

Engineering Economics

Module -2

Present Worth Comparisons

Present worth(PW)

In this method of **comparison**, the cash flows of each alternative will be reduced to time zero by assuming an interest rate i . In most of the practical decision environments, executives will be forced to select the best alternative from a set of competing alternatives.

There are 5 basic methods to have results of selected alternative or to be analysed

Equivalent worth :

- i) Present worth (PW)
- ii) Annual worth (AW)
- iii) Future worth (FW)

Rate of returns

- i) Internal rate of return(IRR)
- ii) External rate of return(ERR)

Conditions for PW comparisions

1. Cash flows are known
2. Cash flows are in constant value Rs.
3. The interest rate is known
4. Comparisions are made with before-tax cash flows
5. Comparisions do not include intangible considerations
6. Comparisions do not include consideration of the availability of funds to implement alternatives

REVENUE-DOMINATED CASH FLOW DIAGRAM

A generalized revenue-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Figure

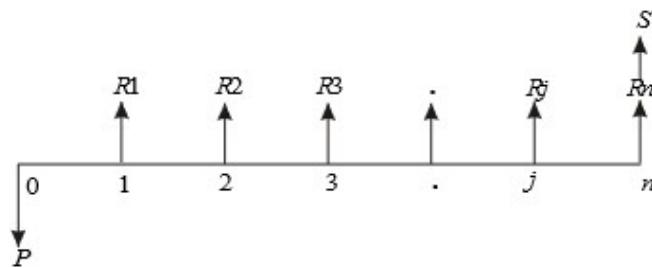


Fig. 4.1 Revenue-dominated cash flow diagram.

In Fig. 4.1,

P represents an initial investment

R_j the net revenue at the end of the jth year.
The interest rate is i, compounded annually.

S is the salvage value at the end of the nth year.

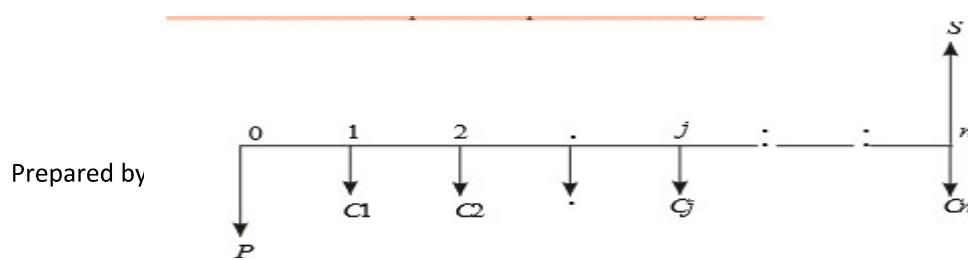
Expenditure is assigned a negative sign and revenues are assigned a positive sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared.

Finally, the alternative with the maximum present worth amount should be selected as the best alternative.

COST-DOMINATED CASH FLOW DIAGRAM

A generalized cost-dominated cash flow diagram to demonstrate the present worth method of comparison is presented in Fig. 4.2.



In Fig. 4.2,

P represents an initial investment

C_j the net cost of operation and maintenance at the end of the j th year

S the salvage value at the end of the n th year

Expenditure is assigned a positive sign and the revenue a negative sign.

If we have some more alternatives which are to be compared with this alternative, then the corresponding present worth amounts are to be computed and compared.

Finally, the alternative with the minimum present worth amount should be selected as the best alternative.

Basic PW comparisions

The present worth of a cash flow overtime is its value today is represented as time ‘0’ in a cash flow diagram.

Two general patterns are,

i) Present worth Equivalence

Example problem

1. The lease on a warehouse amounts to Rs.5000 per month for 5yrs. If payments are made on the first of each month, what is the present worth of the agreement at a nominal annual interest rate of 12% compounded annually.

$$A=5000$$

$$N=5*12=60 \text{ months} \quad i=12\% = 12/12=1\%$$

$$\begin{aligned} PW &= A(P/A, 1\%, 60) \\ &= 5000(P/A, 1\%, 60) \\ &= 5000 * 44.955 \end{aligned}$$

$$\mathbf{PW=2,24,775}$$

ii) Net Present Worth

It has an initial outlay at time 0 followed by a series of receipts and disbursements.

$$\text{Net present-worth} = PW(\text{benefits}) - PW(\text{costs})$$

Example problem

1. A piece of new equipments was proposed by engineers to increase the production of a certain manual welding operation. The investment is Rs.25000 and the equivalent will have salvage value of Rs.5000 at the end of 5yrs. Increased productivity will gain Rs.8000 per year after extra operating costs have been subtracted from the additional production, with rate of interest 20%. Use present worth method.

Ans:

$$\text{Total PW} = \text{PW of cash receipts} - \text{PW of cash outlays}$$

$$= 8000(P/A, 20\%, 5) + 5000(P/F, 20\%, 5) - 25000$$

$$= 8000(2.991) + 5000(0.4019) - 25000 \text{ using table}$$

$$\text{Total PW} = 934.5$$

Problems based on the Concept of PW method of comparison applied to the selection of best alternative.

Problems

1. Alpha industry is planning to expand its production operation. It has identified three different technologies for meeting the goal. The initial outlay and annual revenues with respect to each of the technologies are summarized in Table

below. Suggest the best technology which is to be implemented based on the present worth method of comparison assuming 20% interest rate, compounded annually.

	Initial outlay (Rs.)	Annual revenue (Rs.)	Life (years)
Technology 1	12,00,000	4,00,000	10
Technology 2	20,00,000	6,00,000	10
Technology 3	18,00,000	5,00,000	10

In all technologies, the initial outlay is assigned negative sign and annual revenues are assigned positive sign

Technology 1

$$P=12,00,000$$

$$A=4,00,000$$

$$i=20\%$$

$$n=10 \text{ years}$$

The cash flow diagram of this technology is as shown in Fig. 4.3.

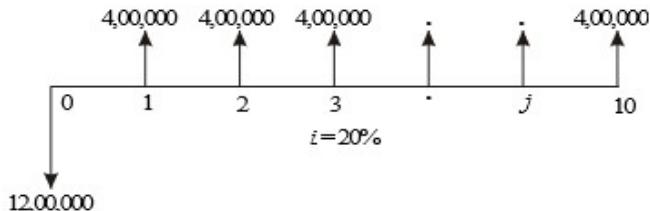


Fig. 4.3 Cash flow diagram for technology 1.

$$PW(20\%) = -12,00,000 + 4,00,000(P/A, 20\%, 10)$$

$$= -12,00,000 + 4,00,000 * 4.192$$

PW(20%) = Rs. 4,76,800

Technology 2

$$P=20,00,000, A=6,00,000, i=20\%, n=10$$

The cash flow diagram of this technology is shown in Fig. 4.4.

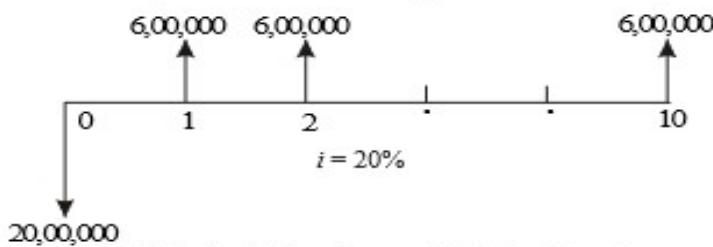


Fig. 4.4 Cash flow diagram for technology 2.

$$\begin{aligned} \text{PW}(20\%) &= -20,00,000 + 6,00,000(P/A, 20\%, 10) \\ &= -20,00,000 + 6,00,000 * 4.192 \end{aligned}$$

PW(20%)=5,15,200

Technology 3

$$P=18,00,000, A=5,00,000, i=20\%, n=10$$

The cash flow diagram of this technology is shown in Fig. 4.5.

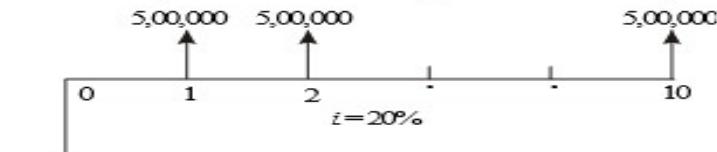


Fig. 4.5 Cash flow diagram for technology 3.

$$\begin{aligned} \text{PW}(20\%) &= -18,00,000 + 5,00,000(P/A, 20\%, 10) \\ &= -18,00,000 + 5,00,000 * 4.1925 \end{aligned}$$

PW(20%)=2,96,250

Technology 2 should be considered.

2. An engineer has two bids for an elevator to be installed in a new building. The details of the bids for the elevators are as follows:

Bid	Engineers estimates		
	Initial cost (Rs.)	Service life (years)	Annual operations and maintainance costs(Rs)
Alpha elevator	4,50,000	15	27000
Beta elevator	5,40,000	15	28500

Determine which bid should be accepted, based on the present worth method of comparison assuming 15% interest rate, compounded annually.

Bid 1 : Alpha elevator

$$P=4,50,000 \quad A=27,000 \quad n=15 \quad i=15\%$$

$$\begin{aligned} PW(15\%) &= 4,50,000 + 27,000 (P/A, 15\%, 15) \\ &= 4,50,000 + 27000 * 5.8474 \end{aligned}$$

$$\mathbf{PW(15\%)=6,07,879.80}$$

The cash flow diagram of bid 1 is shown in Fig. 4.6.

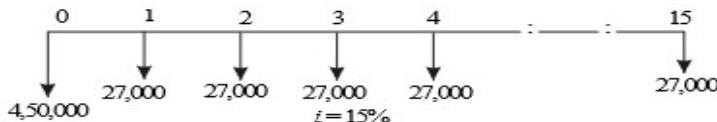


Fig. 4.6 Cash flow diagram for bid 1.

Bid 2 : Beta elevator

$$P=5,40,000 \quad A=28,500 \quad n=15 \quad i=15\%$$

$$PW(15\%)=5,40,000 + 28,500 (P/A, 15\%, 15)$$

$$=5,40,000 + 28,500 * 5.8474$$

$$\text{PW}(15\%)=7,06,650.90$$

The cash flow diagram of bid 2 is shown in Fig. 4.7.

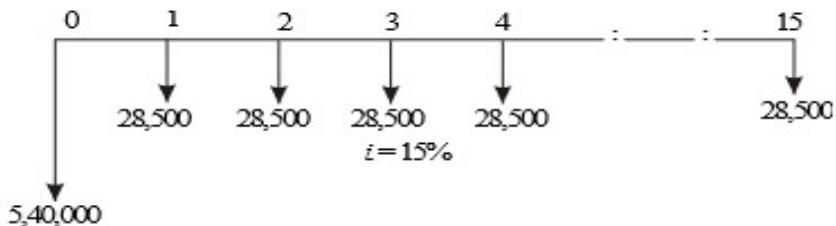


Fig. 4.7 Cash flow diagram for bid 2.

Alpha elevator should be implemented.

3. Investment proposals A and B have the net cash flows as follows:

Proposals	End of years				
	0	1	2	3	4
A(Rs.)	-10,000 6000	3000	3000	7000	
B(Rs.)	-10,000 3000	6000	6000	3000	

Compare the present worth of A with that of B at $i = 18\%$. Which proposal should be selected?

Proposal A

Present worth of A at $i = 18\%$. The cash flow diagram of proposal A is shown in Fig. 4.8.

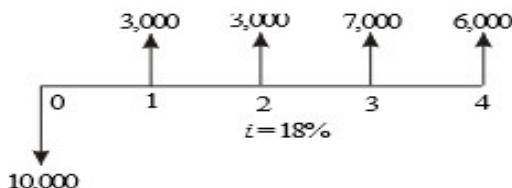


Fig. 4.8 Cash flow diagram for proposal A.

$$PW(18\%) = -$$

$$10,000 + 3000(P/F, 18\%, 1) + 3000(P/F, 18\%, 2) + 7000(P/F, 18\%, 3) \\ + 6000(P/F, 18\%, 4)$$

$$= -10,000 + 3000(0.8475) + 3000(0.7182) + 7000(0.6086) + \\ 6000(0.5158)$$

$$PW(18\%) = \text{Rs. 2052}$$

Proposal B

Present worth of B at $i = 18\%$. The cash flow diagram of the proposal B is shown in Fig. 4.9.

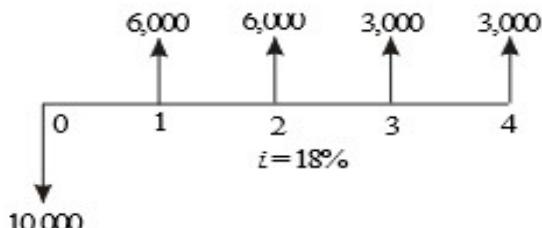


Fig. 4.9 Cash flow diagram for proposal B.

$$PW(18\%) = -10,000 + 6000(P/F, 18\%, 1) + 6000(P/F, 18\%, 2) + \\ 3000(P/F, 18\%, 3) + 3000(P/F, 18\%, 4)$$

$$= -10000 + 6000 (0.8475) + 6000 (0.7182) + 3000(0.6086) + 3000(0.5158)$$

PW(18%) = Rs.2766.2

The PW of proposal B is higher than proposal A . so select proposal B.

4. A granite company is planning to buy a fully automated granite cutting machine. If it is purchased under down payment, the cost of the machine is Rs. 16,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of Rs. 2,00,000 each. Suggest the best alternative for the company using the present worth basis at $i = 18\%$, compounded annually.

Two alternatives,

- i) Down payment = **Rs.16,00,000**
- ii) Down payment of Rs.4,00,000 and 10 annual equal installments of 2,00,000 each.

$$\begin{aligned} \text{PW}(18\%) &= 4,00,000 + 2,00,000 (P/A, 18\%, 10) \\ &= 4,00,000 + 2,00,000 * 4.494 \end{aligned}$$

PW(18%)=12,98,800

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.10.

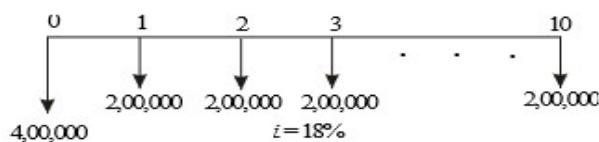


Fig. 4.10 Cash flow diagram for the second alternative.

Second alternative should be selected.

5. A small business with an initial outlay of Rs. 12,000 yields Rs. 10,000 during the first year of its operation and the yield increases by Rs. 1,000 from its second year of operation up to its 10th year of operation. At the end of the life of the business, the salvage value is zero. Find the present worth of the business by assuming an interest rate of 18%, compounded annually.

The cash flow diagram for the small business is depicted in Fig. 4.15.

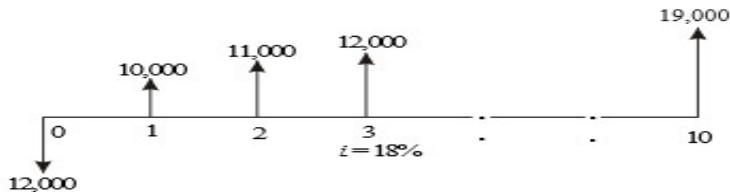


Fig. 4.15 Cash flow diagram for the small business.

$$\begin{aligned} A &= A1 + G (A/G, 18\%, 10) \\ &= 10,000 + 1000 (3.194) \end{aligned}$$

$$A = 13,194$$

$$\begin{aligned} PW(18\%) &= -12,000 + A (P/A, 18\%, 10) \\ &= -12,000 + 13,194 (4.494) \end{aligned}$$

$$\mathbf{PW(18\%) = 47,293}$$

6. A company borrowed 1,00,000 to finance a new product the loan was for 20 years at a nominal interest rate of 8% compounded semiannually. It was to be repaid in 40 equal payments. After one half (1/2) the payments were made, the company decided to pay the remaining balance in one final payment at the end of the 10th year how much was owed?

$$P=1,00,000 \quad N=20 \text{ years} = 20 * 2 = 40 \text{ installments}$$

$i=8\%$ Compounded semi-annually $= 8/2=4\%$

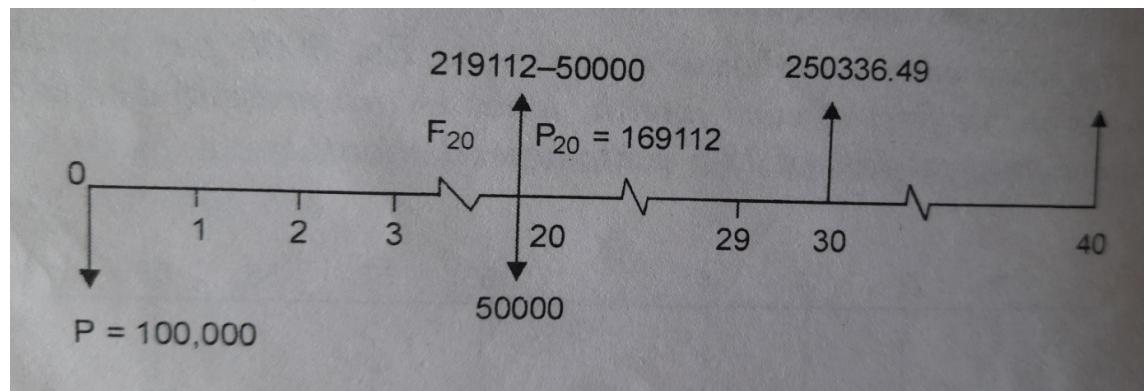
$$\begin{aligned} \text{i) } F_{20} &= P(F/P, 4\%, 20) \\ &= 1,00,000 (2.19112) \\ \mathbf{F_{20}} &= \mathbf{2,19,112} \end{aligned}$$

$$P = 50,000 - 2,19,112$$

$$\mathbf{P=1,69,112}$$

$$\begin{aligned} \text{ii) } F_{30} &= P(F/P, 4\%, 10) \\ &= 1,69,112 * 1.4803 \\ \mathbf{F_{30}} &= \mathbf{2,50,336.49} \end{aligned}$$

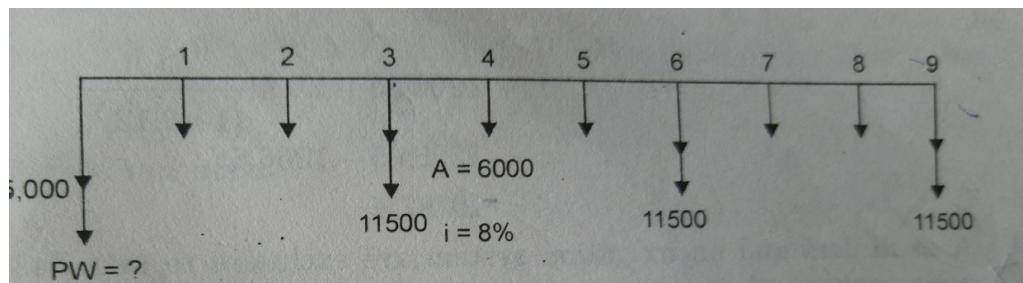
The company owed to finance is **2,50,336.49**



7. A proposed improvement in an assembly line will have an initial purchase and installation cost of Rs.1,75,000. The annual maintenance cost will be Rs.6000. Periodic overhauls once every 3yrs excluding the last year of use, will cost Rs.11,500 each. The improvement will have a usefull life of 9 years at which time it will have no salvage value, what is the

present worth of the 9 year costs of the improvement at $i=8\%$?

$$\begin{aligned}
 PW &= A(P/A, i\%, N) + F(P/F, i\%, N) + F(P/F, i\%, N) + \\
 &\quad F(P/F, i\%, N) \\
 &= 6000(P/A, 8\%, 9) + 11500(P/F, 8\%, 3) + \\
 &\quad 11500(P/F, 8\%, 6) + 11500(P/F, 8\%, 9) + \text{Initial} \\
 &\quad \text{investment} \\
 &= 6000(6.247) + 11500(0.793) + 11500(0.630) \\
 &\quad + 11500(0.5002) + 1,75,000 \\
 \mathbf{PW} &= \mathbf{2,34,598.8}
 \end{aligned}$$



8. A bakery is thinking of purchasing a small delivery truck that has a first cost of Rs.18,000 and is to be kept in service for 6 years. At what time the salvage value is expected to be 2500. Maintenance and operating costs are estimated at Rs.2500 the first year and will increase at a rate of Rs.200/year. Determine the PW of this vehicle using interest rate of 12%.

$$A = A_1 + G \left(\frac{A}{G}, 12\%, 6 \right)$$

$$A = 2500 + 200 (2.17205)$$

$$\mathbf{A=2934.4}$$

$$PW = A(P/A, i\%, N) - F(P/F, i\%, N)$$

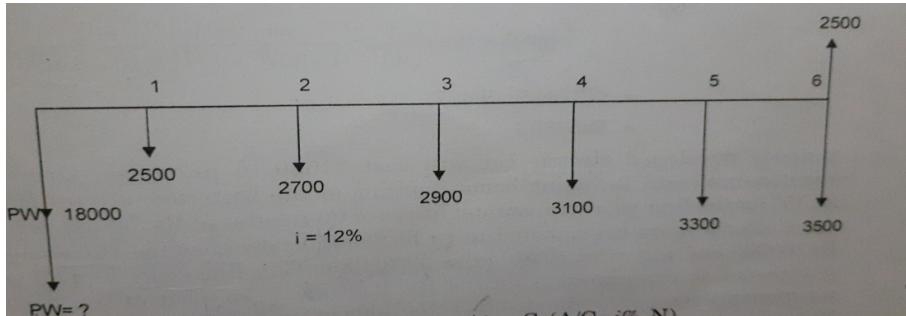
$$= 2934.4 (P/A, 12\%, 6) - 2500 (P/F, 12\%, 6)$$

$$= 2934.4 (4.111) - 2500 (0.5066)$$

$$\mathbf{PW=10,797.9}$$

$$\mathbf{PW \text{ of the vehicle} = 10,797.9 + 18,000}$$

$$= \mathbf{28,797.9}$$



9. A small dam and an irrigation system are exclusive to cost Rs.3,00,000. Annual maintenance and operating costs are expected to be Rs.40000 the first year and will increase at a rate of 10% per year. Determine the equivalent PW of building and operating the system with interest of 10% over a 30 year life.

$$i=10\% \quad N=30$$

$$\begin{aligned} A &= A_1 + G (A/G, i\%, N) \\ &= 40000 + 4000 (A/G, 10\%, 30) \end{aligned}$$

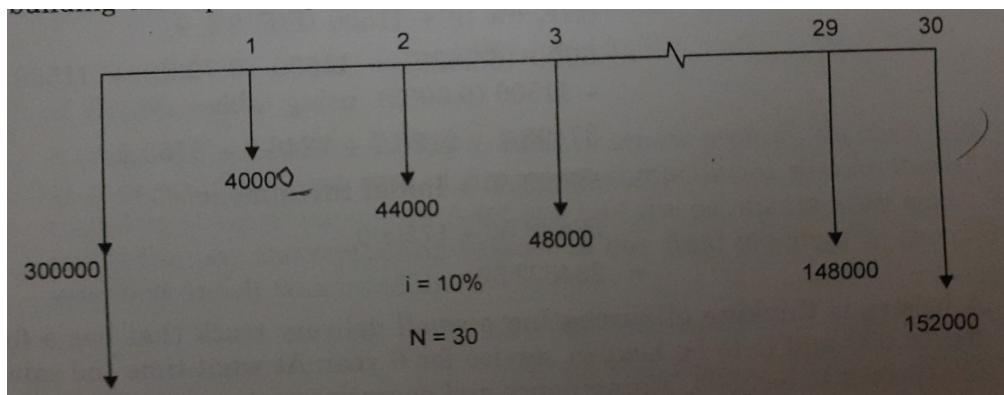
$$A = 72,704.9$$

$$\begin{aligned} PW &= A(P/A, i\%, N) \\ &= 72,704.9 * (P/A, 10\%, 30) \\ &= 72,704.9 * 9.427 \end{aligned}$$

$$PW = 6,85,389.1 + \text{Initial investment}$$

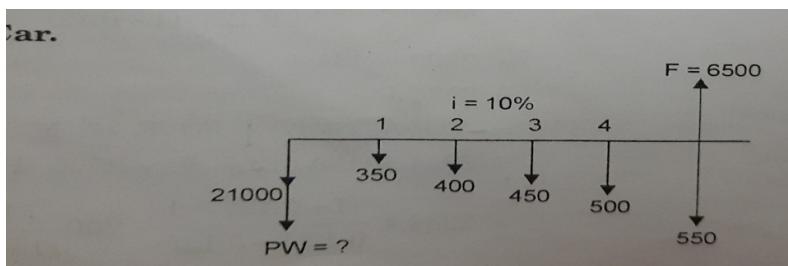
$$PW = 6,85,389 + 3,00,000$$

$$\mathbf{PW=9,85,389}$$



10. A newly developed electric car will cost 21000 to purchase. Operating and maintenance costs is Rs.350 for the first year with annual increase of Rs.50 per year. Salvage value after 5years is Rs.6500. A new gasoline run will cost Rs.16,000 with average 30 miles per gallon. Gasoline costs Rs.1.26 per gallon is expected to increase at a rate of Rs. 0.05 per year each of the next 4years maintenance costs are estimated to be Rs.300 per year.Salvage value is Rs.1500 after

5 years of service. If the vehicles are expected to be driven for 20,000 miles per year, determine which option will have the lower cost over 5 years. Use PW analysis with 10% rate of interest.



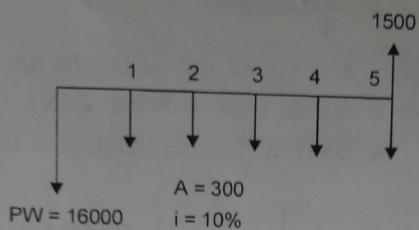
PRESENT WORTH COMPARISONS

39

$$PW = -21000 - [350 + 50(A/G, 10\%, 5)(P/A, 10\%5)] + 6500 (P/F, 10\% 5) \text{ using table}$$

$$PW = -18633.40$$

(b) New gasoline run about



Gasoline cost/year

Expected to run

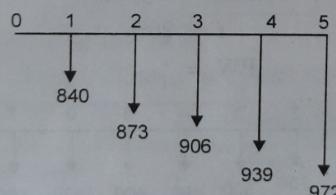
20000 miles per year

30 miles per gallon

$$\therefore \text{No. of gallons} = \frac{20000}{30} \\ = 666.67$$

$$\therefore \text{Cost of gasoline per year is} = 666.67 \times 1.26 \\ = 840$$

It is expected to increase by Rs. 0.05 per litre/year



$$\begin{aligned} PW_{\text{gasoline}} &= [A + G(A/G, i\% N)(P/A, i\% N)] \\ &= [840 + 33(A/G, 10\%5)](P/A, 10\%5) \\ &= [840 + 33 \times (1.810)](3.791) \\ &= 899.7 \times 3.791 \end{aligned}$$

$$PW_{\text{gasoline}} = 3410.9$$

$$\begin{aligned} PW &= -16000 - 300(P/A, 10\%5) - 3410.9 + 1500(P/F, 10\%5) \\ &= -16000 - 300(3.791) - 3410.9 + 931.38 \\ &= -19616.82 \end{aligned}$$

Electric car is having lowest cost.

11. A company is considering constructing a plant to manufacture a proposed new product. The land costs 3,00,000 the building costs 6,00,000. The equipment costs 2,50,000 and 1,00,000 working capital is required. It is expected that the product will result in sales of 7,50,000 per

year for 10 years at which time the land can be sold for 4,00,000 the building for 3,50,000 the equipment for 50,000 and all of the working capital recovered. The annual out-of-pocket expenses for labour, materials and all other items are estimated to be total 4,75,000. If the company requires a minimum return of 25% on projects of comparable risk, determine if it should invest in the new product line use PW methods.

Ans :

$$\text{Investment} = 3,00,000 + 6,00,000 + 2,50,000 + 1,00,000 = 12.5 \text{ lakhs}$$

$$\text{Profit (A)} = 7,50,000 / \text{year for 10 years}$$

$$\text{Salvage value} = 4,00,000 + 3,50,000 + 50,000 + 100,000 = 9,00,000$$

$$\text{Annual expenses} = 4,75,000 / \text{year for 10 years}$$

$$i=25\%, N=10 \text{ years}$$

$$\begin{aligned} \text{PW} &= -P + A (P/A, i\%, N) - A (P/A, i\%, N) + F (P/F, i\%, N) \\ &= -12,50,000 + 7,50,000 (P/A, 25\%, 10) - 4,75,000 \\ &\quad (P/A, 25\%, 10) + 9,00,000 (P/F, 25\%, 10) \\ &= -12,50,000 + 7,50,000 (3.571) - 4,75,000 (3.571) + 9,00,000 (0.108) \\ \text{PW} &= \mathbf{-1,70,775} \end{aligned}$$

12. Determine the PW of the following proposal when MARR is 15%.

	Proposal A
First cost (P)	10,000
Expected life(N)	5 years
Salvage value(F)	-1000
Annual receipts(+A)	8000
Annual expenses(-)	4000

A)	
----	--

$$\begin{aligned}
 PW &= -P + A(P/A, i\%, N) - A(P/A, i\%, N) - F(P/F, i\%, N) \\
 &= -10,000 + 8000(P/A, 15\%, 5) - 4000(P/A, 15\%, 5) - \\
 &\quad 1000(P/F, 15\%, 5) \\
 &= -10,000 + 8000(3.352) - 4000(3.352) - 1000(0.497) \\
 \mathbf{PW} &= \mathbf{2910.8}
 \end{aligned}$$

13. Evaluate machine XYZ on the basis of the present worth method when MARR is 12% pertinent cost data is as follows:

	Machine XYZ
First cost	13,000
Useful life	15 years
Salvage value	3000
Annual operating cost	100
Overhaul end of 5 th year	200
Overhaul end of 10 th year	550

$$\begin{aligned}
 PW &= -P - A(P/A, i\%, N) - F(P/F, i\%, N) - F(P/F, i\%, N) \\
 &\quad + F(P/F, i\%, N) \\
 &= -13,000 - 100(P/A, 12\%, 15) - 200(P/F, 12\%, 5) - \\
 &\quad 550(P/F, 12\%, 10) + 3000(P/F, 12\%, 15) \\
 &= -13,000 - 100(6.811) - 200(0.567) - 550(0.322) + \\
 &\quad 3000(0.1827) \\
 \mathbf{PW} &= \mathbf{-13,422}
 \end{aligned}$$

Assess with Unequal Lives (Present Worth Comparisons with Unequal lives)

1. Repeatability assumptions(LCM method)
2. Co-terminated assumptions/Study period method

Problems

1. Two holidays cottages are under consideration. Compare the present worth of the cost of 24 years service, at an interest rate of 5% when no salvage value.

	Cottage 1	Cottage 2
First cost	Rs. 4500	Rs. 10,000
Estimated life	12 years	24 years
Annual maintenance cost	Rs. 1000	Rs. 720

LCM of 12 and 24 = 24

Cottage 1

$$N=12$$

$$24/12 = 2$$

$$\begin{aligned} PW &= P + A (P/A, i\%, N) + F1(P/F, i\%, N) \\ &= 4500 + 1000 (P/A, 5\%, 24) + 4500(P/F, 5\%, 12) \\ &= 4500 + 1000 (13.79864) + 4500 (0.55684) \end{aligned}$$

$$\mathbf{PW=20,804.42}$$

Cottage 2

$$\begin{aligned} PW &= P + A(P/A, i\%, N) \\ &= 10,000 + 720 (P/A, 5\%, 24) \\ &= 10,000 + 720 (13.79864) \end{aligned}$$

$$\mathbf{PW=19,935.02}$$

Cottage 2 is selected.

2. Two types of trucks are available for transportation use. They are needed for 10 years.

	Truck A	Truck B
First cost	Rs. 10,00,000	Rs. 15,00,000
Estimated annual maintenance cost	Rs. 20,000	Rs. 15,000
Estimated life	5 years	10 years
Estimated salvage value	Rs. 2,00,000	Rs. 5,00,000

3. The following alternatives are available to accomplish an objective of 12 years duration.

	Plan A	Plan B	Plan C
Life cycle (in yrs)	6	3	4
First cost (\$)	2000	8000	10,000
Annual cost (\$)	3200	700	500

4.

Problems to be solved

1. A finance company advertises two investment plans. In plan 1, the company pays Rs. 12,000 after 15 years for every Rs. 1,000 invested now. In plan 2, for every Rs. 1,000 invested, the company pays Rs. 4,000 at the end of the 10th year and Rs. 4,000 at the end of 15th year. Select

the best investment plan from the investor's point of view at $i = 12\%$, compounded annually.

Ans : Plan 1 PW=1,192 Plan 2 PW=1018 Plan 1 is the best one.

2. Novel Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 8,00,000 at the end of the 20th year. Innovative Investment Ltd. accepts Rs. 10,000 at the end of every year for 20 years and pays the investor Rs. 15,00,000 at the end of the 25th year. Which is the best investment alternative? Use present worth base with $i = 12\%$.

**Ans: Novel investment ltd. PW(12%)=Rs.8266
Innovative Investment ltd. PW(12%) = Rs.13,506 .
Innovative investment ltd plan is the best one for the investor.**

3. Autocon company is evaluating three robots for possible use in its assembly operations (only one robot will be purchased). Data associated with these robots are as follows,

	Robot A	Robot B	Robot C
First cost Rs.	55,000	58,000	53,000
Operating & maintenance costs Rs.	3000/year	4500/year	4000/year
Expected income	40,000/year	44,000/year	38,000/year
Estimated salvage value Rs.	4000	6000	4000

Assuming a technological life of 3 years and a desired interest rate of 12% , which robot seems to be preferable

assuming all other factors are equal. Use a net PW evaluation.

- 4.** The following data presents for two feasible alternatives A and B for which revenues and costs are known and which have different lives. If the minimum alternative rate of return is 10% , show which feasible alternative is more desirable by using PW.

	A	B
Investment (first) cost	3500	5000
Annual revenue	1900	2500
Annual cost	645	1383
Useful life	4	8