PROPOSITIONAL LOGIC

MATHEMATICAL LOGIC

- Logic is the foundation of computation.
- We will use logic for multiple purposes:
  - Describing specifications
  - Describing program executions
  - Mathematical guarantees of logic will translate to guarantees of program correctness
  - Decision procedures for logic will be used for verification.

PROPOSITIONAL LOGIC

Is $p \to q \to r \leftrightarrow (p \wedge q) \to r$ valid?

Is $p \wedge \bot \to \neg q \vee \top$ satisfiable?

SYNTAX

Atom	Truth Values - $\bot$: False, $\top$: True Propositional Variables - p,q,r…
Logical Connectives	$\wedge$: and, $\vee$: or, $\neg$: not, $\rightarrow$: implies, $\leftrightarrow$: if and only if(iff)
Literal	Atom or its negation
Formula	A literal or the application of logical connectives to formulae

SEMANTICS

Interpretation I

I : Set of Propositional Variables → { ⊥ , ⊤ }

MODEL OF

Given an interpretation I and Formula F,

$I \models F$	F evaluates to ⊤ under I
$I \not\models F$	F evaluates to ⊥ under I

SEMANTICS: INDUCTIVE DEFINITION

Base Case:

$I \models \top$	
$I \not\models \bot$	
$I \models p$	iff I(p)=$\top$
$I \not\models p$	iff I(p)=$\bot$

Inductive Case:

$I \models \neg F$	iff $I \not\models F$
$I \models F_1 \wedge F_2$	iff $I \models F_1$ and $I \models F_2$
$I \models F_1 \vee F_2$	iff $I \models F_1$ or $I \models F_2$
$I \models F_1 \rightarrow F_2$	iff $I \not\models F_1$ or $I \models F_2$
$I \models F_1 \leftrightarrow F_2$	iff $I \models F_1$ and $I \models F_2$, or $I \not\models F_1$ and $I \not\models F_2$
Other cases ...	

EXAMPLE

$I = \{p : True, q : False\}$ $F = p \wedge q \rightarrow p \vee \neg q$

Is $I \models F$?

1. $I \not\models q$
2. $I \not\models p \wedge q$
3. $I \models p \wedge q \rightarrow p \vee \neg q$

PRECEDENCE OF LOGICAL CONNECTIVES

- We assume the following precedence from highest to lowest:
  - $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
  - Example: $\neg p \wedge q \rightarrow p \vee q \wedge r$ is the same as $((\neg p) \wedge q) \rightarrow (p \vee (q \wedge r))$.
- We assume that all logical connectives associate to the right.
  - Example: $p \rightarrow q \rightarrow r$ is the same as $p \rightarrow (q \rightarrow r)$
- Parenthesis can be used to change precedence or associativity.

SATISFIABILITY AND VALIDITY

- A formula F is satisfiable iff there exists an interpretation I such that $I \vDash F$.
- A formula F is valid iff for all interpretations I, $I \vDash F$.
- A formula F is valid iff $\neg F$ is unsatisfiable.
  - A Decision Procedure for satisfiability is therefore also a decision procedure for validity. How?