Engineering Economics

Module -4

Depreciation and Breakeven Analysis

Depreciation - Causes of depreciation, Basic methods of computing Depreciation charges: Straight line method of depreciation--numerical exercises. Declining balance method of depreciation--numerical exercises, Sum of year's digits method of depreciation-numerical exercises, Sinking fund method of depreciation -numerical exercises
Breakeven Analysis - Introduction to breakeven analysis, methods for lowering the break-even point , Break-even chart, Break-even point, margin of safety ,Numerical exercises on breakeven analysis

Introduction

Any equipment which is purchased today will not work for ever. This may be due to wear and tear of the equipment or obsolescence of technology. Hence, it is to be replaced at the proper time for continuance of any business. The replacement of the equipment at the end of its life involves money. This must be internally generated from the earnings of the equipment. The recovery of money from the earnings of an equipment for its replacement purpose is called *depreciation fund* since we make an assumption that the value of the equipment decreases with the passage of time. Thus, the word "depreciation" means *decrease* in value of any physical asset with the passage of time.

Definition of Depreciation

Depreciation is the decrease in the value of fixed asset due to use/lapse of time.

Reduction in value.

Causes of Depreciation

1. Usage
2. Physical causes
3. Abnormal occurrences
4. Technological development and changes
5. Sudden failure
6. Depletion

Basic methods of Computing Depreciation Charges

Depreciation is a function of time , use , time & use, time & maintenance , time & interest.

1. **Straight line method**
2. **Reducing balance or diminishing balance method**
3. Production based methods – per unit , per hour
4. Repair provision method
5. Annuity method
6. **Sinking fund method**
7. Endowment policy method
8. Re-evaluation method
9. **Sum of digits method.**

Method 1 : Straight line method or Fixed Instalment method or Proportional method

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the lifetime of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the salvage value of the asset. Here, we make an important assumption that inflation(increase in price/fall in the purchasing value of money) is absent.

Let,

Por I or C = first cost of the asset/equipment,

For S or R = salvage/residual value of the asset(scrap value),

N = life of the asset/machine,

B_t = book value of the asset at the end of the period t,

D_t = depreciation amount for the period t.

The formulae for depreciation and book value are as follows:

$$D_t = (P - F)/n \quad or \quad (I-S)/n \quad or \ (C-R)/n$$

$$B_t = B_{t-1} - D_t$$

$$Bt = P - t \times [(P - F)/n] \text{ (for particular value(period) t)}$$

Problems

1. A melting unit for a steel foundry was purchased for Rs. 40,000 . Rs. 10,000 more were spent in its errection and commissioning. The estimated residual value after 10 yrs was Rs.12,000.

(a) Calculate the annual rate of depreciation.

(b) Calculate the book value of the m/c at the end of each year using the straight line depreciation and plot the graph of no. of yrs. v/s the depreciation fund.

(c) Calculate the depreciation fund collected at the end of 8^{th} year.

Solution : P=40,000

Money spent on errection and maintenance =10,000

Total cost of capital(P) = 40,000 + 10,000=50,000

S=12,000 N=10 yrs

NOTE: Book value after the estimated life should be equal to the salvage value.

(a) Depreciation / year = (P-S)/n

= (50,000 – 12,000)/12 = 3800

(b) To calculate book value after each year

No. of years	Cost and balance	Depreciation/yr	Book value after depreciation each year
1	50,000	3800	46,200
2	46200	3800	42400
3	42400	3800	38600
4	38600	3800	34800
5	34800	3800	31000
6	31000	3800	27200
7	27200	3800	23400
8	23400	3800	19600
9	19600	3800	15800
10	15800	3800	12000

(c) Depreciation fund collected at the end of 8^{th} year

= 3800 * 8 = 30,400

2. An investment of Rs. 5000 in a new equipment is expected to have a salvage value of Rs.1000 after a 4yrs life.

(a) Find a straight line depreciation expense.

(b) Plot the graph, the depreciation fund v/s the no. of years.

Solution: P=5000 , S=1000 , N=4

Depreciation / year = (P-S)/n = (5000 – 1000)/4 = 1000

To calculate book value after each year

No. of years	Cost and balance	Depreciation/yr	Book value after depreciation each year
1	5,000	1000	4000
2	4000	1000	3000
3	3000	1000	2000
4	2000	1000	1000

3. A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Determine the depreciation charge and book value at the end of various years using the straight line method of depreciation.

Solution : P=1,00,000 N=8yrs S=20,000

Depreciation / yr = (P-S) / n = (1,00,000 – 20,000)/ 8 = 10,000

No. of years	Cost and balance	Depreciation/yr	Book value after depreciation each year
1	1,00,000	10,000	90,000
2	90000	10000	80000
3	80000	10000	70000
4	70000	10000	60000
5	60000	10000	50000
6	50000	10000	40000
7	40000	10000	30000
8	30000	10000	20000

4. Consider problem 3 and compute the depreciation and the book value for period 5.

Solution : D5 = (P-F)/n = (100000-20000)/8 = 10,000

$$Bt = P - t \times [(P - F)/n]$$

$$= 1,00,000 - 5 * [(10,000)]$$

$$B5 = 50,000$$

Advantages

1. Easy to understand and simple to operate
2. Frequently used in practice
3. Uniform annual charge offers better comparative costs
4. This method requires little work for calculating depreciation amounts.

Disadvantages

1. The fixed assets do not wear out exactly at the same rate during their life.
2. A straight line method in many cases becomes unrealistic.

Method 2 : Reducing Balance Method or Diminishing Balance Method or Percentage on Book value Method

In this method of depreciation, a constant percentage of the book value of the previous period of the asset will be charged as the depreciation amount for the current period. This approach is a more realistic approach, since the depreciation charge decreases with the life of the asset which matches with the earning potential of the asset.

The book value at the end of the life of the asset may not be exactly equal to the salvage value of the asset. This is a major limitation of this approach.

$$\text{Depreciation Percentage 'P'} = 1 - \left(\frac{R}{C}\right)^{1/n}$$

The formulae for depreciation and book value are as follows:

$$D_t = K \times B_{t\text{-}1}$$

$$B_t = B_{t-1} - D_t$$

$$= B_{t-1} - K \times B_{t-1}$$

$$= (1 - K) \times B_{t-1}$$

The formulae for depreciation and book value in terms of P are as follows:

$$\boldsymbol{D_t = K(1 - K)^{t-1} \times P}$$

$$\boldsymbol{B_t = (1 - K)^t \times P}$$

P = first cost of the asset,
F = salvage value of the asset,
n = life of the asset,
B_t = book value of the asset at the end of the period t, K = a fixed percentage, and
D_t = depreciation amount at the end of the period t.

While availing income-tax exception for the depreciation amount paid in each year, the rate K is limited to at the most $2/n$. If this rate is used, then the

corresponding approach is called the ***double declining balance method of depreciation.***

Note: If R is not given or no salvage value, consider it as 1 (not zero).

Problems

1. A machine costs Rs. 10,000 has a scrap value of Rs.400 at the end of 4yrs of its serviceable life. Using Reducing balance:
   (a) Determine the depreciation and book value in each year
   (b) Plot the graph, no. of yrs v/s depreciation fund.
   (c) What is the amount of depreciation collected at the end of 2nd year or after 2yrs.

Solution : (a) Percentage depreciation = $1-\left(\frac{400}{10000}\right)^{1/4}$

=0.552 or 55.2% of amount/year

(b)

No. of years	Cost of capital at beginning of each year	Depreciation/yr	Book value after depreciation each year
1	10,000	10000*0.552=5520	4480
2	4480	4480*0.552=2472.96	2007.04
3	2007.04	2007.04*0.552=1107.88	899.153
4	899.153	899.153*0.552=496.332	402.82

(c) Amount of depreciation fund collected at the end of 2nd year = 1st year + 2nd year

=5520 + 2472.96 = **7992.96 Rs**.

2. A car was purchased for Rs.32,000 and salvage value was established Rs.8000 after 7yrs. Using the reducing balance method.
   (a) Calculate the percentage of depreciation
   (b) Calculate the book value and depreciation in each year
   (c) Plot a graph, no. of yrs v/s depreciation fund

Solution : (a) Percentage depreciation = $1-\left(\frac{8000}{32000}\right)^{1/7}$

= 0.1796 or 17.96% per year

No. of years	Cost of capital at beginning of each year	Depreciation/yr	Book value after depreciation each year
1	32000	32000*0.1796=5747.2	26252.8
2	26252.8	26252.8 * 0.1796 = 4715.0028	21537.7971
3	21537.7971	21537.7971*0.1796=3838.1883	17669.6087
4	17669.6087	17669.6087*0.179=3173.4617	14496.14697
5	14496.14697	14496.14697*0.179=2603.5079	11892.638
6	11892.638	11892.638*0.179=2135.91796	9756.7200
7	9756.7200	9756.7200*0.179=1752.306	8004.41308

3. Two machines are purchased each for Rs.12,000. The estimated useful life of m/c is 5yrs. The estimated scrap value is Rs.2000. For machine A, the straight line method and for B the reducing balance method are used to calculate the depreciation every year. Compare the depreciations charged in each year of both i.e, A and B and plot the graph, the no. of yrs v/s the depreciation fund for both.

Solution :

Depreciation /year for machine A = (C-R)/n

= (12,000-2000)/5 =2000

Percentage depreciation for m/c B = $1-\left(\frac{2000}{12000}\right)^{1/5}$

=0.3011=30.11%

No. of years	Cost of capital at beginning of each year		Depreciation/yr		Book value after depreciation each year	
	m/c A	m/c B	St.line m/c A	R.Bal. m/c B	m/c A	m/c B
1	12,000	12000	2000	3613.2	10000	8386.8
2	10000	8386.8	2000	2525.26	8000	5861.53
3	8000	5861.53	2000	1764.90	6000	4096.62
4	6000	4096.62	2000	1233.49	4000	2863.12
5	4000	2863.12	2000	862.08	2000	2001.04

4. A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. and demonstrate the calculations of the declining balance method of depreciation by assuming 0.2 for *K*.

Solution : Depreciation percentage(k) = 0.2

No. of years	Cost of capital at beginning of each year	Depreciation/yr	Book value after depreciation each year
1	1,00,000	1,00000*0.2=20000	80000
2	80000	80000*0.2=16,000	64000
3	64000	64000*0.2=12800	51200
4	51200	51200*0.2=10240	40960
5	40960	40960*0.2=8192	32768
6	32768	32768*0.2=6553.6	26214.4
7	26214.4	26214.4*0.2=5242.88	20971.52
8	20971.52	20971.52*0.2=4194.3	16777.22

5. Consider problem 4 and calculate the depreciation and the book value for period 5 using the declining balance method of depreciation by assuming 0.2 for *K*.

Solution : $D_t = K(1 - K)^{t-1} \times P$

$D5 = 0.2\ (1\text{-}0.2)^4 * 1{,}00{,}000 = \mathbf{8192}$

$B_t = (1 - K)^t \times P$

$B5 = (1\text{-}0.2)^5 * 1{,}00{,}000 = \mathbf{32{,}768}$

Advantages

1. Simple to understand and calculate
2. Mathematical relationship can be employed to arrive at the appropriate percentage
3. This method is more logical as the largest annual amount of depreciation is charged in the first year where repair and maintenance charges are almost negligible.

Disadvantages

1. It is not simple to fix percentage P accurately.
2. A standard percentage P for all conditions may produce misleading results.

Method 3 : Sum of Digits Method

In this method of depreciation, it is assumed that the book value of the asset decreases at a decreasing rate. If the asset has a life of eight years, first the sum of the years is computed as,

Sum of the years = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

$$= 36 = n(n + 1)/2$$

If ‘n’ is the estimated life of the m/c the rate is calculated for each period as a fraction in which the denominator is always the sum of the series 1,2,3,….n and the numerator for the first period is n , for the second period is n-1 and so on.

The rate of depreciation charge for the first year is assumed as the highest and then it decreases. The rates of depreciation for the years 1–8, respectively are as follows: 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.
For any year, the depreciation is calculated by multiplying the corresponding rate of depreciation with $(P - F)$.

$$D_t = \text{Rate} \times (P - F)$$

$$B_t = B_{t-1} - D_t$$

The formulae for D_t and B_t for a specific year t are as follows:

$$D_t = \frac{n - t + 1}{n(n+1)/2} \quad (P - F)$$

$$B = (P - F)\frac{(n - t)}{n}\quad\frac{(n - t + 1)}{(n + 1)} + F$$

T

Problems

1. The cost of the vehicle is Rs. 19,000 , the scrap value after 5yrs is estimated at Rs.4000. Using the sum of digits method, calculate the depreciation at the end of each year. Plot a graph, the no. of yrs v/s the depreciation fund.

   Solution: No. of years , n= 1,2,3,4,5

   Denominator = 1+2+3+4+5=15

   Depreciation for the 1^{st} year = Rate * (P-F)

   =5/15 * (19000-4000)

   =5000

   Depreciation for 2^{nd} year = 4/15 * 15000

   =4000

   Depreciation for 3^{rd} year = 3/15 * 15000=3000

   Depreciation for 4^{th} year =2/15 * 15000 =2000

   Depreciation for 5^{th} year =1/15 * 15000=1000

2. An investment of Rs. 5900 in new equipment is expected to have a salvage value of Rs.1000 after 4yrs. Using the sum-of-digits method , find out the depreciation for each year and what is the depreciation amount to be collected after 2yrs.

Solution: No.of years = 1,2,3,4

Denominator = 1+2+3+4=10

(a) Depreciation for 1^{st} year=4/10 * (5900-1000) = 1960

Depreciation for 2^{nd} year=3/10 * 4900=1470

Depreciation for 3^{rd} year=2/10 * 4900 =980

Depreciation for 4^{th} year=1/10 * 4900=490

(b) Depreciation amount after 2 years = 1^{st} yr + 2^{nd} year = 1960+1470= **Rs. 3430**

3. A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Demonstrate the calculations of the sum-of-the-years-digits method of depreciation.

   Solution : P=1,00,000 F=20,000 n=8 yrs

   Denominator = n(n+1) / 2 = 8(8+1) / 2 =8 * 9/2 =36

No. of years	Cost of capital at beginning of each year	Depreciation/yr	Book value after depreciation each year
1	1,00,000	8/36 * (80,000)=17777.77	82222.23
2	82,222.22	7/36 * 80,000=15,555.55	66,666.68
3	66,666.68	6/36*80,000=13,333.33	53333.35
4	53333.35	5/36*80,000=11,111.11	42,222.24
5	42,222.24	4/36*80000=8,888.88	33,333.36
6	33,333.36	3/36*80000=6,666.66	26,666.7
7	26,666.7	2/36*80000=4,444.44	22,222.26
8	22,222.26	1/36*80000=2,222.22	20,000.04

4. Consider above problem 3 and find the depreciation and book value for the 5th year using sum-of-years-digits method of depreciation.

Solution

P = Rs. 1,00,000
F = Rs. 20,000
n = 8 years

$$D_t = \frac{n - t + 1}{n(n+1)/2}(P - F)$$

$$D_5 = \frac{8 - 5 + 1}{8(8+1)/2}(1{,}00{,}000 - 20{,}000)$$

$$= \text{Rs. } 8{,}888.88$$

$$B_t = (P - F)\frac{n - t}{n}\frac{n - t + 1}{n + 1} + F$$

$$B_5 = (1{,}00{,}000 - 20{,}000)\frac{8 - 5}{8}\frac{8 - 5 + 1}{8 + 1} + 20{,}000$$

$$= 80{,}000 \times (3/8) \times (4/9) + 20{,}000$$

$$= \text{Rs. } 33{,}333.33$$

Method 4 : Sinking Fund Factor

This method is based on the assumption of setting up a sinking fund in which money accumulates to replace the existing asset at the proper time. At the end of the useful life of the asset, the total amount in depreciation + compound interest should become equal to original cost of the fixed asset.

In this method of depreciation, the book value decreases at increasing rates with respect to the life of the asset

$$\text{Rate of depreciation} = \frac{i(C-R)}{(1+i)^n - 1}$$

Where, 'i' is the rate of interest

'n' is the expected life

'C' is the investment

'R' is the salvage value of the equipment

The loss in value of the asset ($P - F$) is made available an the

form of cumulative depreciation amount at the end of the life of the asset by setting up an equal depreciation amount (A) at the end of each period during the life time of the asset.

$$A = (P - F) \times [A/F, i, n]$$

The fixed sum depreciated at the end of every time period earns an interest at the rate of i% compounded annually, and hence the actual depreciation amount will be in the increasing manner with respect to the time period.

A generalized formula for D_t is

$$D_t = (P - F) \times (A/F, i, n) \times (F/P, i, t - 1)$$

The formula to calculate the book value at the end of period 't' is

$$B_t = P - (P - F)\,(A/F, i, n)\,(F/A, i, t)$$

The above two formulae are very useful if we have to calculate D_t and B_t for any specific period. If we calculate D_t and B_t for all the periods, then the tabular approach would be better.

Problems

1. A machine is purchased for Rs. 10,000 the estimated life of the m/c is 4yrs. and the scrap value is Rs. 400 . The rate of interest on the depreciation fund is 4%. Calculate the book value of the machine at the end of each year using sinking fund method.

Solution : C=10,000 F=400 n=4 i=4%

Rate of depreciation = $\frac{0.04(10{,}000-400)}{(1+0.04)^4-1}$ = **2,260.7118**

Book value of each year = 10,000 – corresponding years balance

No. of years	Balance	Interest at 4%	Annual provision col.2 + col.3	Annual Investment	Balance col 4 + col 5	Book Value 10,000 – col. 6
1	-	-	-	2260.71	2260.71	7739.29
2	2260.71	90.4	2351.13	2260.71	4611.8	5388.2
3	4611.8	184.47	4796.27	2260.71	7056.98	2943.01
4	7056.98	282.27	7339.25	2260.71	9599.96	400.03

2.A m/c is purchased for Rs. 40,000 . The estimated life of the m/c is 5yrs. The scrap value is Rs. 15,000. The rate of interest on the depreciation is 10%. Calculate the book value of the m/c at the end of each year using Sinking fund method and plot a graph of the number of years v/s depreciation fund.

Solution : C=40,000 F=15,000 n=5 i=10%

Rate of depreciation = $\frac{0.1(40,000-15,000)}{(1+0.1)^5-1}$ = **4094.93**

Book value of each year = 40,000 – corresponding years balance

No. of years	Balance	Interest at 10%	Annual provision col.2 + col.3	Annual Investment	Balance col 4 + col 5	Book Value 40,000 – col. 6
1	-	-	-	4094.93	4094.93	35905.06
2	4094.93	409.49	4504.42	4094.93	8599.35	31400.64
3	8599.35	859.93	9459.28	4094.93	13554.21	26445.78
4	13554.21	135.54	14909.63	4094.93	19004.56	20995.43
5	19004.56	190.04	19194.6	4094.93	23289.53	16710.47

3. A company has purchased an equipment whose first cost is Rs. 1,00,000 with an estimated life of eight years. The estimated salvage value of the equipment at the end of its lifetime is Rs. 20,000. Use sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution: C=1,00,000 F=20,000 n=8 i=12%

Rate of depreciation = $\frac{0.12(1,00,000-20,000)}{(1+0.12)^8-1}$ = **6,504.22**

Book value of each year = 1,00,000 – corresponding years balance

No. of years	Balance	Interest at 10%	Annual provision col.2 + col.3	Annual Investment	Balance col 4 + col 5	Book Value 1,00,000 – col. 6
1	-	-	-	6504.22	6504.22	93,495.78
2	6504.22	650.422	7154.642	6504.22	13,658.862	86,341.138
3	13658.862	1365.8862	15,024.7482	6504.22	21,528.9682	78,471.0318
4	21,528.9682	2152.8968	23,681.865	6504.22	30,186.085	69,813.915
5	30,186.085	3018.6085	33,204.6935	6504.22	39,708.9135	60,291.0865
6	39,708.9135	3970.8913	43,679.8048	6504.22	50,184.0248	49,815.9752
7	50,184.0248	5018.4024	55,198.4272	6504.22	61,702.6472	38,297.3528
8	61,702.6472	6170.2647	67,872.9119	6504.22	74,377.1319	25,622.8681

4. Consider above problem 3 and compute D_5 and B_7 using the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

Solution:

$P = \text{Rs. } 1{,}00{,}000$
$F = \text{Rs. } 20{,}000$
$n = 8 \text{ years}$
$i = 12\%$

$$D_t = (P - F)\,(A/F, i, n)\,(F/P, i, t-1)$$
$$D_5 = (P - F)\,(A/F, 12\%, 8)\,(F/P, 12\%, 4)$$
$$= (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 1.574$$
$$D5 = \text{Rs. } 10{,}237.30$$

$$B_t = P - (P - F)\,(A/F, i, n)\,(F/A, i, t)$$
$$B_7 = P - (P - F)\,(A/F, 12\%, 8)\,(F/A, 12\%, 7)$$
$$= 1{,}00{,}000 - (1{,}00{,}000 - 20{,}000) \times 0.0813 \times 10.089$$
$$B7 = 34{,}381.10$$

Requirements of a Depreciation method

1.Provide for the recovery of inverted capital as rapidly as is consistent with economic facts involved.

2.Not be too complex.

3. Ensure that the book value will be reasonably close to the market value at any time.

4.Be accepted by the Internal Revenue Service(IRS), if the method is also to be used for determining federal income tax.