MODEL CHECKING

- Exhaustive exploration of the state-space of a program.
  - If an error state is not reached, then model checking outputs safe.
  - If an error state is reached, then the path to the error state can be reconstructed, resulting in a counterexample.
- Model Checking for sequential programs comes in many variants:
  - Concrete Model Checking
  - Symbolic Model Checking
  - Bounded Model Checking
  - Abstract Model Checking

CONCRETE MODEL CHECKING

```
ConcreteModelChecking(Γ_c,P)
  worklist := {(l_0,σ)|σ ∈ P};
  reach := ∅;
  while worklist ≠ ∅ do{
    Choose (l,σ) ∈ worklist;
    worklist := worklist \ {(l,σ)};
    if ((l,σ) ∉ reach) then
    {
      reach := reach ∪ {(l,σ)};
      foreach ((l,c,l′) ∈ T)
        worklist := worklist ∪ {(l′,σ′)|σ′ ∈ sp({σ},c)};
    }
  }
  if ((l_err,_) ∈ reach) then
    return UNSAFE
  else
    return SAFE
```

CONCRETE MODEL CHECKING

WITH COUNTEREXAMPLE GENERATION

```
ConcreteModelChecking(Γ_c,P)
  worklist := {(l_0,σ)|σ ∈ P}; parents := λx.NR;
  reach := ∅;
  while worklist ≠ ∅ do{
    Choose (l,σ) ∈ worklist;
    worklist := worklist \ {(l,σ)};
    if ((l,σ) ∉ reach) then
    {
      reach := reach ∪ {(l,σ)};
      foreach ((l,c,l′) ∈ T ∧ (l′,σ′) ∈ sp({σ},c)){
        worklist := worklist ∪ {(l′,σ′)};
        parents((l′,σ′)) := (l,σ);
      }
    }
  }
  if ((l_err,_) ∈ reach) then
    return UNSAFE
  else
    return SAFE
```

SYMBOLIC MODEL CHECKING

```
SymbolicModelChecking(Γ_c,P)
  worklist := {(l_0,P)};
  reach(l_0) := P;
  foreach (l ∈ L\{l_0}) reach(l) := false;
  while worklist ≠ ∅ do{
    Choose (l,F) ∈ worklist;
    worklist := worklist \ {(l,F)};
    if (reach(l) ⇏ F) then
    {
      reach(l) := reach(l) ∨ F;
      foreach ((l,c,l') ∈ T)
        worklist := worklist ∪ {(l',sp(F,c))};
    }
  }
  if (reach(l_err) ≠ false) then
    return UNSAFE
  else
    return SAFE
```

BOUNDED MODEL CHECKING

- Concrete/Symbolic model checking for a finite number of steps
  - Unroll loops in the program for a fixed number of iterations, and then do concrete/symbolic model checking on the resultant program.
- Alternatively, we can apply Static Single Assignment (SSA) transformation on the unrolled program, and directly encode the BMC problem in FOL.

ABSTRACT MODEL CHECKING

- All the previous approaches to model checking have severe limitations:
  - Concrete and Symbolic Model Checking may not terminate and are in general computationally expensive.
  - Bounded Model Checking can only be used to find bugs, and not for verification.
- Let’s bring back abstraction!
  - Consider a sound Abstract Interpretation framework $(D, \leq, \alpha, \gamma, \hat{F})$.

ABSTRACT MODEL CHECKING

```
AbstractModelChecking(Γc,P)
  worklist := {(l0,α(P))};
  reach := ∅;
  while worklist ≠ ∅ do{
    Choose (l,d) ∈ worklist;
    worklist := worklist \ {(l,d)};
    if ( ∄(l,d') ∈ reach . d ≤ d') then
    {
      reach := reach ∪ {(l,d)};
      foreach ((l,c,l') ∈ T)
         worklist := worklist ∪ {(l',d') | d' = f̂c(d)};
    }
  }
  if ((lerr,d) ∈ reach ∧ d ≠ ⊥ ) then
    return UNSAFE
  else
    return SAFE
```

ABSTRACT MODEL CHECKING

WITH COUNTEREXAMPLE GENERATION

```
AbstractModelChecking(Γ_c,P)
  worklist := {(l_0,α(P))}; parents := λx.NR;
  reach := ∅;
  while worklist ≠ ∅ do{
    Choose (l,d) ∈ worklist;
    worklist := worklist \ {(l,d)};
    if ( ∄(l,d') ∈ reach . d ≤ d') then
    {
      reach := reach ∪ {(l,d)};
      foreach ((l,c,l') ∈ T){
         worklist := worklist ∪ {(l',f̂_c(d))};
         parents((l',f̂_c(d))) := (l,d);
      }
    }
  }
  if ((l_err,d) ∈ reach ∧ d ≠ ⊥ ) then
    return UNSAFE
  else
    return SAFE
```

PREDICATE ABSTRACTION

- The predicate abstraction domain is parameterized by a fixed, finite set of predicates P.
  - Each predicate is a formula over the program variables.
  - Example: $P = \{x \leq 1, y = 0, x + y \leq -1\}$
- There are two predicate abstraction domains:
  - Boolean Predicate Abstraction
  - Cartesian Predicate Abstraction

CARTESIAN PREDICATE ABSTRACTION

- The abstract domain is $\mathbb{P}(P) \cup \{ \perp \}$
- The partial order relation $\sqsubseteq$ is defined as follows:
  - $\forall s \in \mathbb{P}(P) \,.\, \perp \sqsubseteq s$
  - $\forall s_1, s_2 \in \mathbb{P}(P) \,.\, s_1 \sqsubseteq s_2 \Leftrightarrow s_1 \supseteq s_2$
- Top element is $\emptyset$, bottom element is $\perp$
- Example: $P = \{x \leq 1, y = 0, x + y \leq -1\}$. Which of the following are true?
  - $\{x \leq 1\} \sqsubseteq \{x \leq 1, x + y \leq -1\}$
  - $\{x + y \leq = 1, y = 0\} \sqsubseteq \{y = 0\}$
  - $\{x \leq 1\} \sqsubseteq \emptyset$

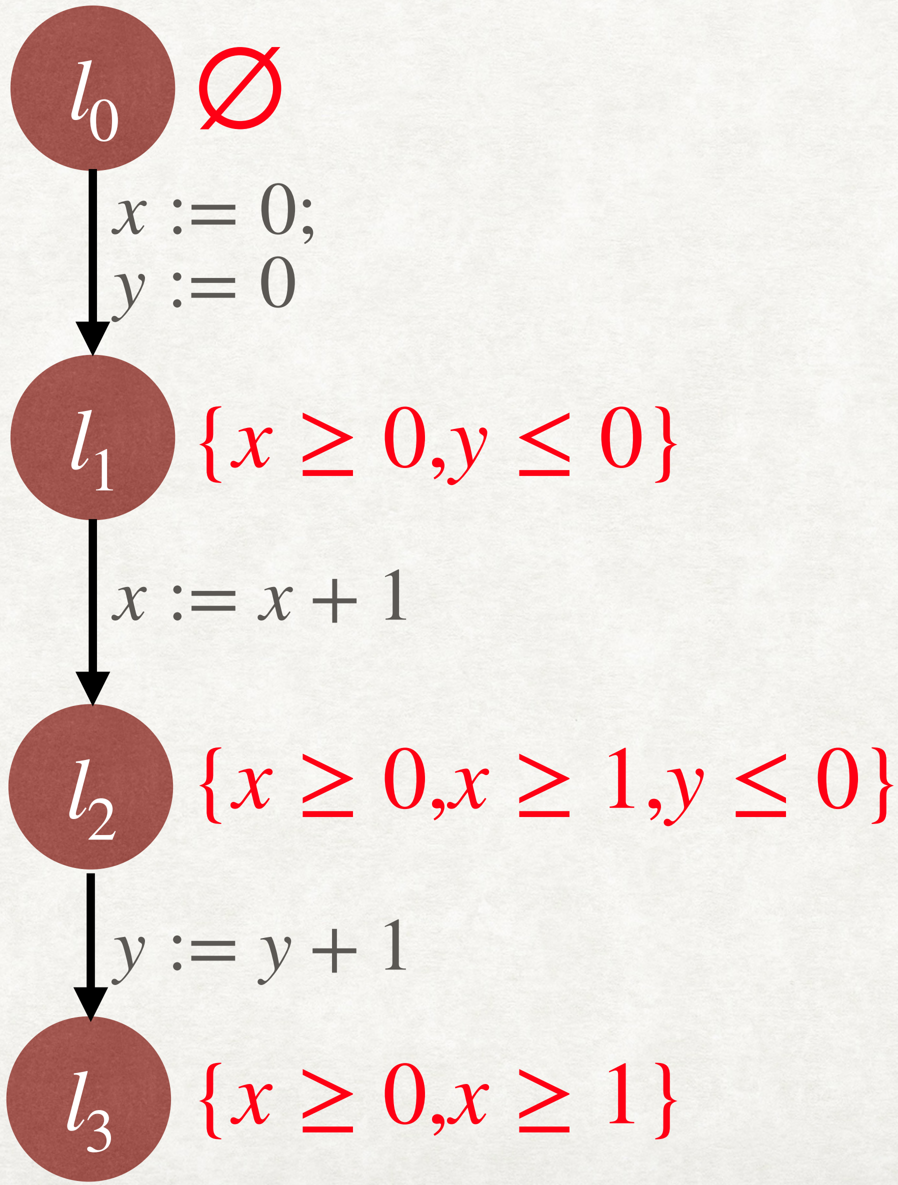
CARTESIAN PREDICATE ABSTRACTION

- Abstraction function: $\forall c \in \mathbb{P}(State) \,.\, c \neq \emptyset \Rightarrow \alpha(c) = \{p \in P \mid \forall \sigma \in c \,.\, \sigma \vDash p\}$
  - $\alpha(\emptyset) = \bot$
- Concretization function: $\forall s \in \mathbb{P}(P) \,.\, \gamma(s) = \{\sigma \mid \sigma \vDash \bigwedge_{p \in s} p\}$
  - $\gamma(\bot) = \emptyset$
- Examples $P = \{x \leq 1, y = 0, x + y \leq -1\}$
  - $\alpha(\{(0,0)\}) = \{x \leq 1, y = 0\}$
  - $\alpha(\{(0,0), (-1,-1)\}) = \{x \leq 1, x + y \leq -1\}$
  - $\alpha(x \leq 0) = \{x \leq 1\}$
- Homework: Prove that $(\mathbb{P}(State), \subseteq) \underset{\gamma}{\overset{\alpha}{\rightleftarrows}} (\mathbb{P}(P) \cup \{\bot\}, \sqsubseteq)$ is an Onto Galois Connection.

ABSTRACT MODEL CHECKING

WITH CARTESIAN PREDICATE ABSTRACTION

$$P = \{x \geq 0, y \leq 0, x \geq 1\}$$



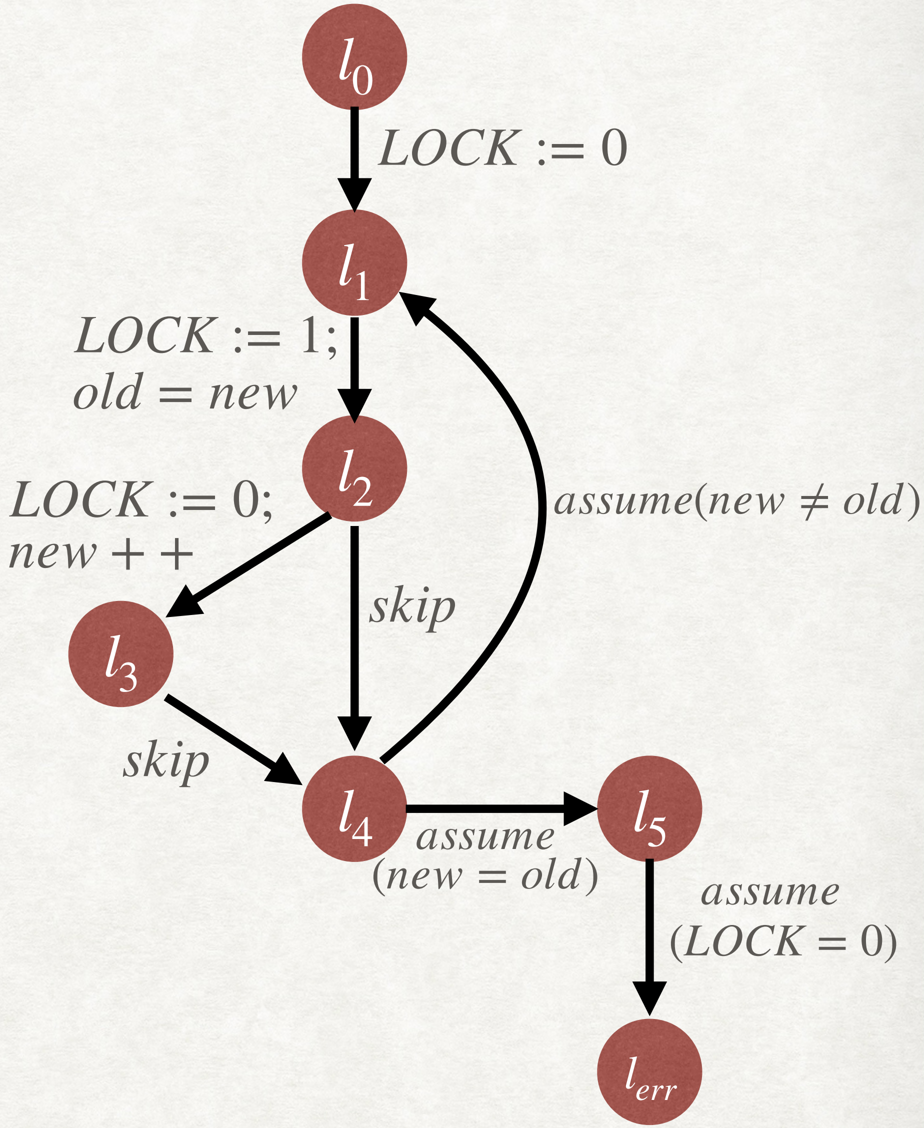
VERIFICATION USING CARTESIAN PREDICATE ABSTRACTION

```
0:  LOCK = 0;
1:  do {
      LOCK = 1;
      old = new;
2:    if (*) {
3:      LOCK = 0;
        new++;
      }
4:  } while (new != old);
5:  if (LOCK==0)
6:    error();
    LOCK = 0;
```

Software Model Checking. Ranjit Jhala and Rupak Majumdar. ACM Computing Surveys(2009)

VERIFICATION USING CARTESIAN PREDICATE ABSTRACTION

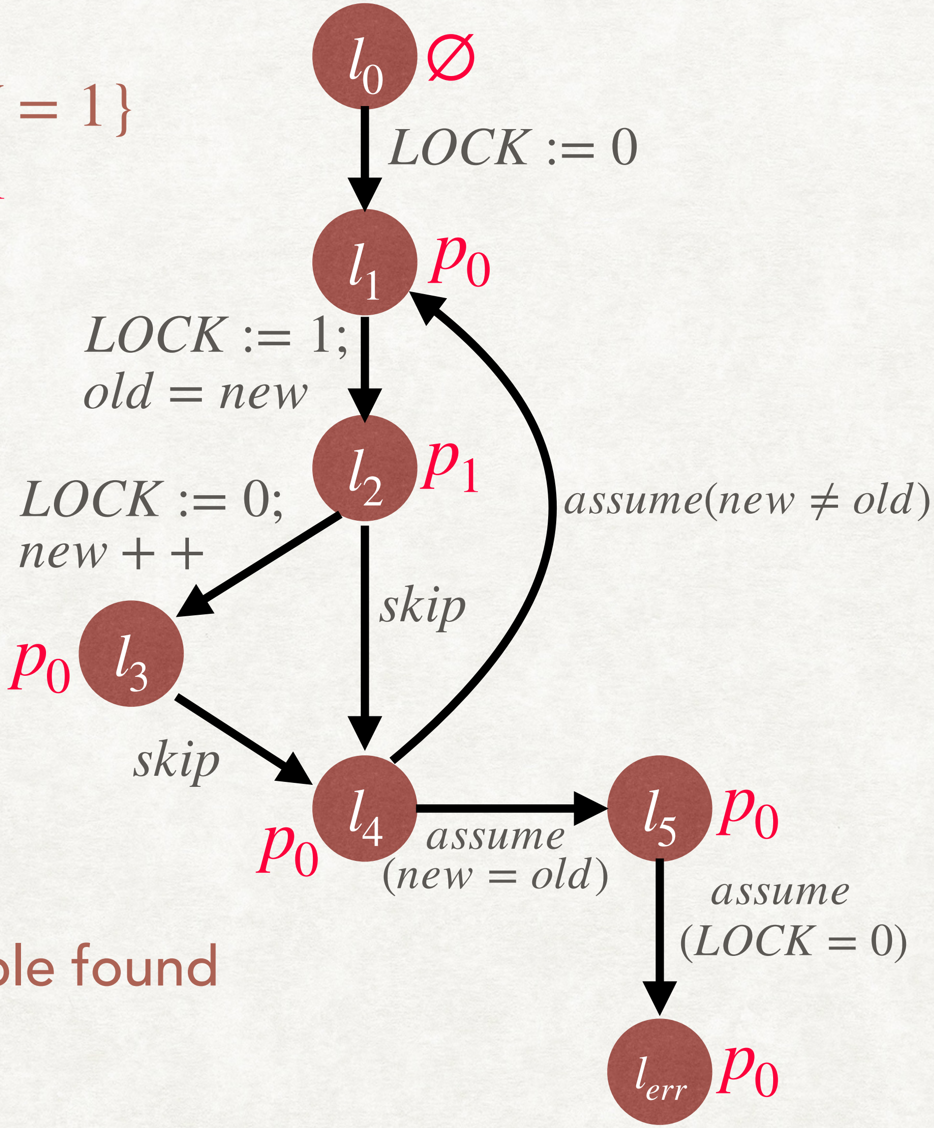
```
0:   LOCK = 0;
1:   do {
       LOCK = 1;
       old = new;
2:     if (*) {
3:       LOCK = 0;
         new++;
       }
4:   } while (new != old);
5:   if (LOCK==0)
6:     error();
     LOCK = 0;
```



Software Model Checking. Ranjit Jhala and Rupak Majumdar. ACM Computing Surveys(2009)

VERIFICATION USING CARTESIAN PREDICATE ABSTRACTION

$P = \{\underbrace{LOCK = 0}_{p_0}, \underbrace{LOCK = 1}_{p_1}\}$



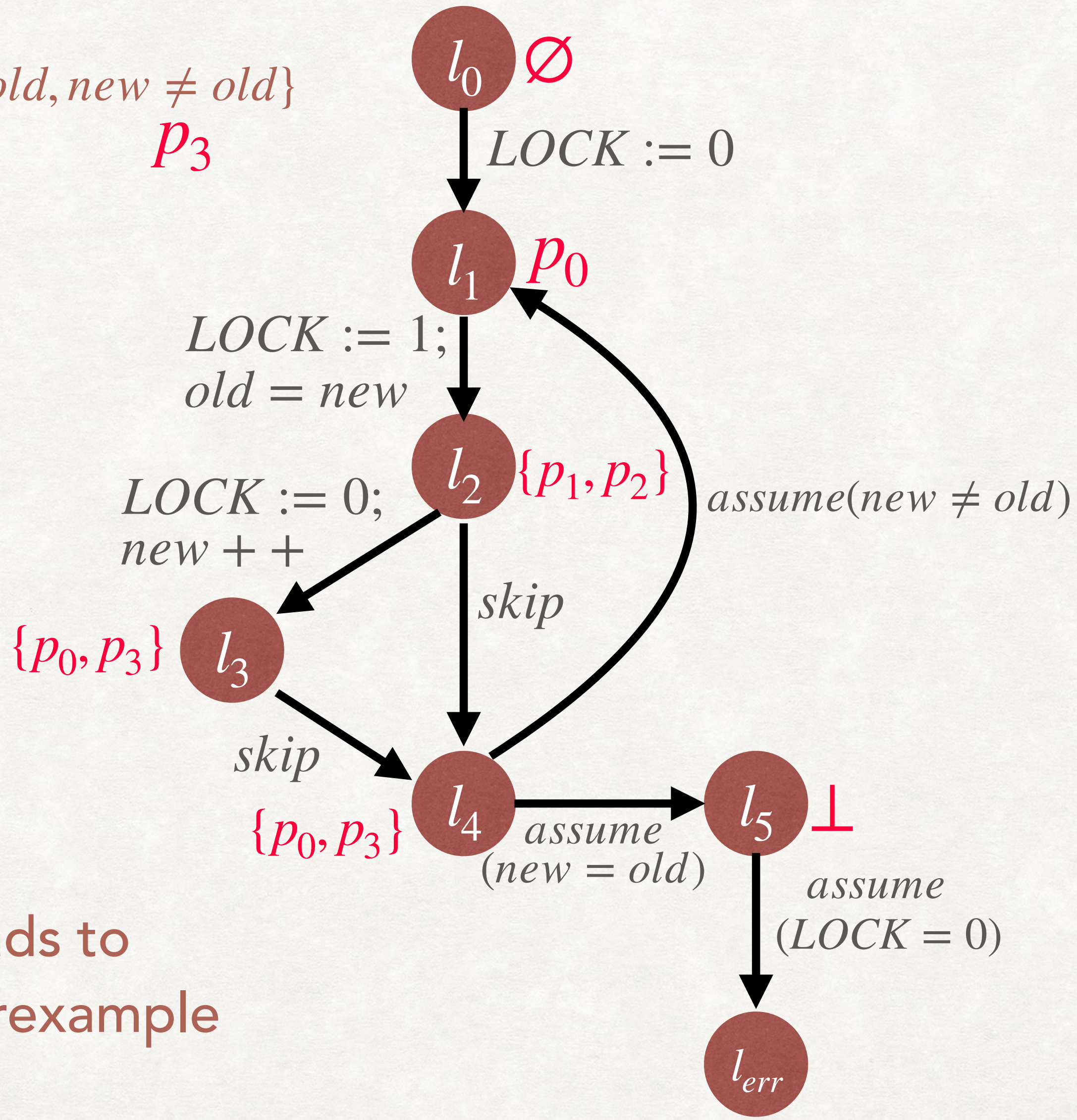
Spurious Counterexample found

Software Model Checking. Ranjit Jhala and Rupak Majumdar. ACM Computing Surveys(2009)

VERIFICATION USING CARTESIAN PREDICATE ABSTRACTION

$P = \{LOCK = 0, LOCK = 1, new = old, new \neq old\}$

p_0 p_1 p_2 p_3



Abstraction Refinement leads to elimination of spurious counterexample

Software Model Checking. Ranjit Jhala and Rupak Majumdar. ACM Computing Surveys(2009)

ABSTRACTION REFINEMENT

- Given two abstract domains $(D_1, \leq_1, \alpha_1, \gamma_1)$ and $(D_2, \leq_2, \alpha_2, \gamma_2)$, we say that D_2 refines D_1 if $\forall c \in \mathbb{P}(State) \,.\, \gamma_2(\alpha_2(c)) \subseteq \gamma_1(\alpha_1(c))$.
- Intuitively, D_2 introduces lower over-approximation during abstraction, leading to more refined abstractions.

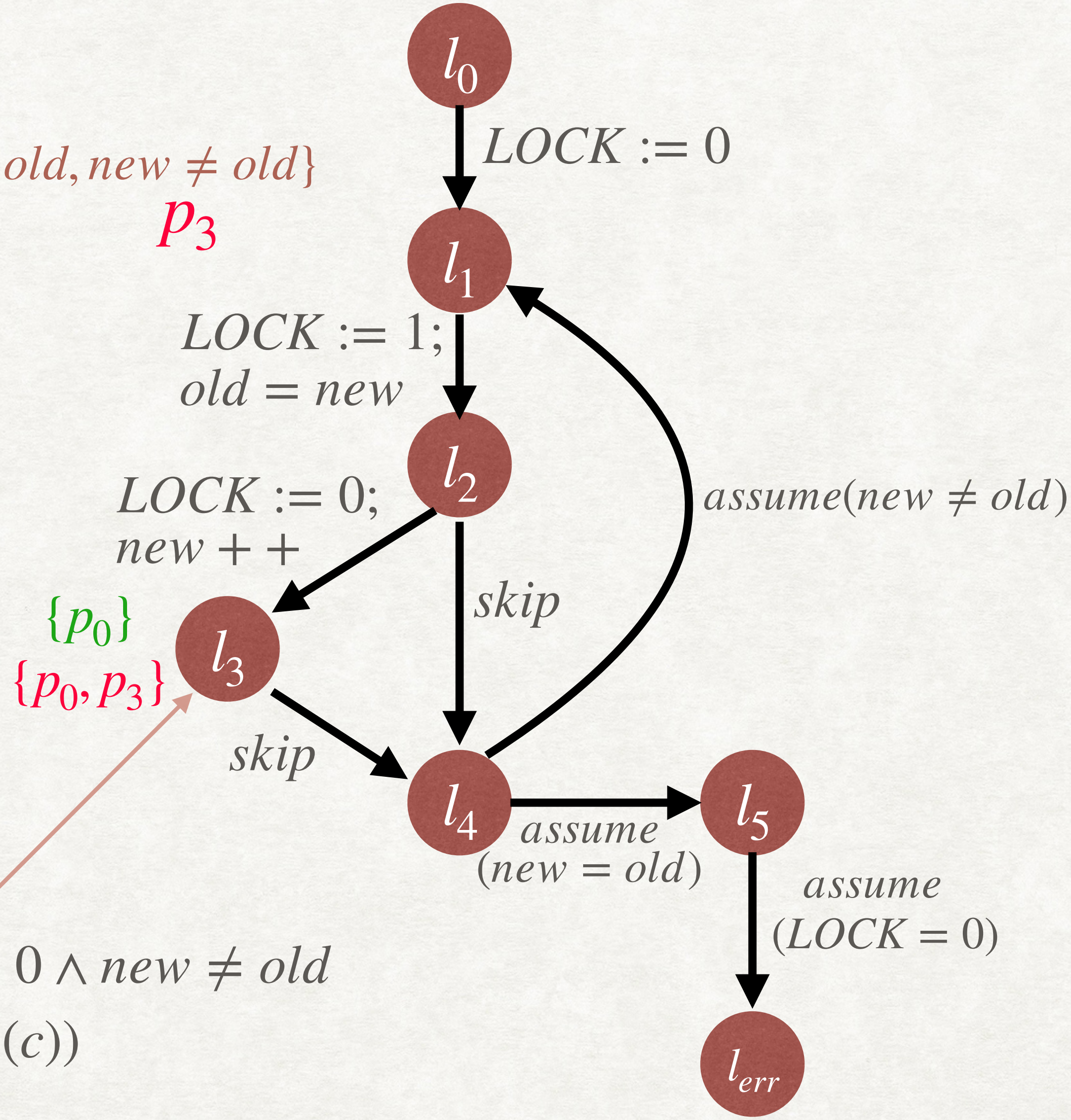
ABSTRACTION REFINEMENT: EXAMPLE

$\quad p_0 \qquad\qquad p_1$

$P_1 = \{LOCK = 0, LOCK = 1\}$

$P_2 = \{LOCK = 0, LOCK = 1, new = old, new \neq old\}$

$\quad p_0 \qquad\qquad p_1 \qquad\qquad p_2 \qquad\qquad p_3$



Concrete state $c : LOCK = 0 \land new \neq old$

$\gamma_2(\alpha_2(c)) \subseteq \gamma_1(\alpha_1(c))$

ABSTRACTION REFINEMENT

- Given two abstract domains $(D_1, \leq_1, \alpha_1, \gamma_1)$ and $(D_2, \leq_2, \alpha_2, \gamma_2)$, we say that D_2 refines D_1 if $\forall c \in \mathbb{P}(State) \,.\, \gamma_2(\alpha_2(c)) \subseteq \gamma_1(\alpha_1(c))$.
- Intuitively, D_2 introduces lower over-approximation during abstraction, leading to more refined abstractions.
- Homework: Given sets of predicates P_1 and P_2 such that $P_1 \subseteq P_2$, prove that the abstract domain $\mathbb{P}(P_2) \cup \{ \perp \}$ refines $\mathbb{P}(P_1) \cup \{ \perp \}$

FINDING REFINEMENTS

- If verification fails with set of predicates P, then we can consider the counterexample, which is a path from the initial location to the error location.
- We can check if the counterexample is valid or spurious.
  - Can be checked by executing the path concretely or symbolically.
- If the counter example is spurious, then we can deduce new predicates which make the counter example infeasible.

TRACE FORMULA

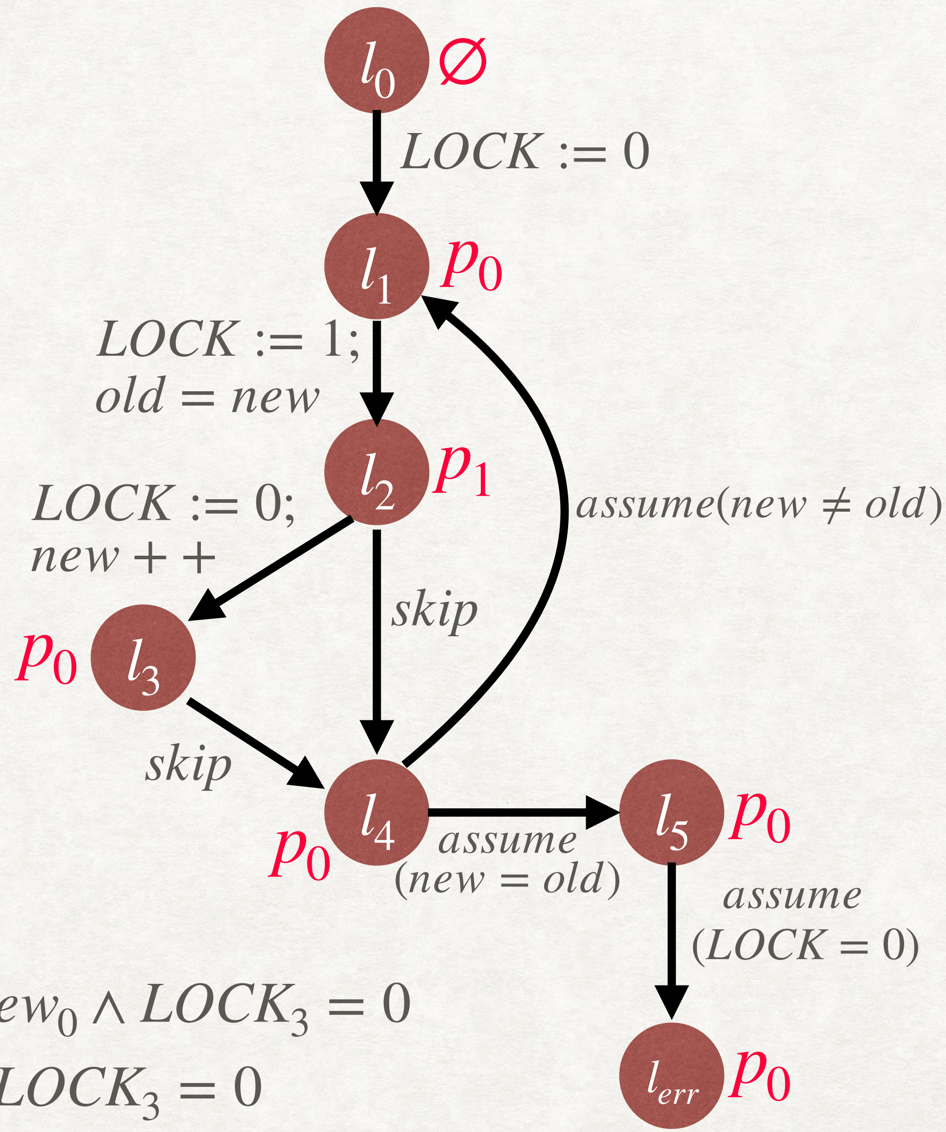
- Given a counterexample $l_{i_0}, l_{i_1}, \ldots, l_{i_n}$ (where $i_0 = 0$ and $i_n = err$), assume that $\forall j \,.\, (l_{i_j}, c_{i_{j+1}}, l_{i_{j+1}}) \in T$. We can symbolically execute the path by constructing its trace formula:

$$\bigwedge_{i=0}^{n-1} \rho(c_{i_{j+1}})[V_{i_j}/V, V_{i_{j+1}}/V']$$

- Here, $\rho(c_{i_j})$ is the encoding of the operational semantics of c_{i_j} in FOL.

TRACE FORMULA : EXAMPLE

$$P = \{\underbrace{LOCK = 0}_{p_0}, \underbrace{LOCK = 1}_{p_1}\}$$



$$LOCK_1 = 0 \wedge LOCK_2 = 1 \wedge old_1 = new_0 \wedge LOCK_3 = 0$$
$$\wedge\, new_1 = new_0 + 1 \wedge new_1 = old_1 \wedge LOCK_3 = 0$$