Lyapunov-based control for switched power converters

Kartikeya Veeramraju

***Abstract*— Switch Mode Power Supplies (SMPS) are a mature technology and deal with various power control problems ranging from systems connected to power grids to more sensitive applications such as medical and aerospace engineering. The control problems in the SMPS field often deal with regulation around a set point and stabilization of the state variables around the set point. However, as the systems are generally complicated to deal with, due to the multiple state variables and varying system topologies due to switching actions, the design of controllers for these systems is generally done by small-signal averaging and linearization of the nonlinear plant. In this work, instead of using a linear controller for the regulation problem, a nonlinear control law is derived using the Lyapunov stability theory. The concept of energy-in-the-increment is used for designing the Lyapunov functions. A theorem providing proof of global stability of controllers is presented, and extensions to the small signal-averaged model are provided. Finally, an adaptive controller for handling uncertain nominal state values is presented. The control laws designed by Lyapunov theory work effectively for both small signal and large signal perturbations.**

I. INTRODUCTION

Many power electronic control techniques involve the control of power converters whose dynamic models are linearized at an operating point and are regulated at a said value. This form of control makes the system linear and easy to come up with conventional transfer function based or state feedback based control laws that can be implemented in both continuous and discrete systems. However, the process of linearization is disadvantageous as the linear model that a designer comes up with, is only valid at the singular operating point about which the system is linearized and does not hold for other operating points due to the modification of the system's parameters. Also, the system is only capable of regulating the converter's output along the single operating point for small signal perturbations and cannot necessarily regulate the system against large signal perturbations [2].

In the previous available literature, large-signal control is classified into two schemes [1] namely,

1) Sliding mode schemes [3]–[5]
2) State space averaging scheme [6]–[9]

II. BACKGROUND

In this section the concept of energy in the increment will be used to prove the stability of dc-dc converters and will be shown to give a Lyapunov function that can be associated to the systems behavior.

A. Energy in the increment

If a switching converter has ideal dc voltage sources, switches, passive resistors and reactive elements and a nominal voltage $\tilde{v}(t)$ and current value $\tilde{i}(t)$ are given for a particular duty value, then the deviations from the particular set of nominal state variables is given by,

$$\begin{aligned} \delta i(t) &= i(t) - \tilde{i}(t) \\ \delta v(t) &= v(t) - \tilde{v}(t) \end{aligned} \tag{1}$$

By using the statement of Tellegen's Theorem [10], which states that *the sum of powers in all components of an electrical circuit at any given instant of time equates to zero.*

$$\sum_{j=1}^{b} v_j(t) \cdot i_j(t) = 0 \tag{2}$$

As the statement of Tellegen's theorem is extremely general, it can also be applied for each mode of a power converter and therefore can be generally written as,

$$\begin{aligned} \sum_{dcsources} \delta v \delta i + \sum_{switches} \delta v \delta i + \sum_{res} \delta v \delta i \\ + \sum_{ind} \delta v \delta i + \sum_{cap} \delta v \delta i = 0 \end{aligned} \tag{3}$$

As ideal dc voltage or current sources have one of the δv or δi terms set to zero, ideal switches on the other hand are only subjected to either zero voltage or zero current and thus their individual sum tends to zero. The inductance and capacitance's summation terms equate to the rate of change of stored energy in the increment. Due to incremental passivity of resistors, and therefore describes the dissipated power along the system's trajectory. The time rate of change of stored energy in the increment is given by,

$$\frac{d}{dt}\mathbf{V}(\delta x) = \sum_{ind} \delta v \delta i + \sum_{cap} \delta v \delta i \tag{4}$$

By Tellegen's theorem,

$$\begin{aligned} \sum_{ind} \delta v \delta i + \sum_{cap} \delta v \delta i + \sum_{res} \delta v \delta i = 0 \\ \frac{d}{dt}\mathbf{V}(\delta x) + \sum_{res} \delta v \delta i = 0 \\ \frac{d}{dt}\mathbf{V}(\delta x) = -\sum_{res} \delta v \delta i \leq 0 \end{aligned} \tag{5}$$

Equation 5 holds due to the disspiative nature of the resistors. Therefore the energy in the increment is a Lyapunov function and is stable in the sense of Lyapunov due to a negative semi-definite result. As the circuit does not have any resistors in the inductive and capacitive due to ideal assumptions. However, practical circuits have ESR and ESL in the network elements paving a way to assert asymptotic stability [11].

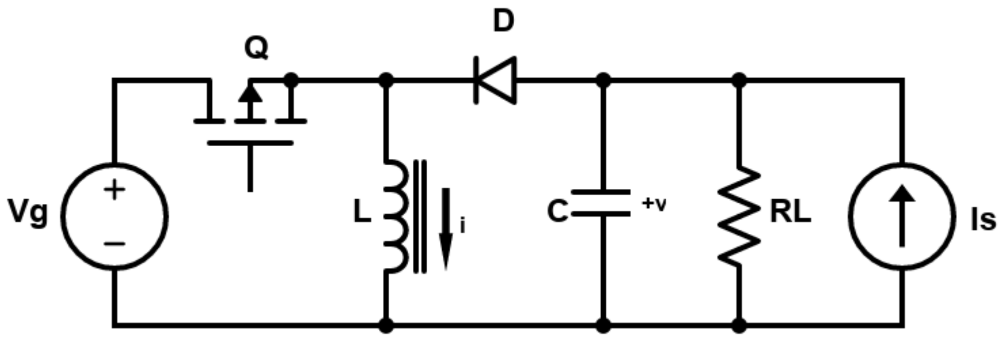

Fig. 1: Buck-boost converter circuit diagram

This leads to the therorem showing stability in the large, which states,

Theorem 1: If a switching converter is constructed from ideal switches, ideal dc sources, incrementally passive resistors and reactive elements that are strictly relatively passive and that its averaged model has an equilibrium point, then the equilibrium point is stable in the large [1]

B. System model description

Before proceeding with the converter's control, the converter's model is required and is given in the state space averaged form as,

$$\dot{x} = Ax + (Bx + b)d \tag{6}$$

where $x = \begin{bmatrix} i - i_n & v - v_n \end{bmatrix}$ and d is the duty ratio from its nomianl value ($d = d_t - d_n$), where d_t is the total duty value and d_n is the nominal duty value. The value d is therefore the perturbation duty value that actively controls the satte variables at their nominal set points.

$$\begin{aligned} A &= \begin{bmatrix} 0 & (1-d_n)/L \\ -(1-d_n)/C & 0 \end{bmatrix} \\ B &= \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \\ b &= \begin{bmatrix} (V_s - v_n)/L \\ i_n/C \end{bmatrix} \\ Q &= \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix} \end{aligned} \tag{7}$$

III. Methodology

In this section, a Lyapunov based controller is designed for a buck boost converter based on the background concepts covered in the previous sections.

A. Design of Lyapunov based controllers

For coming up with a control law for a dc-dc converter, the concept of energy in the increment described earlier will be very useful. From Equation 4 one can write,

$$\begin{aligned} \frac{d}{dt}\mathbf{V}(\delta x) &= \sum_{ind} \delta v \delta i + \sum_{cap} \delta v \delta i \\ \mathbf{V}(\delta x) &= \int \left[\sum_{ind} \delta v \delta i + \sum_{cap} \delta v \delta i \right] \\ &= \sum_{ind} \frac{1}{2} \delta i_k^T L_k \delta i_k + \sum_{cap} \frac{1}{2} \delta v_k^T C_k \delta v_k \end{aligned} \tag{8}$$

By Equation 8, one can write for a single inductor, and single capacitor as in the case of a conventional buck-boost converter that,

$$\begin{aligned} V &= \frac{1}{2} L (i - i_n)^2 + \frac{1}{2} C (v - v_n)^2 \\ &= \frac{1}{2} x^T Q x \end{aligned} \tag{9}$$

Finding the derivative of the Lyapunov function in Equation 9 to the system trajectory in Equation 6,

$$\begin{aligned} \frac{d}{dt}\mathbf{V}(x) &= \frac{1}{2} x^T (QA + A^T Q) x \\ &\quad + \frac{1}{2} \left(x^T (QB + B^T Q) x + 2 b^T Q x \right) d \end{aligned} \tag{10}$$

Since $QA + A^T Q = 0$ and $QB + B^T Q = 0$ would finally lead to the simplified derivative of the Lyapunov along the system's trajectory given by,

$$\frac{d}{dt}\mathbf{V}(x) = (b^T Q x) d \tag{11}$$

By investigating Equation 11 one can simply write a few control laws that can stabilize the states at required set points. For example, a very simple control law for making $\dot{\mathbf{V}}(x)$ negative definite would be to select $d = -\alpha b^T Q x$, for any real and positive α. One can also apply a saturation constraint over the existing control to prevent unnecessary over driving of the actuation signal by,

$$d = f(x) = \begin{cases} -\alpha y, & -d_n \leq \alpha y \leq 1 - d_n \\ -d_n, & -\alpha y < -d_n \\ 1 - d_n, & -\alpha y > 1 - d_n \end{cases} \tag{12}$$

where y can be written as,

$$y = (V_s - v_n)(i - i_n) + i_n(v - v_n) \\ = (V_s - v)(i - i_n) + i(v - v_n) \tag{13}$$

The block diagram representation to be while implementing in PLECS®/Simulink® blocksets is shown in Fig. 2

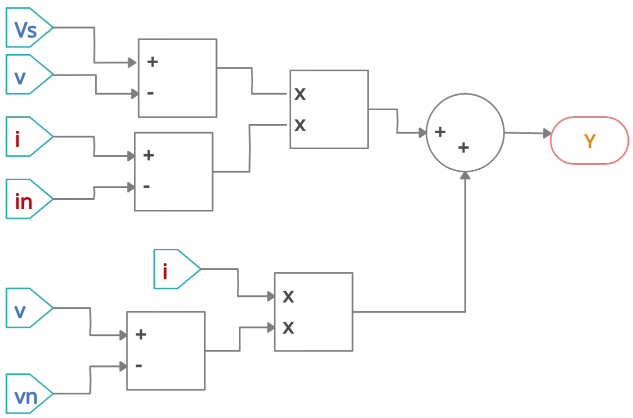

Fig. 2: Lyapunov based controller block diagram

Finally applying Equation 13 in Equation 12.

$$\frac{d}{dt} = yd = \begin{cases} -\alpha y^2, & -d_n \leq \alpha y \leq 1 - d_n \\ -d_n y, & -\alpha y < -d_n \\ (1 - d_n)y, & -\alpha y > 1 - d_n \end{cases} \tag{14}$$

When the control enters the saturation region, i.e. $-\alpha y < -d_n$ or $-\alpha y > 1 - d_n$ the $\dot{\mathbf{V}}(x)$ computes to $-d_n^2/\alpha$ and $-(1 - d_n)^2/\alpha$ respectively, which are negative definite and so the the actuation effort $-\alpha y$ enters the unsaturated region. And when it enters the unsaturated region, the system becomes negative definite except at $y = 0$. Also $y \equiv 0$ would mean that $d \equiv 0$ and that would make $d_t = d_n$, indicating that the converter is regulating the system at the nominal duty value and therefore is also regulating the the voltage at the required set point. As the state variables in Equation 6 are defined as deviations of v and i from the nominal, i.e. $x_1 = i - i_n$ and $x_2 = v - v_n$, from Equation 13, for $y \equiv 0$ would require $x_1 \equiv 0$ and also $x_2 \equiv 0$, thus forcing v back to the nominal v_n. So, by *LeSalle's Invariant Set Theorem*, as y and x form an invariant set, it can be concluded that the system is *asymptotically stable*.

B. Feedforward duty computation

When implementing the controller practically, the total duty value d_t is comprised of the controller controlled duty d and the nominal duty d_n. Therefore making the total duty $d_t = d_n + d$. Whenever there are changes in the input voltage, the nominal duty value has to change for readjusting the duty quickly and therefore gives rise to the necessity of employing the additional feedforward term that computes d_n, given for a buck-boost converter by,

$$\frac{v_n}{v_g} = \frac{d_n}{1 - d_n} \tag{15}$$

The value d_n is fixed for a given ratio of input to output voltage and must be added to the Lyapunov actutation duty d to finally construct the total control duty value d_t.

C. Effects on stability in non-ideal circuit conditions

If non ideal circuit parameters are considered for the inductors and capacitors by introducing ESR's, a stronger case for stability can be made as they would add more terms in the $\sum_{res} \delta v \delta i$ term of Equation 3. Additionally, considering resistances in the various parts of the converter would cause a change in the A,B,b matrices (as the previous matrices in Equation 7 were for a lossless assumption). Additionally, $QA + A^T Q \neq 0$ and $QB + B^T Q \neq 0$ would hold due to the lossy nature and ultimately cause more and more terms of $-x^T Rx$ form to start appearing in the derivative of the Lyapunov along the system's nominal trajectory in Equation 14.

IV. RESULTS AND DISCUSSIONS

In this section the control of a buck-boost converter modeled in PLECS® environment is used to run the Lyapunov based controller developed in the previous section using the concept of energy-in-the-increment. The system is run under various large-signal perturbations and the results are discussed. For the rest of the discussions the parameters of the buck-boost converter are as given in Table I.

TABLE I: Parameters of buck-boost converter

Parameter	Value
C	5.4 µF
L	0.18 mH
R	∞
d_n	3/8
V_s	15 V
I_s	2 A
v_n	-9 V
i_n	3.2 A

A. Unperturbed system dynamics

The converter is first switched with an initial voltage and current values given to the states. The voltage and current states are initialized at 1 V and 1 A and a constant α value of 0.001 is used. The phase plane plots for the system response is shown in Fig. 3 and the time domain response is plotted in Fig. 4. The Lyapunov controller brings the voltage to the desired value of ~ -9 V. The phase plane plot is acquired at a switching frequency of 1 MHz to remove the large ripple effects present in inductor current and capacitor voltage waveforms and get a relatively smooth graph. However, the shape of the plot was found to be almost the same in the cases where the converter was switched at 50 kHz or 1 MHz. The Lyapunov function plot which is computed by Eq.9 in Fig. 4 shows the decay of the Lyapunov function showing steadily dissipating system. Therefore the system is also shown to be asymptotically stable. The controlled duty d value is shown to be perturbing along the nominal d_n causing d_t to take the profile as shown in Fig. 4's last row.

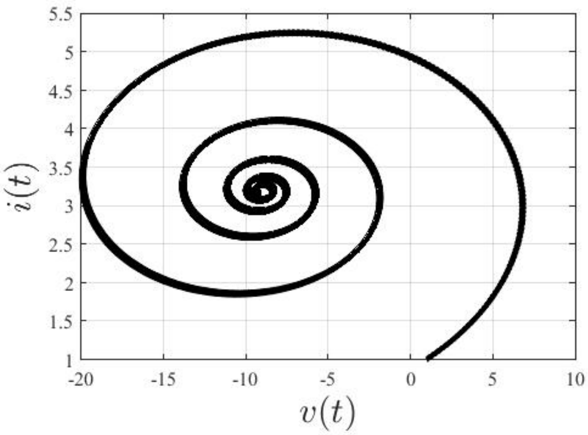

Fig. 3: Phase plane response of buck-boost converter run at 1 MHz upon startup in the absence on any load current or source voltage perturbations

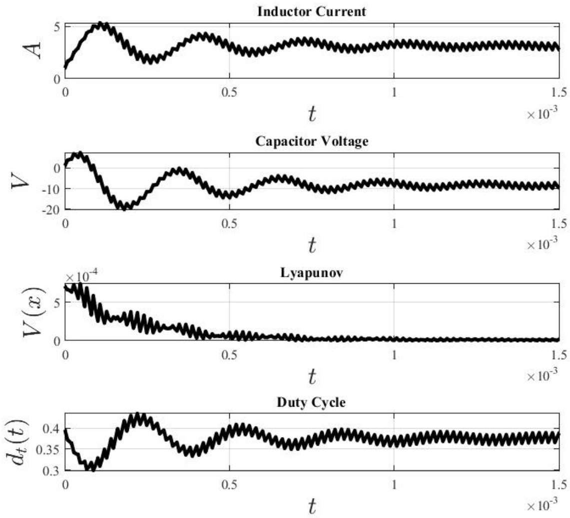

Fig. 4: Time domain response of buck-boost converter run at 50 kHz upon startup in the absence on any load current or source voltage perturbations

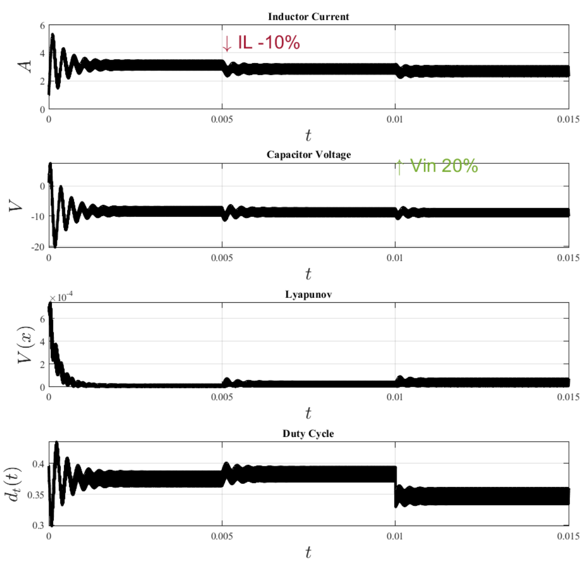

Fig. 5: Time domain response of buck-boost converter run at 50 kHz on a voltage rise of 20% and a load current fall of 10% as depicted by the event markers

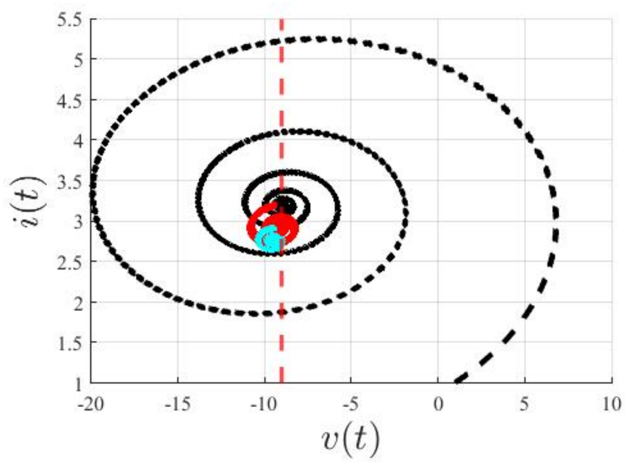

Fig. 6: Phase plane response of buck-boost converter run at 1 MHz on a voltage rise of 20% and a load current fall of 10% initialized at $X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}$

B. Perturbed system dynamics

In the next iteration, the converter with the controller is run under load current and source voltage perturbations as shown in Fig. 5 and the resulting phase plane trajectory is shown in Fig. 6. The dotted red line in the phase plane trajectory shows the nominal voltage given as the setpoint, which is in this case $-9\,\mathrm{V}$. The phase trajectory is initialized at $X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and the controller as a part of its start up sequence, brings the voltage to the desired output voltage value of $-9\,\mathrm{V}$. The three events are color coded and the phase plane trajectory shows the paths of the state variables. The controller stabilizes the operating point of the converter at the set point of $-9\,\mathrm{V}$ and keeps it on the reference line (dotted red line in Figure 6).

V. CONCLUSIONS

This project discusses a Lyapunov based controller that is suitable for controlling dc-dc converters without being restrained by the limitations of small signal approximations. To this effect, the concept of energy-in-the-increment is shown to provide a Lyapunov candidate function that proves the stability of the system at large. Then a system controller is designed by evaluating the derivative of the Lyapunov along the system's trajectory. It is also shown that adding a saturation feature to the controller, proved the asymptotic stability of the closed loop system using LeSalle's invariant set theorem. A case is made for higher stability of the closed loop system is made when nonideal system with parasitic resistances is considered, which would make the system more dissipative and hence more stable. Finally, a simulation model for a buck-boost converter is built on PLECS® and the converter is shown to be stable during startup transients and during large signal load current and source voltage perturbations. The stability of the system is discussed in both phase plane and time domain perspectives and the controller is shown to be successfully regulating the converter's output voltage at the desired set point value with satisfactory performance.

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