FUNCTIONS

Mathematics allows for no hypocrisy and no vagueness- Stendhal (pen name of Marie-Henri Beyle), French writer.

Instructor: John Karuitha

Defining a function.

- A function is a mathematical construct (think of it as an object or machine) that takes in a value $\mathtt{x}$ and outputs a value $\mathtt{y}$ in a one-to-one correspondence.
- A function relates **each** element in a set $\mathtt{x}$ to **exactly one** element in another set $\mathtt{y}$.
- Note each element in $\mathtt{x}$ must map to only one element in $\mathtt{y}$.
- Each element in $\mathtt{y}$ must NOT have a corresponding element in $\mathtt{x}$
- The values in $\mathtt{x}$ constitute the **domain**; those in y make up the **range**.

Example: Not a function. Why?

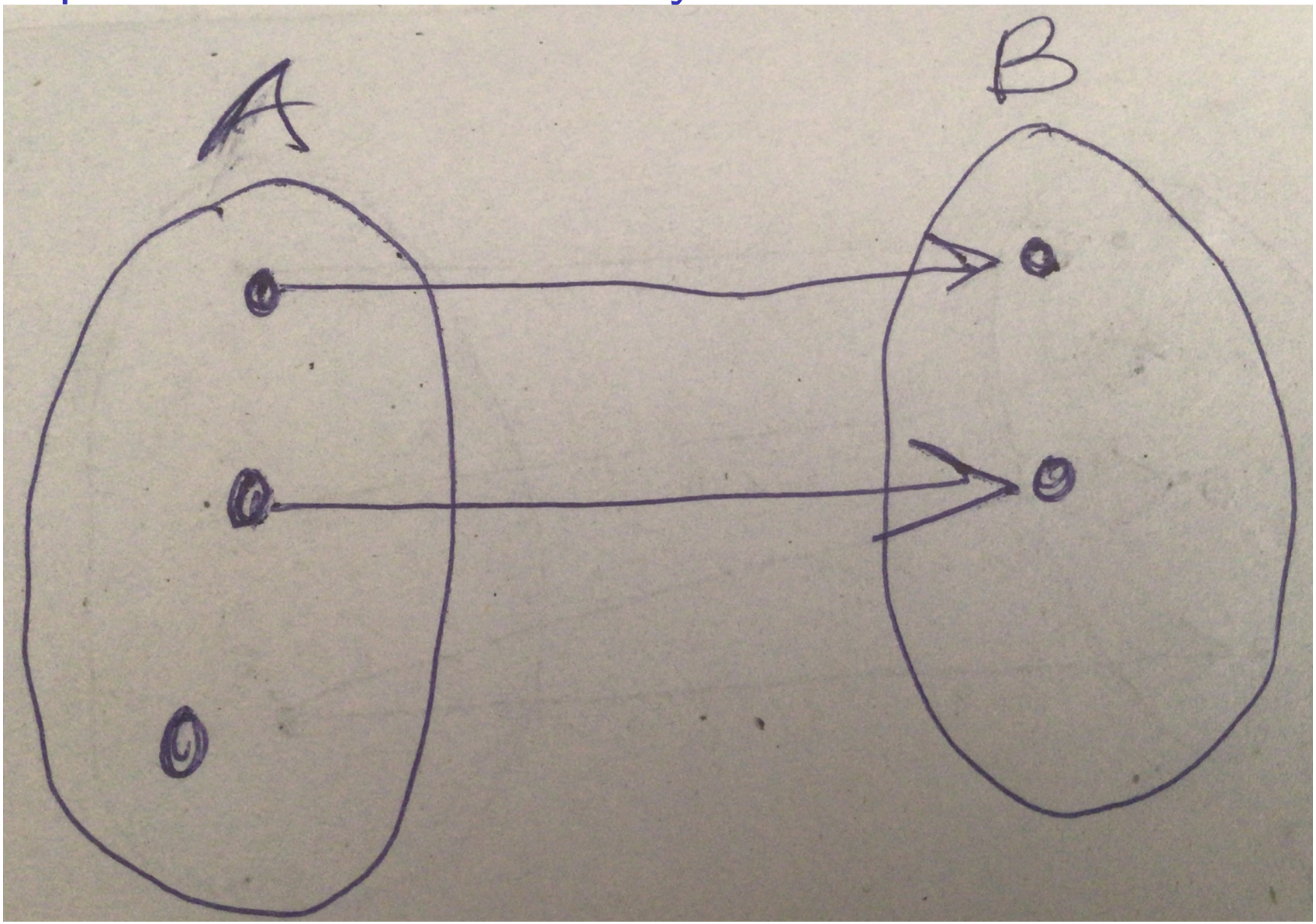


Figure 1: Not a function

Example: Not a function. Why?

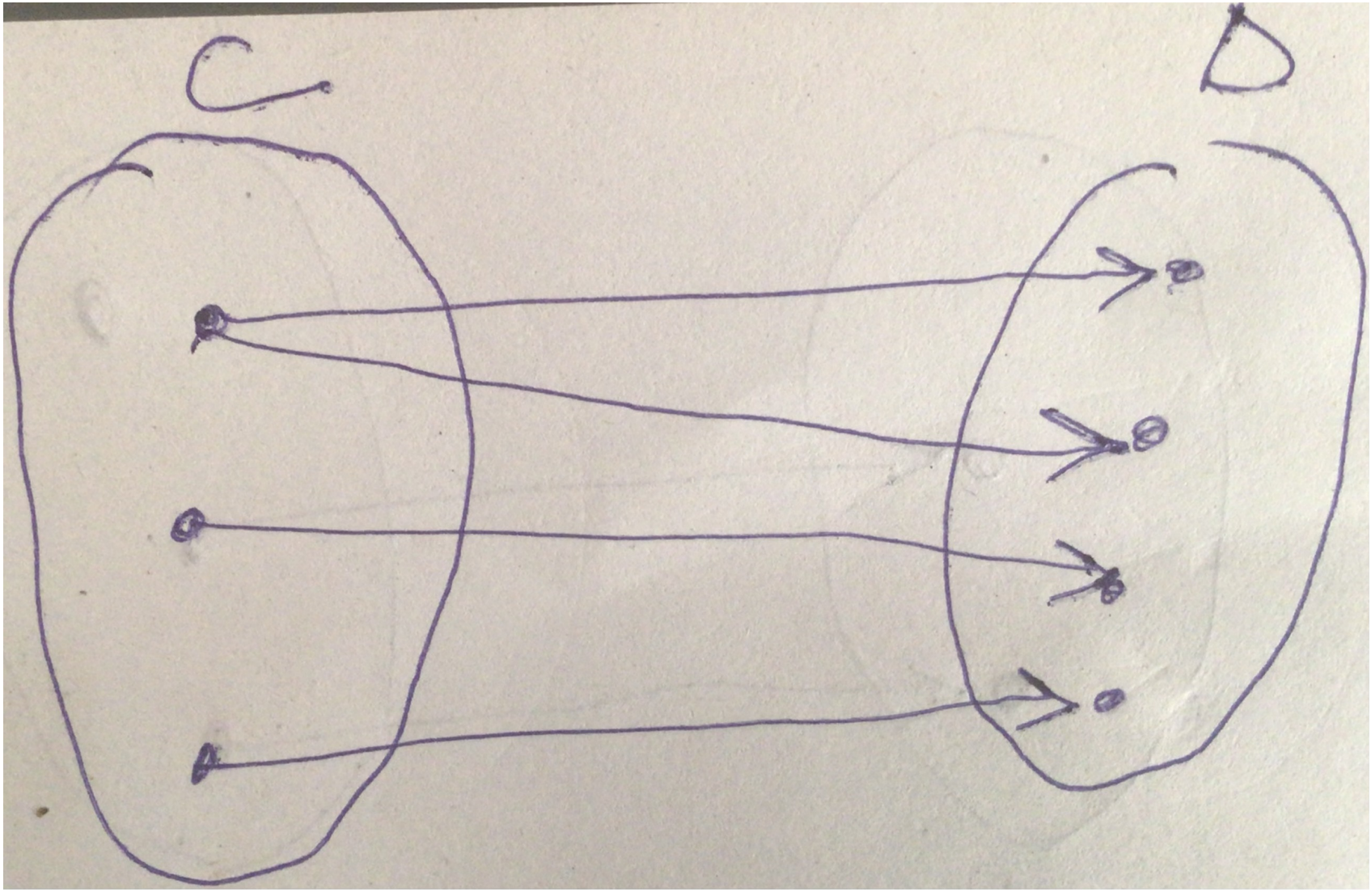


Figure 2: Not a function

Example: Is a function. Why?

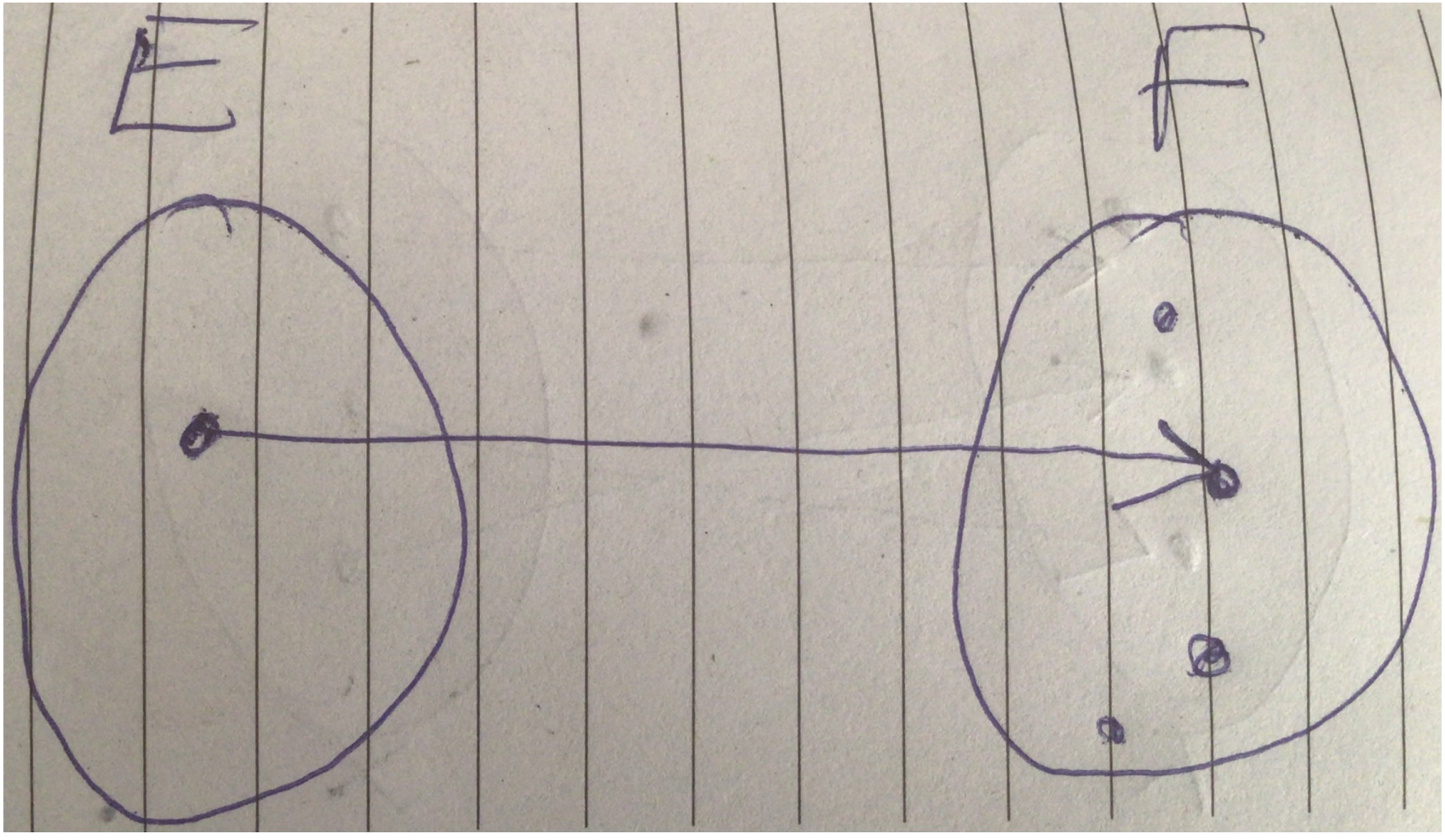


Figure 3: Is a function

Example: Is a function. Why? Obviously

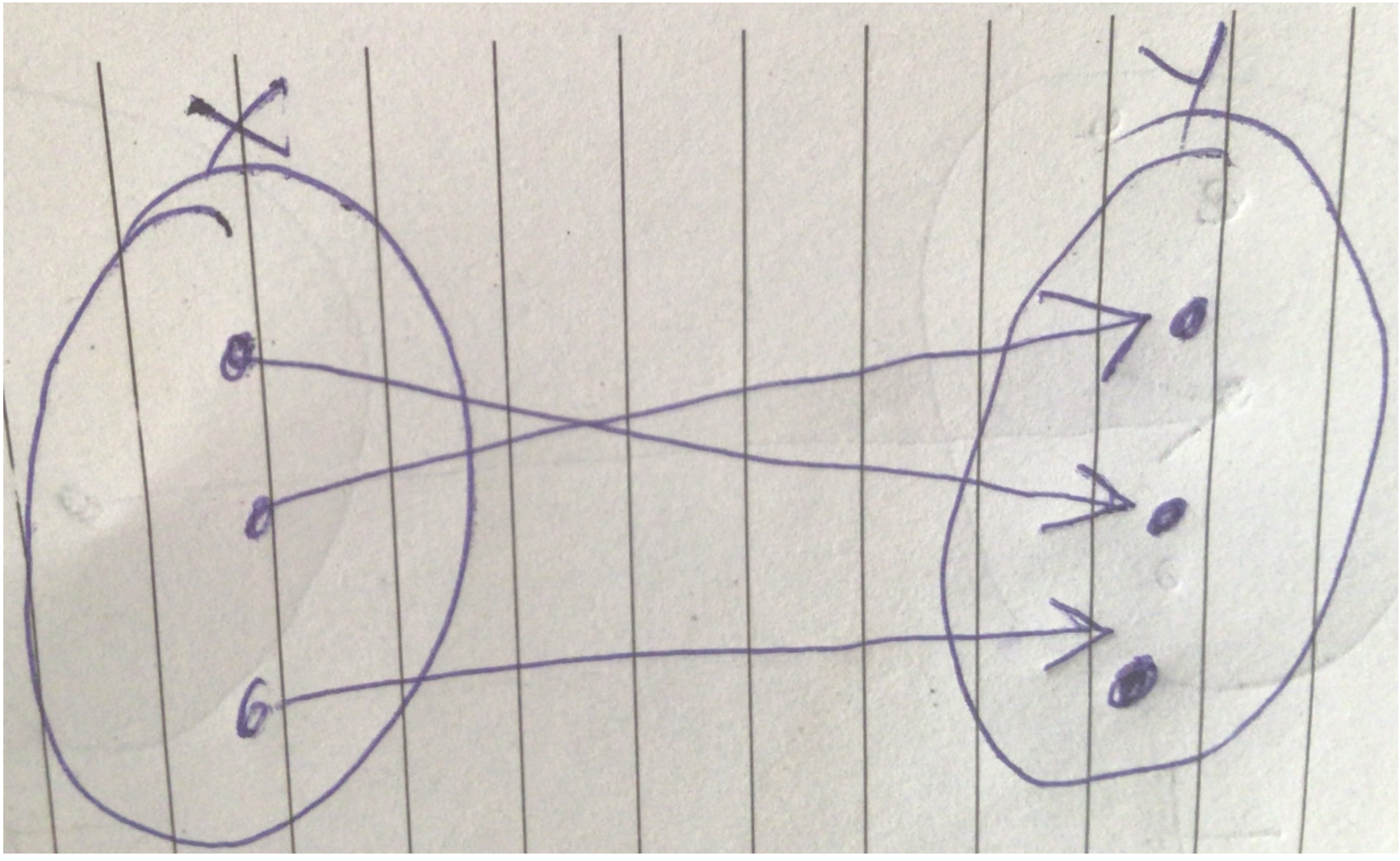


Figure 4: Is a function

Example: Is a function. Why? Obviously

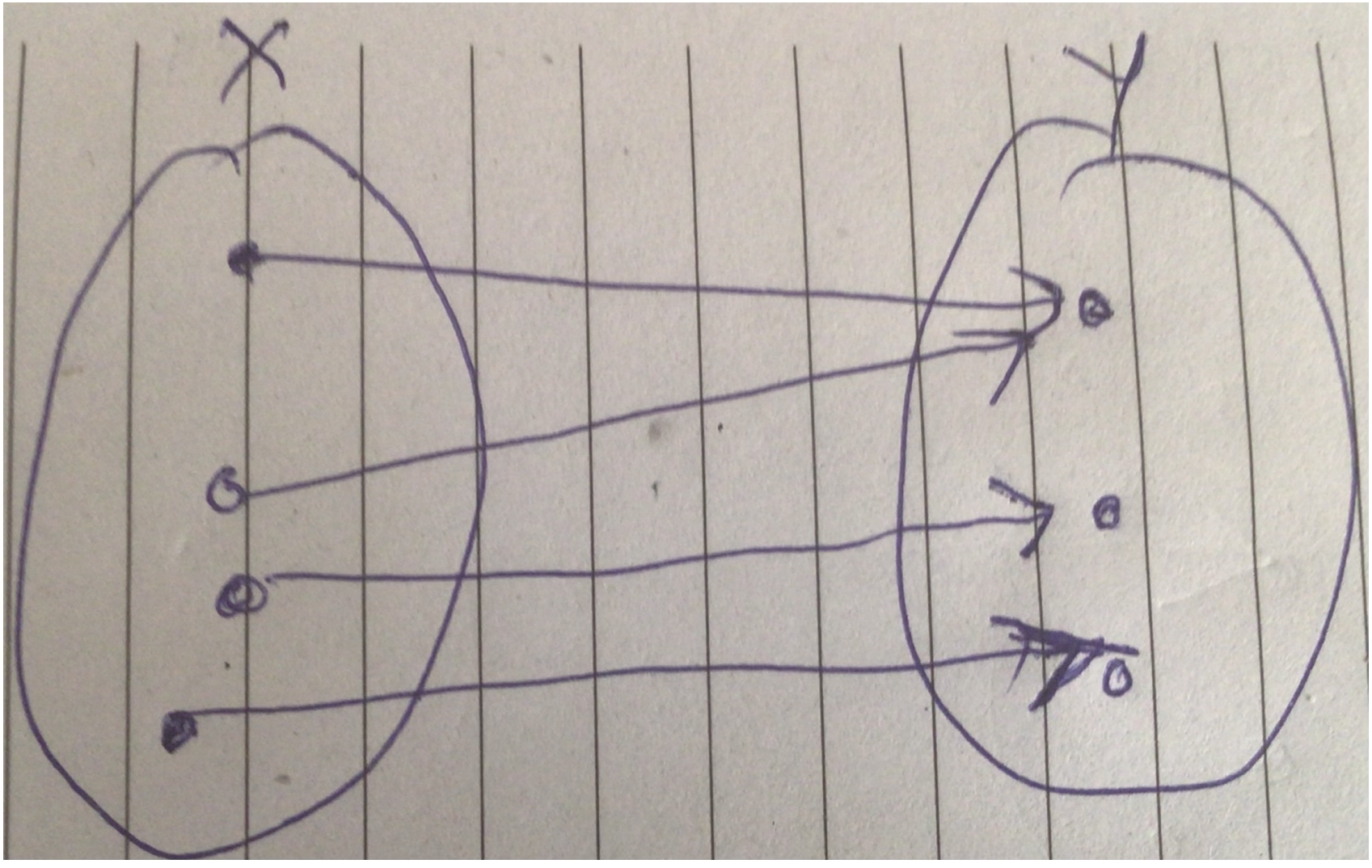


Figure 5: Is a function

Defining a Function

- The letters x and y are commonly used in maths, although these can be replaced with any other letters in the alphabet.
- You will commonly see a function written as;

$y = f(x)$

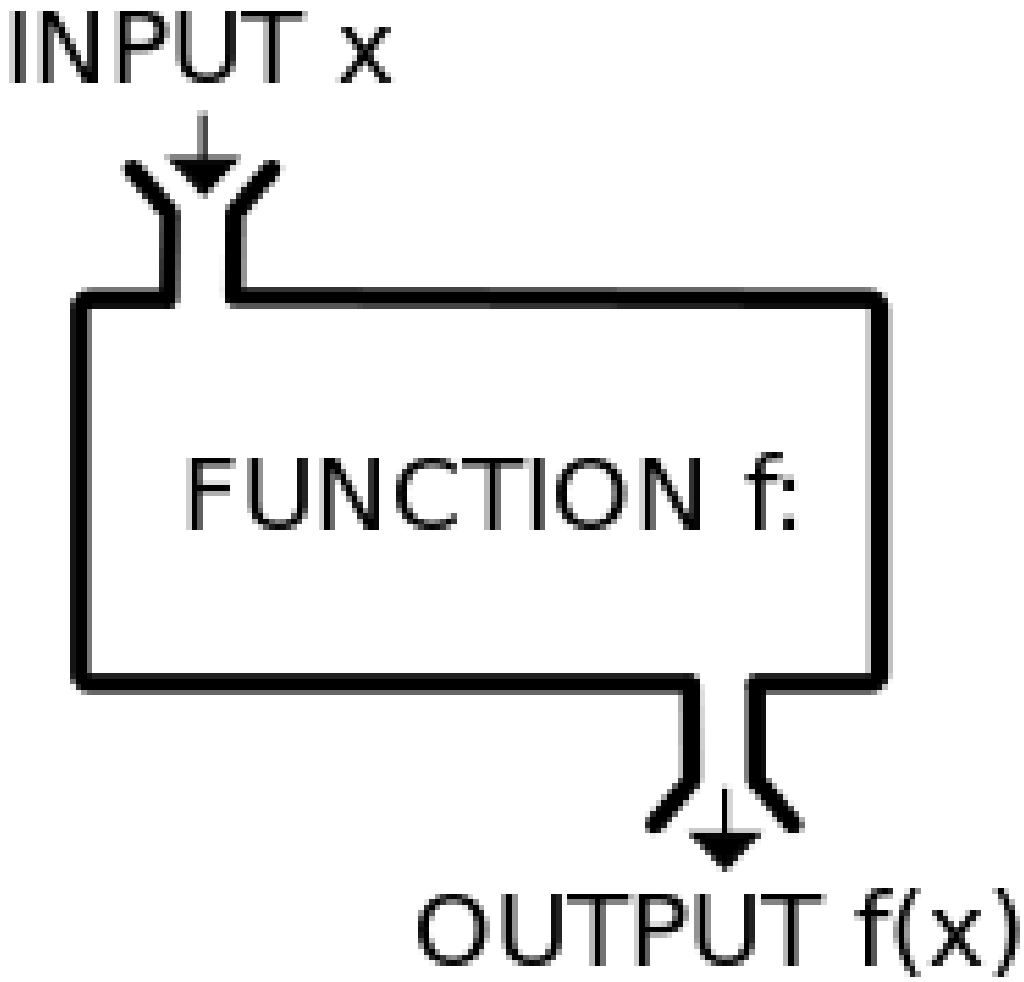
$y = f(x)$ means that y is a function of x. In other words, x can be used to represent y.

Defining a function.

- Consider this; $y = 2x + 3$. The moment we know x, we can compute y.
- Are there two or more values of x that yield the same value of y in the equation above?
- The answer is no. This is what we mean by "one-to-one" correspondence.
- Each x that goes into the machine yields one and only one y
- In a situation where we one input of x yielding two or more values of y, then our "machine" is no longer a function but a relation.

Defining a function in a picture.

- This is a function [Source:]



Working with $y = 2x + 3$.

- When we have $x = -5$, then $y = 2 * -5 + 3$ equals -7.
- When we have $x = 0$, then $y = 2 * 0 + 3$ equals 3.
- And so on.
- REPEAT: If you have a case where you put in one value of x and get two or more values of y, then that is not a `function` but a `relation`.

Examples of NOT functions.

- The equation $(x-h)^2+(y-k)^2=R^2$ represents a circle whose center is {h,k} and a radius of R.
- For instance, $x^2+y^2=49$ is a circle centred at {0,0}, with a radius of 7.
- What happens when $x=0$. We have $y^2=49$ giving $y=7$ or $y=-7$.
- This goes against the definition of a function. There is no one-to-one correspondence.
- Research online on other equations that are not functions.

Visualizing our circle.

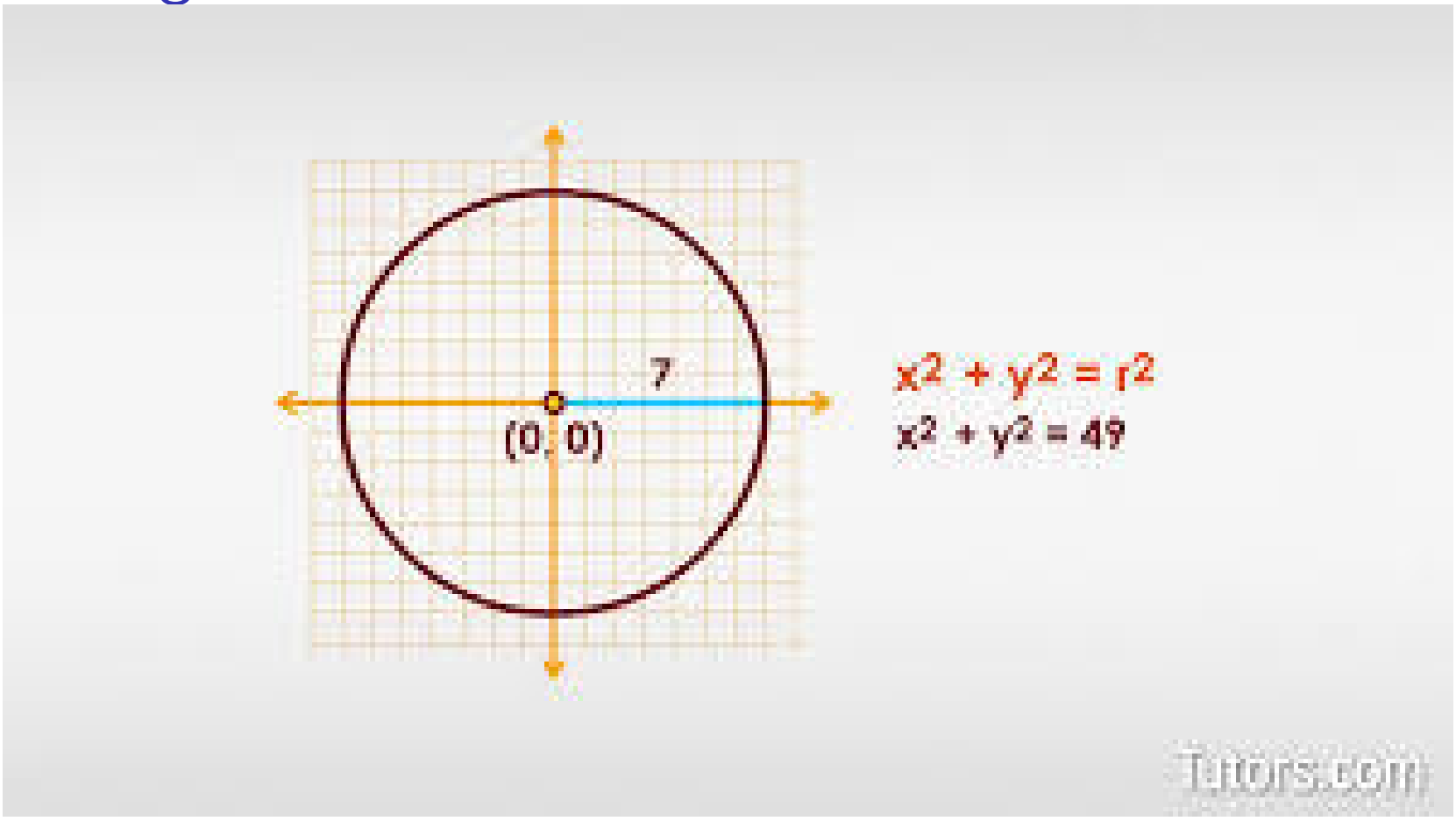


Figure 7: my-circle

- You can see there is no one to one correspondence.
- x = 0, gives two values of y.

Questions to ponder.

- The quadratic equation is represented as;

$y = ax^2 + bx + c$

- Is this a function?
- Research the equation of oval shapes. Is this a function?
- Ask yourself, can I input a value x and end up with two or more values of y?
- if yes, this is not a function. Otherwise, it is a function.
- Does $y^2 = x$ express a function of x? Hint: Rewrite as $y = f(x)$.

Domain and Range of a function

- The set of values that `x` in a function can take is referred to as the `domain`.
- Given the domain, the set of values that `y` can take is the `range`.

Example; what is the domain and range of the function $y = x^2$

Solution:

- There are no limits on the values x can take so the `domain` is any negative, positive number, and zero.
- Note that fractions are also admissible. So any number and zero consist of the range. The set of all possible numbers here refers to `real numbers`- all those numbers present in the number line, including zero.
- However, the `range` is the set if all numbers greater than or equal to zero. Why?
- What is the square of a negative number?

Exercises: Domain and Range

Find the domain and range of each of;

- $y = 2x + 3$
- $y = 4x^2$
- $y = \sqrt{(4 - x)}$
- $y = 1 + \frac{1}{x-2}$
- $y = log(x)$, use log to base 10.

Exercises: Graphing a function

Graph each of the functions in the example above.

Exercises

if $y = f(x) = 2x^3 - 3x^2 + 1$ find the following;

(i) $f(3)$, (ii) $f(-1)$, (iii) $f(a)$, and (iv) $f(-a)$

The relationship between the number of individual income tax returns filed electronically and the number of years after 1995 can be described by the equation

$y = f(x) = 5.091x + 11.545$

where y is in millions and x is the number of years after 1995.

(a) Find f(8).
(b) Write a sentence that explains the meaning of the result in (a).

Functions covered in this course.

- Linear functions, inequalities and equations.
- Quadratic and other special functions.
- Exponential and logarithmic functions.