<u>Financial Engineering Lab (MA374)</u>

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Lab - 09

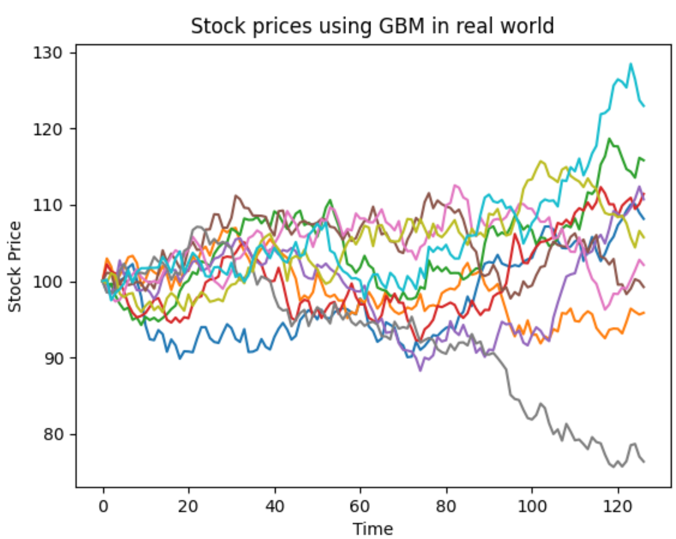
To run the code type **python3 180123021_Kartikeya_Singh_q1.py** into the terminal.

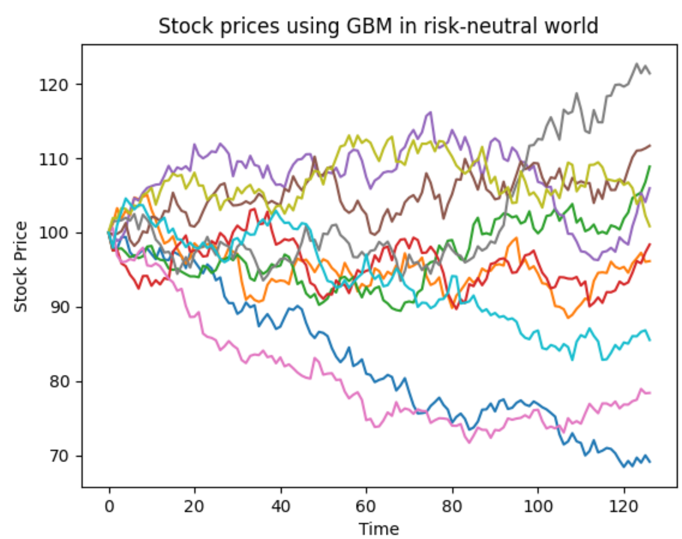
Question 1

In the GBM model, the stock prices vary as -

$$ds(t) = \mu s(t)dt + \sigma s(t)dw(t).$$

Using this model, 10 different paths of an asset are simulated.

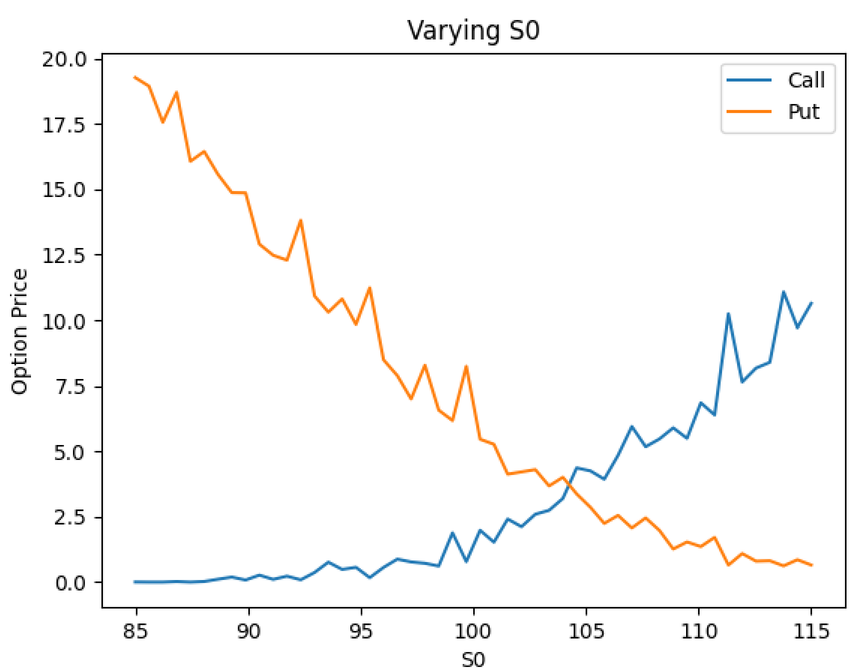


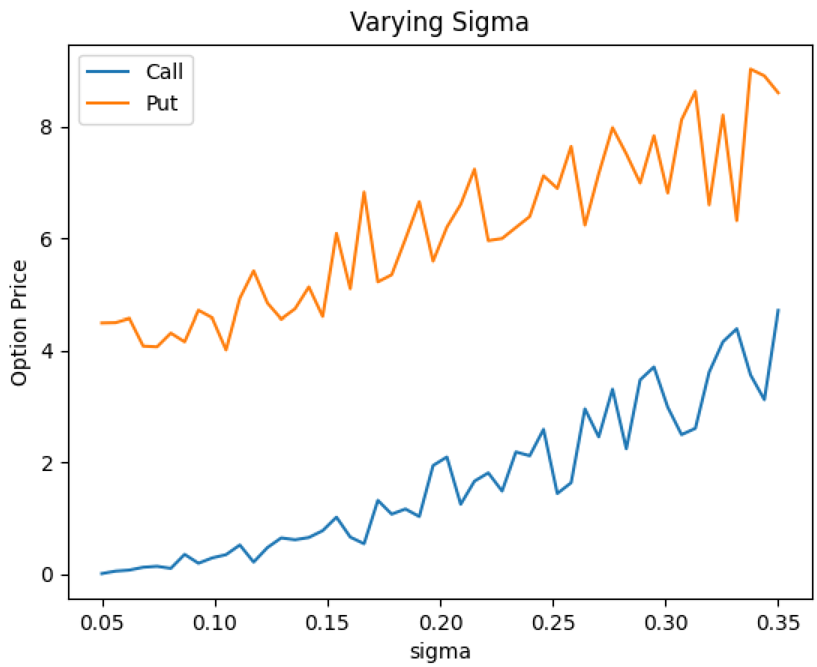


The prices of an Asian option are calculated using Monte-Carlo Simulations. The calculated prices are given by -

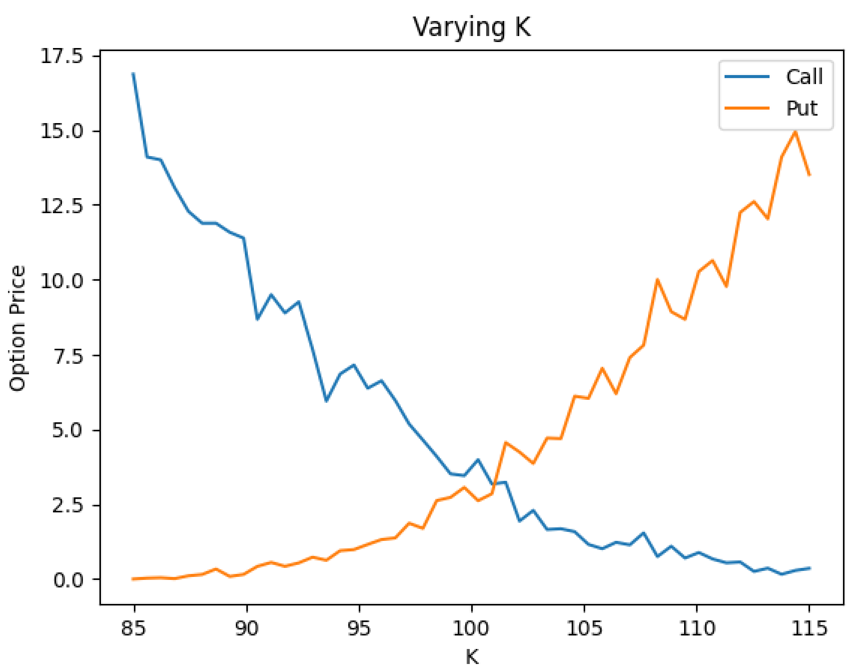
- Call Price for K = 90 is 10.720745
- Put Price for K = 90 is 0.294879
- Call Price for K = 105 is 1.223124
- Put Price for K = 105 is 5.472169
- Call Price for K = 110 is 0.740202
- Put Price for K = 110 is 10.052774

The sensitivity of the Option Price is measured against the initial price (S_0), volatility (σ) and the Strike Price (K). The graphs are -





Varying Sigma
Call
Put
Option Price
sigma
0
2
4
6
8
0.05
0.10
0.15
0.20
0.25
0.30
0.35



Varying K
Call
Put
Option Price
K
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
85
90
95
100
105
110
115

Question 2

The method of antithetic variates is used to reduce the variance.

$\theta = E[Y] = E[g(X)]$

where θ is the quantity we want to estimate ,

we can generate two sample Y_1 and Y_2 s.t. the new unbiased estimator of θ is

$$\hat{\theta} = \frac{Y_1 + Y_2}{2}$$

Hence we have

$$Var(\theta) = \frac{var(Y_1) + var(Y_2) + 2Cov(Y_1, Y_2)}{4}$$

It is obvious that we could get a variance reduction if we have the two samples negatively correlated.

If $X \sim \mathcal{N}(0, 1)$ then we can apply the following algorithm

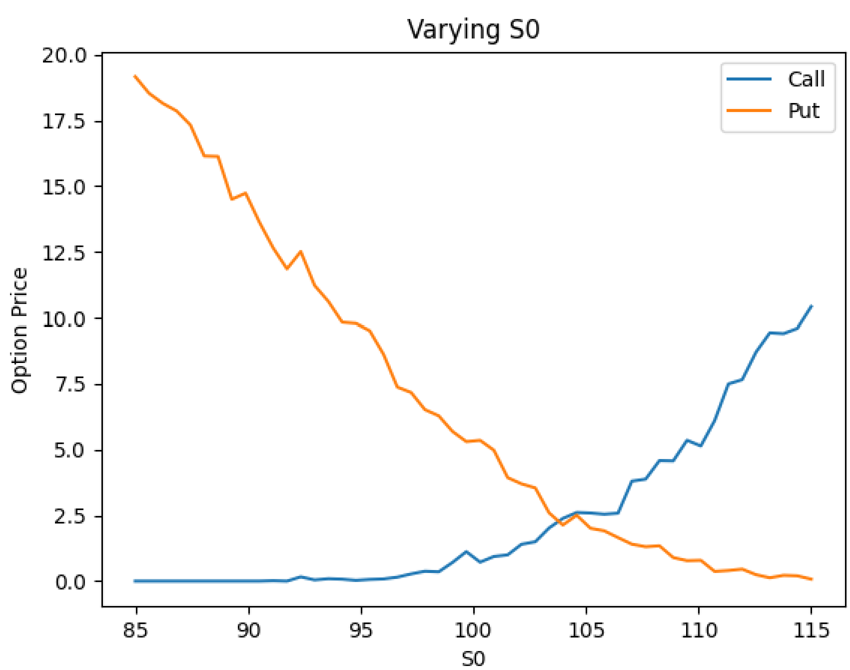
$$\hat{\theta} = \frac{1}{n} \sum_{i=1}^{N} \frac{g(X_i) + g(-X_i)}{2} \text{ with i.i.d. } X_i \sim \mathcal{N}(0, 1)$$

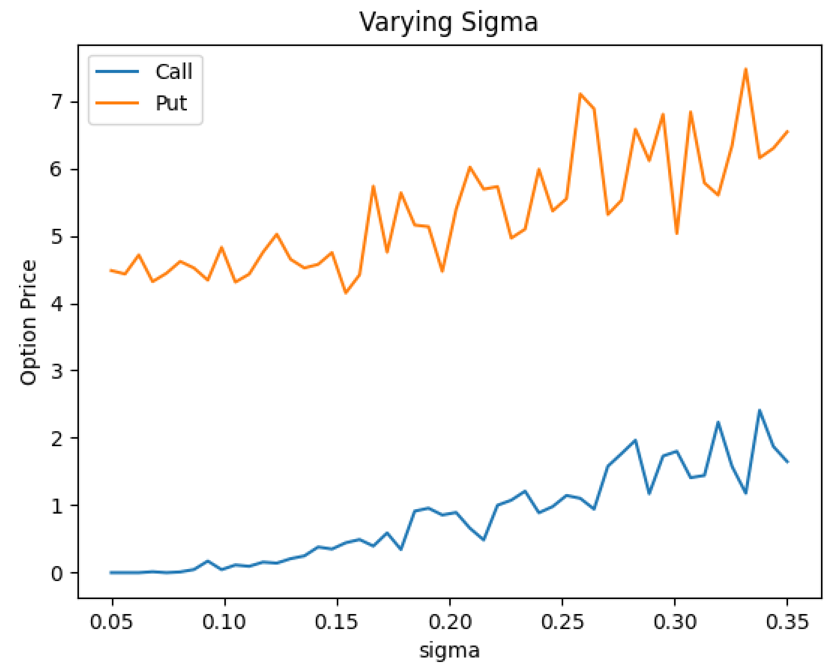
It can be observed that the variance of the option prices has reduced using the method of antithetic variates.

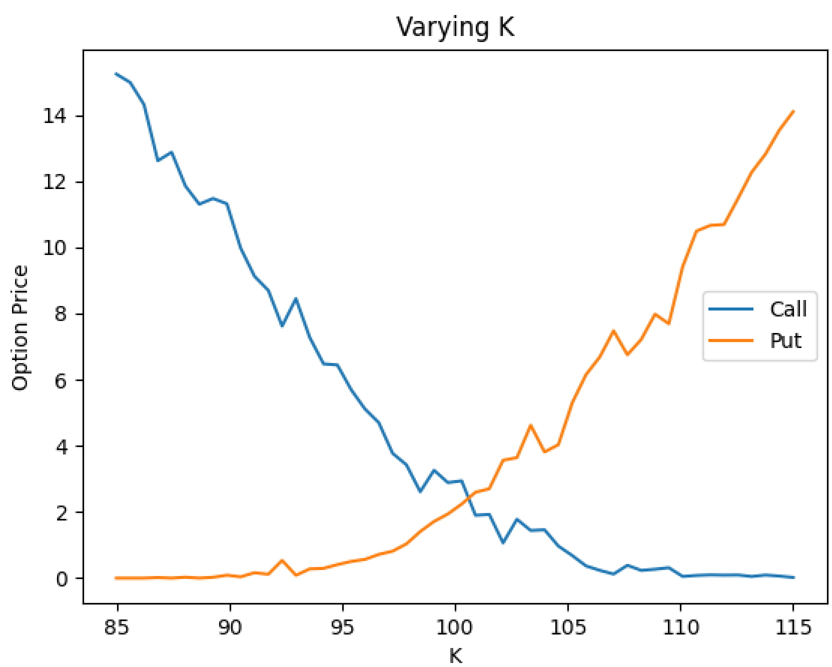
Reduction of Variance using Antithetic Variates

K	Type of Option	Variance	Variance after reduction
90	Call	56.794188	35.417917
90	Put	2.695878	0.116244
105	Call	16.490622	4.937906
105	Put	30.604956	22.447588
110	Call	3.073649	0.864429
110	Put	50.097366	32.490456

The sensitivity of option prices is analysed and the graphs are -




Varying Sigma
Call
Put
Option Price
sigma
0
1
2
3
4
5
6
7
0.05
0.10
0.15
0.20
0.25
0.30
0.35


Varying K
Call
Put
Option Price
K
0
2
4
6
8
10
12
14
85
90
95
100
105
110
115