors in the above expression for PW_B results in the following;

PW_B =– 4000000+545000×7.9427 +5000×37.3506+1000000×0.4440

PW_B = - 4000000 + 4328772 + 186753 + 444000

PW_B = Rs.959525

Thus from the comparison of equivalent present worth of the alternatives, it is evident that the construction firm should select Company-B Dozer over Company-A Dozer, as it shows higher positive equivalent present worth i.e. $PW_B > PW_A$.

Comparison by future worth method:-

In the following example, the comparison of three mutually exclusive alternatives by future worth method will be illustrated.

Problem 6: A construction contractor has three options to purchase a dump truck for transportation and dumping of earth at a construction site. All the alternatives have the same useful life. The cash flow details of all the alternatives are presented as follows;

Option-1: Initial purchase price = Rs.2500000, Annual operating cost Rs.45000 at the end of 1^{st} year and increasing by Rs.3000 in the subsequent years till the end of useful life, Annual income = Rs.120000, Salvage value = Rs.550000, Useful life = 10 years.

Option-2: Initial purchase price = Rs.3000000, Annual operating cost = Rs.30000, Annual income Rs.150000 for first three years and increasing by Rs.5000 in the subsequent years till the end of useful life, Salvage value = Rs.800000, Useful life = 10 years.

Option-3: Initial purchase price = Rs.2700000, Annual operating cost Rs.35000 for first 5 years and increasing by Rs.2000 in the successive years till the end of useful life, Annual income = Rs.140000, Expected salvage value = Rs.650000, Useful life = 10 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 8% per year.

Solution:

The cash flow diagram of Option-1 is shown in figure 3.16.

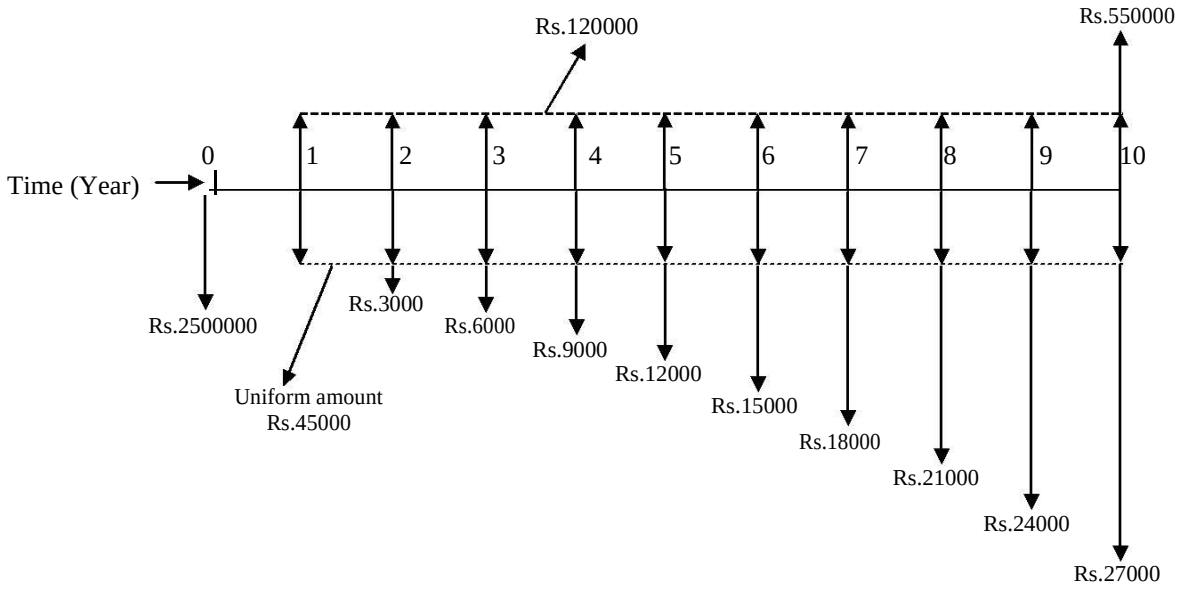


Fig. 3.16 Cash flow diagram of Option-1 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

The equivalent future worth (in Rs.) of Option-1 is determined as follows;

$$FW_1 = -2500000(F/P,8\%,10) - 45000(F/A,8\%,10) - 3000(F/G,8\%,10) + 120000(F/A,8\%,10) + 550000$$

$$FW_1 = -2500000(F/P,8\%,10) + (120000 - 45000)(F/A,8\%,10) - 3000(F/G,8\%,10) + 550000$$

Now putting the values of different compound interest factors in the above expression for FW_1 results in the following;

$$FW_1 = -2500000 \times 2.1589 + 75000 \times 14.4866 - 3000 \times 56.0820 + 550000$$

$$FW_1 = -5397250 + 1086495 - 168246 + 550000$$

FW_1 = - Rs.3929001

The cash flow diagram of Option-2 is shown in figure 3.17

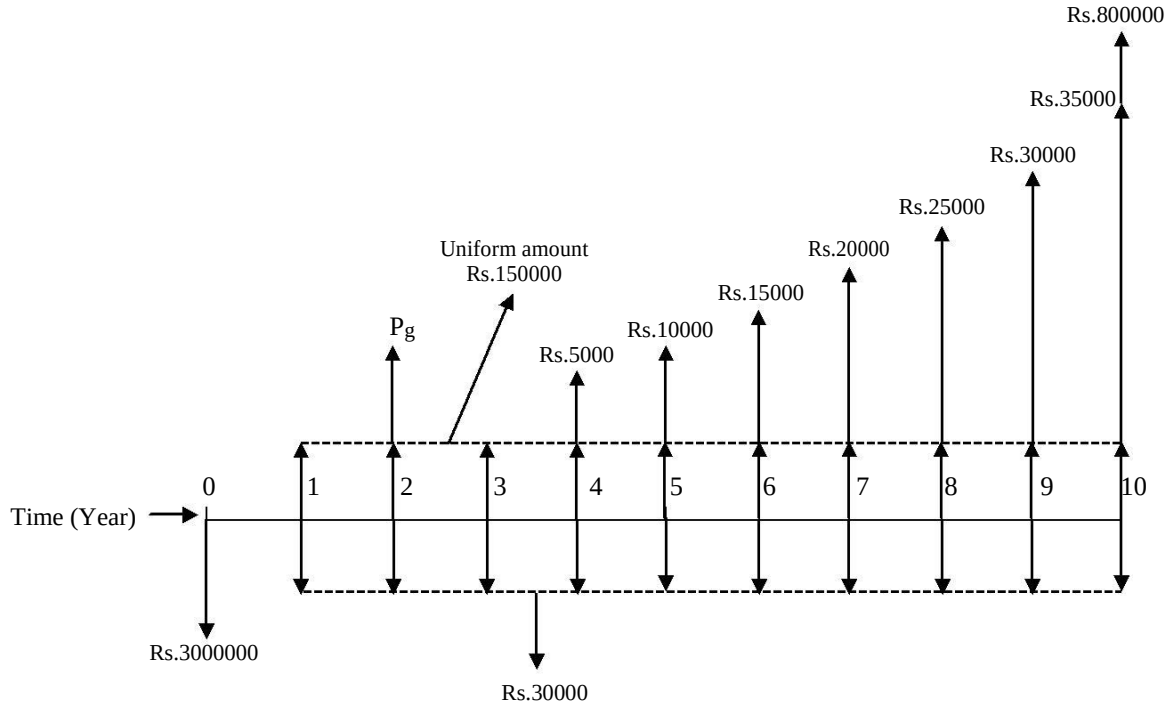


Fig.3.17 Cash flow diagram of Option-2 with annual income split into uniform base amount and gradient amount (shown for ready reference)

The equivalent present worth of the gradient series (of the annual income) starting from end of year „4" will be located at the end of year „2". The future worth of this amount at end of year „10" will be obtained by multiplying the equivalent present worth „P_g' (shown in Fig. 2.8) at the end of year „2" with the single payment compound amount factor *(F/P, i, n)*.

The equivalent future worth (in Rs.) of Option-2 is determined as follows;

$$FW_2 = -3000000(F/P,8\%,10) - 30000(F/A,8\%,10) + 150000(F/A,8\%,10) + P_g(F/P,8\%,8) + 800000$$

Now replacing P_g with *G (P/G, i, n)* i.e. 5000(*P/G, 8%, 8*) in the above expression;

$$FW_2 = -3000000(F/P,8\%,10) + (150000 - 30000)(F/A,8\%,10) + 5000(P/G,8\%,8)(F/P,8\%,8) + 800000$$

It may be noted here that, in the above expression, 5000*(P/G, 8%, 8) (F/P, 8%, 8)* can be replaced by 5000*(F/G, 8%, 8)* and will result in the same value.

Now putting the values of different compound interest factors in the above expression;

$$FW_2 = -3000000 \times 2.1589 + 120000 \times 14.4866 + 5000 \times 17.8061 \times 1.8509 + 800000$$

FW_2 = - 6476700 + 1738392 + 164787 + 800000

FW_2 = - Rs.3773521

The cash flow diagram of Option-3 is shown in figure 3.18

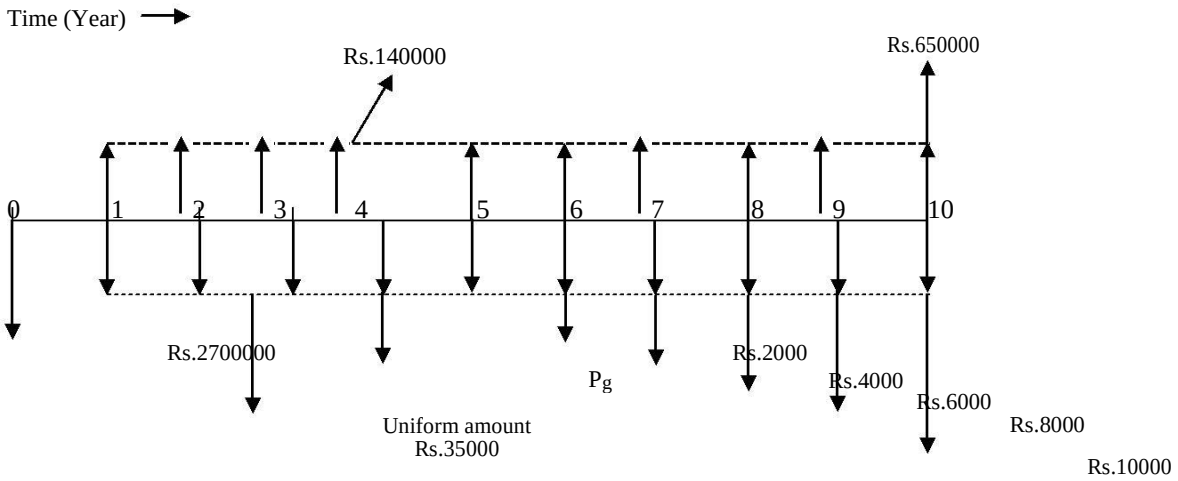


Fig. 3.18 Cash flow diagram of Option-3 with annual operating cost split into uniform base amount and gradient amount (shown for ready reference)

For the annual operating cost, the equivalent present worth of the gradient series starting from end of year „6" will be located at the end of year „4". The future worth of this amount at end of year „10" will be determined by multiplying the equivalent

present worth „P_g' (shown in Fig. 2.10) at the end of year „4" with the single payment compound amount factor *(F/P, i, n).*

The equivalent future worth (in Rs.) of Option-3 is determined as follows;

$$FW_3 = -2700000(F/P,8\%,10) - 35000(F/A,8\%,10) - P_g(F/P,8\%,6) + 140000(F/A,8\%,10) + 650000$$

Now replacing P_g with *G (P/G, i, n)* i.e. 2000(*P/G, 8%, 6*) in the above expression;

$$FW_3 = -2700000(F/P,8\%,10) + (140000 - 35000)(F/A,8\%,10) - 2000(P/G,8\%,6)(F/P,8\%,6) + 650000$$

In the above expression, 2000*(P/G, 8%, 6) (F/P, 8%, 6)* can also be replaced by 2000*(F/G, 8%, 6).*

Now putting the values of different compound interest factors in the above expression;

$$FW_3 = -2700000 \times 2.1589 + 105000 \times 14.4866 - 2000 \times 10.5233 \times 1.5869 + 650000$$

$$FW_3 = -5829030 + 1521093 - 33399 + 650000$$

FW_3 = - Rs.3691336

Comparing the equivalent future worth of all the three alternatives, it is evident that Option-3 shows lowest negative equivalent future worth as compared to other options. Thus Option-3 will be selected for the purchase of the dump truck. This outcome obtained by future worth method is same as that obtained from the present worth method i.e. Option-3 is the most economical alternative.

Pay-back comparision:

payback method does not consider the present value of cash flows. Under this method, an investment project is accepted or rejected on the basis of payback period. Payback period means the period of time that a project requires to recover the money invested in it. The payback period of a project is expressed in years and is computed using the following formula:

Formula of payback period:

$$\text{Payback period} = \frac{\text{Investment required for a project}}{\text{Net annual cash inflow}}$$

Pay back peroid=(Required investment/Annual receipts-annual disbursements)

Pay back=(investment-salvage) / Operating Advantage/year=(I-S)/OA/year

Problem1: The lake city bank is considering a purchase of a data processing storage unit which will cost Rs 20,000 and will last 20 years and then have a guaranteed salvage value of Rs 2000.It will generate savings of Rs 4000/year before depreciation but necessiates that Rs 1000 of the savings must be paid in taxes.If Management insists on a 5 year pay off period.Does this investment qualify.?

Solution:

Given: Investment I=Rs 20,000

Expected life =20 years

Salvage value=Rs 2000

O.A/year= Rs 4000

Taxes paid=Rs 1000/year

OA/year after taxes=OA/year-taxes=4000-1000=Rs 3000

Pay off period= 5 years

Payback period is given by…. Paypack period = (I-S)/OA per year= (20000-2000)/3000= 6 years.

Conclusion: The investment doesnot meet the management criteria as the payback period is more than the required one.