Leontief Input-Output matrix

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The Leontief Input-Output Matrix

Introduction

- Used in economic planning.
- Especially in centrally planned economies.
- Vasilly Leotief.

The concept

- Suppose you produce 100 kgs of maize, and consume half of it.
- How much can you sell? 100 - 0.5(100) = 50
- Suppose you produce X kgs and consume 0.7 (a fraction of X).
- How much can you sell? X - 0.7X
- Suppose you produce X kgs and consume A (A is a fraction of X).
- How much can you sell? X - AX = X(1 - A).
- What is the equivalent of ZERO in matrix algebra?
  - If you want to show that there is nothing in matrix algebra, how would you write it? It has only Zeros. NULL matrix. It is a matrix that has zeros as elements.
- What is the equivalent of ONE in matrix algebra? The IDENTITY matrix. It has ones i the main diagonal, and zeros everywhere else.

```
## Example of a NULL matrix
matrix(c(0,0,0,0,0,0,0,0,0), nrow = 3)
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

```
## Example of an identity matrix
matrix(c(1,0,0,0,1,0,0,0,1), byrow= TRUE, nrow = 3)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

- What is the symbol for Identity matrix? I
- X(I - A) ### NB: This represents a surplus or excess produce that you can sell out.

- We call this the surplus or the final demand, D.
- Hence we can write, D = X(I - A), where A is the amount consumed internally.
- Let us try to solve for X.
- $X = \frac{D}{(I-A)} = \frac{1}{(I-A)} * D = (I-A)^{-1} * D$

$X = (I-A)^{-1} * D$

- NB: X is what you need to produce.
- A is the fraction of the produce you use internally.
- D is the final demand or surplus. The amount you can sell outside after satisfying your internal demand.
- Given A and D, can you solve for X? This is the central idea of the Leontief input-Output matrix.

Examples in the slides:

```
I <- matrix(c(1,0,0,0,1,0,0,0,1), byrow= TRUE, nrow = 3)
I
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
A <- matrix(c(0.5,0.1,0.1,0.2,0.5,0.3,0.1,0.3,0.4), byrow= TRUE, nrow = 3)
A
```

```
##      [,1] [,2] [,3]
## [1,]  0.5  0.1  0.1
## [2,]  0.2  0.5  0.3
## [3,]  0.1  0.3  0.4
```

```
W = I - A
W
```

```
##      [,1] [,2] [,3]
## [1,]  0.5 -0.1 -0.1
## [2,] -0.2  0.5 -0.3
## [3,] -0.1 -0.3  0.6
```

```
W_inverse <- solve(W)

D = matrix(c(85,65,0), byrow = FALSE, nrow = 3)
D
```

```
##      [,1]
## [1,]   85
## [2,]   65
## [3,]    0
```

```
W_inverse %*% D
```

```
##      [,1]
## [1,]  300
## [2,]  400
## [3,]  250
```