Lab Assignment 3

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QUESTION – 1

Computationally investigate the motion of a pendulum and a spring-mass system as discussed in the class (without any damping). Draw phase plots to explain your observations.

ASSUMPTIONS:

1) Air drag is neglected.
2) Effect of air density is neglected.
3) The displacement is very small.

INITIALIZATIONS:

1) Initial Theta = 3.14/18
2) g = Gravitational Acceleration = 9.8
3) Length l = 1
4) Initial angular velocity init_w = 0
5) Mass of body for spring = 3
6) Spring constant k = 10

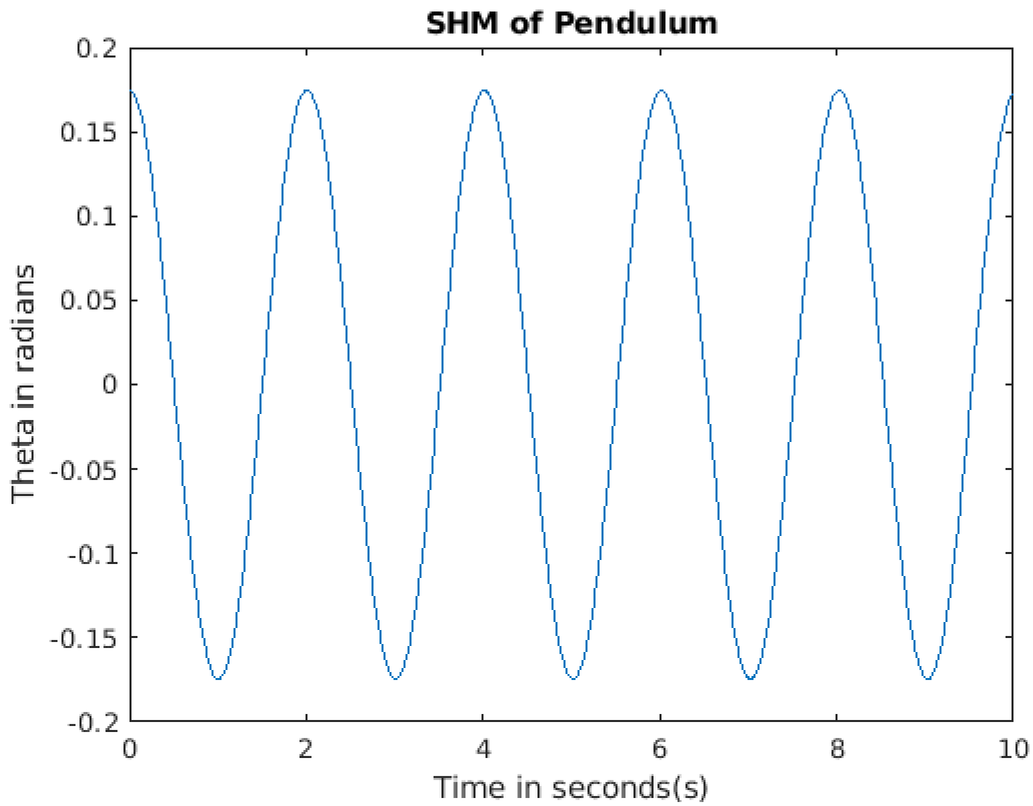
ANALYTICAL SOLUTION:

$$dw/dt = -g*\theta/l$$

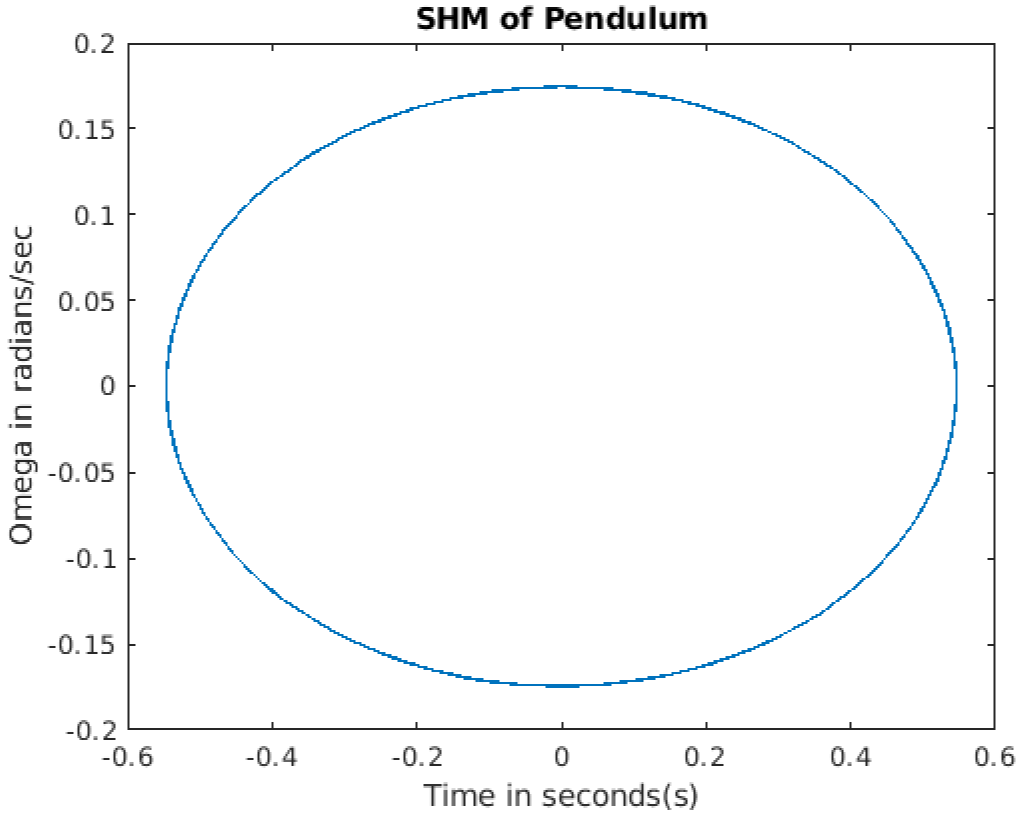
$$d\theta/dt = w$$

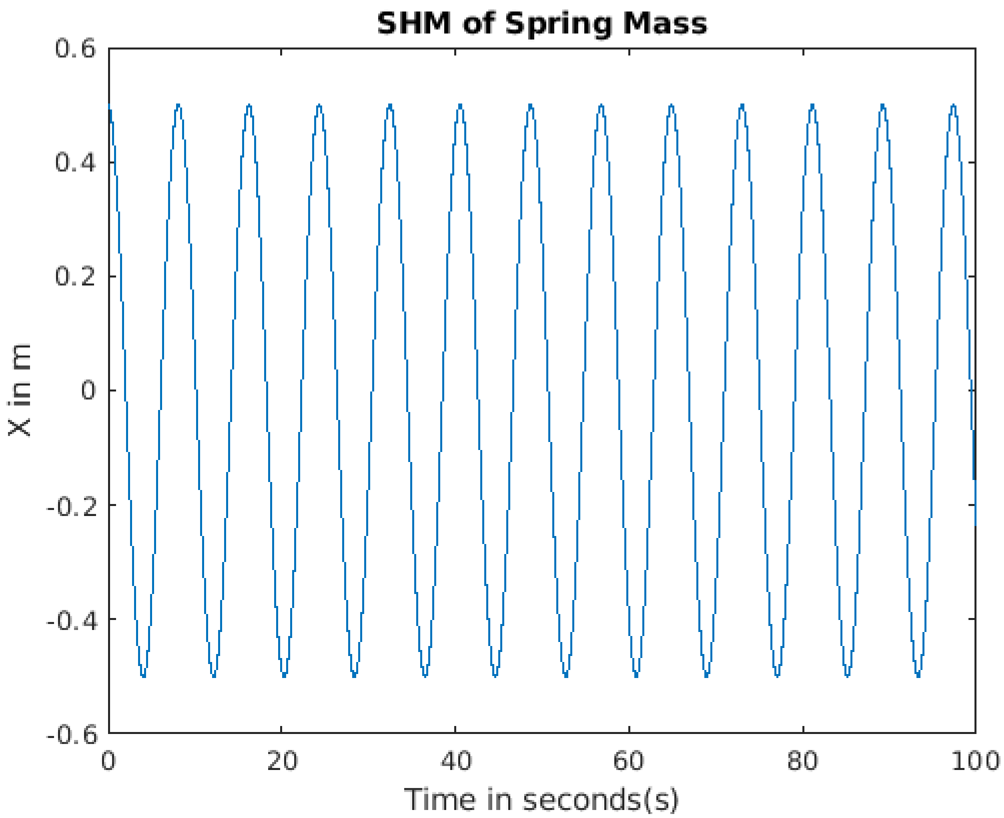
COMPUTATIONAL MODEL:

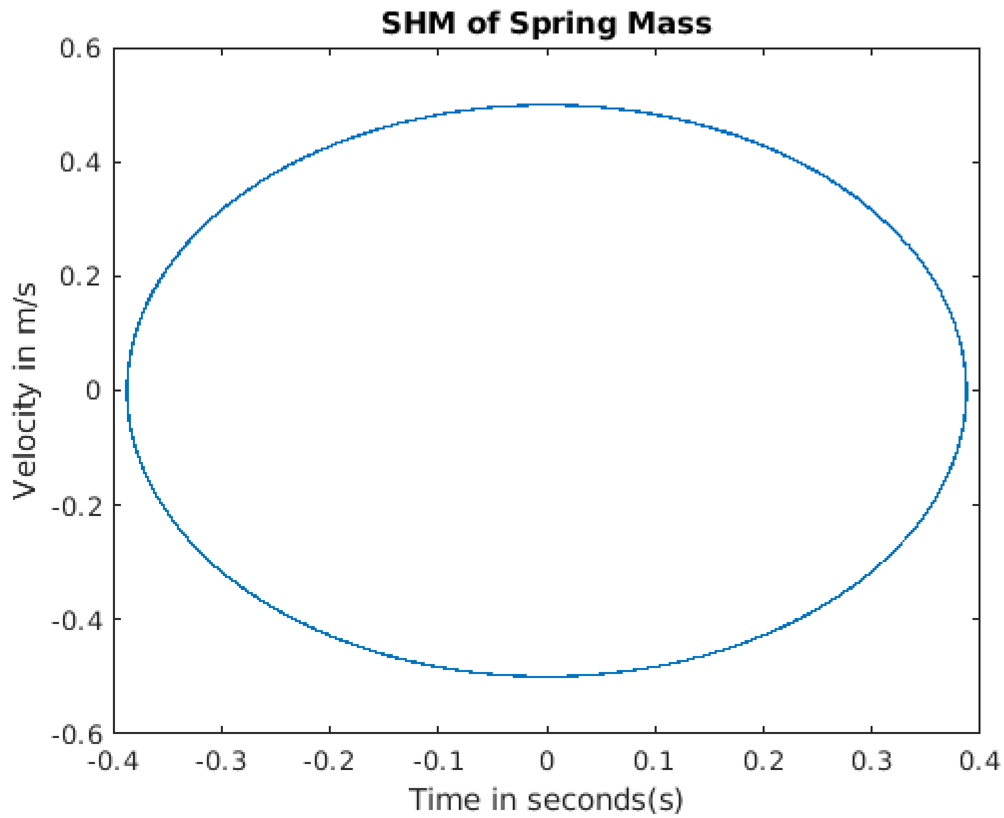
The matrix d1dt has the values: d1dt(1) = angular displacement = theta and d1dt(2) = angular velocity = w. We used @ode45 to calculate the differential equations.



SHM of Pendulum
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
Theta in radians
0
2
4
6
8
10
Time in seconds(s)


SHM of Pendulum
Omega in radians/sec
Time in seconds(s)
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


SHM of Spring Mass
X in m
Time in seconds(s)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
20
40
60
80
100



OBSERVATIONS:

1) The phase circle is complete in both the graphs, i.e. the motion is periodic.
2) Both the plots have a sinusoidal wave-form for displacement and velocity (angular displacement and angular velocity for the Pendulum) .

QUESTION – 2

a) Investigate (computationally) the cannon-shell trajectories ignoring both air drag and the effect of air density. Compare your result with exact solutions. Acceleration due to gravity depends on altitude; include this effect in your computational model by making some rational assumption

ASSUMPTIONS:

1) Variation in gravitational acceleration with height.
2) Air drag is neglected
3) Effect of air density is neglected
4) Target at same height as that of canon

INITIALIZATIONS:

1) Initial angle θ = 30∘
2) v_ini = 10 m/s
3) Radius of earth 6400km
4) Acceleration due to gravity go = 9.8 m/s^2

ANALYTICAL SOLUTION:

Acceleration due to gravity changes with height,

$$g = g_0 (1 - 2h/R)$$

Velocity in x direction

$$v_x = v_0 \cos(\theta)$$

Velocity in y direction

$$v_y = v_0 \sin(\theta)$$

Acceleration in x

$$a_x = \frac{d^2 x}{dt^2} = 0$$

Acceleration in y

$$a_y = \frac{d^2 y}{dt^2} = -g$$

Displacement in x

$$x = v_x * t$$

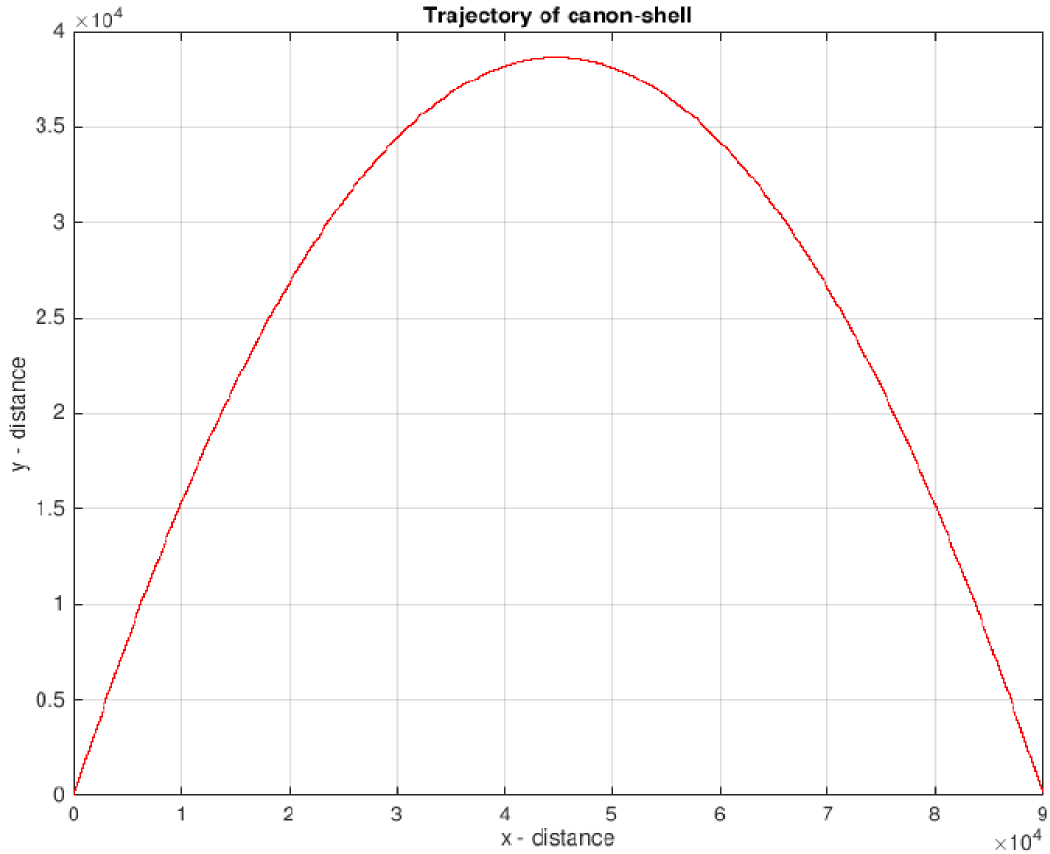
Displacement in y

$$x_y = v_y * t + \frac{1}{2} * g * t^2$$

COMPUTATIONAL MODEL:

The matrix d0 has the values:

d0(1) has displacement in the x-direction and d0(2) has displacement in the y-direction. We used @ode45 to calculate the differential equations. The velocity along the x-direction remains constant, whereas the velocity along the y-direction changes. So, the acceleration along the x-direction is zero.



OBSERVATIONS:

1. The first observation that we make is that the velocity and the distance travelled by the object of mass m is independent of its mass whenever we neglect the effect of atmosphere.

2. The range travelled by the object will be maximum when the angle of projection is 45◦.

3. Since the acceleration changes with height, the velocity obtained by the body will decrease till it reaches the maximum height and then will start increasing till it reaches the maximum velocity.

(b)Investigate the trajectory of the canon shell including both air drag (proportional to square of velocity) and reduced air density at high altitudes. Perform your calculation for different firing angles; and determine the value of the angle that gives the maximum range.

$$F_{drag} = -b v^2$$

Density varies as follows

$$\rho = \rho_0 e^{\frac{-y}{y_0}}$$

y is the altitude $y_0 = 1000\,m$

ASSUMPTIONS:

1. Gravitational acceleration is constant with height.

2. Drag force is proportional to v^2

INITIALIZATIONS:

1) Initial angle, θ = 30∘
2) Initial velocity, v_0 = 750 m/s
3) Radius of the Earth = 6400e3
4) Acceleration due to gravity = g_0 = 9.8 m/s²
5) Velocity at any point of time $v = \sqrt{v_x^2 + v_y^2}$
6) Velocity in x direction $v_x = v_0 \cos(\theta)$
7) Velocity in y direction $v_y = v_0 \sin(\theta)$
8) Air drag constant = b
9) F_d air drag force = bv^2
10) Change in atmospheric density $\rho = \rho_0 e^{\frac{-y}{y_0}}$
11) Initial atmospheric density $\rho_0 = 1.225$

ANALYTICAL SOLUTION:

$$F_{dt} = \frac{\rho}{\rho_0} * F_d$$

$$\frac{b}{m} = 4e-5$$

Thus the acceleration in x

$$a_x = \frac{-b}{m} * e^{\frac{x}{x_0}} * v * v_x$$

Acceleration in y

$$a_y = -g - \frac{b}{m} * e^{\frac{x}{x_0}} * v * v_y$$

Displacement in x

$$x = v_0 \cos(\theta) t$$

Displacement in y

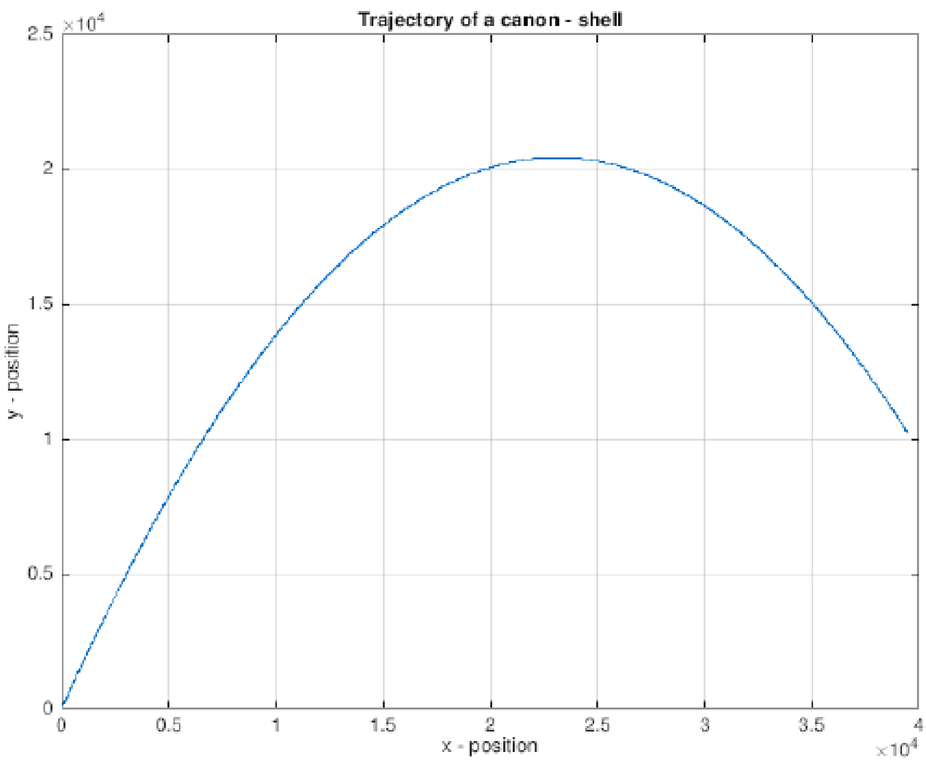
$$y = v_0 \sin(\theta) * t - \frac{1}{2}(a_y) * t^2$$

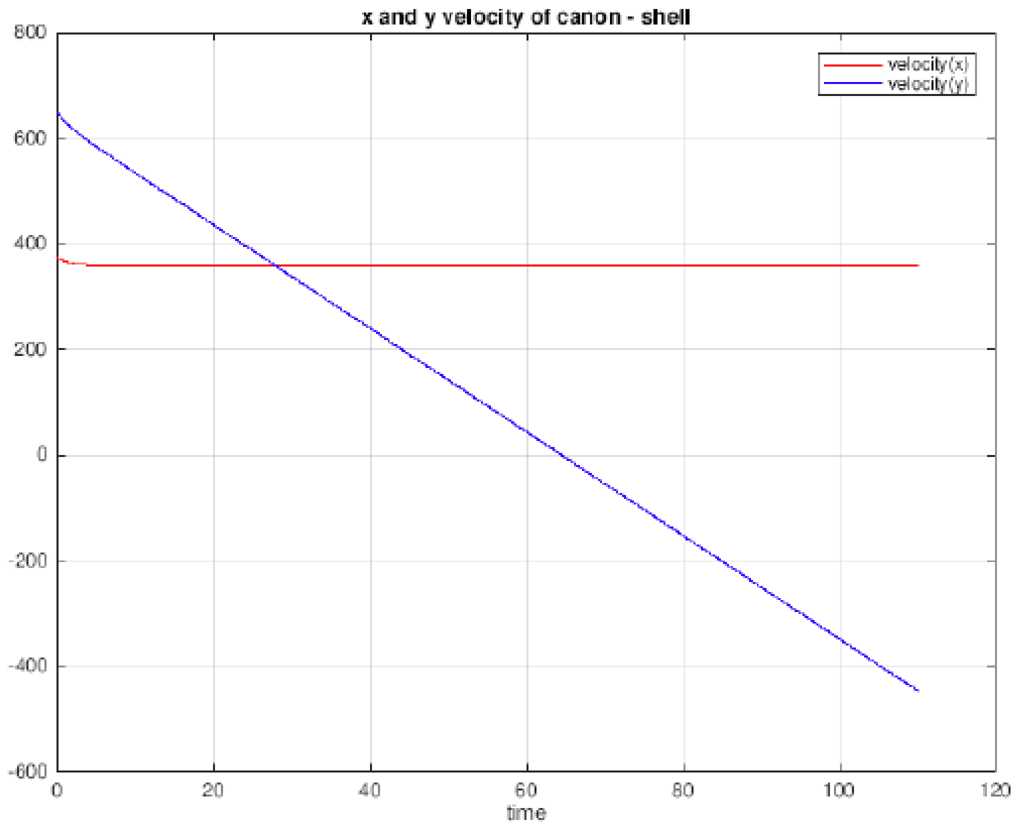
COMPUTATIONAL MODEL:

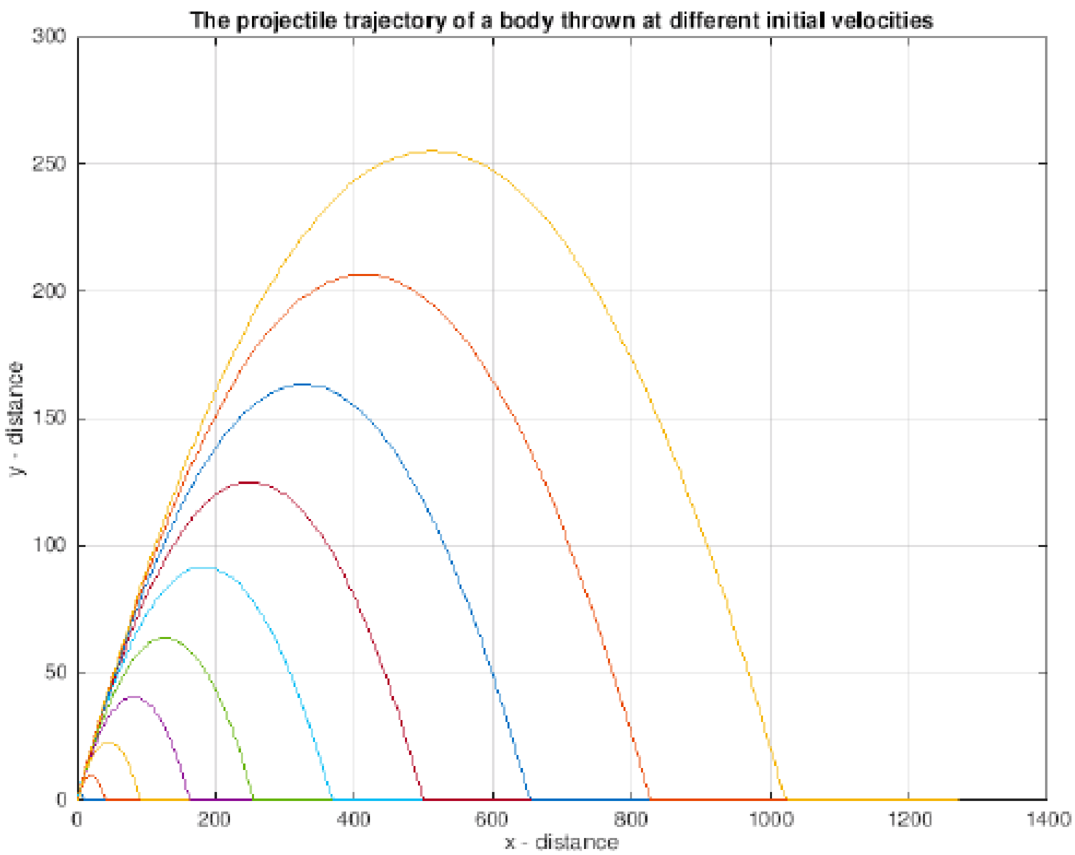
The matrix d0 has the values :

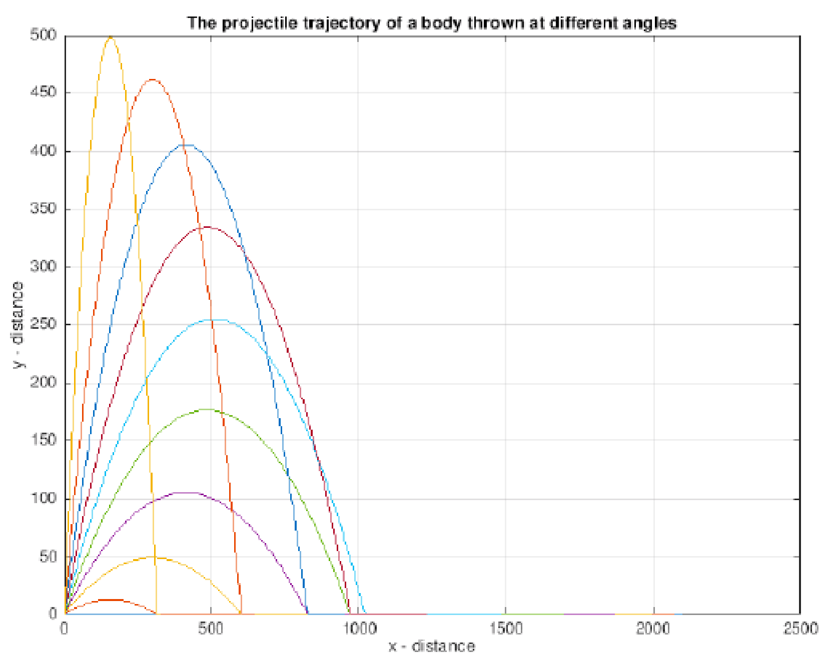
d0(1) has displacement in the x-direction and d0(2) has displacement in the y-direction. We used @ode45 to calculate the differential equations. The velocity along the x-direction remains constant, whereas the velocity along the y-direction changes. So, the acceleration along the x-direction is zero.

GRAPHS:




x and y velocity of canon - shell
velocity(x)
velocity(y)
800
600
400
200
0
-200
-400
-600
0
20
40
60
80
100
120
time





OBSERVATIONS:

The first observation that we make is that the velocity and the distance travelled by the object of mass m is dependent on its mass due to the presence of air drag force.

(c) Generalize the program so that it can deal with situations where the target is at a different altitude (higher or lower) than the canon. Investigate for both the cases. How the minimum firing velocity to hit a target varies as the altitude of the target varies.

ASSUMPTIONS:

1) No effect of air drag
2) Ignore effect of air density
3) Density of Air remains constant with height
4) Gravitational Acceleration changes with altitude, altitude is very small compared to radius of earth

INITIALIZATIONS:

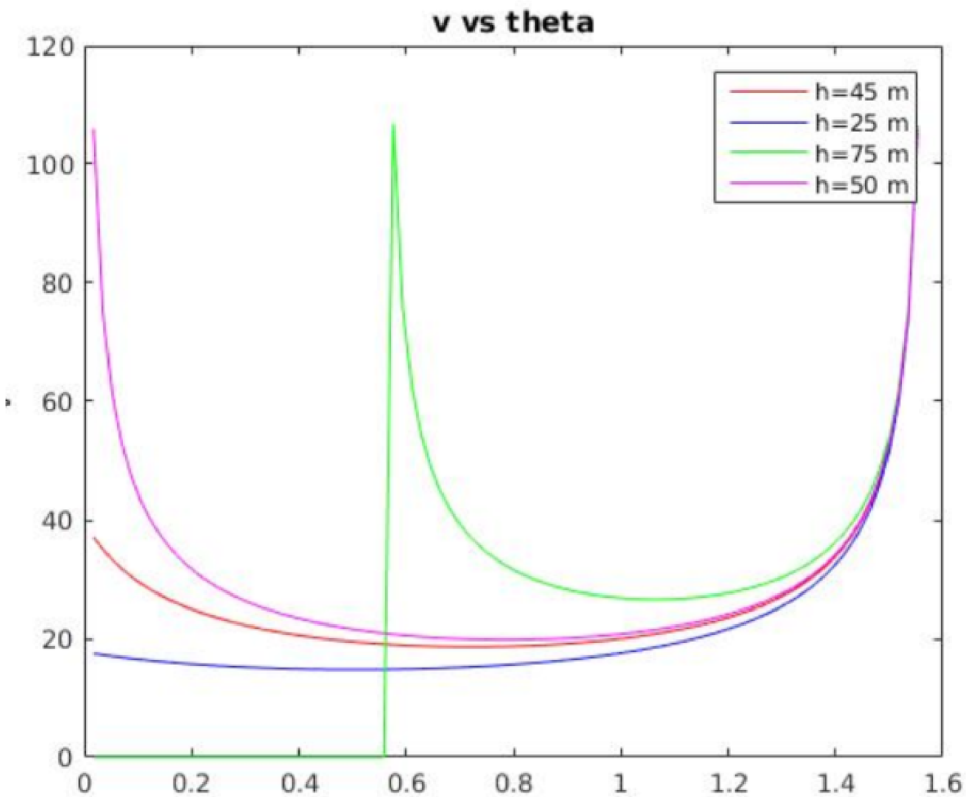
1) ch = height of the cannon
2) h = height of the target
3) x = distance between the target and the cannon

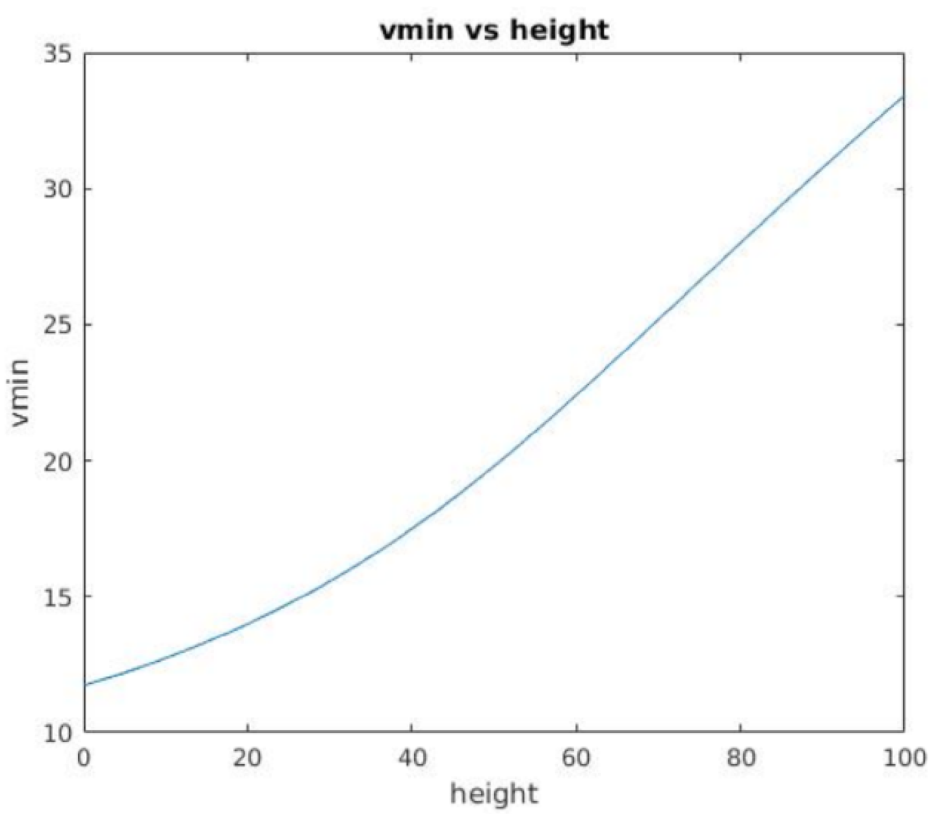
SOLUTION:

$$v = \sqrt{\frac{g x^2}{2\cos^2\theta\left(x tan\theta - h + ch\right)}}$$

In this way the minimum velocity could be found given a height and distance of a cannon from the target for a firing angle.

GRAPHS:


v vs theta
h=45 m
h=25 m
h=75 m
h=50 m
120
100
80
60
40
20
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6


vmin vs height
vmin
height
35
30
25
20
15
10
0
20
40
60
80
100

OBSERVATION:

For altitude greater than height of cannon; initially for very small angles, the object can never reach the target.

Then there will be an angle, where you will need infinite velocity to reach the target.

Now as you increase the angle the velocity required decreases and reaches a minimum.

Beyond this point, if you increase the angle, velocity required again increases.

For altitude less than height of cannon, object will reach the target at angle 0. However, if you increase the angle, there will be a point where minimum velocity is required. Beyond this, velocity increases.

QUESTION - 3

Bicycle problem:

(a) Rewrite the bicycle problem/code as discussed in the class. Investigate the effect of rider's power, mass and frontal area on the ultimate velocity.

Generally, for a rider in the middle of a group the effective frontal area is about 30% less than the rider at the front. How much less energy does a rider in the group expend than one at the front (assuming both moving at 12.5 m/s).

ASSUMPTIONS:

1. Constant power is provided to the bicycle by the cyclist.
2. The force acting on the bicycle is constant as the velocity is less.
3. The surface on which the bicycle is moving is friction-less.

INITIALIZATIONS :

1) Initial position, init_x = 0
2) Initial velocity, init_v = 10
3) Power = 400
4) Mass of the system(person + bicycle) = 60
5) Drag coefficient, c = 0.5
6) Acceleration, a = 0.25
7) Air density = 1.225

ANALYTICAL SOLUTION:

$F = m * dv/dt$

$P = dE/dt$

$P = mv * dv/dt$

Solving or v gives us,

$v = v_{init}^2 + 2Pt/m$

But this is incorrect as the velocity keeps on increasing, we deal with this by subtracting a drag factor,

$F_{drag} = -c\rho A v^2$

For moderate velocity we have,

$dv/dt = P/mv - C\rho A v^2$

Reaching the terminal velocity, dv/dt = 0,

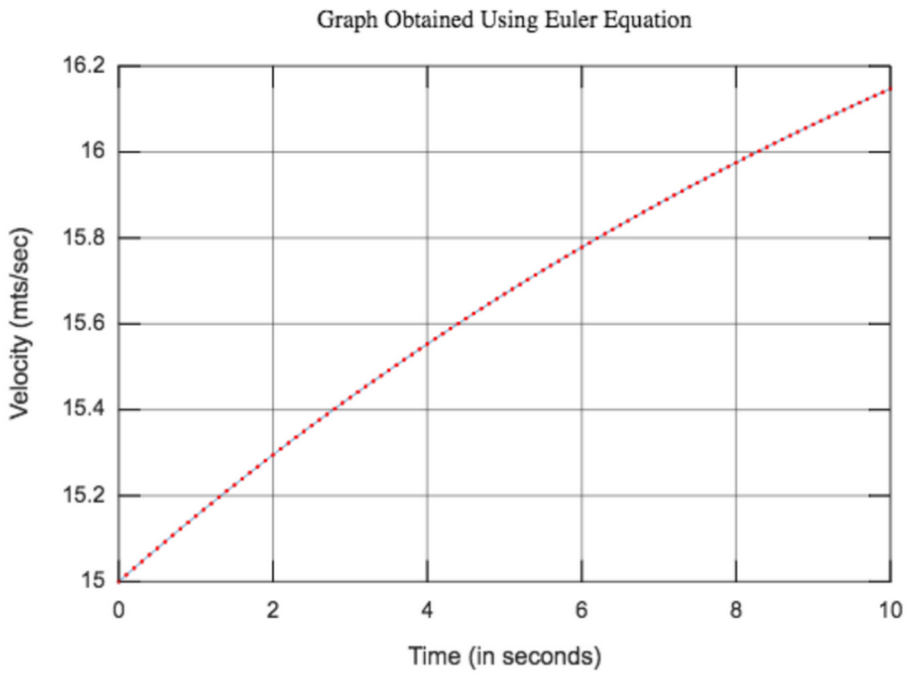
$$P/mv = (-1/2 * \rho_{air} A v^2)/m$$

$$P = 1/2 * \rho_{air} A v^3$$

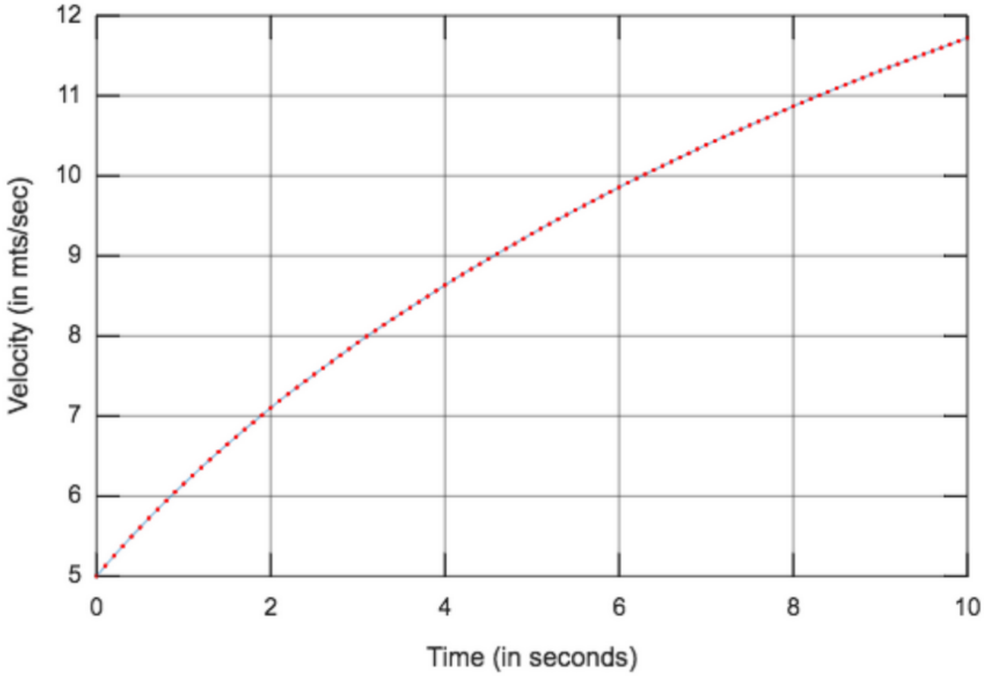
Therefore,power will be directly proportional to Frontal Area. So if the rider at the middle has effective frontal area 30% less than the rider at Front then Power expend by rider at the middle will be 30% less than the Power expended by rider at the front.

COMPUTATIONAL MODEL:

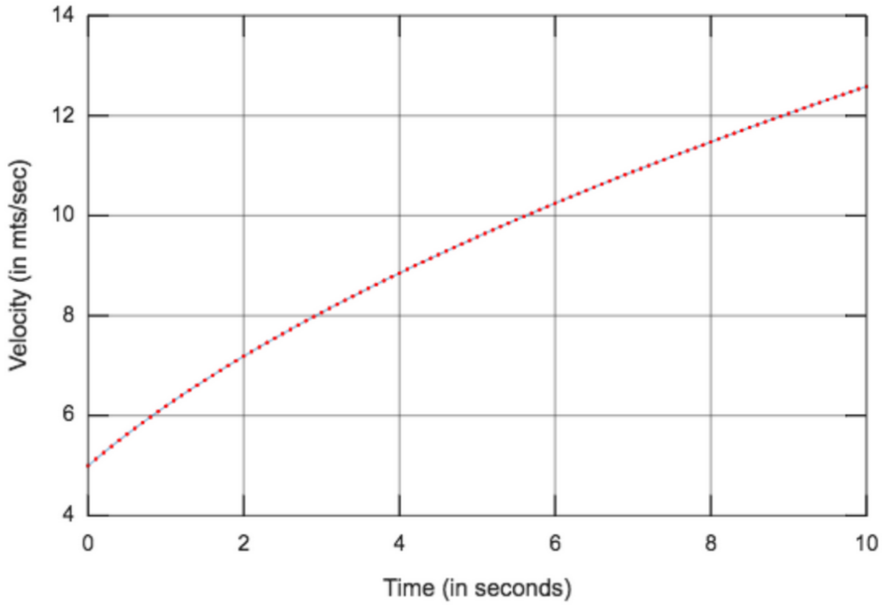
For computational analysis, we use matlab’s ode45 differential equation solver. Change in velocity with respect to time is calculated by solving differential equation where in the step size is taken as 0.1 with initial velocity of 10, frontal area = 0.33, Power = 400, mass = 60 and the time interval of simulation is 100.

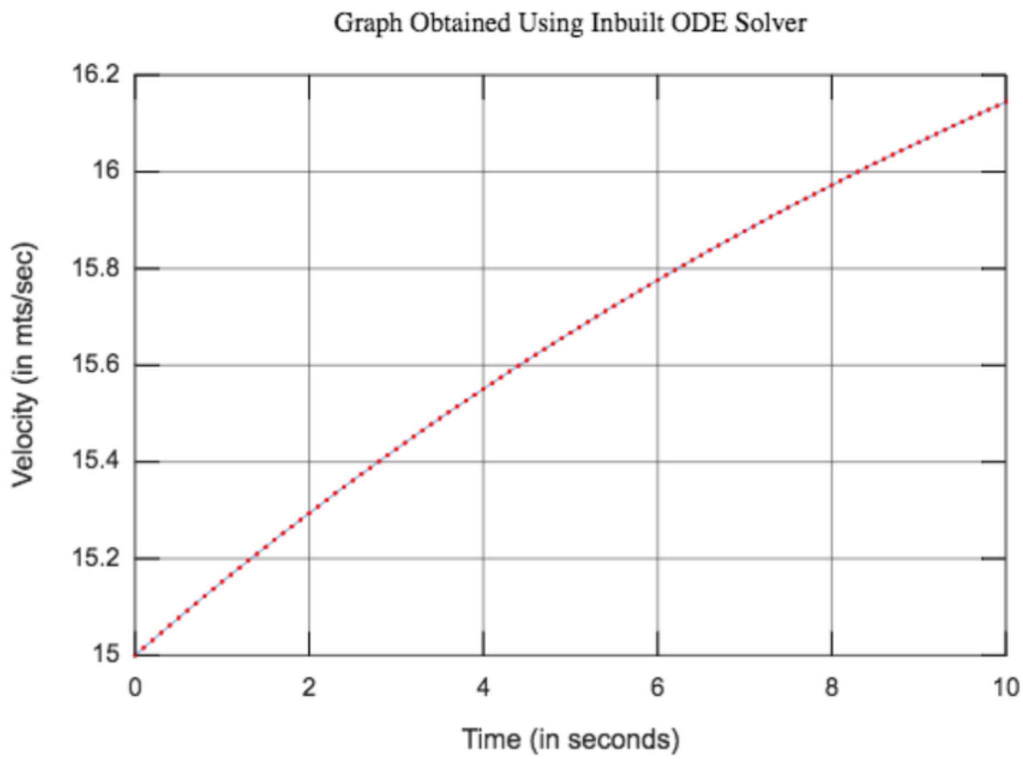

Graph Obtained Using Euler Equation
Velocity (mts/sec)
Time (in seconds)
16.2
16
15.8
15.6
15.4
15.2
15
0
2
4
6
8
10

Graph Obtained Using Inbuilt ODE Solver



Graph of Analytical Solution





OBSERVATIONS :

1. Here if the initial velocity is greater than terminal velocity then the graph is decreasing as the v^2 terms will dominate. So the ode solver is given initial velocity less than terminal velocity and Euler's graph is given more velocity.
2. If you give a much higher initial velocity then drag will be even bigger as it depends on v^2. So you will not be able to peddle and drag will overpower you and the velocity slowly decreases until a point when both cancels each other.
3. As we can see if we increase mass allowing all other variables be constant, terminal velocity is not changed by much, however, time required to attain terminal velocity is increased .
4. Investigation of the effect of rider's power, mass and frontal area on the terminal velocity :

Power	Terminal Velocity

400	12.5719
200	9.9784
100	7.9234

As we can see that if we increase power, the terminal velocity increases.

Mass	Terminal Velocity
75	12.5719
100	12.5707
200	12.5145

As we can see, even if we increase mass, the terminal velocity does not change significantly.

Frontal Area	Terminal Velocity
0.2	14.8525
0.33	12.5719
0.75	9.5622

(b) Run your code (case (a) discussed during class) with initial v=0; observe the output and give possible explanation. Explain why it is important to give a non-zero initial velocity.

As shown, we formulated a formula of acceleration, in which there is a term where we divide acceleration with velocity. But if we take initial velocity as zero, then at time zero the acceleration becomes infinite so velocity will also jump directly to infinity.

We have taken power as constant (Power = Force * velocity) but if our assumption is true then at velocity equals to 0, the force becomes infinity, which is not possible. So we must have some positive initial velocity.

(c) As discussed in the class, we have assumed that the bicyclist maintains a constant power. What about the assumption when the bicycle has a very small velocity? (instantaneous power=product of force and velocity).

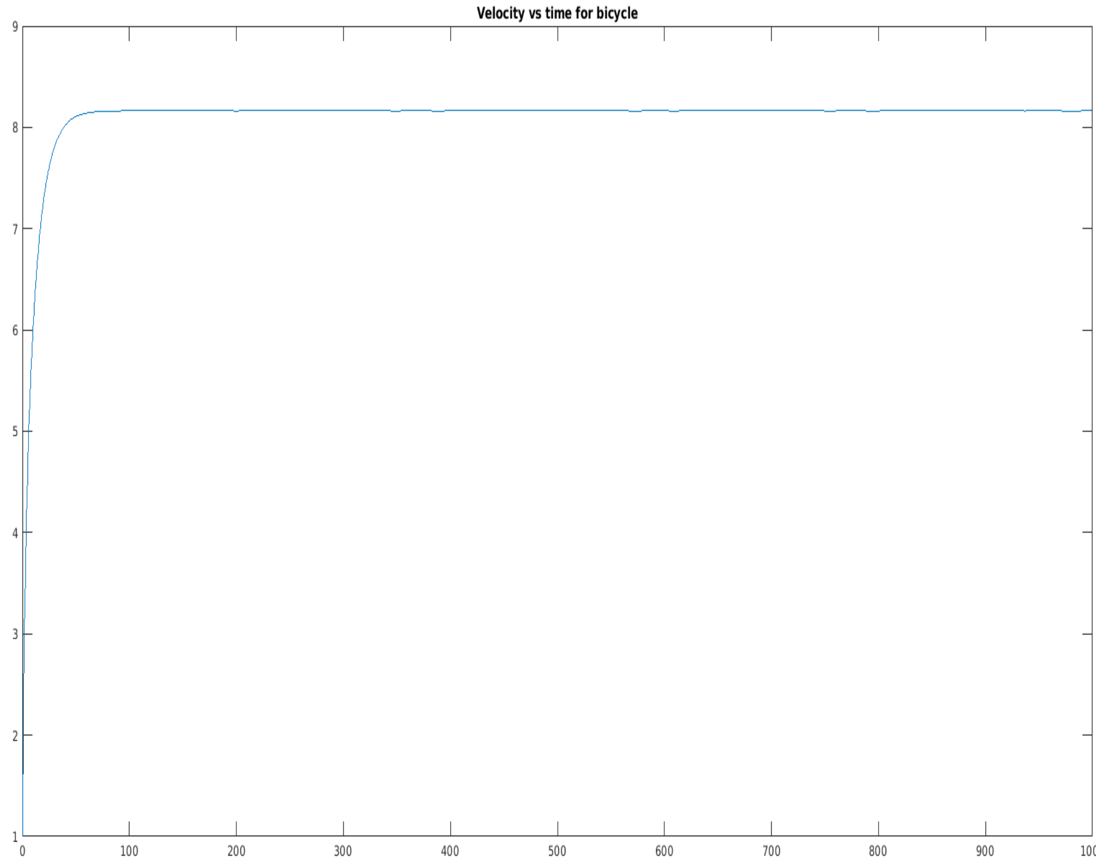
Here if we take initial velocity 0.001 m/s then according to equation dv/dt = P/mv = F/m . We have constant power hence to achieve such small velocity we need too large force which is not possible in real life situation.

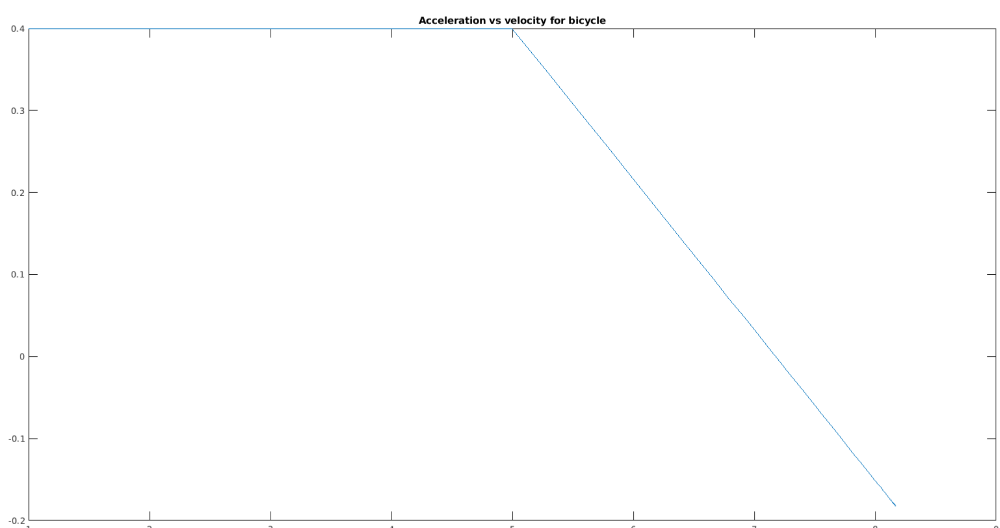
(d) At low velocities it is more realistic to assume, that the rider is able to exert a constant force. That means for small "v" there is a constant force, which means eqn is dv/dt=F0/m.

ASSUMPTIONS: Same as question 3(a)

INITIALIZATIONS:

1) M = mass = 50
2) P = power = 100
3) A = Area = 0.33
4) Init_x = 0, init_v = 1, init_a = 0


Velocity vs time for bicycle
9
8
7
6
5
4
3
2
1
0
100
200
300
400
500
600
700
800
900
1000


Acceleration vs velocity for bicycle
0.4
0.3
0.2
0.1
0
-0.1
-0.2
1
2
3
4
5
6
7
8
9

OBSERVATIONS:

1) Here, as seen from the graph, acceleration is constant for low velo. and decreases linearly with increase in velocity.
2) This scenario will also work for zero initial velocity and low initial velocities as force is considered constant during that time.