ISTA 311 Homework 4

Due: Thursday, November 7, 2019

Complete the problems below. "LN" means the lecture notes. Submit this homework handwritten on a separate sheet or sheets of paper.

If you collaborate with another student on this assignment, please note the name of your collaborator(s) on your paper.

1. A species of fish makes an annual migration up a river. On a particular day, the probability that the fish have begun their migration is 0.1. If the migration has not begun, then the probability of observing a fish in any given 10-minute period is 0.4. If the migration has begun, no fish will be observed.
   (a) If you wait at the river for 10 minutes and observe no fish, what is the probability that the migration has begun?
   (b) How long must you wait and observe no fish before you are at least 50% confident that the migration has begun?

2. A car insurance company classifies its customers as low-risk, average-risk, and high-risk drivers; these categories correspond to the safest 20% (low-risk), the riskiest 20% (high-risk), and the middle 60% (average-risk).

   It is estimated that a low-risk driver has a 5% chance of being involved in an accident in any given year; an average risk driver, 15%, and a high-risk driver, 40%.
   (a) According to the company's model, what is the probability that a randomly selected driver will be involved in an accident in 2020?
   (b) Suppose you signed up for insurance with this company at the beginning of 2019, and you were in an accident this year. According to the company's model, what is the probability that you will be in an accident in 2020? *(Hint: use Bayes' theorem to calculate the probability that you belong to each of the three categories, and then repeat your calculation from part (a).)*
   (c) Your friend was also in an accident this year, but they have held an insurance policy with the same company since 2015 (with no accidents previously). Would the company make the same prediction for your friend as for you? Why or why not? (You don't need to do the full calculation.)

3. The makers of MagiCola claim that 4 out of 5 people prefer their cola to a generic brand, GeneriCola. You decide to carry out a blind taste test with three of your friends.
   (a) The company's hypothesis can be stated probabilistically as saying: the probability that a randomly selected person prefers MagiCola over GeneriCola is 0.8.

   State a hypothesis probabilistically expressing the claim that people cannot tell the difference between MagiCola and GeneriCola.
   (b) You and your two friends carry out a blind taste test. In the test, all three of you prefer MagiCola. Calculate the Bayes factor quantifying how much this result favors the company's claim over the other hypothesis.
   (c) What if all three of you preferred GeneriCola? Calculate the Bayes factor in this scenario. Does it make sense to conclude, based on this Bayes factor, that the hypothesis from part (a) is correct?