INTERVAL ABSTRACT DOMAIN

- $I = \{[a, b] \mid a, b \in \mathbb{R} \cup \{-\infty, \infty\}\} \cup \{\perp\}$
  - $D = V \to I$
  - Also called Box abstract domain.
- $[a_1, b_1] \sqsubseteq [a_2, b_2] \Leftrightarrow a_2 \leq a_1 \wedge b_1 \leq b_2$, $\forall d \in I \,.\, \perp \sqsubseteq d$
  - Is $(I, \sqsubseteq)$ a lattice?
  - Is $(I, \sqsubseteq)$ a complete lattice?
  - Maximal element?
- $[a_1, b_1] \sqcup [a_2, b_2] = ???$

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- $(D, \sqsubseteq)$: $\forall d_1, d_2 \in D \,.\, d_1 \sqsubseteq d_2 \Leftrightarrow \forall v \in V \,.\, d_1(v) \sqsubseteq d_2(v)$

INTERVAL ABSTRACT DOMAIN

ABSTRACTION AND CONCRETIZATION FUNCTION

- $\alpha : \mathbb{P}(State) \to D, \gamma : D \to \mathbb{P}(State)$
- $\alpha(c) = d$
  - $d(v) = [min\{\sigma(v) \,|\, \sigma \in c\}, max\{\sigma(v) \,|\, \sigma \in c\}]$
- $\gamma(d) = \{\sigma \mid \forall v \in V \,.\, d(v) = [a, b] \Rightarrow a \leq \sigma(v) \leq b\}$
- Is $(\mathbb{P}(State), \subseteq) \underset{\gamma}{\overset{\alpha}{\rightleftarrows}} (D, \sqsubseteq)$ a Galois Connection?
  - Is it an Onto Galois Connection?

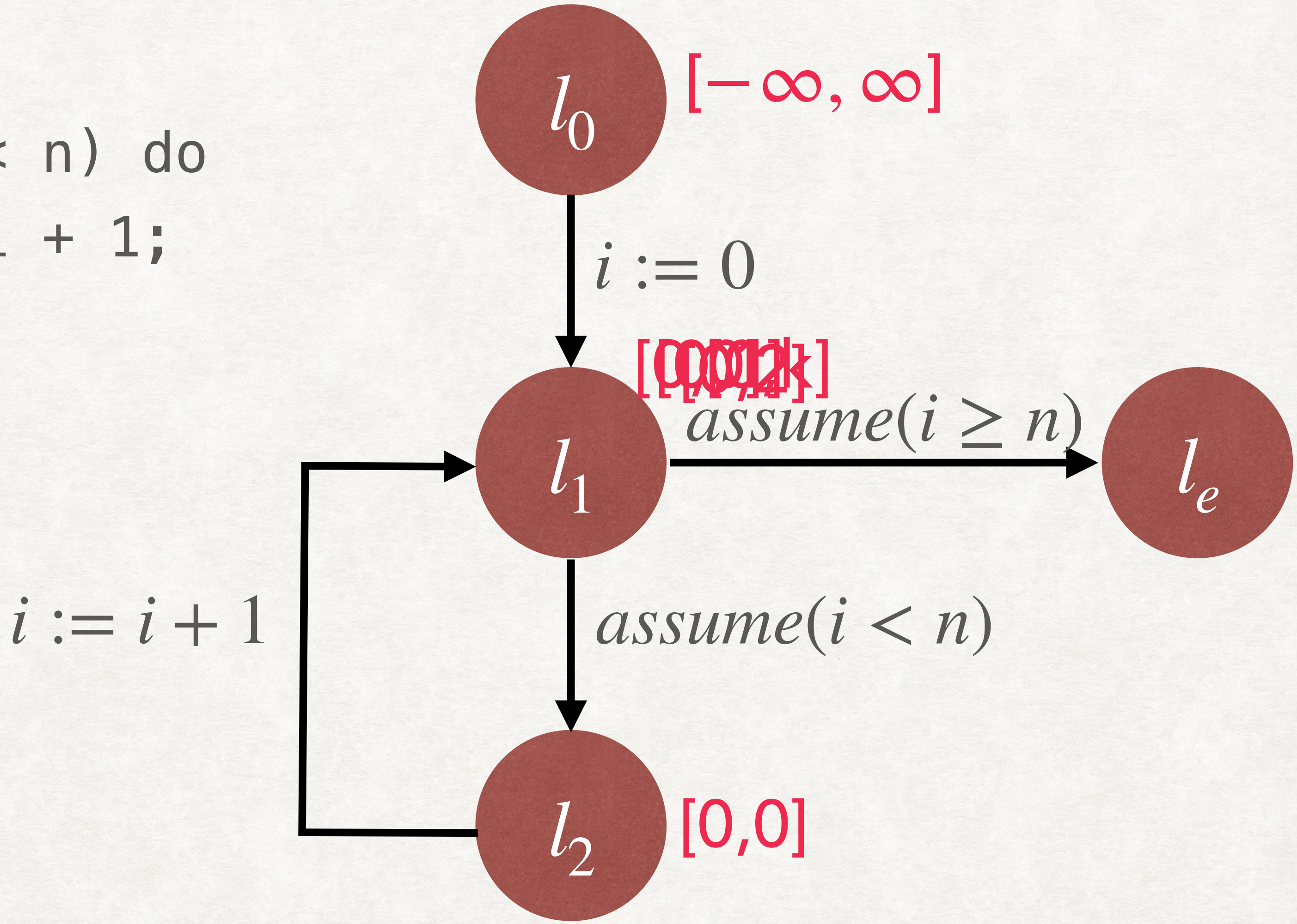
INTERVAL ABSTRACT DOMAIN

ABSTRACT TRANSFER FUNCTION

- Consider $c : x := x + y$
  - We can use interval arithmetic for $\hat{f}_c$
- Assuming $d(x) = [l_x, u_x], d(y) = [l_y, u_y]$
  - $\hat{f}_c(d) = d[x \mapsto [l_x + l_y, u_x + u_y]]$
- Is $\hat{f}_c$ monotonic?
  - Is it distributive?

USING INTERVAL DOMAIN

```
i := 0;
while(i < n) do
    i := i + 1;
```



Interval Abstract Domain does not satisfy ACC, hence Kildall's Algorithm may not terminate

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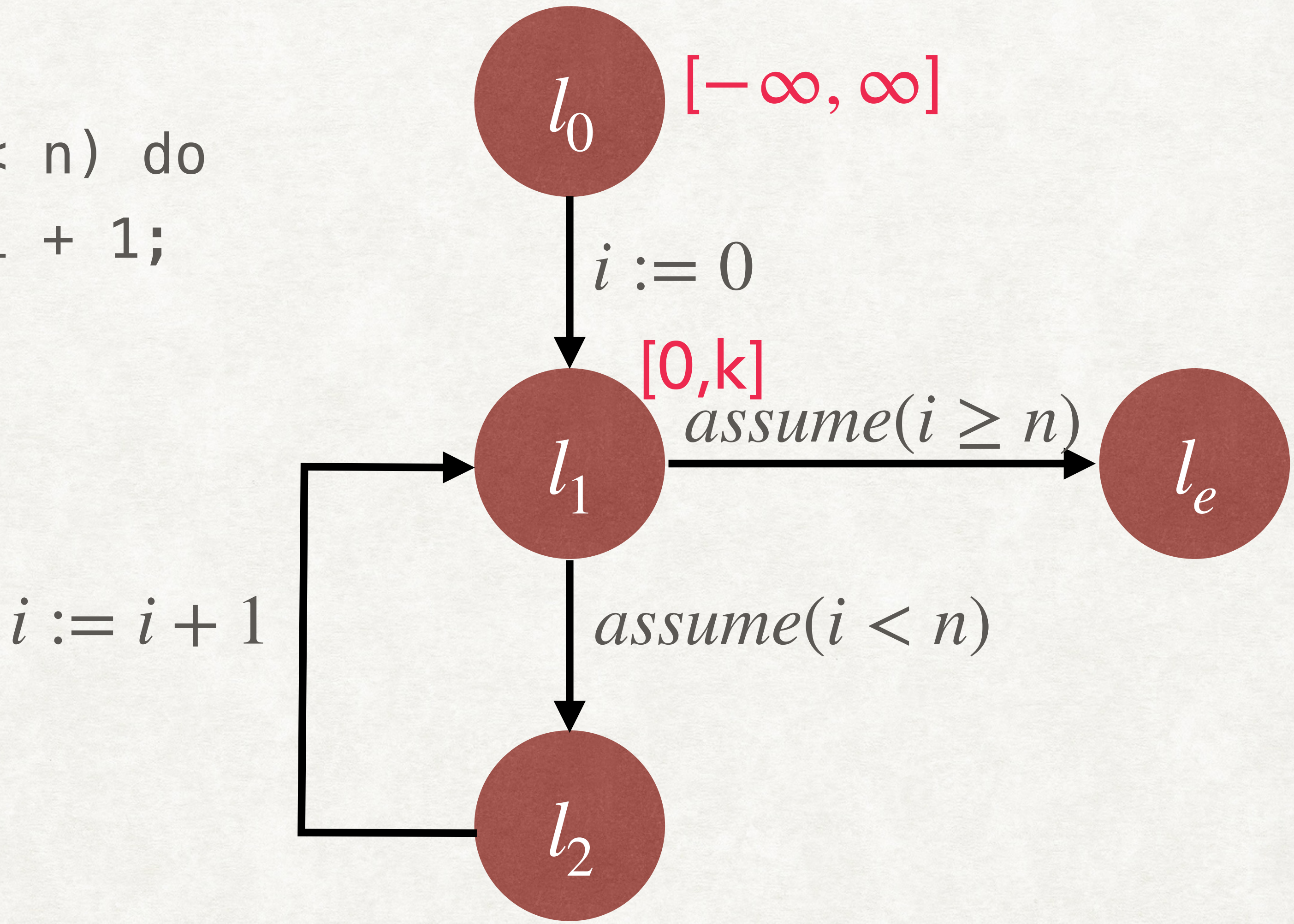

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WIDENING

- A widening function $\nabla : D \times D \to D$ on a poset $(D, \leq)$ satisfies the following properties:
  - $\forall x, y \in D \,.\, x \sqcup y \leq x \nabla y$
  - For an increasing chain $x_0, x_1, \ldots$, the increasing chain $y_0, y_1, \ldots$ where $y_0 = x_0$ and $y_n = y_{n-1} \nabla x_n$ eventually stabilizes.

WIDENING FOR THE INTERVAL DOMAIN

- We can define the widening operator for interval domain as follows:
  - $[a, b] \triangledown \perp = [a, b]$
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- Examples
  - $[1,2] \triangledown [0,2] = ???$

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KILDALL'S ALGORITHM WITH WIDENING

```
AbstractForwardPropagate(Γ_c,P)
  S := {l_0};
  μ̂_K(l_0) := α(P);
  μ̂_K(l) := ⊥, for l ∈ L\{l_0};
  while S ≠ ∅ do{
     l := Choose S;
     S := S \ {l};
     foreach (l,c,l') ∈ T do{
        F := f̂_c(μ̂_K(l));
        if ¬(F ≤ μ̂_K(l')) then{
           μ̂_K(l') := μ̂_K(l') ▽ F;
           S := S ∪ {l'};
        }
     }
  }
```

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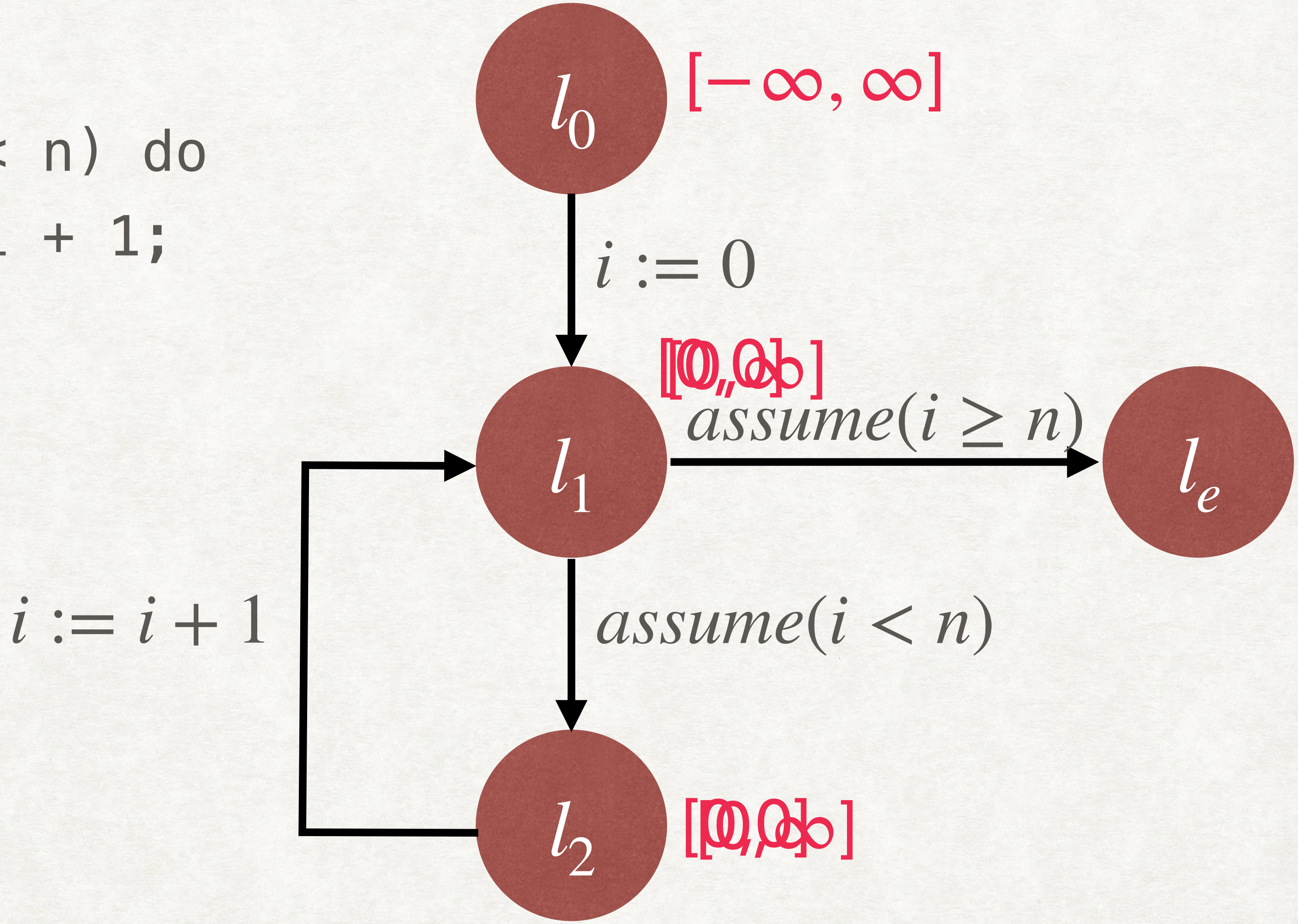

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