ISTA 311 Written Homework 2

Due: Thursday, October 3 by end of class

Complete the problems below. "LN" means the lecture notes. This homework should be submitted **on a separate sheet (or sheets) of paper**.

You are encouraged to collaborate with other students on this assignment. If you do so, please note the name of your collaborator at the beginning of the assignment.

1. LN Chapter 4, Problem 5
2. (Blyth's paradox.)[1] Suppose you are playing a game with three spinners, labeled A, B, and C. These spinners:
   - Spinner A always gives a value of 3.
   - Spinner B gives values of 2, 4, or 6 with probabilities 0.56, 0.22, and 0.22 respectively.
   - Spinner C gives a value of 1 with probability 0.51, and a value of 5 with probability 0.49.

   (a) You play the game with one friend. You get to pick a spinner first, then your opponent picks another spinner. You both spin; highest number wins. Which spinner should you pick to maximize your chances of winning? (Hint: compare each pair to calculate each spinner's probability of winning vs. the others.)

   (b) Same as part A, but you play the game with two friends. You still get to pick the first spinner; which spinner should you pick?
3. We choose one of the words in the following sentence uniformly at random and then choose one of the letters of that word (again uniformly at random):

   `THE QUICK BROWN FOX JUMPED OVER THE GATE`

   (a) What is the probability that the chosen letter is `O`?

   (b) What is the probability that the chosen word has at least 4 letters?

   (c) What is the probability that the chosen letter is `O`, given that the chosen word has at least 4 letters?
4. Suppose that $P(A) = 0.4$ and $P(B) = 0.7$. Making no further assumptions on A or B, explain why $0.1 \leq P(A \cap B) \leq 0.4$.
5. We choose one of the letters in the following sentence in two different ways:

   `SOME DOGS ARE BROWN`

   Method 1: choose the letter uniformly at random from all letters (spaces don't count).
   Method 2: choose a word uniformly at random, then select a letter uniformly at random from that word.

   Do the two methods give the same probability of choosing the letter `R`? Justify your answer.
6. Jane must get at least 3 of the four problems on the exam correct to get an A. She has been able to do 80% of the problems on old exams, so she assumes that the probability she gets any problem correct is .8. She also assumes that the results on different problems are independent.

   (a) What is the probability Jane gets an A?

   (b) If Jane gets the first question correct, what is the probability she gets an A?

[1] Colin Blyth was the first to name and describe Simpson's paradox, in an article published in 1972 (referencing Simpson's article of 1951). This is another "paradox" described by Blyth.