Assignment 6

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Apr 9, 2019

DOUBLE PENDULUM

1.1 Problem Statement:

Derive the Euler-Lagrange DEs for double pendulum as discussed during the lecture.

• Using the above equations to computationally investigate the dynamics of double pendulum.

• Choose different set of initial conditions (mass, lengths etc.) and study the effect on the dynamics.

• Plot the movement in x-y space. (make conclusions about the parametric plot)

• Plot time vs thetas (generalized coordinates).

• With the help of a plot prove that your code/ implementation is accurate.

• What are the conclusions if the initial angular displacements are small?

• What are the conclusions if the initial angular displacements are large?

• Introduce animation in your implementation as shown during the lecture and study the system.

• How do chaotic orbits change with different initial conditions?

1.2 Assumptions:

1) Gravitational acceleration is constant.
2) Effect of air drag force is not taken into account.
3) Strings are assumed to be massless.
4) Energy of the system remains constant and conserved.

1.3 Initializations:

1) m_1, m_2 mass of the first and second pendulums respectively.
2) L_1, L_2 length of the string attached to the first and second pendulum respectively
3) g acceleration due to gravity.
4) Θ_1, Θ_2 angular displacements of the first and second pendulum.
5) ω_1, ω_2 angular velocity of the first and second pendulum respectively.

1.4 Analytical Solution:

The horizontal component of the first pendulum is:

$$x_1 = L_1 \sin sin\ (\theta_1)$$

The horizontal component of the second pendulum:

$$x_2 = L_1 sin\,(\theta_1) + L_2 sin(\theta_2)$$

The vertical component of the first:

$$y_1 = -L_1 cos(\theta_1)$$

Vertical component of the second:

$$y_2 = -L_1 cos\,(\theta_1) - L_2 cos(\theta_2)$$

Differentiating the above four equations gives us

$$= L_1 cos\theta_1 \dot{\theta}_1$$

$$\dot{x}_2 = L_1 cos\theta_1 \dot{\theta}_1 + L_2 cos\theta_2 \dot{\theta}_2$$

$$\dot{y}_1 = L_1 sin\theta_1 \dot{\theta}_1$$

$$\dot{y}_2 = L_1 sin\theta_1 \dot{\theta}_1 + L_2 sin\theta_1 \dot{\theta}_2$$

The kinetic energy is:

$$K.E. = \tfrac{1}{2} m_1 \left(\dot{x}_1^2 + \dot{y}_2^2\right) + \tfrac{1}{2} m_2 (\dot{x}_1^2 + \dot{y}_2^2)$$

Substituting values from above we get

$$K.E. = \tfrac{1}{2}m_1\dot{\theta}_1^{\,2}L_1^2 + \tfrac{1}{2}m_2(\dot{\theta}_1^{\,2}L_1^2 + \dot{\theta}_2^{\,2}L_2^2 + 2\theta_1 L_1\theta_2 L_2 cos(\theta_1 - \theta_2))$$

Potential energy is:

$$U = -(m_1 + m_2)gL_1 cos(\theta_1 - \theta_2) - m_2L_2gcos\theta_2$$

The lagrangian would thus be:

L = K.E. – U

$$L = \tfrac{1}{2}m_1\left(\dot{x}_1^2 + \dot{y}_2^2\right) + \tfrac{1}{2}m_2\left(\dot{x}_1^2 + \dot{y}_2^2\right) + (m_1 + m_2)gL_1 cos(\theta_1 - \theta_2) - m_2L_2gcos\theta_2$$

Applying euler – lagrange equations that we get:

$$(m_1 + m_2)L_1^2\ddot{\theta}_1 + (m_2)L_1L_2\ddot{\theta}_2\, cos(\theta_1 - \theta_2) + (m_2)L_1L_2\dot{\theta}_1^{\,2}\, cos(\theta_1 - \theta_2) + gL_1(m_1 + m_2)si$$

Simplifying for $\ddot{\theta}_1$ and $\ddot{\theta}_2$

We get:

$$\ddot{\theta}_1 = \frac{-(m_2)L_2\ddot{\theta}_2\, cos(\theta_1 - \theta_2) - (m_2)L_2\dot{\theta}_1^{\,2}\, cos(\theta_1 - \theta_2) + g(m_1 + m_2)sin\theta_1}{(m_1 + m_2)L_1}$$

$$\ddot{\theta}_2 = \frac{-L_1\ddot{\theta}_1 cos(\theta_1 - \theta_2) + L_1\dot{\theta}_1^2 sin(\theta_1 - \theta_2) - gsin(\theta_2)}{L_2}$$

1.5 Computational Model:

Certain expressions can be assumed as constants as they get covered in the assumptions made at the beginning of the document.

$$a = (m_1 + m_2)L_1$$

$$b = m_2L_2 \cos cos\ (\theta_1 - \theta_2)$$

$$c = m_2L_1cos(\theta_1 - \theta_2)$$

$$d = m_2L_2$$

$$e = -m_2L_2(\omega_2)sin(\theta_1 - \theta_2) - g(m_1 + m_2)sin(\theta_1)$$

$$f = -m_2L_1(\omega_1)sin(\theta_1 - \theta_2) - gm_2sin(\theta_2)$$

Thus rewriting the equations, we get:

$$a\frac{d\omega_1}{dt} + b\frac{d\omega_2}{dt} = e$$

$$c\frac{d\omega_1}{dt} + d\frac{d\omega_2}{dt} = f$$

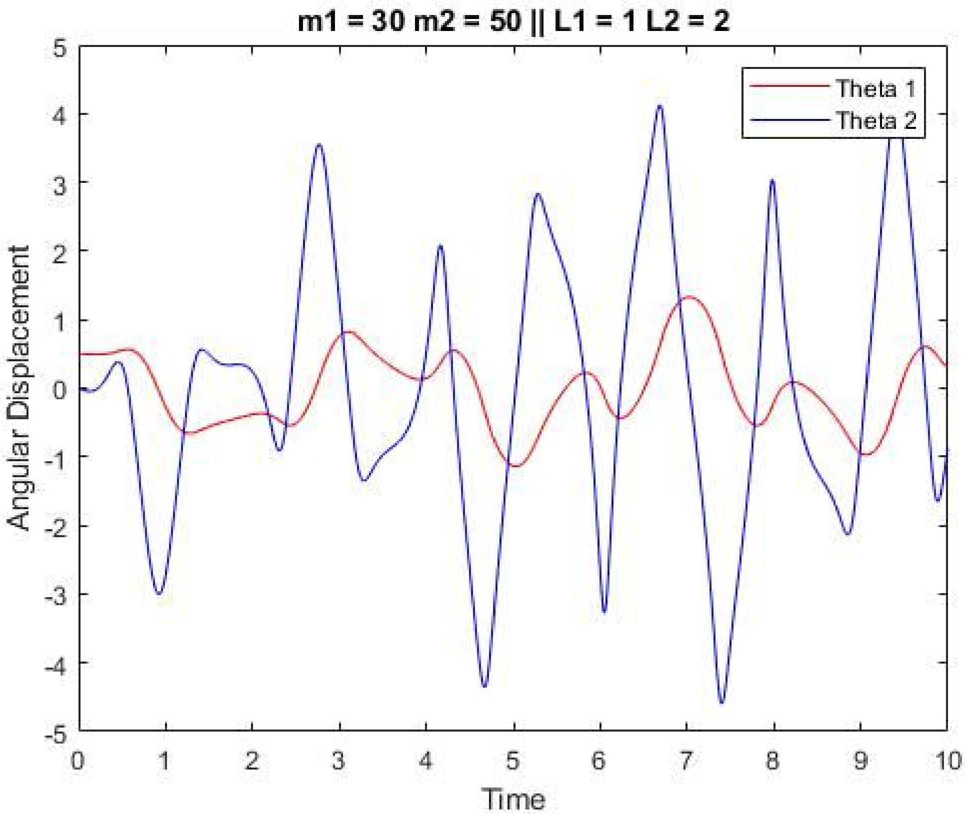
Solving the above equations gets us:

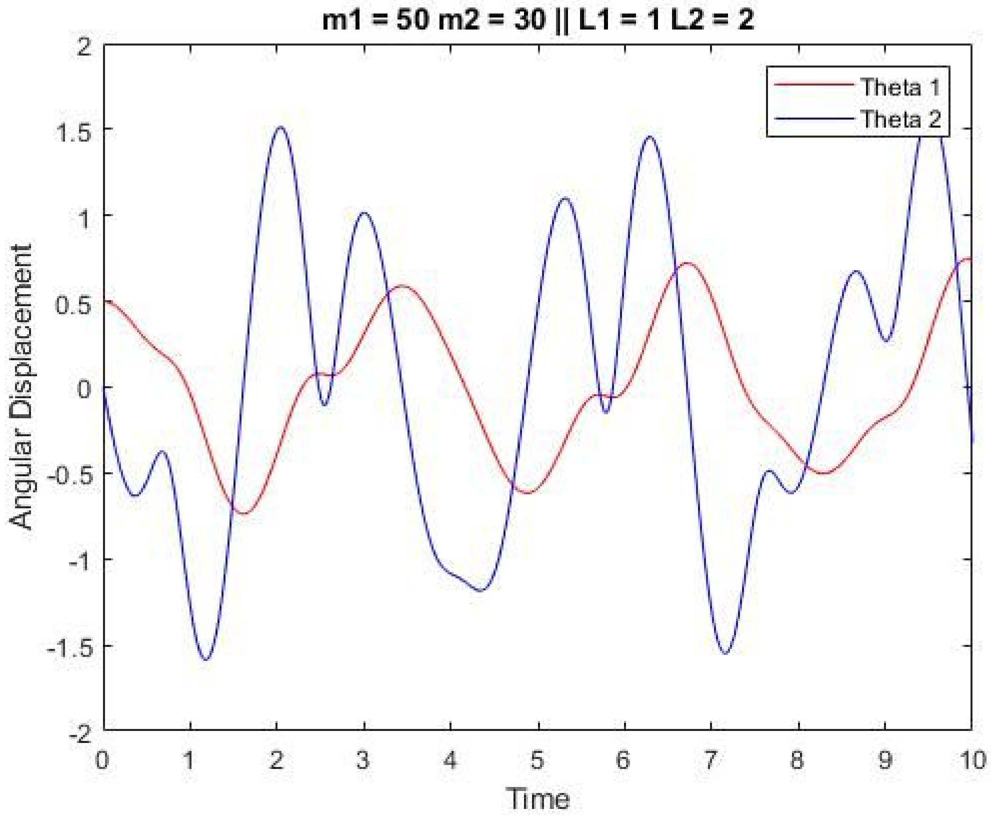
$$\frac{d\omega_1}{dt} = \frac{ed-bf}{ad-cd}$$

$$\frac{d\omega_2}{dt} = \frac{af-ce}{ad-cb}$$

And from here we can make various observations as following:

1.6 Results:

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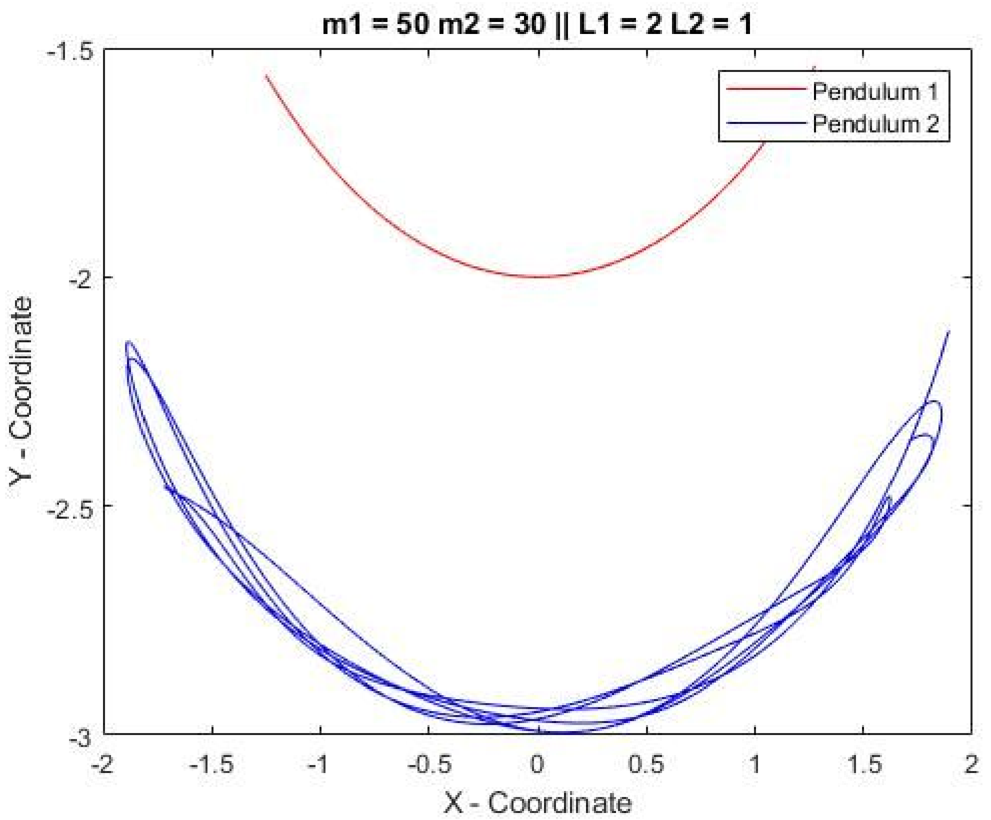
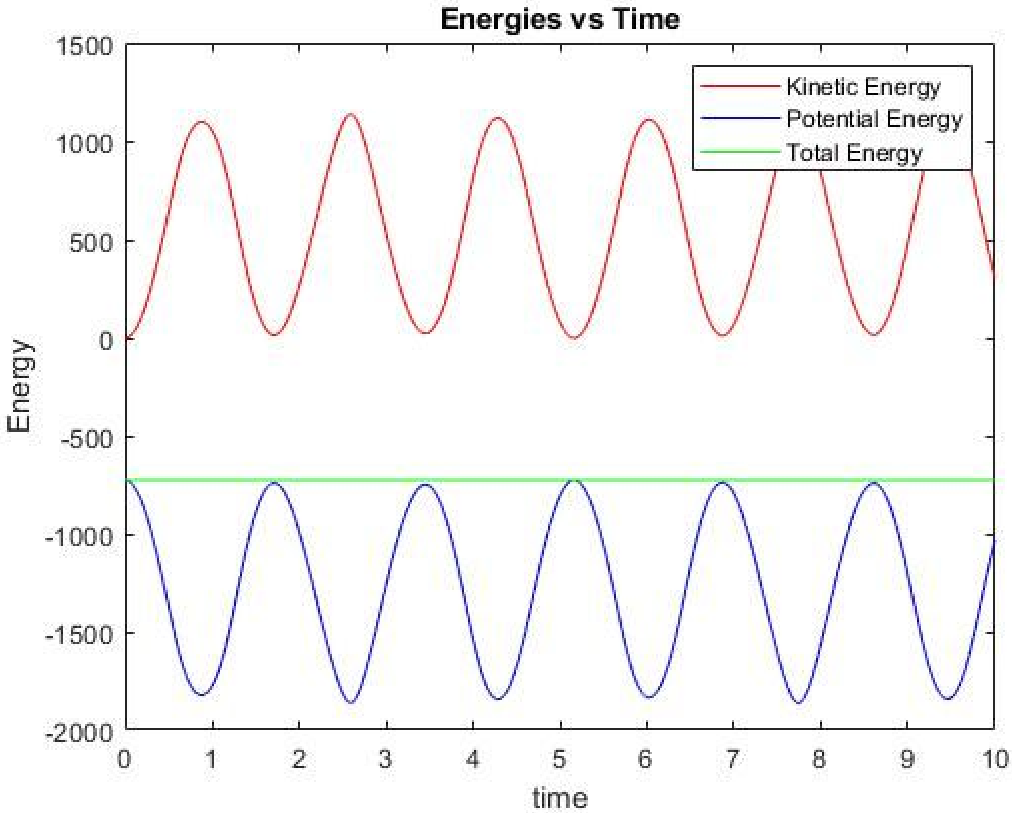
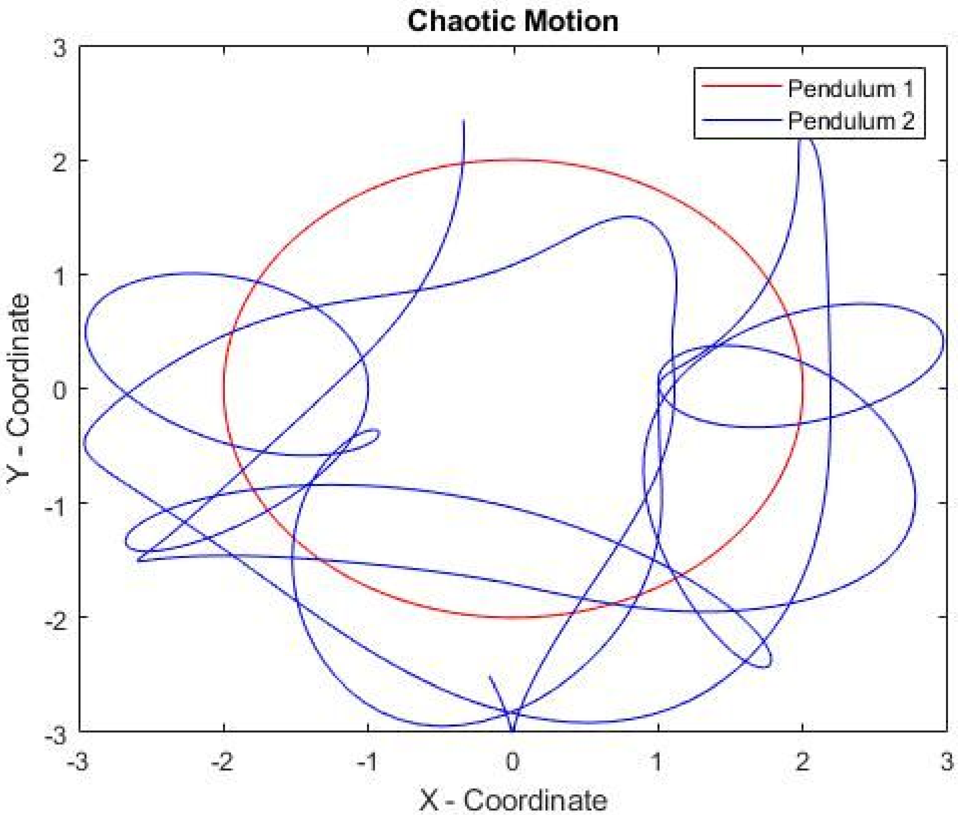
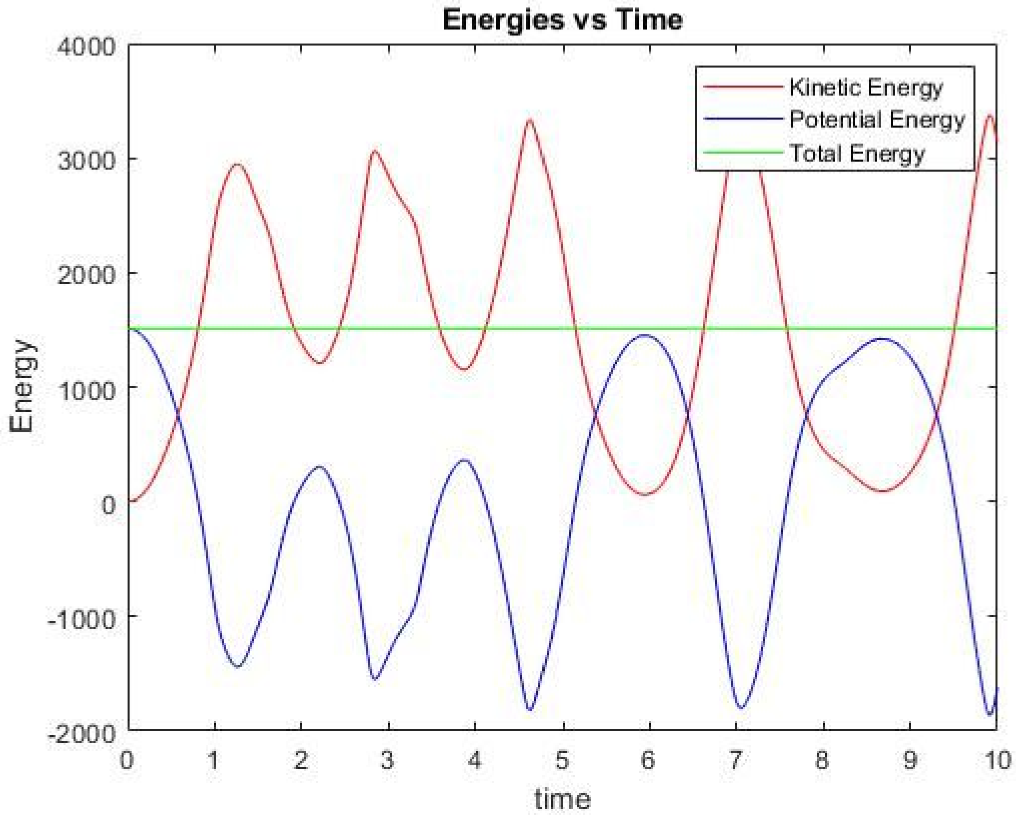
Theta 1
Theta 2
Angular Displacement
Time


m1 = 50 m2 = 30 || L1 = 2 L2 = 1
Pendulum 1
Pendulum 2
Y - Coordinate
X - Coordinate


Energies vs Time
Kinetic Energy
Potential Energy
Total Energy
Energy
time
1500
1000
500
0
-500
-1000
-1500
-2000
0
1
2
3
4
5
6
7
8
9
10


Chaotic Motion
Pendulum 1
Pendulum 2
Y - Coordinate
X - Coordinate
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3


Energies vs Time
Kinetic Energy
Potential Energy
Total Energy
Energy
time
4000
3000
2000
1000
0
-1000
-2000
0
1
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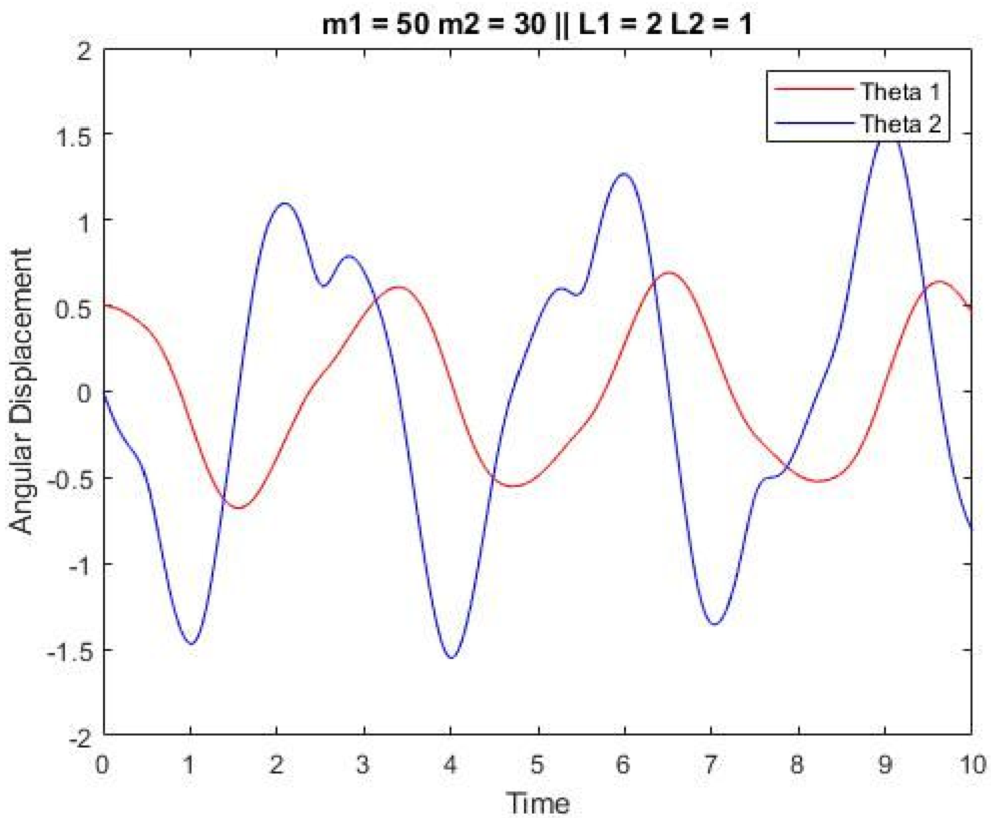
3
4
5
6
7
8
9
10


m1 = 50 m2 = 30 || L1 = 2 L2 = 1
Theta 1
Theta 2
Angular Displacement
Time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
1
2
3
4
5
6
7
8
9
10

1.7 Observations:

CASE 1: If the initial angular displacements are small

The body may perform SHM or roughly a periodic motion, but the energy of the system would be less compared to a more general case.

CASE 2: If the initial angular displacements are large:

Chaotic motion is performed by the body with the possibility that the second or both pendulums perform circular motion. The energy would be more with respect to the general case.

CHAOTIC MOTION:

This only becomes apparent for large systems. When the energy is small, we can use small angle approximation to get motion of system. But when energy is beyond this point, systems starts getting more and more complex and for certain energies, chaos may develop in the system.

Effect on chaotic orbits with different initial conditions:

- Mass and length only affect the energy of the system. Bigger mass stops pendulum from going higher and orbits are thus small and vice-versa. Longer string would have its effect such that the orbits are larger.
- If initial velocities are large, orbits would be chaotic and circular.
- If initial displacements are large, orbits would be smoother and more circular and vice-versa.