Monte Carlo Simulations (MA323) Lab 1

Name - Kartikeya Singh
Roll no - 180123021

Question 1

(Run python3 180123021_Kartikeya_Singh_q1.py, 2 CSV files containing the first 100 terms of each sequence is generated which could be opened by spreadsheet software like MS-Excel)
(If a CSV file with the same name (180123021_q1_part1_output.csv/ 180123021_q1_part2_output.csv) exists in the folder it must be deleted before running the command otherwise the data would be appended to the existing file)

Outputs

Case 1: a = 6, b = 0, m = 11

$X_0 = 0$
Sequence - 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
Repetition Period = 1

$X_0 = 1$
Sequence - 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, …
Repetition Period = 10

$X_0 = 2$
Sequence - 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, ...
Repetition Period = 10

$X_0 = 3$
Sequence - 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, ...
Repetition Period = 10

$X_0 = 4$
Sequence - 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, ...
Repetition Period = 10

$X_0 = 5$
Sequence - 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, ...
Repetition Period = 10

$X_0 = 6$

Sequence - 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, …
Repetition Period = 10

$X_0 = 7$
Sequence - 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, ...
Repetition Period = 10

$X_0 = 8$
Sequence - 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, ...
Repetition Period = 10

$X_0 = 9$
Sequence - 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, ...
Repetition Period = 10

$X_0 = 10$
Sequence - 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, ...
Repetition Period = 10

Case 2: a = 3, b = 0, m = 11

$X_0 = 0$
Sequence - 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...
Repetition Period = 1

$X_0 = 1$
Sequence - 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, …
Repetition Period = 5

$X_0 = 2$
Sequence - 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, ...
Repetition Period = 5

$X_0 = 3$
Sequence - 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, ...
Repetition Period = 5

$X_0 = 4$
Sequence - 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, ...
Repetition Period = 5

$X_0 = 5$

Sequence - 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, ...
Repetition Period = 5

$X_0 = 6$
Sequence - 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, …
Repetition Period = 5

$X_0 = 7$
Sequence - 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, ...
Repetition Period = 5

$X_0 = 8$
Sequence - 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, ...
Repetition Period = 5

$X_0 = 9$
Sequence - 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, ...
Repetition Period = 5

$X_0 = 10$
Sequence - 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, ...
Repetition Period = 5

Observations

1) Case 1 (a = 6, b = 0, m = 11) has a period of 1 for $X_0 = 0$ and a period of 10 for $X_0 \neq 0$
2) Case 2 (a = 3, b = 0, m = 11) has a period of 1 for $X_0 = 0$ and a period of 5 for $X_0 \neq 0$
3) Case 1 (a = 6) is preferred for the Linear Congruence Generator as it has a higher period, hence a higher degree of randomness

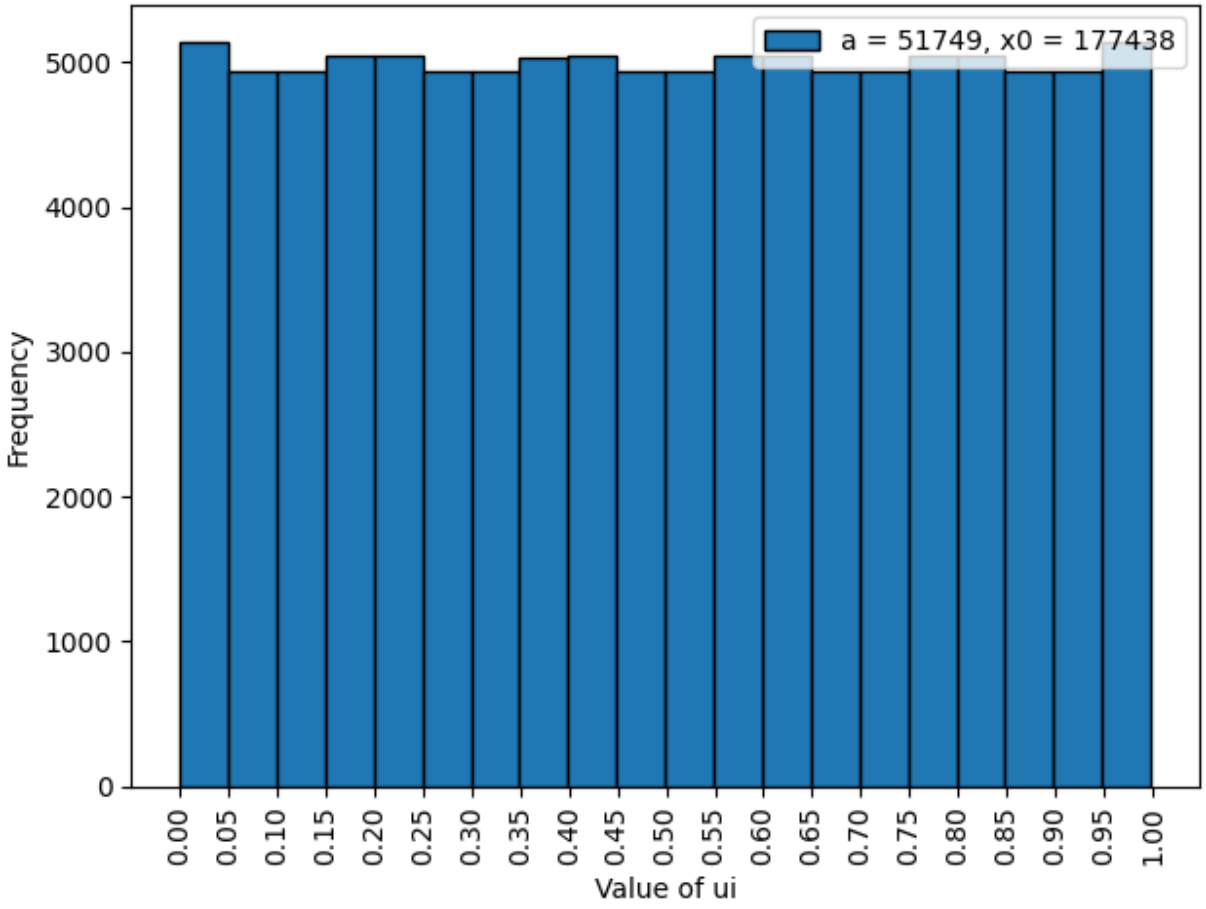
Question 2

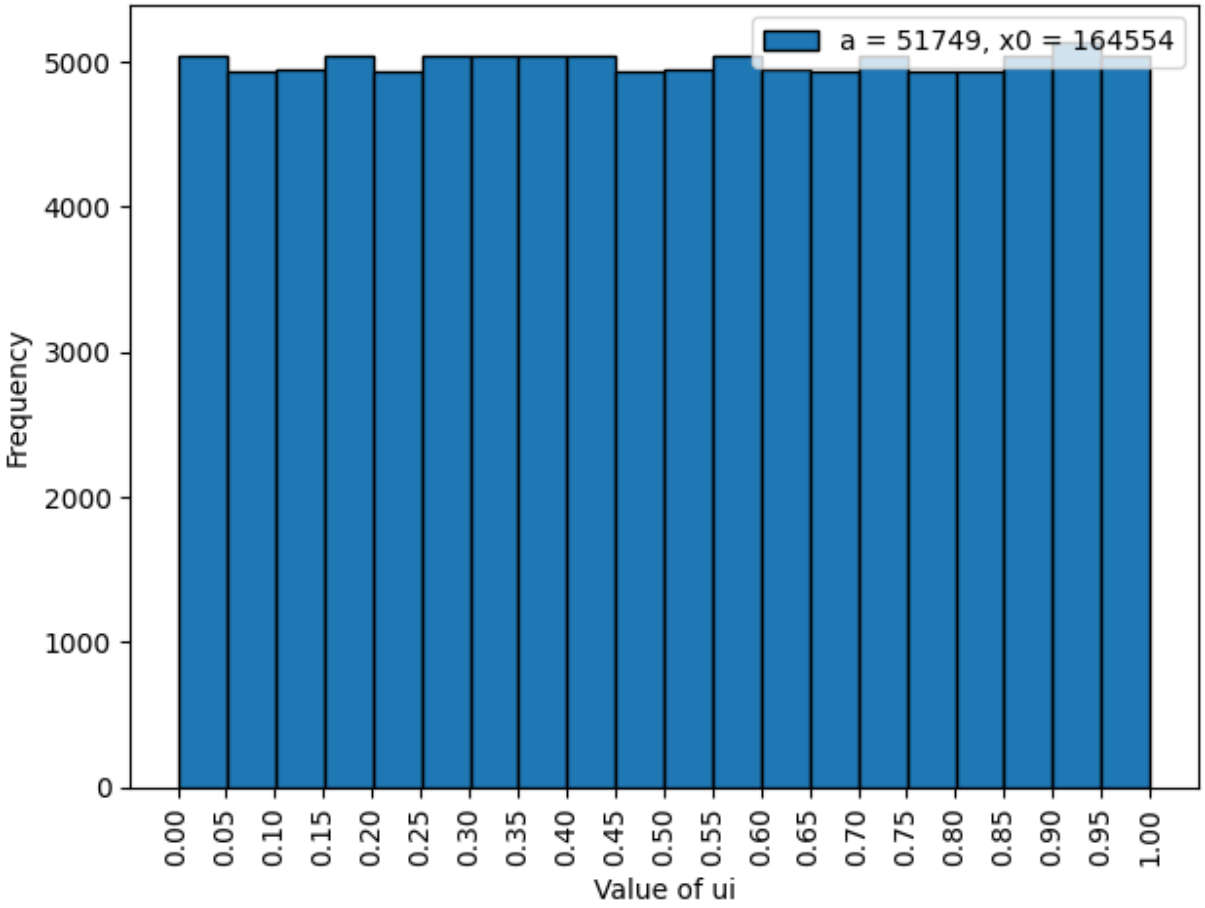
(Run python3 180123021_Kartikeya_Singh_q2.py, a CSV file containing the frequencies in various ranges and 10 images containing the plots are generated, the images might not be same as the ones shown below as the seed(X_0) is randomly generated)
(If a CSV file with the same name (180123021_q2_output.csv) exists in the folder it must be deleted before running the command otherwise the data would be appended to the existing file)

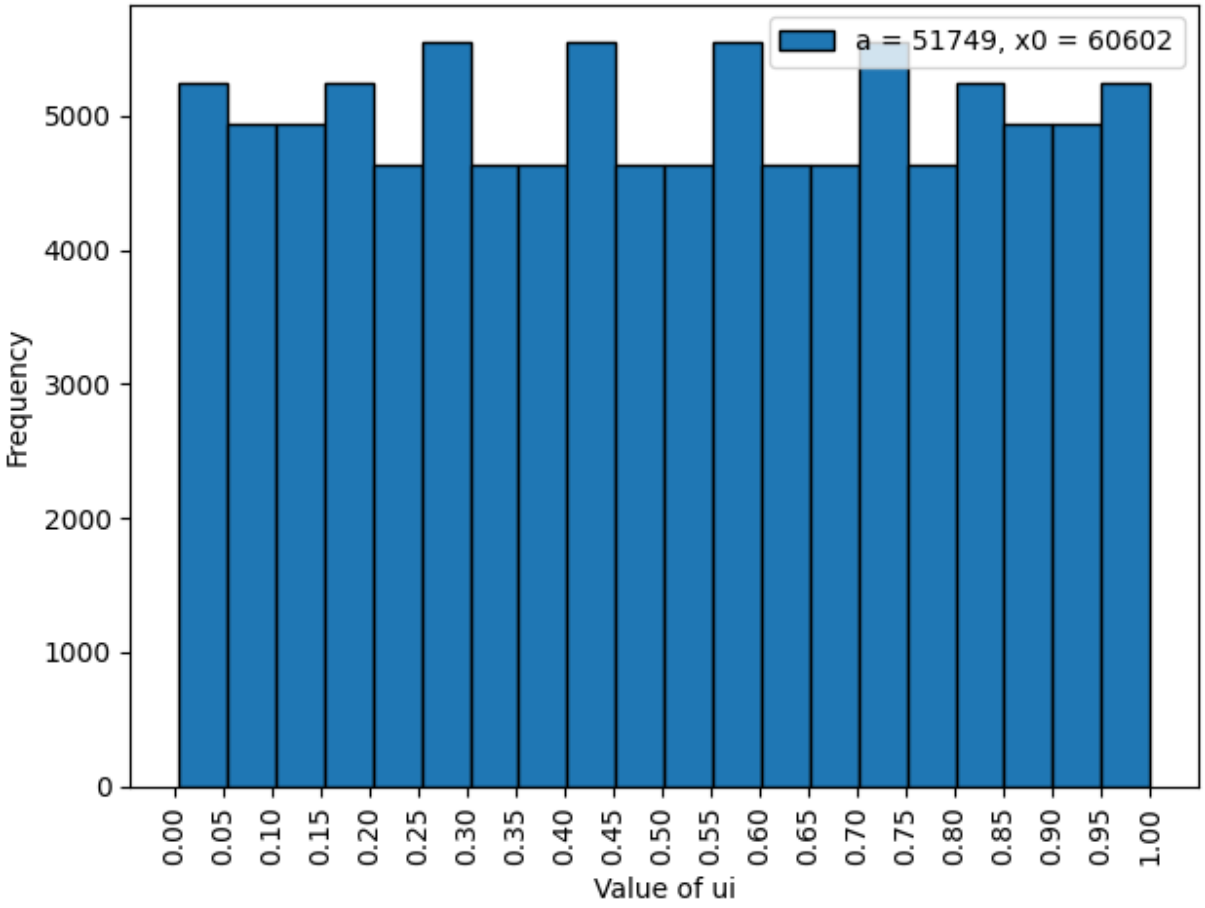
Outputs

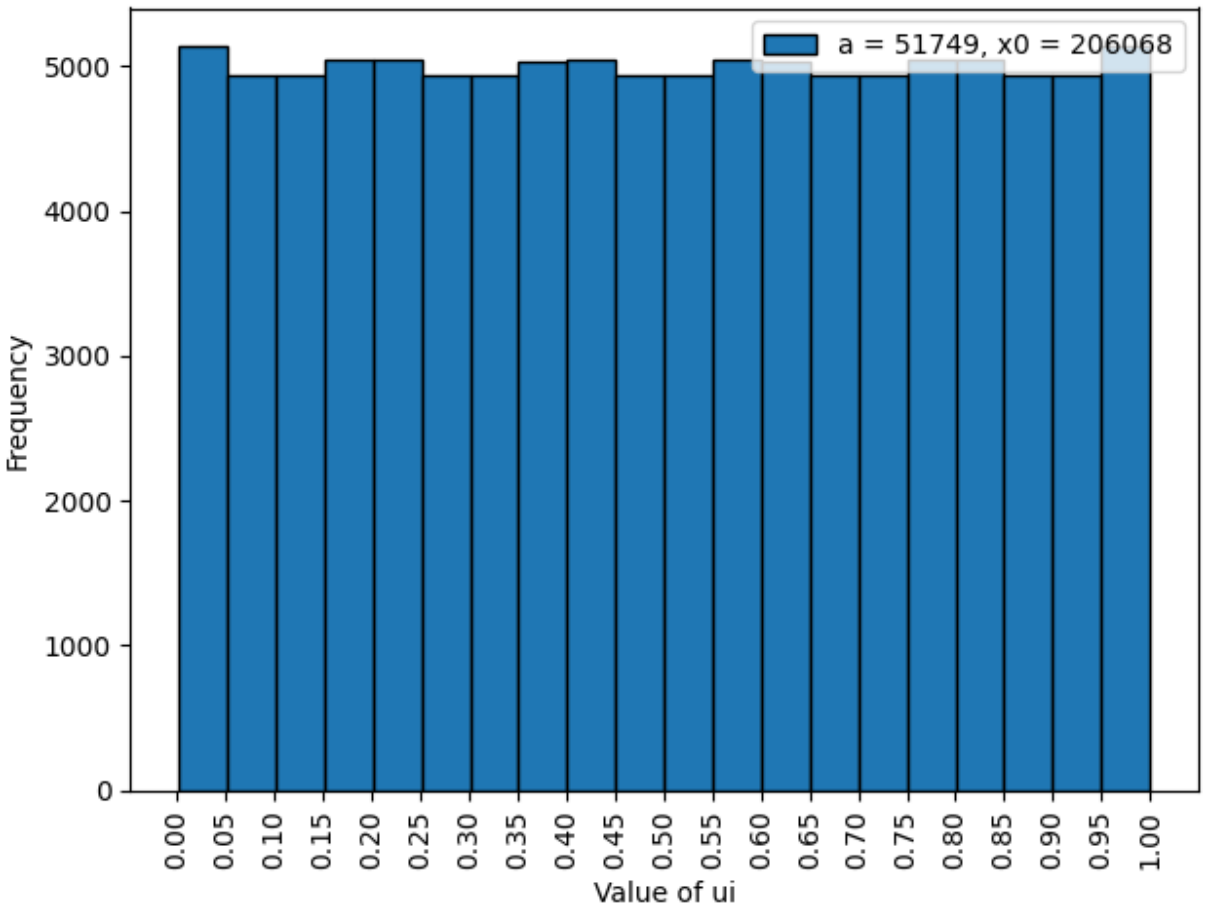
a = 1597 and 51479 (5 values of X_0 each), b = 3436, m = 244944

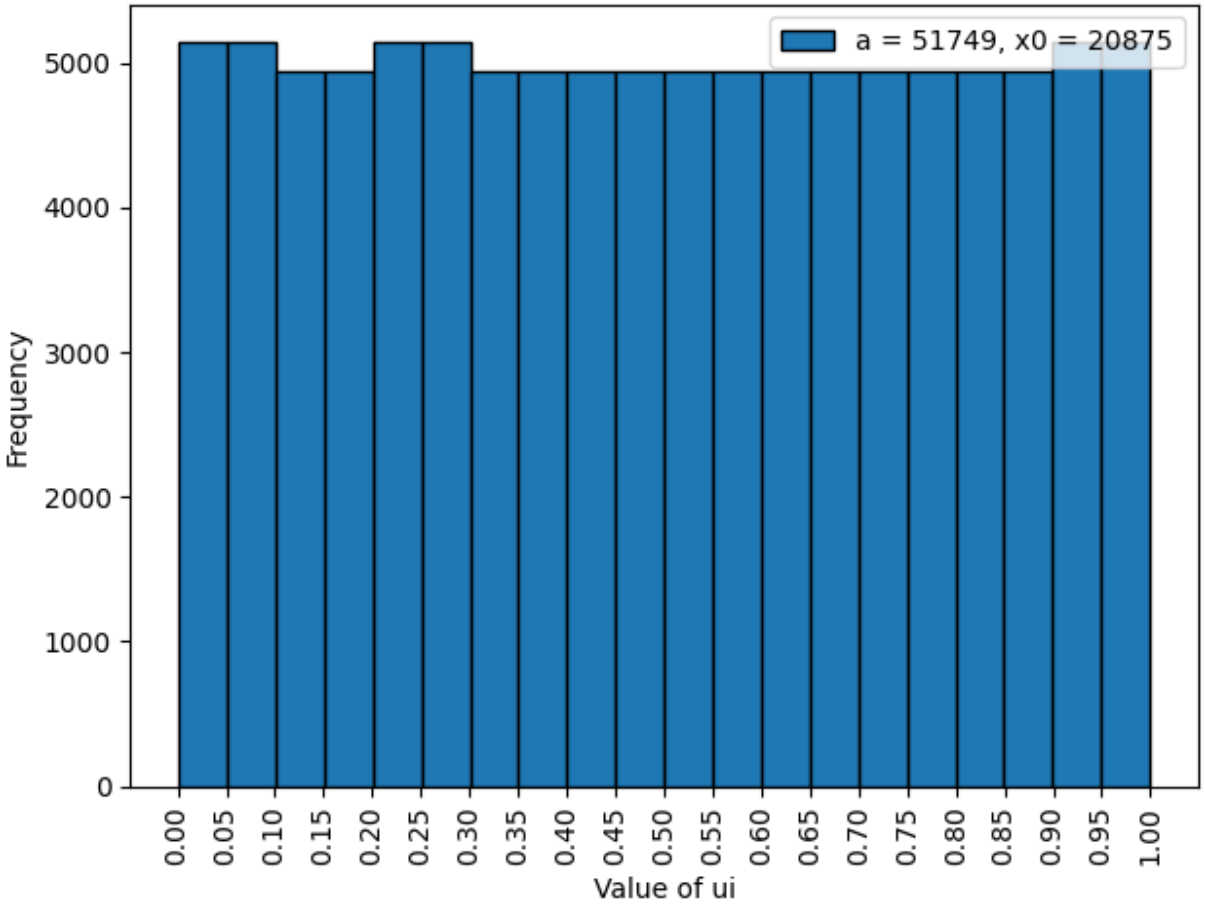
5 distinct values of X_0 are generated randomly and first 100000 elements of the sequence are generated and the frequencies between various ranges are plotted as a histogram.

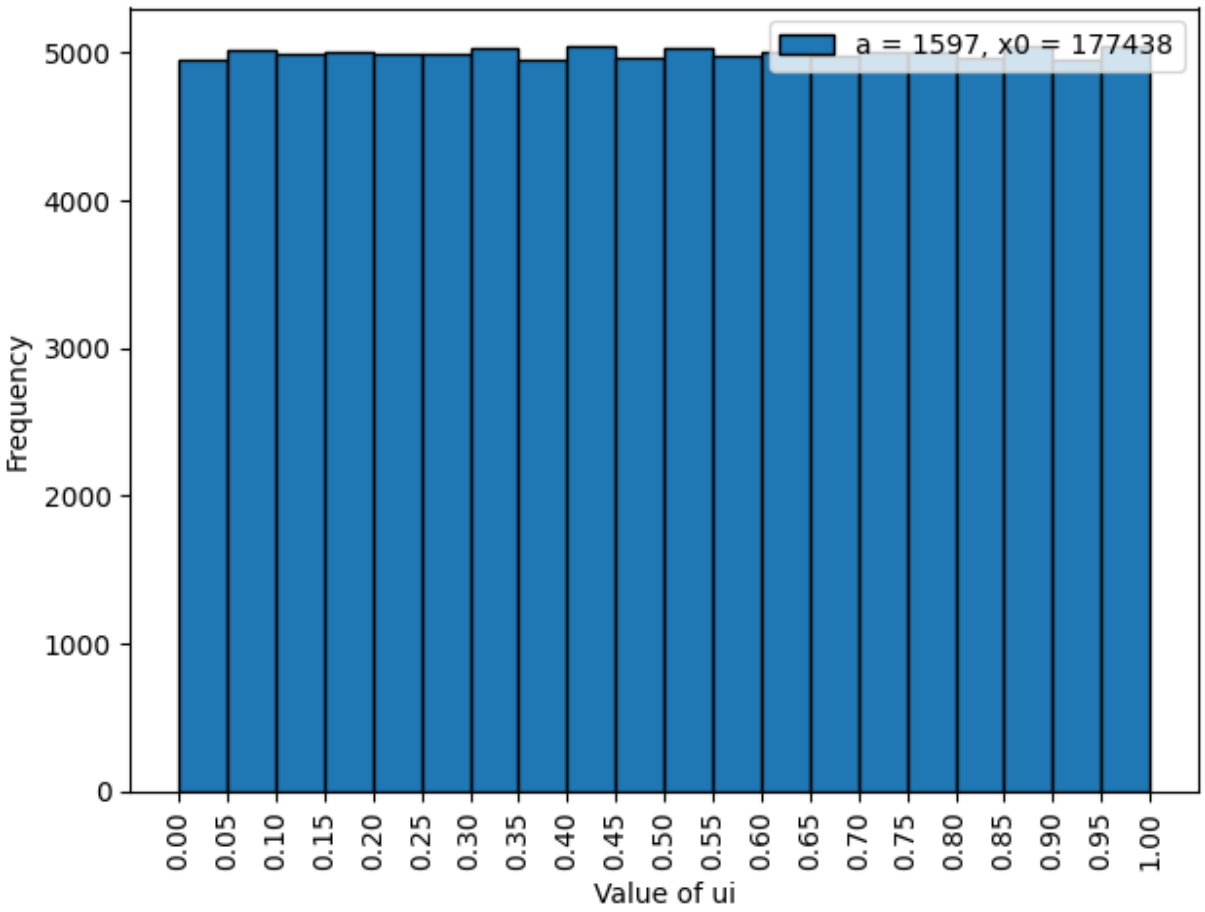


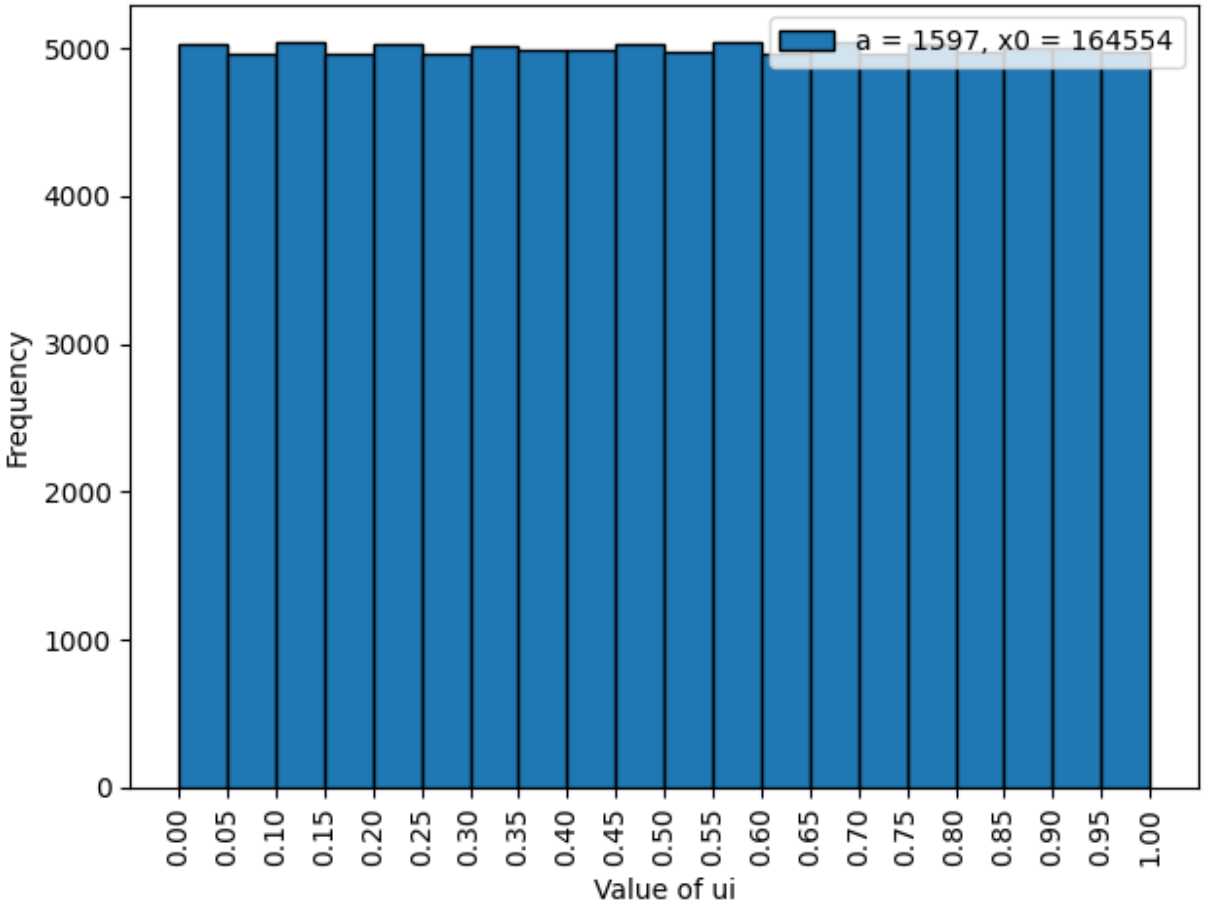

a = 51749, x0 = 164554
Frequency
5000
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui

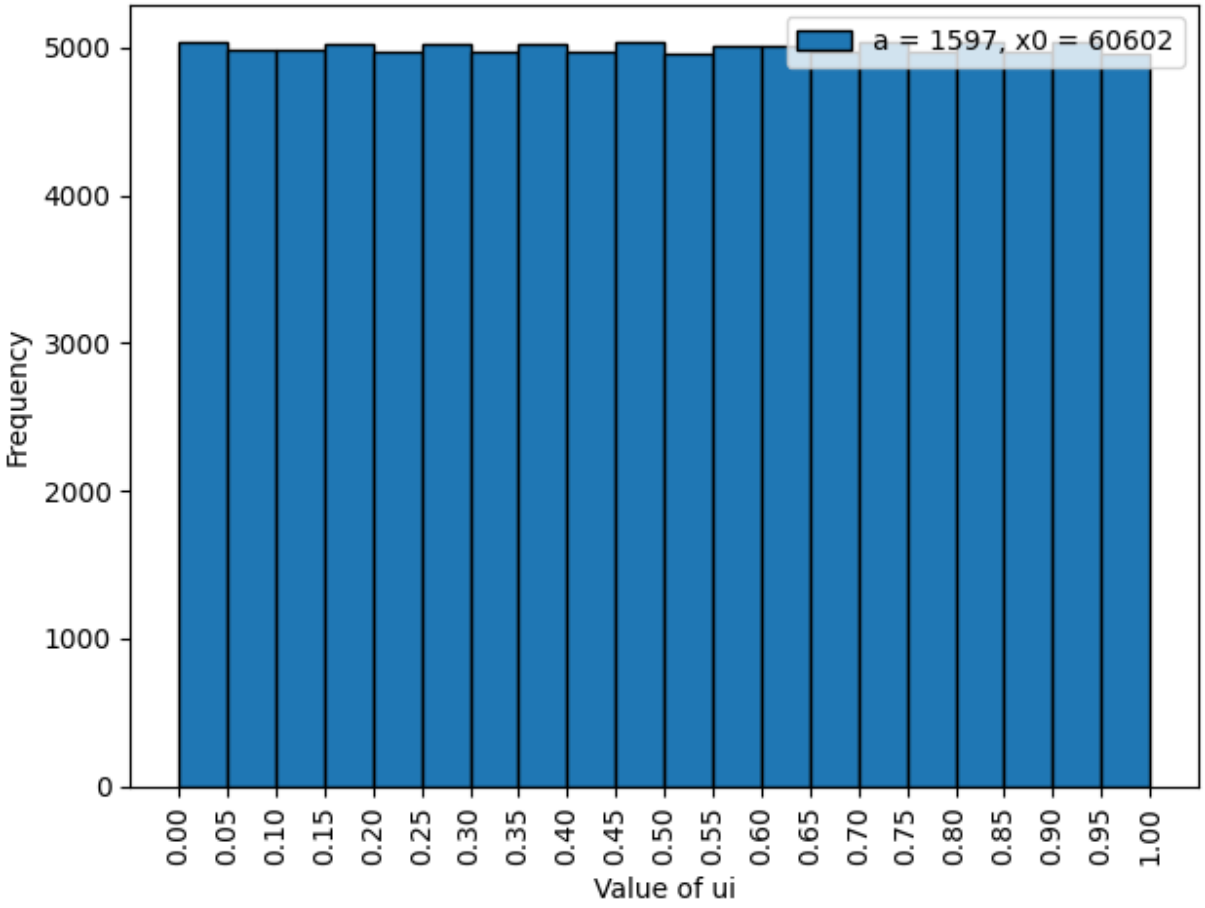

a = 51749, x0 = 60602
Frequency
0
1000
2000
3000
4000
5000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui

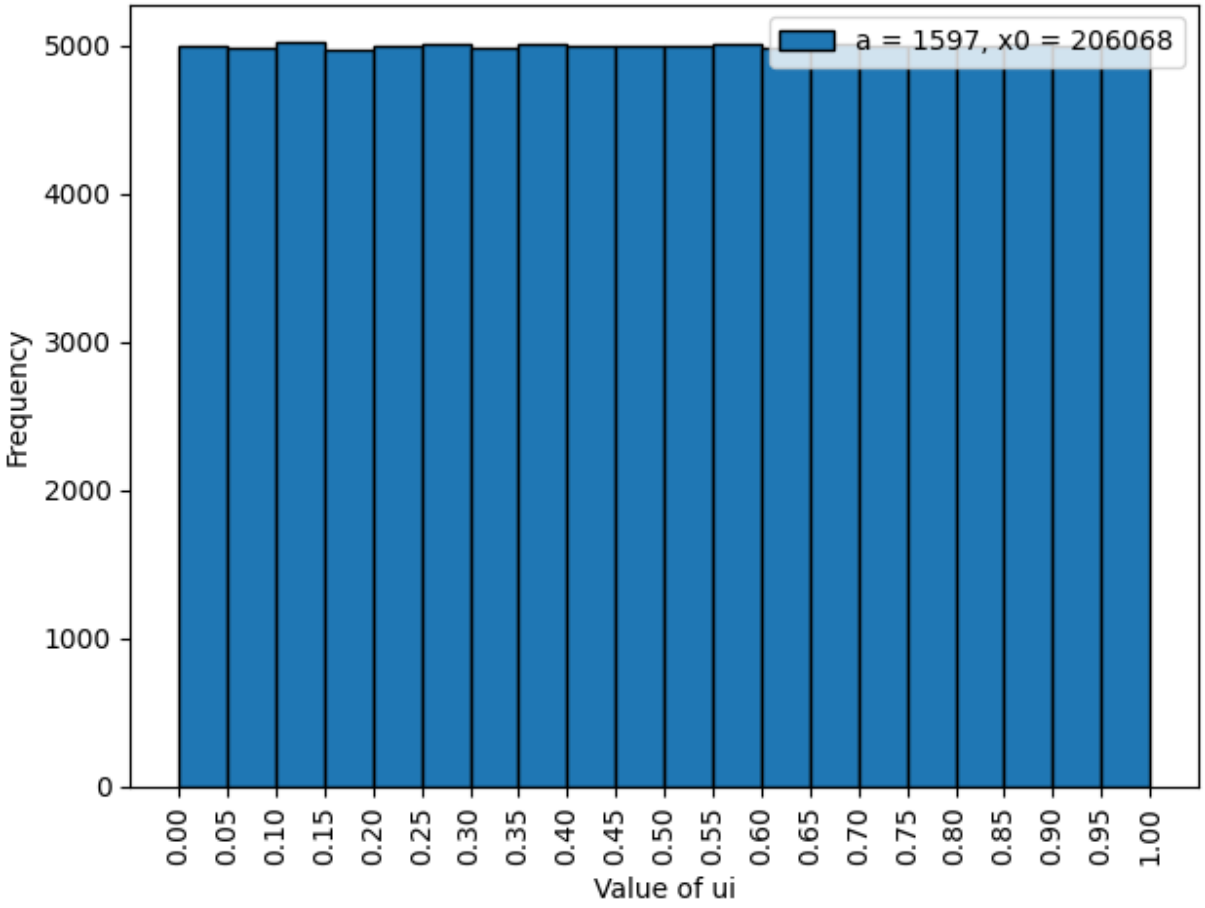

a = 51749, x0 = 206068
Frequency
5000
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui

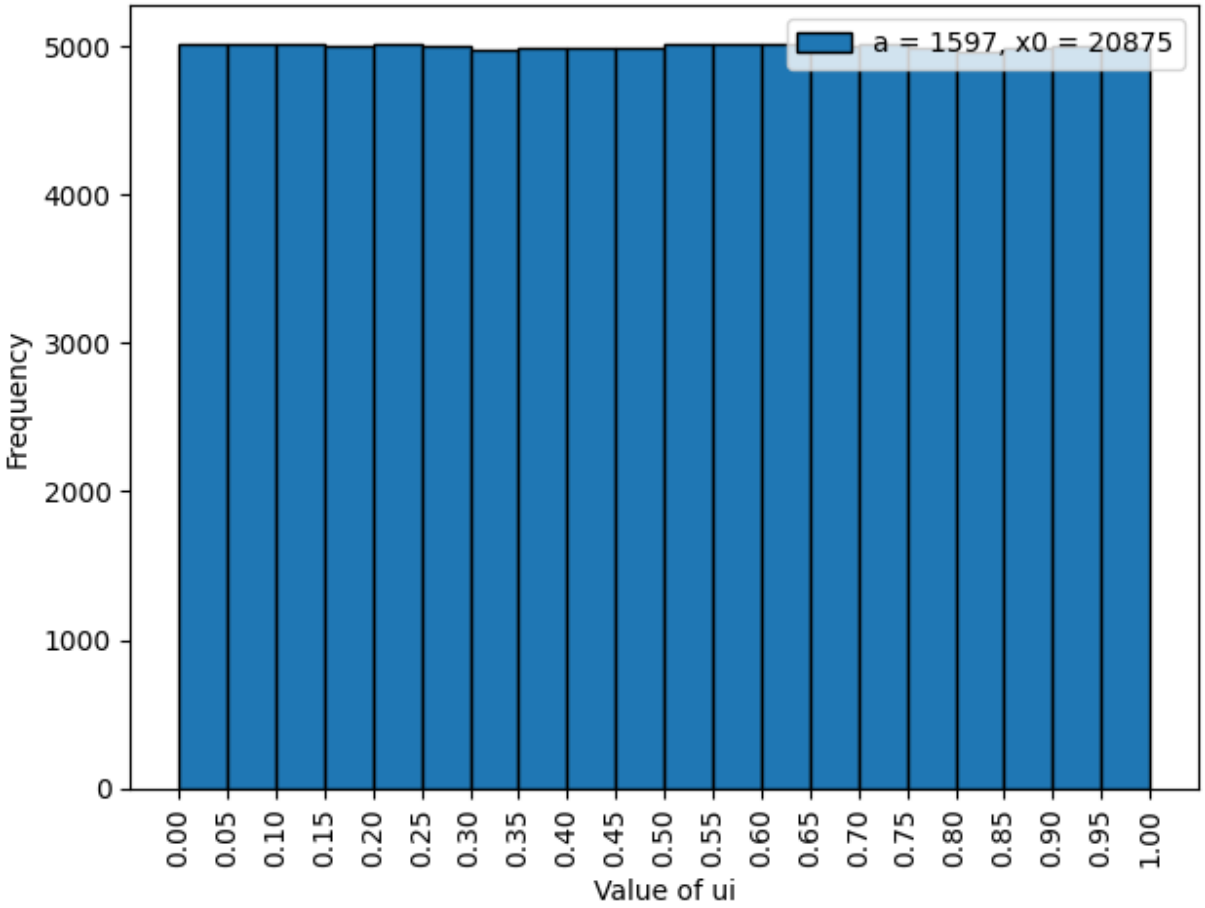

a = 51749, x0 = 20875
Frequency
5000
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui


a = 1597, x0 = 177438
5000
4000
3000
2000
1000
0
Frequency
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui


a = 1597, x0 = 164554
5000
4000
3000
2000
1000
0
Frequency
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui


a = 1597, x0 = 60602
5000
4000
3000
2000
1000
0
Frequency
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui


a = 1597, x0 = 206068
5000
4000
3000
2000
1000
0
Frequency
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui



Observations

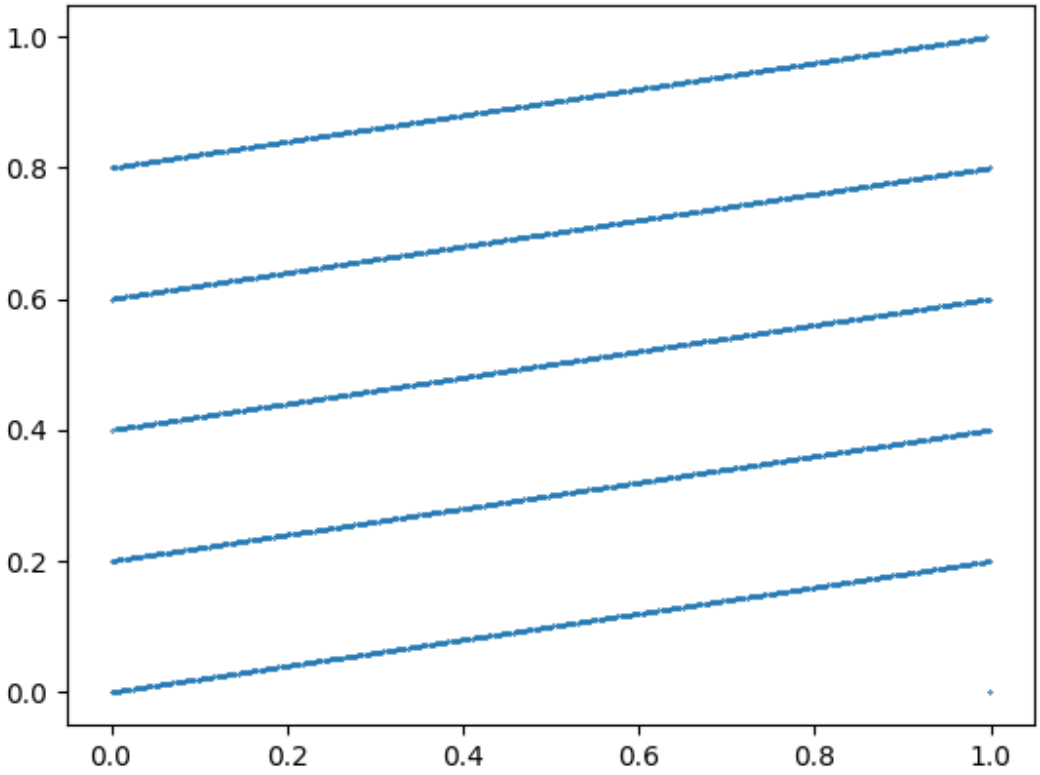
1) The frequency of elements in various ranges is almost equal indicating that the values generated are nearly uniformly distributed

Question 3

(Run python3 180123021_Kartikeya_Singh_q3.py, the plot shown below is generated)

Outputs

The first 10000 elements of the sequence (u_n) is generated for a = 1229, b = 1, m = 2048, and X_0 chosen randomly and the values of (u_{i-1}, u_i) are plotted on a scatter plot. The result obtained is -



Observations

1) The points (u_{i-1}, u_i) lie on parallel lines
2) This shows the fact that the numbers generated by a linear congruence generator are not completely random (had it been completely random the plot would be uniformly scattered over the 2-D plane