

Time Series Analysis & Forecasting Using R

The forecasters' toolbox-Part 2

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Outline

- 1 Residual diagnostics
- 2 Evaluating point forecast accuracy
- 3 Time Series Cross Validation (TSCV)
- 4 Evaluating prediction interval accuracy
- 5 Lab session 6

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Forecasting residuals

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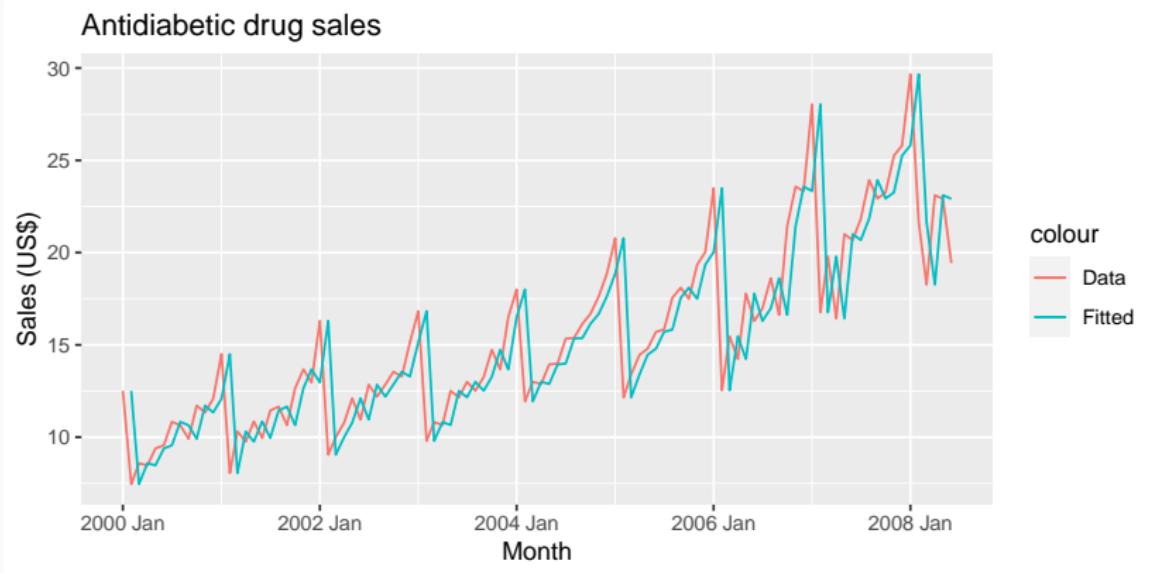
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- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
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Useful properties (for prediction intervals)

- 3 $\{e_t\}$ have constant variance.
- 4 $\{e_t\}$ are normally distributed.

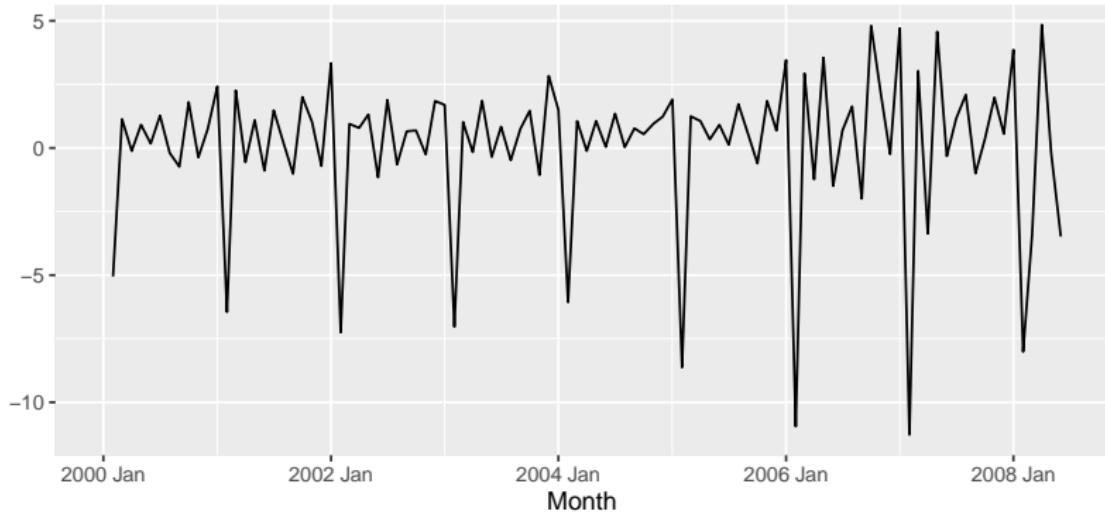
Example: Antidiabetic drug sales



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```
augment(fit) %>%
  autoplot(.resid) + xlab("Month") + ylab("") +
  ggtitle("Residuals from naïve method")
```

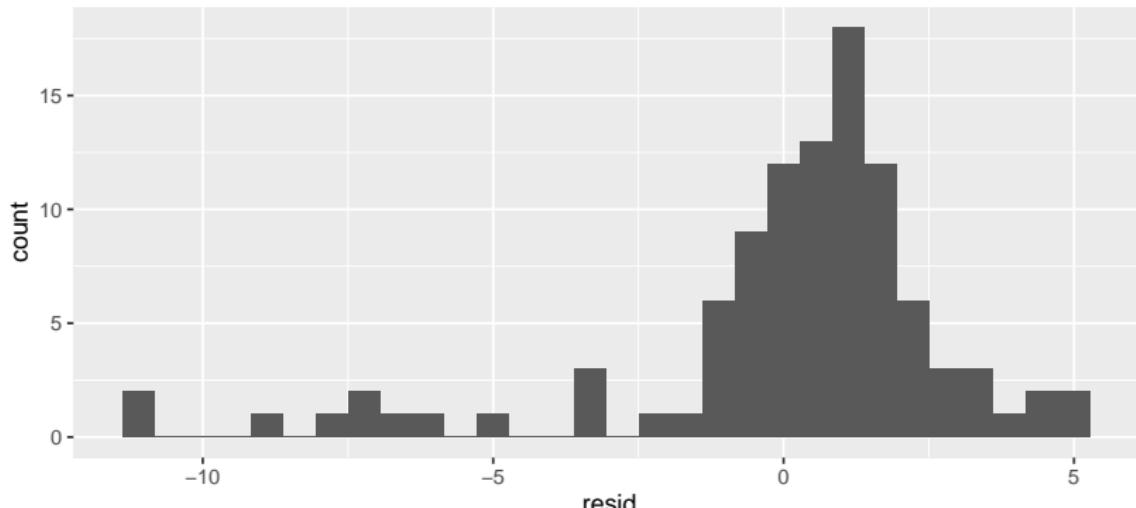
Residuals from naïve method



Example: Antidiabetic drug sales

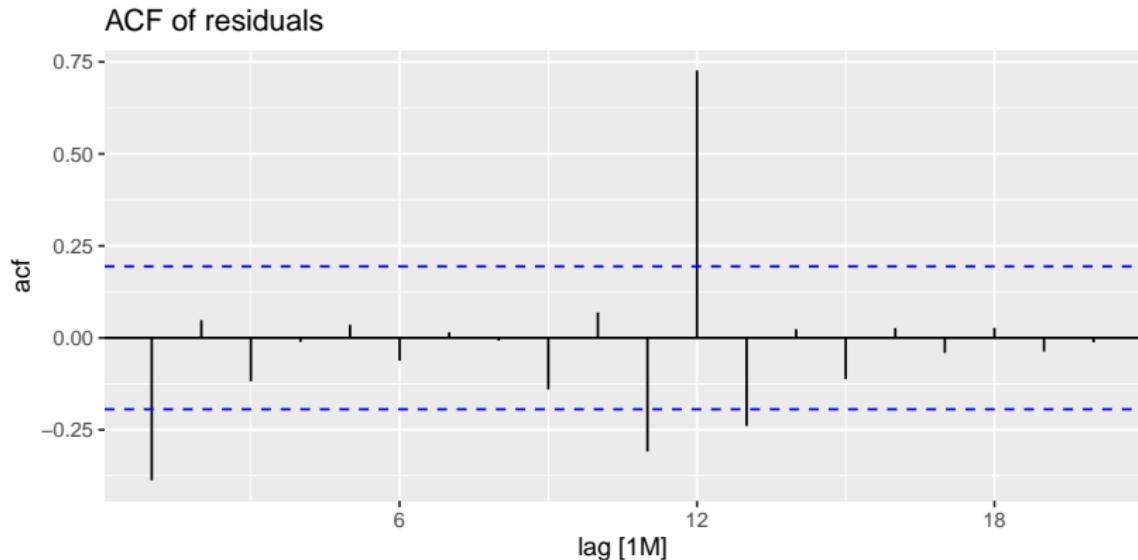
```
augment(fit) %>%
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 30) +
  ggtitle("Histogram of residuals")
```

Histogram of residuals



Example: Antidiabetic drug sales

```
augment(fit) %>% ACF(.resid) %>%
  autoplot() + ggtitle("ACF of residuals")
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

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Box-Pierce test

$$Q = T \sum_{k=1}^h r_k^2$$

where h is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T + 2) \sum_{k=1}^h (T - k)^{-1} r_k^2$$

where h is max lag being considered and T is number of observations.

- Preferences: $h = 10$ for non-seasonal data,
 $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

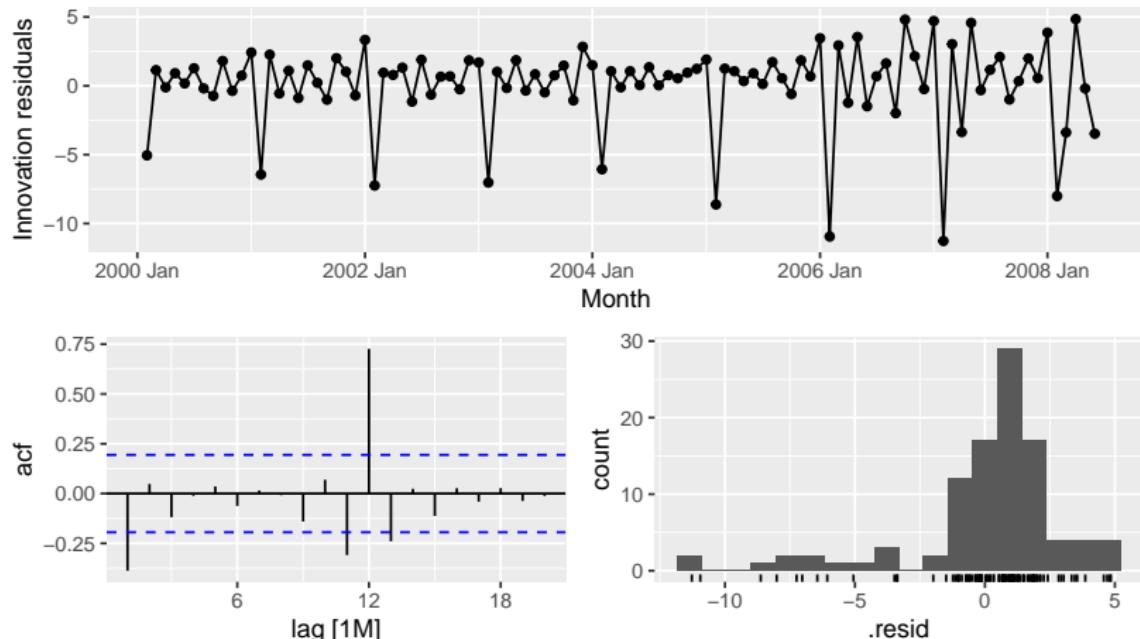
- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.

```
augment(fit) %>% features(.resid, ljung_box, lag=10, dof=0)
```

```
## # A tibble: 1 x 3
##   .model      lb_stat lb_pvalue
##   <chr>        <dbl>     <dbl>
## 1 NAIVE(Cost)  20.7     0.0232
```

gg_tsresiduals function

```
fit %>%  
  gg_tsresiduals()
```



Outline

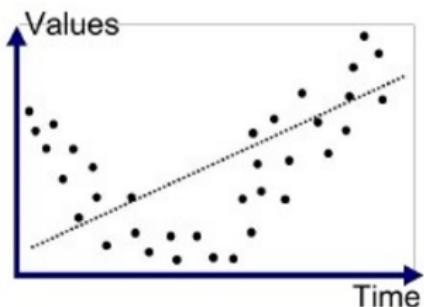
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Evaluating point forecast accuracy

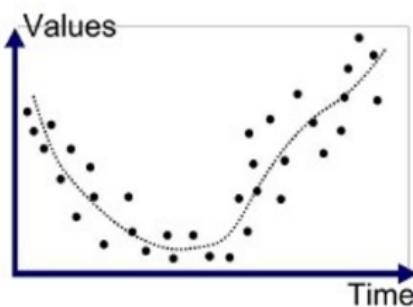
Evaluate forecast accuracy

- Residual diagnostic is not a reliable indication of forecast accuracy
- A model which fits the training data well will not necessarily forecast well
- A perfect fit can always be obtained by using a model with enough parameters
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data

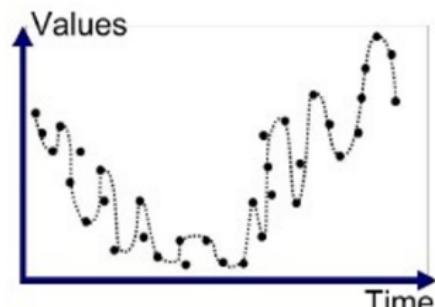
Fitting



Underfitted



Good Fit/R robust



Overfitted

Evaluate forecast accuracy

The accuracy of forecasts can only be determined by considering how well a model performs on new data that were not used when fitting the model

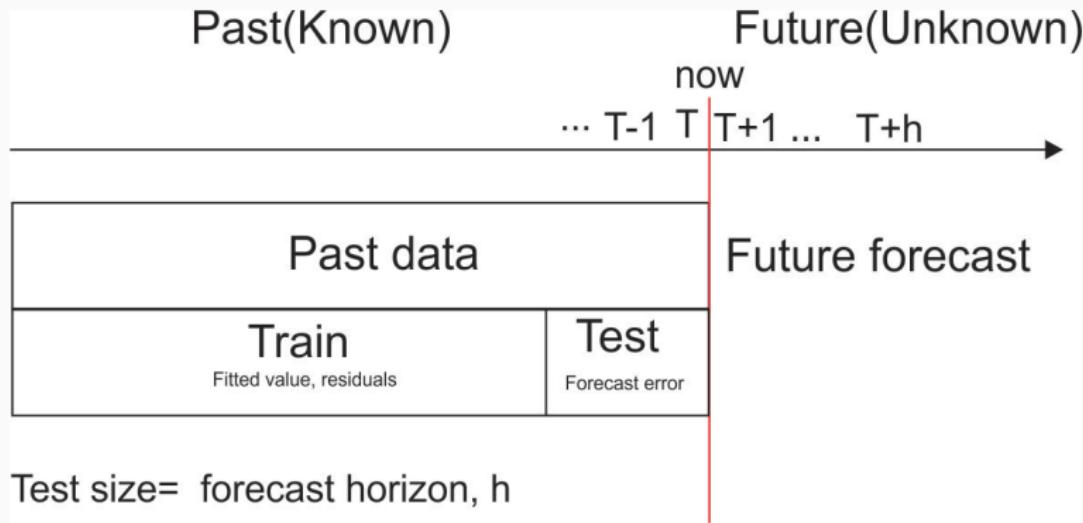
Forecast accuracy evaluation using test sets

- We mimic the real life situation
- We pretend we don't know some part of data(new data)
- It must not be used for *any* aspect of model training
- Forecast accuracy is based only on the test set

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Training and test series



Split the data

Use functions in `dplyr` and `lubridate` such as
`filter`, `filter_index`, `slice`

```
# Filter the year of interest
antidiabetic_drug_sale %>%
  filter_index("2006"~.)
```

```
## # A tsibble: 30 x 2 [1M]
##       Month   Cost
##       <mth> <dbl>
## 1 2006 Jan  23.5
## 2 2006 Feb  12.5
## 3 2006 Mar  15.5
## 4 2006 Apr  14.2
```

Forecast errors

Forecast “error”: the difference between an observed value and its forecast

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy

y_{T+h} = $(T + h)$ th observation, $h = 1, \dots, H$

$\hat{y}_{T+h|T}$ = its forecast based on data up to time T .

$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$

MAE = $\text{mean}(|e_{T+h}|)$

MSE = $\text{mean}(e_{T+h}^2)$

RMSE = $\sqrt{\text{mean}(e_{T+h}^2)}$

MAPE = $100\text{mean}(|e_{T+h}|/|y_{T+h}|)$

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- MAE, MSE, RMSE are all scale dependent
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t , and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|)/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

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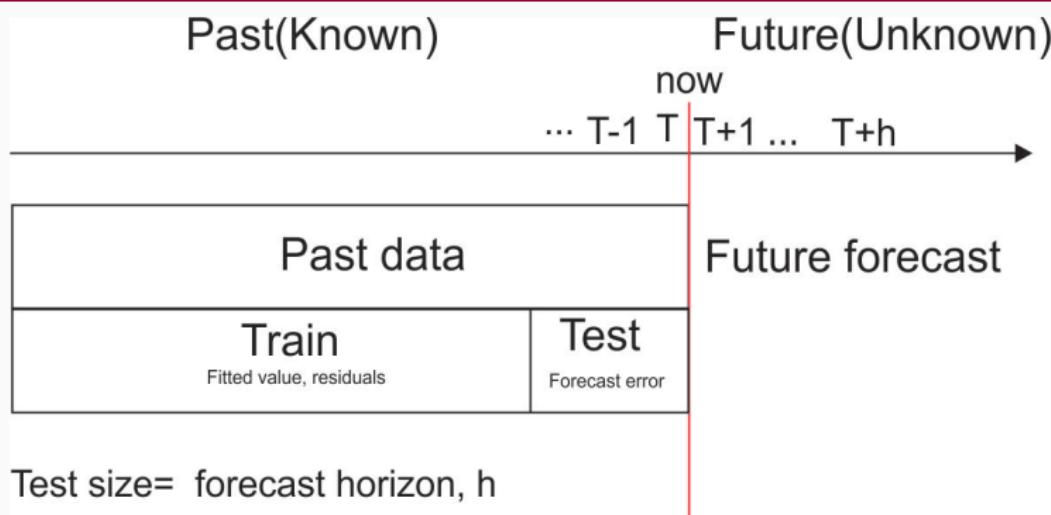
$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

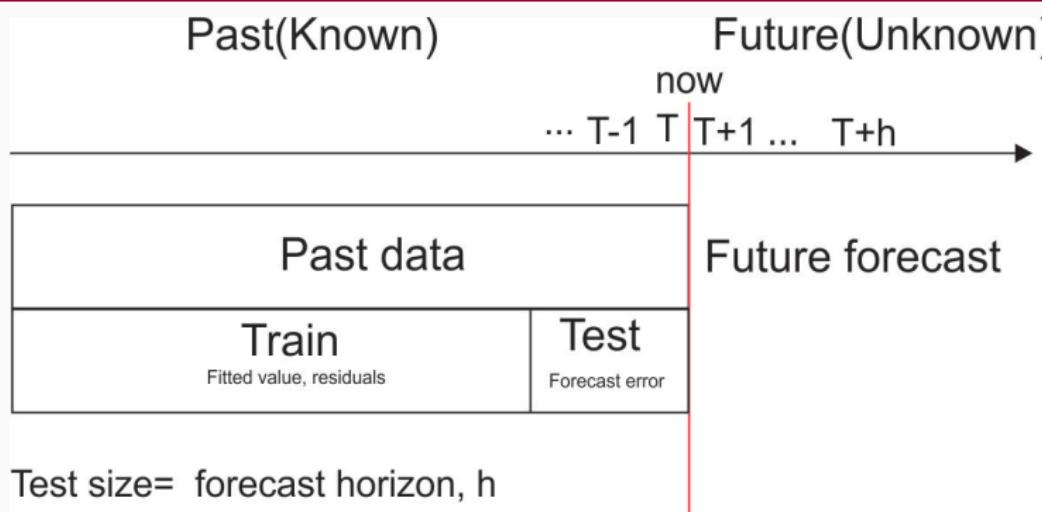
Poll: true or false?

- 1 Good point forecast models should have normally distributed residuals.
- 2 A model with small residuals will give good forecasts.
- 3 The best measure of forecast accuracy is MAPE.
- 4 Always choose the model with the best forecast accuracy as measured on the test set.

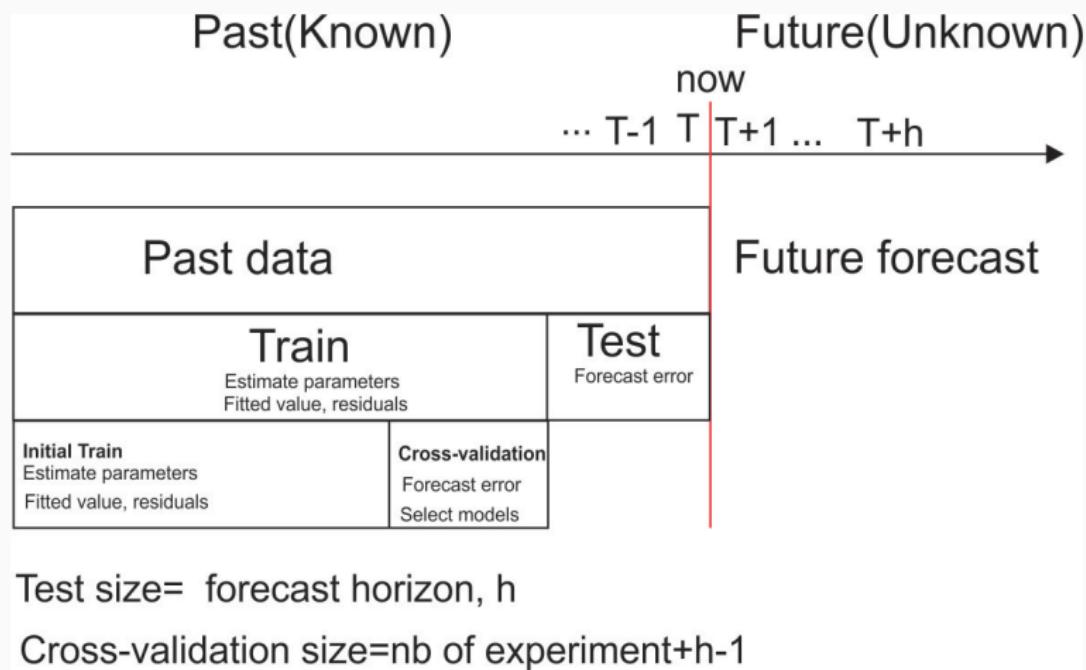
Issue with train/test split without time series cross-validation



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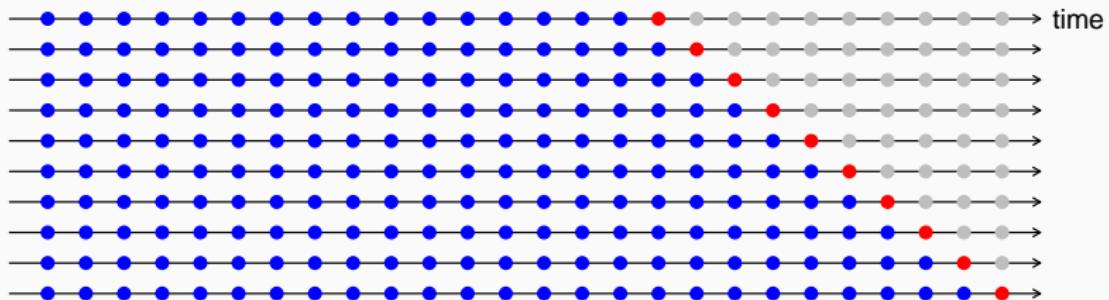


Time series cross-validation



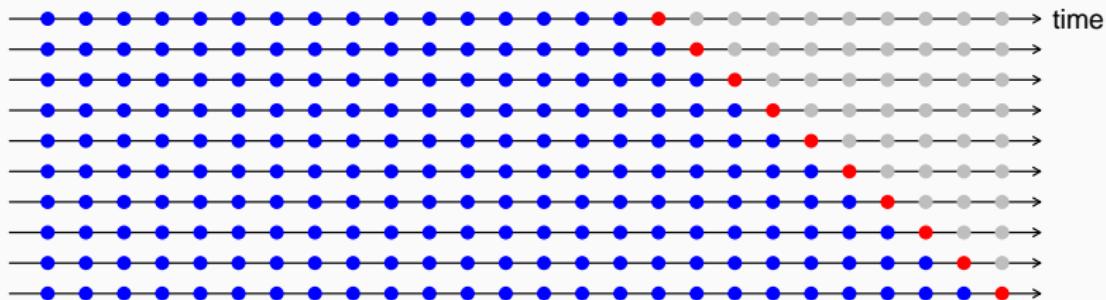
Time series cross-validation

Time series cross-validation



Time series cross-validation

Time series cross-validation



- Forecast accuracy averaged over test sets.

Creating the rolling training sets

There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

Time series cross-validation

Stretch with a minimum length of 24, growing by 1 each step.

```
forecast_horizon <- 12

test <- antidiabetic_drug_sale %>% filter_index(as.character(
  max(antidiabetic_drug_sale$`id`)))

train <- antidiabetic_drug_sale %>%
  filter_index(. ~ as.character(max(antidiabetic_drug_sale$`id`)))
drug_sale_tcsv <- train %>% slice(1:(n()-forecast_horizon))
stretch_tsibble(.init = 24, .step = 1)

## # A tsibble: 2,209 x 3 [1M]
## # Key:     .id [47]
##       Month  Cost   .id
##       <mth> <dbl> <int>
## 1 2000 Jan 12.5     1
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
drug_fit_tr <- drug_sale_tcsv %>%
  model(snaive=SNAIVE(Cost))
```

```
## # A mable: 47 x 2
## # Key:     .id [47]
##     .id    snaive
##   <int>  <model>
## 1     1 <SNAIVE>
## 2     2 <SNAIVE>
## 3     3 <SNAIVE>
## 4     4 <SNAIVE>
## # ... with 43 more rows
```

Time series cross-validation

Produce 8 step ahead forecasts from all models.

```
drug_fc_tr <- drug_fit_tr %>%
  forecast(h=forecast_horizon) %>%
  group_by(.id,.model) %>%
  mutate(h=row_number()) %>%
  as_fable(response="Cost",distribution="Cost") |>
  ungroup()
```

```
## # A fable: 564 x 6 [1M]
## # Key:      .id, .model [47]
##     .id .model   Month       Cost .mean     h
##     <int> <chr>    <mth>     <dist> <dbl> <int>
## 1     1 snaive 2002 Jan   N(14, 1.7) 14.5     1
## 2     1 snaive 2002 Feb   N(8, 1.7)  8.05    2
## 3     1 snaive 2002 Mar   N(10, 1.7) 10.3    3
```

Time series cross-validation

```
# Cross-validated  
drug_fc_tr %>%  
  accuracy(antidiabetic_drug_sale)
```

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Winkler score

Winkler proposed a scoring method to enable comparisons between prediction intervals:

- it takes account of both coverage and width of the intervals.

Winkler score

$$W(l_t, u_t, y_t) = \begin{cases} u_t - l_t & \text{if } l_t < y_t < u_t \\ (u_t - l_t) + \frac{2}{\alpha}(l_t - y_t) & \text{if } y_t < l_t \\ (u_t - l_t) + \frac{2}{\alpha}(y_t - u_t) & \text{if } y_t > u_t \end{cases}$$

Prediction interval accuracy

```
# Compute interval accuracy
drug_fc_tr %>%
  accuracy(antidiabetic_drug_sale,
  measures = interval_accuracy_measures) %>%
  mutate(Method = paste(.model, "method")) %>%
  select(Method, winkler) %>%
  gt::gt() %>%
  gt::as_latex()
```

Method	winkler
snaive method	7.596448

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Lab session 6

- 1 use filter_index() function to subset data into train and test
 - 2 Create folds/windows for time series cross validation
- **Hint:** use stretch_tsibble(.init = 4*365, .step = 1)
- 3 Specify model and train model on each fold/window
 - 4 Forecast for 42 days
 - 5 Compute RMSE and MAE to evaluate point forecast
8.1 compute overall accuracy
8.2 compute accuracy over each forecast horizon
8.3 compute accuracy for each .id
 - 6 Evaluate the prediction intervals using Winkler score
 - 7 Which model is the most accurate one?
 - 8 Apply the most accurate model to all time series

Recap

- 1 First, import your data and prepare them using `tsibble` function.
- 2 Visualise and see whether your series contains key features
- 3 Determine how much of your data you want to allocate to training, and how much to testing; the sets should not overlap.
- 4 Subset the data to create a training set, which you will use as an argument in your forecasting function(s). Optionally, you can also create a test set to use later.
- 5 Compute forecasts of the training set using whichever forecasting function(s) you choose, and set `h` equal to the number of values you want to forecast.

Recap

- 6 Use residual diagnostic based on residuals in the training set to see whether all information is captured by models.
- 7 Create different windows to evaluate forecast accuracy using time series cross validation
- 8 Train model to each window
- 9 To calculate the forecast accuracy, use the `accuracy()` function with the `fable` as the first argument and original data as the second.
- 10 Pick a measure in the output to evaluate the forecast(s); a smaller error indicates higher accuracy.
- 11 Finally, produce forecast using the selected approach for real future.