Model Reference Adaptive Control for Voltage Stabilization in a DC-DC Dual Active Bridge Converter

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***Abstract*—This project implements a Model Reference Adaptive Controller (MRAC) for output voltage tracking application in a Dual Active Bridge (DAB) converter. First, a large-signal model for the converter is formulated for a conventional phase shift modulation scheme. Then, the Generalized Average Modeling (GAM) technique is used as it is difficult to directly handle the zero average nature of the leakage inductor current in the modeling process. The resulting third-order model using GAM is reduced into a first-order model that only captures the capacitor voltage's dc dynamics. The resulting simpler first-order model is then utilized for formulating adaptation laws for a direct MRAC scheme that tracks a reference, despite no information about the passive component values or load resistance value. However, the ripple of the output voltage is a part of the unmodeled dynamics of the DAB converter and presents itself as a bounded noise input to MRAC, which is sensitive to noise, driving the closed-loop system into instability. Therefore, a dead zone modification is done on the existing control laws to impart robustness to the adaptive controller.**

***Index Terms*—Dual Active Bridge converter, Robust Adaptive Control, Model Reference Adaptive Control, Reduced Order Model, Generalized Averaged Model**

I. Introduction

The Dual Active Bridge converter is a versatile power electronic converter topology used in high power dc-dc conversion when galvanic isolation and bidirectional power transfer are necessary. Its main attractive features compared to other bidirectional topologies are [1], [2]:

- Simple systematic structure
- Simple control structure
- High power density
- Low device stress
- Ability to soft switch

Due to these attractive features, the DAB is extensively used in microgrid architectures, aerospace systems [3], distribution networks as a Solid State Transformer (SST) [4].

In an aerospace environment, due to stringent specifications, the stability of the output voltage must always be guaranteed. Many linear control methods exist for the DAB control [5], [6]. However, linear control techniques may not work against parametric drift-induced effects. In this regard, Model reference Adaptive Control is an effective technique as it adjusts the system response based on continuously adjusting for parametric variations.

II. Background

In this section, the GAM will be derived for the DAB converter. A figure of the dc-dc DAB is given in Figure 1. To simplify the model of the converter, only the output capacitor and leakage inductor will be considered. The dynamic model of the converter is written as

$$L_t \frac{di_t(t)}{dt} = v_s q_1(t) - R_o i_t(t) - \frac{N_1}{N_2}\frac{R}{R+R_c} q_2(t) v_c(t) \quad (1)$$

$$C\frac{dv_c(t)}{dt} = \frac{N_1}{N_2}\frac{R}{R+R_c} q_2(t) i_t(t) - \frac{1}{R+R_c} v_c(t) \quad (2)$$

where the two differential equations pertain to the leakage inductance L_t, the filter capacitance C, where the equivalent series resistance of C and L_t is given by R_c, and R_t. The load is characterized as a pure resistance R connected across the capacitor C. The two bridges of the DAB are modulated by switching functions $q_1(t)$ and $q_2(t)$. The inductor current and capacitor voltage states are represented by $i_t(t)$ and $v_c(t)$ respectively. R_o is an intermediate-term given by

$$R_o = \frac{R_t R + R_t R_c + R R_c \frac{N_1^2}{N_2^2}}{R+R_c} \quad (3)$$

A. Generalized Average Model

In order to translate to the GAM model, the derivative and product terms can be translated to the frequency domain. The frequency-domain components of interest are the fundamental switching frequency component and the dc component. The capacitor voltage predominantly has dc components, so fundamental switching terms are dropped. Similarly, in the leakage inductor, the dc terms are dropped as it has a zero average, and only switching frequency terms are considered. The derivative and product terms used to translate to GAM are

$$\left\langle \frac{dx(t)}{dt} \right\rangle_k = \frac{d\left\langle x(t)\right\rangle_k}{dt} + jk\omega \langle x\rangle_k(t) \quad (4)$$

$$\langle x(t)y(t)\rangle_k = \sum_{p=-\infty}^{p=+\infty} \langle x\rangle_{k-p}(t)\langle y\rangle_p(t) \quad (5)$$

The k^{th} Fourier coefficient of modulation term q_2 is given by

$$\langle q_2\rangle_k(t) = -j\frac{2}{k\pi}e^{-jkd\pi} \quad (6)$$

where $d = \phi/\pi$. Considering i_t and v_c Fourier coefficient expansions to be

$$\langle i_t\rangle_1 = i_{t\Re}(t) + j i_{t\Im}(t) \quad (7)$$

$$\langle v_c\rangle_1 = v_c(t) \quad (8)$$

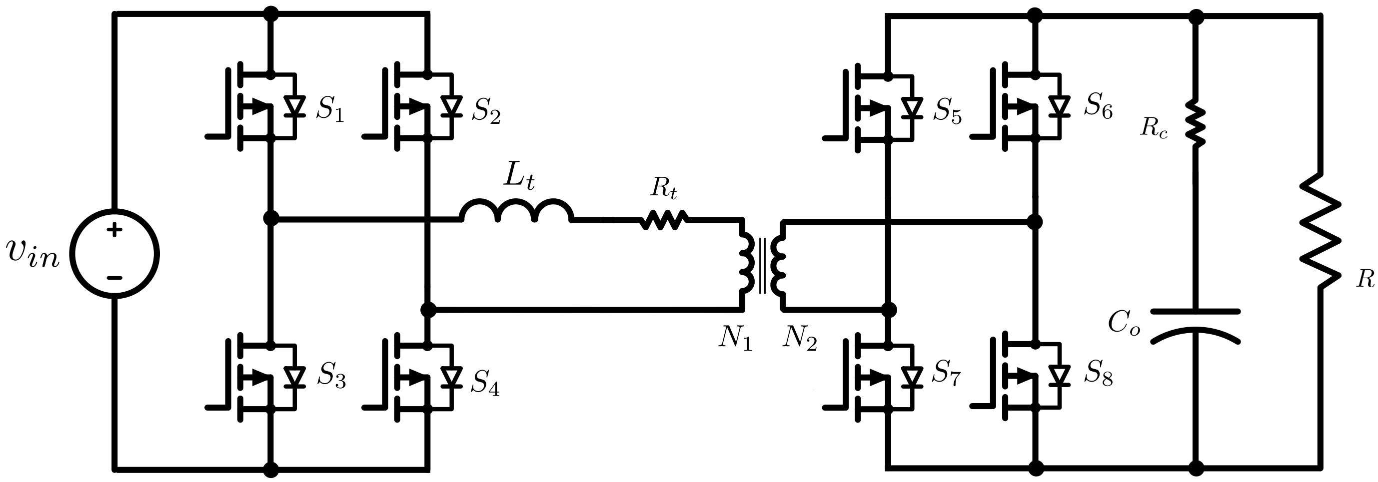

Fig. 1: Schematic of a dc-dc DAB converter

where $\Re$, $\Im$ represent the real and imaginary parts of the state. The final GAM model for the DAB can then be written as

$$\begin{aligned}\frac{di_{t\Re}(t)}{dt} &= -\frac{R_o}{L_t}i_{t\Re}(t) \\ &+ \omega i_{t\Im}(t) + \frac{2}{\pi}\frac{R}{(R+R_c)L_t}v_c(t)\sin(d\pi)\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{di_{t\Im}(t)}{dt} &= -\frac{R_o}{L_t}i_{t\Im}(t) - \omega i_{t\Re}(t) \\ &+ \frac{2}{\pi}\frac{R}{(R+R_c)L_t}v_c(t)\cos(d\pi) - \frac{2}{\pi}\frac{v_s}{L_t}\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{dv_c(t)}{dt} &= -\frac{4}{\pi}\frac{R}{(R+R_c)C}\sin(d\pi)i_{t\Re} \\ &- \frac{4}{\pi}\frac{R}{(R+R_c)C}\cos(d\pi)i_{t\Im} - \frac{v_c}{(R+R_c)C}\end{aligned} \tag{11}$$

B. Reduced Order Approximation

The GAM of DAB as shown in the system (9), (10) and (11), is rather complicated for control, as there are three state equations to deal with. However, the capacitor voltage remains relatively slow when compared to the leakage inductor's state. Therefore, the steady-state solution of $\langle i_t\rangle_1$ is obtained by solving (9), (10) and is given as

$$\langle i_t\rangle_1 = \frac{R_o - j\omega L_t}{R_o^2+(\omega L_t)^2}\left(\dot{q}_1\frac{N_2}{N_1}v_s - \dot{q}_2\frac{R}{R+R_c}\right) \tag{12}$$

The steady state solution (12) when replaced in (11) gives the final reduced order model given by

$$\begin{aligned}\frac{dx}{dt} &= \frac{8}{\pi^2}\frac{R}{(R+R_c)C}\frac{N_2 v_s}{N_1\mid Z\mid}\cos(d\pi-\gamma) \\ &+ -\frac{x}{(R+R_c)C} - \frac{8}{\pi^2}\frac{R^2}{(R+R_c)^2C}\frac{\cos(\gamma)}{\mid Z\mid}\end{aligned} \tag{13}$$

where $\mid Z\mid = \sqrt{R_o^2+(\omega L_t)^2}$, and $\gamma = \arctan(\omega L_t/R_o)$ and as the frequency of switching is in several kilo hertz, $\gamma \approx \pi/2$.Therefore the plant reduces to

$$\frac{dx}{dt} = -\frac{x}{(R+R_c)C} + \frac{8}{\pi^2}\frac{R}{(R+R_c)C}\frac{N_2V_{in}}{N_1\mid Z\mid}\sin(\phi) \tag{14}$$

which can be reduced to the model

$$\dot{x} = f(x) + gu \tag{15}$$

with,

$$\begin{cases} f(x) = -\frac{x}{(R+R_c)C} = -a_p x \\ g = \frac{8}{\pi^2}\frac{R}{(R+R_c)C}\frac{N_2V_{in}}{N_1|Z|} \\ u = \sin(\phi)\end{cases} \tag{16}$$

Now, with the reduced-order model, the MRAC control formulation can be done in the subsequent sections.

III. Methodology

In this section, the model (15) will be used to formulate a direct MRAC controller for output voltage tracking applications. The direct MRAC controller has three parts: the control law, the reference model, and the adaptation laws. The necessary condition for the stability of the closed-loop system is that the reference model be stable. As the DAB model in (15) is a first-order system, even the reference model needs to be a first-order system. The reference model is therefore

$$\dot{y}_m = -a_m y_m + b_m r(t) \tag{17}$$

where a_m and b_m are the reference model parameters, y_m, the state of the reference model, that is used for error generation and $r(t)$ is the reference value fed to the plant that the DAB must ultimately track. The control law is given by

$$u = a_r(t)r(t) + a_x(t)x(t) \tag{18}$$

where $a_r(t)$, $a_x(t)$ are estimates of the control parameters for compensating the unknowns of the system. It is also assumed that none of the DAB parameters are known. That is the value of C, R, R_c, R_t, v_{in}, $\mid Z\mid$, N_2, and N_1 are unknown, which makes $f(x)$, and g unknown quantities. However, the sign of g is positive as all passive components are known to have a positive real value and input voltage is also positive. The error $e = x(t) - y_m(t)$ of the model. The error dynamics are then given by

$$\dot{e}(t) = a_m e(t) + g\tilde{a}_x(t)x(t) + g\tilde{a}_r(t)r(t) \tag{19}$$

where $\tilde{a}_r(t) = a_r(t) - b_m/g$ and $\tilde{a}_x(t) = a_x(t) - (a_p - a_m)/g$. The model matching condition is satisfied when $\tilde{a}_r(t)$, $\tilde{a}_x(t)$ are both zero. Now a Lyapunov candidate can be chosen to formulate the adaptive control laws. One such candidate can be

$$V = \frac{e^2}{2} + \frac{|\, g \,|}{2\gamma}(\tilde{a}_x^2 + \tilde{a}_r^2) \tag{20}$$

where γ is a positive constant that serves as a tuning parameter to control the rate of adaptation. The proof of the Lyapunov stability is given below.
Proof:

$$\begin{aligned}
\dot{V} &= e(t)\dot{e}(t) + \frac{|\, g \,|}{\gamma}(\tilde{a}_x(t)\dot{\tilde{a}}_x(t) + \tilde{a}_r(t)\dot{\tilde{a}}_r(t)) \\
&= e(t)(a_m e(t) + g\tilde{a}_x(t)x(t) + g\tilde{a}_r(t)r(t)) \\
&+ \frac{|\, g \,|}{\gamma}(\tilde{a}_x(t)\dot{\tilde{a}}_x(t) + \tilde{a}_r(t)\dot{\tilde{a}}_r(t)) \\
&= a_m e^2(t) + \tilde{a}_x(t)\left(ge(t)x_p(t) + \frac{|\, g \,|}{\gamma}\dot{\tilde{a}}_x(t)\right) \\
&+ \tilde{a}_r(t)\left(ge(t)r(t) + \frac{|\, g \,|}{\gamma}\dot{\tilde{a}}_r(t)\right)
\end{aligned}$$

The last two terms can be canceled out to zero to get a negative semi-definite result for $\dot{V}$. Realizing that $g =|\, g \,|\, \text{sgn}(g)$, the second term is written as

$$e(t)\,|\, g \,|\, \text{sgn}(g)x(t) = -\frac{|\, g \,|}{\gamma}\dot{\tilde{a}}_x \tag{21}$$

and for the last term,

$$e(t)\,|\, g \,|\, \text{sgn}(g)r(t) = -\frac{|\, g \,|}{\gamma}\dot{\tilde{a}}_r \tag{22}$$

Therefore two adaptive laws are obtained

$$\dot{\tilde{a}}_x = -\gamma \text{sgn}(g)e(t)x(t) \tag{23}$$
$$\dot{\tilde{a}}_r = -\gamma \text{sgn}(g)e(t)r(t) \tag{24}$$

Now, by Barbalat's lemma, one can show that $e(t)$ converges asymptotically to zero. However, the convergence of $\tilde{a}_r(t)$ and $\tilde{a}_x(t)$ cannot be guaranteed unless Persistency of Excitation (PE) condition is not guaranteed. The ramifications of the parameter estimate not being bounded will be discussed in the next section, and a robust modification will make relaxations to the PE conditions to the existing adaptive control laws.

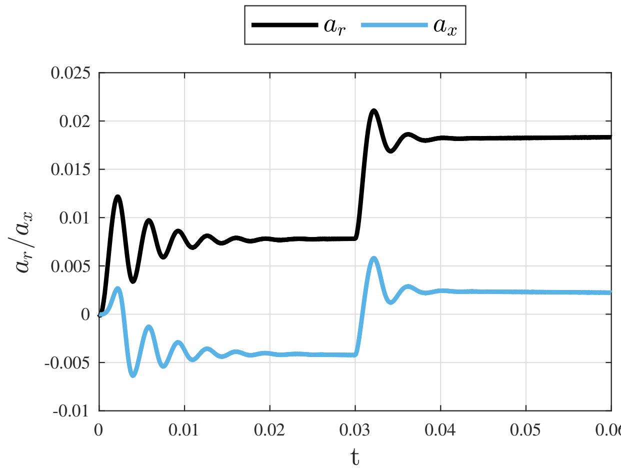

Fig. 2: Parameter updates $a_r(t)$, and $a_x(t)$ for the standard adaptive scheme

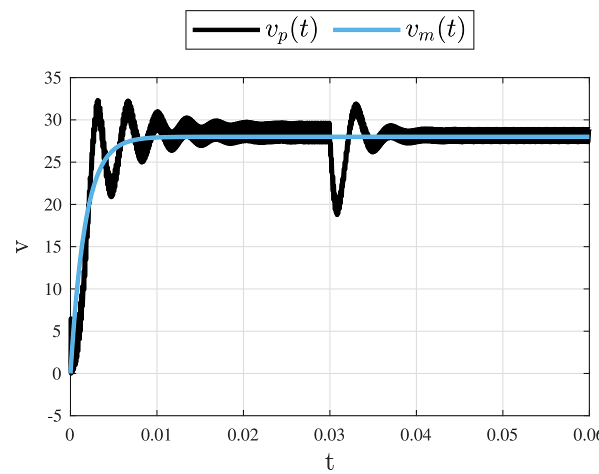

Fig. 3: Output voltage and reference model tracking characteristics for the standard adaptive scheme

IV. Results and Discussions

This section performs three tests to investigate the viability of using MRAC for DAB. The DAB is run at a set reference value in the first test, and its tracking ability is inspected. In the next test, the voltage reference to be tracked is changed, and the DAB tracking performance is shown again. In the third test, a robust adaptive control strategy will counter the ripple effects that serve as a bounded noise input to the MRAC.

All the simulations in this section are performed on Simulink® running a PLECS® file where a dc-dc DAB's model is created. The MRAC controller is constructed using Simulink® blockset, and the control input is given to a MATLAB® script block implementing the conventional phase-shift PWM scheme.

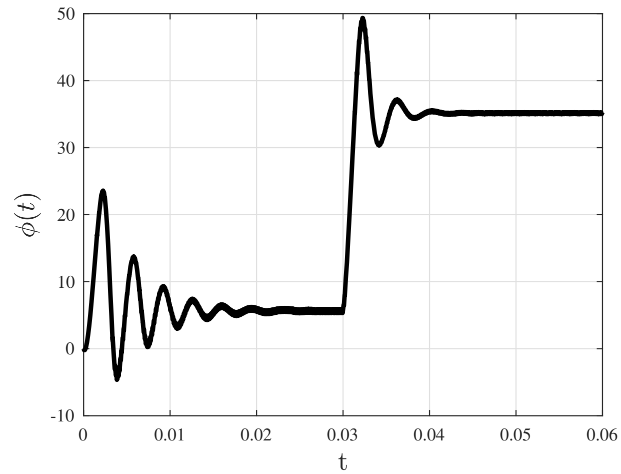

Fig. 4: Phase angle control $\phi(t)$ for standard adaptive control scheme

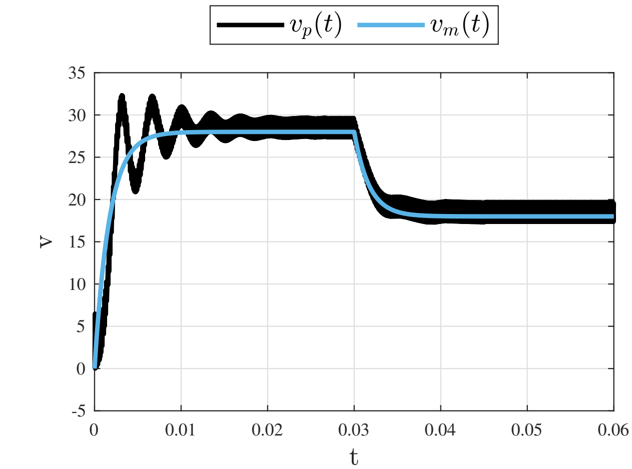

Fig. 5: DAB converter's response to voltage reference change for standard adaptive control

A. Load Step Change

The DAB is run with the MRAC for the parameters shown in Table I. The load is changed suddenly from $0.5\,\mathrm{kW}$ to $1\,\mathrm{kW}$. A γ value of 0.04 is used for adaptation parameter and the reference model parameters a_m, and b_m are both set to 500 each. The results for the tracking operation is shown in Figures 2,3 , and 4 describing the parametric updates, the plant state, and phase values respectively. The DAB is observed to be tracking the reference voltage satisfactorily despite load voltage perturbations.

TABLE I: Parameters for Load Perturbation Study

Parameter	Value
f_{sw}	$10\,\mathrm{kHz}$
v_{in}	$270\,\mathrm{V}$
V_o	$28\,\mathrm{V}$
C_o	$3\,\mathrm{mF}$
R_c	$4\,\mathrm{m\Omega}$
TF N_2/N_1	$5/1$
R_t	$4\,\mathrm{m\Omega}$
L_{lk}	$5\,\mathrm{mH}$
P_1	$0.5\,\mathrm{kW}$
P_2	$1\,\mathrm{kW}$

B. Voltage Reference Tracking

In this test, the reference voltage is changed from $28\,\mathrm{V}$ to $18\,\mathrm{V}$ as a step input, and the DAB is made to track the slowly changing reference voltage of the reference model. A γ value of 0.04 is used for adaptation parameter and the reference model parameters a_m, and b_m are both set to 500 each. Figure 5 shows the satisfactory response of the DAB to the changing voltage reference.

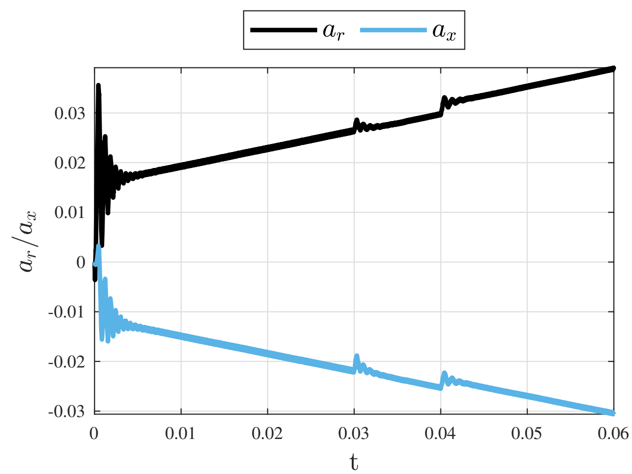

Fig. 6: Output voltage induced parametric drift for higher values of γ

C. Improving the speed of response

If the speed of response of the DAB is to be improved, two parameters are to be altered, namely, the time constant of the reference model described by $\tau = 1/a_m$ and the adaptation parameter γ. However, increasing the adaptation parameter γ does not ensure a fast response as the Lyapunov proof does not ensure the stability of the parameter estimates. The consequence of using high values of γ is that the output voltage ripple in the DAB capacitor acts as a bounded disturbance and causes the well-known parameter drift. For lower values of γ, the value is not as pronounced and causes negligible parameter drift. The parameter drift when γ is increased from 0.04 to 1.5 and reference model parameters (a_m, and b_m) changed from 500 to 1000 each is shown in Figure 6.

The ripple induces noise in the control input as shown in Figure 7. The noise also gets progressively higher due to the drifting parameters, signifying instability in closed-loop control for $t \to \infty$.

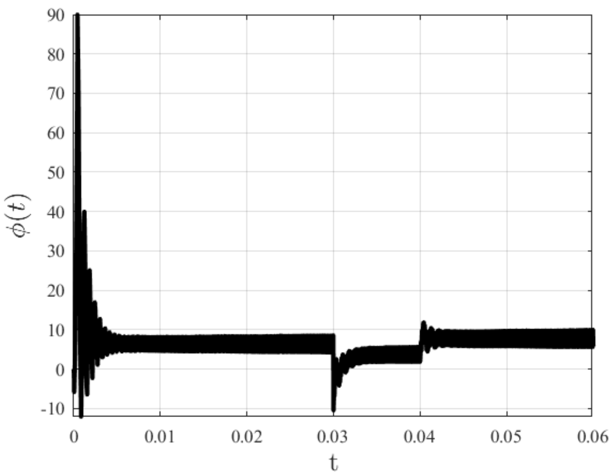

Fig. 7: Noisy control signal due to parametric drift

In order to prevent this unwanted behavior, robust adaptive control techniques can be used. The techniques relax the persistency of excitation requirements to guarantee parametric convergence. As the ripple is available via simulations, a trial and error based method like the dead zone modification [7]–[9] technique can be used. For this method, the adaptation laws are modified as,

$$\frac{d}{dt}a_r(t) = \begin{cases} -\text{sgn}(g)\gamma e_m(t)r(t) & \|e_m\| > e_{bound} \\ 0 & \|e_m\| \leq e_{bound} \end{cases} \quad (25)$$

$$\frac{d}{dt}a_x(t) = \begin{cases} -\text{sgn}(g)\gamma e_m(t)x(t) & \|e_m\| > e_{bound} \\ 0 & \|e_m\| \leq e_{bound} \end{cases} \quad (26)$$

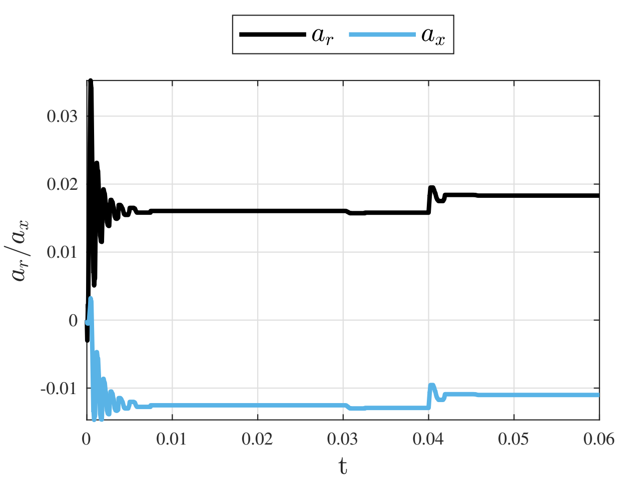

Fig. 8: Deadzone modified MRAC parametric estimates

The bound on error e_{bound} is set to be just above the ripple in order to ensure that the ripple does not contribute to the adaptation. Therefore, when the error is within the acceptable limits of the ripple, the adaptation process is frozen. The result of applying the deadzone modified adaptation laws for $\gamma = 1.5$, and $a_m = b_m = 1000$ is shown in Figures 8,9 , and 10 describing the parametric updates, the plant state, and phase values respectively. The voltage reference is changed from 28 V to 18 V at 0.03 s and the load is changed from 0.5 kW to 1 kW at 0.04 s in this test.

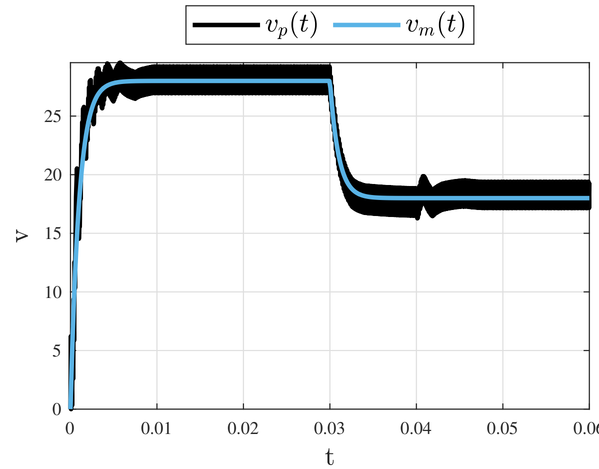

Fig. 9: Deadzone modified MRAC voltage performance

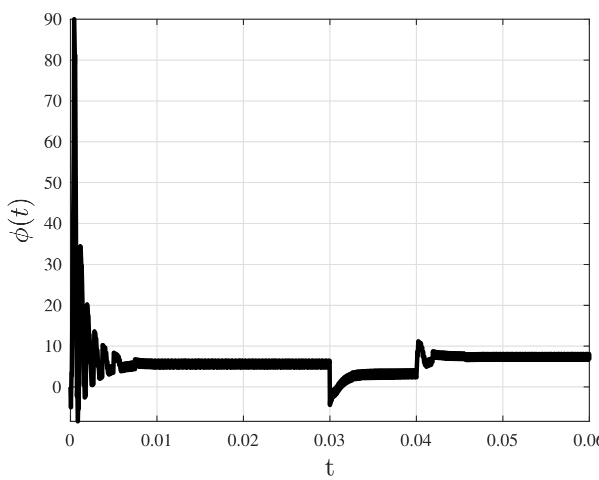

Fig. 10: Deadzone modified MRAC phase angle shift

Therefore, higher γ and a_m values can speed up the response of the closed-loop system, provided the noise effects are curbed via robust control modifications to already derived adaptive laws. The plant is also found to track the reference model more tightly without any overshoots due to higher γ values, which were possible due to the assurance of the robust scheme.

V. Conclusions

This article deals with an MRAC strategy for voltage tracking applications in a DAB converter. MRAC is an attractive feature as it does not need any plant quantitative values to stabilize the closed-loop system. However, qualitative aspects about the plant and the reference model, such as passivity, are to be satisfied for the stability of MRAC. It is found that the ripple in the capacitor as a consequence of the loading and switching effects manifests as unmodeled dynamics to the simple first-order plant. This leads to the well-known parametric drift problem in a closed-loop system where the PE condition is not satisfied. In order to counteract the undesirable effects of noise, it is found that a robust adaptive controller is an essential feature to ensure the stability of the control laws in the presence of bounded noise, which is often present in power electronic applications.

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Appendix A
MATLAB® code

The project files are available in the GitLab® Website. Please note that the project uses the PLECS™ blockset extension on Simulink®. Kindly ensure that the PLECS® package is available. The code successfully was run on MATLAB® version R2021a

```
close all
 clear all
  clc


  Vin = 270; %input voltage
  Vout = 28;
  Vout1 = 28;%target output voltage 1
  Vout2 = 18;%taget output voltage 2
  P1 = 0.5e+3; % power demand 1
  P2 = 1e+3; %power demand 2
  n_pri = 1; %primary turns
  n_sec = 5; %secondary turns
  %Llk = 50e-6*(n_pri/n_sec)^2; %leakage inductance
  Llk = 5e-6;
  Co = 3e-3; %output capacitance

  f = 10e+3; %switching frequency
  Rload1 = Vout1^2/P1; %P1; Vout1 load resistance required
  Rload2 = Vout1^2/(P2); %P2; Vout2 load resistance required
  %Rload = 10;
  Rc = 4e-3; %Capacitance ESR
  Rt = 4e-3; %Leakage inductance ESR

  gamma = 1.5; %controller gain parameter; Required value is somewhere around 0.04
 % Higher values of gamma lead to faster parametric convergence in general

  %ref model
  am = 1000; %reference model am
  bm = 1000; %reference model bm

  ar_0 = 0; %ar parameter estimate's inital state
  ax_0 = 0; %ax parameter estimate's initial state




open_loop = 1;
closed_loop = 0;

%mode = open_loop;
mode = closed_loop;

phi_init = 15;

activate_controller_time = 0;

Ro = (Rload1*Rt + Rc*Rt + Rload1*Rc)/(Rload1 + Rc);
Xl = 2*pi*f*Llk;
atan(Xl/Ro) %this value must be around pi/2 otherwise affine model f(x)+ g(x)u
```

```
%will not be as simple as the one given in the main paper
%sim('DAB.slx');

voltage_step_time = 0.03;
load_step_time = 0.04;
TSIM = 0.06;
min_step_size = 5e-8;

deadzoneenable = 0;
deadband = 1.5; %set the allowable error that arrests adaptation

sim('DAB_MRAC_new.slx');

run('plot_sim_data.m')
```