Bond and Common Share Valuation

Lakehead University

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Outline of the Lecture

- Bonds and Bond Valuation
- The Determinants of Interest Rates
- Common Share Valuation

Bonds and Bond Valuation

A corporation's long-term debt is usually involves interest-only loans.

If, for example, a firm wants to borrow \$1,000 for 30 years and the actual interest rate on loans with similar risk characteristics is 12%, then the firm will pay a total of \$120 in interest each year for 30 years and repay the \$1,000 loan after 30 years.

The security that guarantees these payments is called a bond. A bond may involve more than one interest payment during a year.

In the above example, interest payments could be as follows:

- one payment of \$120 per year;
- two payments of \$60 per year;
- four payments of \$30 per year;
- any arrangement such that a total of \$120 in interest is paid each year.

Bonds and Bond Valuation

With a single interest payment per year, the timing of cash flows to the lender is as follows:

Year	0	1	2	3	...	29	30
Interest		\$120	\$120	\$120		\$120	\$120
Principal							\$1,000

With semiannual payments, the timing is:

Year	0	0.5	1	1.5	2	2.5	3	...	29	29.5	30
Interest		\$60	\$60	\$60	\$60	\$60	\$60		\$60	\$60	\$60
Principal											\$1,000

Bonds and Bond Valuation

A bond is characterized by the following items:

Face Value (F): The amount of principal to be repaid at the bond's maturity date.

Coupon Rate (i): The fraction of F paid in interest each year.

Maturity (T): The number of years until the face value is repaid.

Number of Payments (m): Number of interest payments in a year.

Bonds and Bond Valuation

The annual coupon payment of a bond is then

$$C = i \times F.$$

If the bond makes m payments per year, each coupon payment is

$$C_m = \frac{C}{m} = \frac{iF}{m}$$

and there are $m \times T$ of these payments over the life of the bond.

A bond is usually issued at par, i.e. it sells for $\$F$ when issued.

If, for example, the face value of a bond is \$1,000, an investor pays \$1,000 for the bond when issued.

As time evolves, the return required by buyers of bonds with similar characteristics changes. This required return depends on the market interest rates.

Market interest rates determine the yield to maturity of a bond, which is the annual return to an individual buying the bond at its market price and holding it until maturity.

Bonds and Bond Valuation

The yield to maturity of a bond is an APR, not an EAR.

Let y denote the yield to maturity of a bond, which is also the yield to maturity of bonds with similar risk characteristics.

The market interest rate of a bond between each coupon payment is then $r_m = y/m$.

Bonds and Bond Valuation

What is the price of a bond with a face value $F = \$1,000$, a coupon rate $i = 12\%$ and a time to maturity $T = 30$ years if the bond makes annual interest payments and the rate of return on securities with similar characteristics (yield to maturity) is 10%?

Year	0	1	2	3	…	29	30
Interest		\$120	\$120	\$120		\$120	\$120
Principal							\$1,000

Bonds and Bond Valuation

The price of this bond is

$$
\begin{aligned}
P &= \frac{120}{.10}\left(1-\left(\frac{1}{1.10}\right)^{30}\right) + \frac{1,000}{(1.10)^{30}} \\
&= 120 \times 9.4269 + 1,000 \times 0.0573 \\
&= \$1,188.53
\end{aligned}
$$

Bonds and Bond Valuation

What if the bond makes semiannual coupon payments?

Each coupon payment is then $120/2 = \$60$, the bond makes $2 \times 30 = 60$ coupon payments and the rate of return between payments is $10\%/2 = 5\%$.

Year	0	0.5	1	1.5	2	2.5	3	…	29	29.5	30
Interest		\$60	\$60	\$60	\$60	\$60	\$60		\$60	\$60	\$60
Principal											\$1,000

Bonds and Bond Valuation

The price of this bond is

$$
\begin{aligned}
P &= \frac{60}{.05}\left(1-\left(\frac{1}{1.05}\right)^{60}\right) + \frac{1,000}{(1.05)^{60}} \\
&= 1,135.75 + 53.54 \\
&= \$1,189.29
\end{aligned}
$$

Bonds and Bond Valuation

More generally, the price of a bond making m coupon payments per year over T years when the yield to maturity is y is

$$
\begin{aligned}
P &= \frac{C_m}{r_m}\left(1-\left(\frac{1}{1+r_m}\right)^{Tm}\right) + \frac{F}{(1+r_m)^{Tm}} \\
&= \frac{iF/m}{y/m}\left(1-\left(\frac{1}{1+y/m}\right)^{Tm}\right) + \frac{F}{(1+y/m)^{Tm}} \\
&= \frac{iF}{y}\left(1-\left(\frac{1}{1+y/m}\right)^{Tm}\right) + \frac{F}{(1+y/m)^{Tm}}
\end{aligned}
$$

Bonds and Bond Valuation

Note that

$$P \begin{cases} > F & \text{if } i > y, \\ = F & \text{if } i = y, \\ < F & \text{if } i < y. \end{cases}$$

Bonds and Bond Valuation

A bond is said to

- sell at a premium when $P > F$;
- sell at par when $P = F$;
- sell at a discount when $P < F$.

Zero-Coupon Bonds

A zero-coupon bond is a bond that does not make coupon payments, i.e. it is a bond with a 0% coupon rate.

The price of a zero-coupon bond with $1,000 face value and 22 years to maturity when the return on similar bonds is 6% is

$$P = \frac{1,000}{(1.06)^{22}} = \$277.51.$$

Clearly, a zero-coupon bond always sells at a discount.

Perpetual Bonds

If a bond has no maturity date, its price is

$$P = \frac{C_m}{r_m},$$

where C_m is the size of a coupon payment and r_m is the discount rate between payments.

Note that this is the formula for a perpetuity.

Bonds and Bond Valuation

A bond is normally issued at par, i.e. bonds with a coupon rate of 12% are issued when the yield to maturity of similar bonds is 12%.

If the face value of the bond is $1,000, its price at the time it is issued is $1,000.

If, later on, the yield to maturity of similar bonds increases above (decreases below) 12%, then the bond price will be less (more) than $1,000.

Bonds and Bond Valuation

Knowing the face value F, the coupon rate i, the time to maturity T and the number of coupon payments per year m, we can compute P when y is known and vice versa.

Most often, we know the price at which a bond trades and we use the bond features (F, i and T) to determine its yield to maturity.

Most Canadian bonds make semiannual coupon payments.

Bond prices are usually quoted as a percentage of face value.

If a bond with a face value of \$5,000 is quoted at 97.02, this means that the bond price is

$$97.02\% \times 5,000 \; = \; \$4,851.$$

If this bond has a coupon rate of 8%, makes semiannual payments and has 21 years to maturity, can we find its yield to maturity?

The price of the bond is

$$4,851 \; = \; \frac{200}{y/2}\left(1-\left(\frac{1}{1+y/2}\right)^{42}\right) \; + \; \frac{5,000}{(1+y/2)^{42}}.$$

What do we know about y? It has to be more than 8% since the bond is selling at a discount.

With a computer, we find 8.29%. It is also possible to approximate y.

When $y = 8\%$, the bond price is \$5,000.

When $y = 9\%$, the bond price is \$4,532.

To find the y, we set

$$\frac{y-8}{9-8} = \frac{4,851-5,000}{4,532-5,000},$$

which gives 8.32%.

Bonds and Bond Valuation

The Current Yield

The current yield is defined as the annual interest payment divided by the bond price, i.e.

$$\text{Current yield} = \frac{C}{P}$$

Interest Rate Risk

A bond trader may be interested in two types of gains:

- Interest income
- Capital gain arising from an increase in the bond price

The risk associated with bond price changes arising from changes in market interest rates is called interest rate risk.

Interest Rate Risk

Changes in market interest rates affect bond prices.

Let P_0 denote the bond price when $y = y_0$, and suppose that y changes to y_1, inducing a new bond price P_1. The sensitivity of a bond price to a change in interest rate is defined as

$$\frac{|P_1 - P_0|}{P_0} = \frac{|\Delta P|}{P_0},$$

i.e. the percentage increase or decrease in the bond price.

Interest Rate Risk

The sensitivity of a bond price to changes in y depends on the bond's characteristics. With respect to i and T remember the following results:

1. All other things being equal, the lower i, the greater the interest rate risk;
2. All other things being equal, the greater T, the greater the interest rate risk;

Determinants of Interest Rates

The interest rate or required return represents the cost of money.

It is the compensation for lending money.

When money is lent or borrowed, this cost is referred to as interest rate.

When considering the sale or purchase of ownership interest, such as common shares, this cost is referred to as required return.

Determinants of Interest Rates

Real versus Nominal Rates

The nominal rate of interest is the rate charged by the supplier of funds and paid by the demander of funds.

For example, the rate specified on a credit card is a nominal rate.

But if you pay 17% in nominal interest, what are you *really* paying?

Similarly, if you earn a 17% nominal return on some investment, what is your *real* gain?

Determinants of Interest Rates

Real versus Nominal Rates

What is real?

Goods and services are real.

Why do people work to earn money?

To purchase goods and services (for themselves or for others).

Thus the ultimate return on an investment or the ultimate cost of borrowing money should be measured in term of goods and services or, more specifically, in terms of purchasing power.

Determinants of Interest Rates

Real versus Nominal Rates

Suppose an asset provides a nominal return of 10% over a period during which the average price of goods and services increases by 5%.

That is, the inflation rate during this period is 5%.

$100 invested in this asset at the beginning of the period returns $110 at the end of the period but what is the return on this asset in terms of, say, apples?

Determinants of Interest Rates

Real versus Nominal Rates

An inflation rate of 5% means that the average price of goods and services at the end of the period is 1.05 times what it was at the beginning of the period.

Hence, what could have been purchased for $1 at the beginning of the period costs $1.05 at the end of the period.

Inflation erodes the purchasing power of money.

Real versus Nominal Rates

For instance, let p_0 denote the price of an apple at time 0, i.e. at the beginning of the period and let q_0 denote the number of apples that can be purchased with \$100 at time 0. That is,

$$q_0 = \frac{100}{p_0}.$$

How many apples can \$110 buy at time 1, i.e. at the end of the period?

Real versus Nominal Rates

Let q_1 denote the number of apples that can be purchased with \$110 at time 1. Since the price of an apple at time 1 is $1.05p_0$,

$$q_1 = \frac{110}{1.05p_0}.$$

Determinants of Interest Rates

Real versus Nominal Rates

In terms of apples, the return on the investment described above is then

$$\frac{q_1 - q_0}{q_0} = \frac{110/1.05p_0 - 100/p_0}{100/p_0} = \frac{1.10}{1.05} - 1.$$

This return can be referred to as the real return on the investment.

Determinants of Interest Rates

Real versus Nominal Rates

More generally, if i, π and r denote the nominal return on an investment, the inflation rate and the real rate of return on the investment, respectively, then

$$r = \frac{1+i}{1+\pi} - 1.$$

That is,

$$(1+r)(1+\pi) = 1+i \quad \Rightarrow \quad 1+r+\pi+r\pi = 1+i.$$

Real versus Nominal Rates

The last equation can be rewritten as

$$r = i - \pi - r\pi.$$

Since r and π are usually small fractions, the term $r\pi$ is often negligible and thus

$$r \approx i - \pi.$$

The nominal rate of return on an investment depends on inflation and on the riskiness of the investment.

The nominal rate a company must pay to borrow can be expressed as the risk-free rate plus a risk premium.

Determinants of Interest Rates

A rate of return often used as the risk-free rate is the 3-month T-bill rate.

Let R_f denote the risk-free rate, IP the inflation premium, k^* *the* real rate of interest, RP_j the risk premium on security j and k_j the nominal return on security j. Then

$$k_j = R_f + RP_j = \underbrace{k^* + IP}_{R_f} + RP_j.$$

Determinants of Interest Rates

Inflation is measured as the change in the consumer price index (CPI).

The inflation premium considered when setting nominal rates is the expected inflation rate over the life of the asset, which may be different from the rate of inflation experienced in the immediate past.

Determinants of Interest Rates

Risk Premium and Issuer Characteristics

For a given issuer of debt, the nominal rate is given by

$$k_j = k^* + IP + RP_j.$$

How is RP_j determined?

A Note on Real and Nominal Rates

The yield to maturity of a bond, or its market interest rate, is a nominal rate.

It is a nominal rate because it is used to discount nominal payments.

Nominal rates should be used to discount nominal payments.

Real rates should be used to discount real payments.

A Note on Real and Nominal Rates

Example

Consider a $1,000 bond making annual coupon payments of $120. If there are 20 years left until maturity, the nominal cash flows of this bond are

	0	1	2	3	...	19	20
Interest		$120	$120	$120		$120	$120
Principal							$1,000

A Note on Real and Nominal Rates

Example (Continued)

If the bond's yield to maturity is 8%, then its price is

$$P = \frac{120}{.08}\left(1 - \left(\frac{1}{1.08}\right)^{20}\right) + \frac{1,000}{(1.08)^{20}} = \$1,393.$$

A Note on Real and Nominal Rates

Example (Continued)

If the annual inflation rate is expected to be 4% over the next 20 years, then the bond's real cash flows are

	0	1	2	3	…	19	20
Interest		\$115.38	\$110.95	\$106.68		\$56.96	\$54.77
Principal							\$456.39

discounted at the real interest rate $r = \frac{1.08}{1.04} - 1 = 3.8\%$, the present value of these cash flows is also \$1,393.

The Term Structure of Interest Rates

The term structure of interest rates refers to the relationship between time to maturity and yields for a particular category of bonds at a particular time.

Ideally, other factors such as the risk of default are held constant across the bonds represented in a yield curve.

The term structure of interest rates has three basic components:

Real Rate of Interest: Does not really affect the shape of the term structure, mostly affects the overall level of interest rates.

Inflation Premium: Future inflation stongly affects the shape of the term structure.

Interest Rate Risk Premium: Interest rate risk increases with time to maturity.

Theories to Explain the Term Structure of Interest Rates

Expectation Hypothesis: The yield curve reflects investor expectations about future interest rates and inflation.

Liquidity Preference Theory: Investors require a premium for tying up funds for longer periods. Short-term securities are perceived as less risky than long-term securities. Borrowers are willing to pay a premium to obtain funds for long periods.

The Term Structure of Interest Rates

Theories to Explain the Term Structure of Interest Rates

Market Segmentation Theory: The market for loans is segmented on the basis of maturity and the supply and demand of funds within each segment determine the prevailing interest rates.

Interest Rates in an International Environment

Compare these two alternatives:

- invest \$1 in the U.S. a the rate $i^{\$}$ and enter a forward contract that exchanges US\$ for Swiss Francs (SF) at the rate $F^{\mathrm{SF}/\$}$.
- Exchange US\$ for SF today at the spot rate $S^{\mathrm{SF}/\$}$ and invest it is Switzerland at the rate i^{SF}.

Interest Rates in an International Environment

Both investments should return the same number of SF in the end, i.e.

$$\left(1+i^{\$}\right)F^{\mathrm{SF}/\$} = \left(1+i^{\mathrm{SF}}\right)S^{\mathrm{SF}/\$}$$

Interest Rates in an International Environment

Let $F_n^{\mathrm{SF}/\$}$ denote n-day forward rate for the SF/$ exchange rate.

Let i^c denote annual interest rate in (currency c)-denominated deposits.

The spot and forward rates are considered to be at interest rate parity if

$$\frac{F_n^{\mathrm{SF}/\$}}{S^{\mathrm{SF}/\$}} = \frac{1+i^{\mathrm{SF}}\times\frac{n}{360}}{1+i^{\$}\times\frac{n}{360}}$$

Common Stock Valuation

Consider a stock that promises to pay a $1 dividend one year from now.

If the stock price in one year is expected to be $25, then the overall cash flow to a stockholder one year from now is expected to be $26.

If the return an investor requires to invest in such a stock is 12%, how much will he accept to pay for the stock?

Common Stock Valuation

Answer:

$$\text{PV of the stock cash flows} = \frac{26}{1.12} = \$23.21,$$

and thus this investor would never pay more than $23.21 for this stock.

Similarly, a stockholder with the same beliefs would never sell the stock for less than $23.21, and thus this value must be the equilibrium stock price (without transaction costs).

Common Stock Valuation

More generally, let

$$\begin{aligned} P_0 &\equiv \text{today's stock price;} \\ P_1 &\equiv \text{stock price one year from now;} \\ D_1 &\equiv \text{dividend payment one year from now;} \\ k &\equiv \text{required return on this type of investment.} \end{aligned}$$

Then

$$P_0 = \frac{D_1 + P_1}{1+k}.$$

Common Stock Valuation

Let P_t, D_t denote the stock price and dividend in year t.

What is P_1?

$$P_1 = \frac{D_2 + P_2}{1+k}.$$

What is P_2?

$$P_2 = \frac{D_3 + P_3}{1+k}.$$

Common Stock Valuation

Therefore,

$$\begin{aligned}
P_0 &= \frac{D_1 + P_1}{1+k} \\
&= \frac{D_1 + \frac{D_2+P_2}{1+k}}{1+k} = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{P_2}{(1+k)^2} \\
&= \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{\frac{D_3+P_3}{1+k}}{(1+k)^2} = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \frac{P_3}{(1+k)^3} \\
&\vdots \\
&= \sum_{t=1}^{T} \frac{D_t}{(1+k)^t} + \frac{P_T}{(1+k)^T}.
\end{aligned}$$

Common Stock Valuation

If

$$\lim_{T\to\infty} \frac{P_T}{(1+k)^T} = 0,$$

then

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}.$$

That is, the price of a stock is the present value of its future dividend payments into perpetuity.

Special Cases

Zero Growth

Suppose $D_t = D$ for all t. Then

$$P_0 = \sum_{t=1}^{\infty} \frac{D}{(1+k)^t} = \frac{D}{k}.$$

Special Cases

Constant Growth

Suppose $D_t = (1+g)D_{t-1}$ for all t.

$$\begin{aligned} P_0 &= \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \\ &= \frac{D_1}{1+k} + \frac{(1+g)D_1}{(1+k)^2} + \frac{(1+g)^2 D_1}{(1+k)^3} + \cdots \\ &= \frac{D_1}{k-g} \text{ if } g < k. \end{aligned}$$

Special Cases

Two-Stage Growth

Suppose $g = g_1$ for the first T years, after which $g = g_2$ into perpetuity. Then

$$
\begin{aligned}
P_0 &= \frac{D_1}{1+k} + \frac{(1+g)D_1}{(1+k)^2} + \frac{(1+g)^2 D_1}{(1+k)^3} + \ldots + \frac{(1+g)^{T-1}D_1}{(1+k)^T} + \frac{P_T}{(1+k)^T} \\
&= \frac{D_1}{k-g_1}\left(1-\left(\frac{1+g_1}{1+k}\right)^T\right) + \frac{D_{T+1}/(k-g_2)}{(1+k)^T} \\
&= \frac{D_1}{k-g_1}\left(1-\left(\frac{1+g_1}{1+k}\right)^T\right) + \frac{(1+g_2)(1+g_1)^{T-1}D_1}{(k-g_2)(1+k)^T}.
\end{aligned}
$$

Components of the Required Return

Rearranging

$$P_0 = \frac{D_1}{k-g},$$

we obtain

$$k = \frac{D_1}{P_0} + g,$$

where $\frac{D_1}{P_0}$ can be referred to as the dividend yield and g can be referred to as the capital gains yield.

Growth Opportunities

Consider a firm that never invests its earnings in new projects, i.e. it pays all of its earnings as dividends. Suppose, moreover, that earnings are expected to be constant forever. For such a firm,

$$P_0 = \frac{D}{k} = \frac{\text{EPS}}{k}.$$

Growth Opportunities

Suppose now that the firm retains some of its earnings to invest in new projects. More specifically, suppose the firm always retains a fraction b of its earnings each year and suppose the new projects the firm invests in generate a constant return k_i each year. That is,

$$\text{EPS}_t = \text{EPS}_{t-1} + k_i \times b\text{EPS}_{t-1},$$

and thus

$$\text{EPS}_t = (1 + k_i b)\text{EPS}_{t-1}.$$

Growth Opportunities

The growth rate in earnings being $k_i b$, the growth rate in dividends is

$$\frac{D_t - D_{t-1}}{D_{t-1}} = \frac{(1-b)\text{EPS}_t - (1-b)\text{EPS}_{t-1}}{(1-b)\text{EPS}_{t-1}} = \frac{\text{EPS}_t - \text{EPS}_{t-1}}{\text{EPS}_{t-1}} = k_i b.$$

The stock price is then

$$P_0 = \frac{D_1}{k - k_i b} = \frac{(1-b)\text{EPS}_1}{k - k_i b}.$$

Growth Opportunities

If $k_i > k$, then

$$P_0 = \frac{(1-b)\text{EPS}_1}{k - k_i b} > \frac{(1-b)\text{EPS}_1}{k - kb} = \frac{(1-b)\text{EPS}_1}{(1-b)k} = \frac{\text{EPS}_1}{k},$$

so we could say that

$$P_0 = \frac{\text{EPS}_1}{k} + \text{NPVGO},$$

where NPVGO stands for net present value of growth opportunities. If $k_i > k$, then NPVGO > 0.

Growth Opportunities

If, on the other hand, $k_i < k$, then

$$P_0 < \frac{\text{EPS}_1}{k}.$$

Since

$$P_0 = \frac{\text{EPS}_1}{k} + \text{NPVGO},$$

we would have NPVGO < 0 in this case.

Estimating the Growth Rate

We found earlier that the growth rate in dividends was $k_i \times b$.

What is k_i?.

The return on the firm's investments can be approximated by firm's return on equity, and thus the growth rate in dividends can be approximated by

$$\text{ROE} \times b = \text{ROE} \times (1 - \text{Payout ratio}).$$