Linear_Equations_and_Functions

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Linear equations and functions: General form

The general form of linear functions is, $y = ax + b$,

- You may also have come across $y = mx + c$.
- We are just changing the letters but the two are equivalent.
- a/ m is the slope/ gradient.
- b/ c represents the intercept.

Linear equations and functions; the x-axis and y-axis

- The sign of the slope determines the direction of the line.
- If the slope is positive (m $>$ 0), the line will slope upwards from bottom left.
- If the slope is negative (m $<$ 0), the line will trend downwards from the top left.
- There are two special lines;
  - The y-axis, which is written as $x = 0$ which is vertical- Slope is undefined.
  - The x-axis, written as $y = 0$ that is horizontal- Slope $= 0$

Linear equations and functions; the intercepts

- The x-intercept is the point at which the line touches the x-axis.
- The y-intercept is the point at which the line touches the y-axis.
- To find the y-intercept(s) of the graph of an equation, set $x = 0$ in the equation and solve for y. Note: A function of x has at most one y-intercept.
- To find the x-intercept(s), set $y = 0$ and solve for x.

Intercepts: Exercises

Example 1 Find the intercepts and graph the following.

- $3x + y = 9$
- $x = 4y$

Example 2 A business property is purchased for Ksh. 122,880 and depreciated over a period of 10 years. Its value y is related to the number of months of service x by the equation. Find the x-intercept and the y-intercept and use them to sketch the graph of the equation.

$4096xLinearequationsandfunctions; rateofchange/slopeofaline. + 4y = 491,520$

Find the x-intercept and the y-intercept and use them to sketch the graph of the equation.

Linear equations and functions; rates of change/ Slopes

- The slope of a line is represented as $slope = \frac{changeiny}{changeinx}$.
- An easier formula to remember for slope is $slope = \frac{rise}{run}$
- Watch this YouTube link for the different types of slopes
- https://www.youtube.com/watch?v=WQrz8Yljr_E
- $x = 0$ is the y-axis that has undefined slope. Other vertical lines like $x = 1$, $x = 2$ and so on, have an undefined slope.
- $y = 0$ is the x-axis that has a slope of zero. Other horizontal lines like $y = 1$, $y = 2$, and so on, have a slope of zero.

Linear equations and functions; rates of change/ intercept-Exercises.

- Find the slope of

a. line passing through and (4, 3).

b. line passing through (3, 0) and (4, -3).

c. Establish the equations of the lines in section (a) and (b).

Linear equations and functions; rates of change/ intercept- Some applications

- Two distinct non-vertical lines are parallel if and only if their slopes are equal.
- A line with l slope m, where , is perpendicular to line k if and only if the slope of k is $-\frac{1}{m}$. (The slopes are negative reciprocals.). In other words, if two lines are perpendicular, the product of their slopes is -1.

Linear equations and functions; rates of change/ intercept: Exercises

a. Find the slope of the line through (4, 6) and (28,6).

b. If a line has slope $m_1 = 0$, then the line is _____. If a line has an undefined slope, then the line is _____.

c. Suppose that line 1 has slope $m_1 = 5$ and line 2 has slope m_2.

- If line 1 is perpendicular to line 2, find m_2.
- If line 1 is parallel to line 2, find m_2.

Linear equations and functions: The point-slope form

The equation of the line passing through the point (x_1, y_1) and with slope m can be written in the point-slope form

$y - y_1 = m(x - x_1)$

Example Write the equation for each line that passes through (1, -2) and has

a. *Slope* $= \frac{2}{3}$: solution: here, $y_1 = -2$ and $x_1 = 2$, therefore;

$y - (-2)) = \frac{2}{3}(x - 1)$

$y + 2 = \frac{2}{3}x - \frac{2}{3})$

$y = \frac{2}{3}x - \frac{8}{3})$

b. *Slope* $= -4$

c. Undefined slope.

d. Line (2,3) also on the line.

Linear equations and functions: Summary

1. General forms: $ax + by + c = 0$
2. point slope form $(y - y_1) = m(x - x_1)$
3. Slope-intercept form, $y = mx + c$
4. $\frac{x}{a} + \frac{y}{b} = 1$, a = x intercept, b = y-intercept.
5. Verical line, $x = a$
6. Horizontal line $y = b$

Linear equations and functions: Applications and exercises

1. The Kenya National Bureau of statistics data indicate that the average price p of digital television sets can be expressed as a linear function of the number of sets sold N (in thousands). In addition, as N increased by 1000, p dropped by Ksh. 10.40, and when 6485 (thousand) sets were sold, the average price per set was Ksh. 504.39.

- Write an expression of the form $y = f(x)$ for this scenario.
- Write the equation of the line determined by this information.

Linear equations and functions: Applications and exercises

2. The percent p of high school seniors who smoke cigarettes can be described by

$p = 85.79 - 2.39t$

Where t is the number of years past 1975.

a. Find the slope and p-intercept of this equation.

b. Write a sentence that interprets the meaning of the slope as a rate of change.

c. Write a sentence that interprets the meaning of the p-intercept.

Hint: p is your y, while t is x.

Additional resources

https://www.youtube.com/watch?v=jx3-K4uVBow