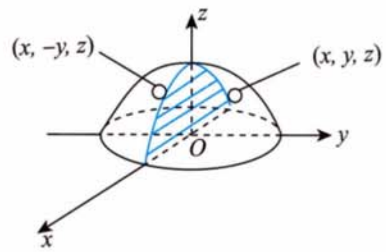


图 18-1

关于其他坐标面对称的情况与此类似．

(2) 轮换对称性．

在直角坐标系下，若把 x 与 y 对调后，Ω 不变，则 $\iiint\limits_{\Omega} f(x, y, z)\mathrm{d}x\mathrm{d}y\mathrm{d}z = \iiint\limits_{\Omega} f(y, x, z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$，这就是**轮换对称性**．

$\mathrm{d}v=\mathrm{d}x\mathrm{d}y\mathrm{d}z$ 具有交换律，可两两交换，

如 $x^2+y^2+\frac{z^2}{2}\leqslant 1$

用轮换对称性就是为了相加后，被积函数简单

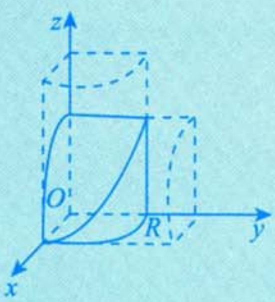
关于其他情况与此类似．

如 $\Omega=\left\{(x, y, z)\middle|x^2+y^2+z^2\leqslant R^2\right\}$，则 $\iiint\limits_{\Omega} f(x)\mathrm{d}x\mathrm{d}y\mathrm{d}z = \iiint\limits_{\Omega} f(y)\mathrm{d}x\mathrm{d}y\mathrm{d}z = \iiint\limits_{\Omega} f(z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$ 可以化简计算．

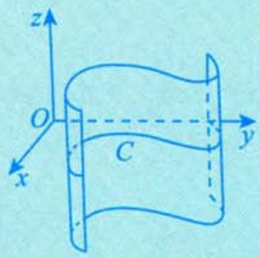
具体应用见后面的例子．

注 空间区域 Ω 大观．

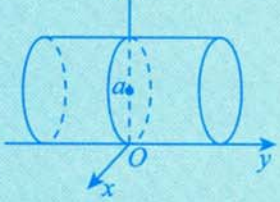
考生应能熟练画出以下图形，并时常翻之，看之，动手画图．



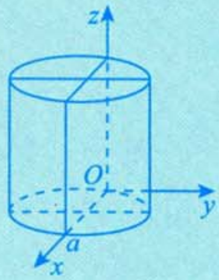
$$\begin{cases} x^2+y^2=R^2, \\ x^2+z^2=R^2 \end{cases}$$



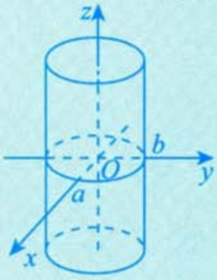
$F(x,y)=0$



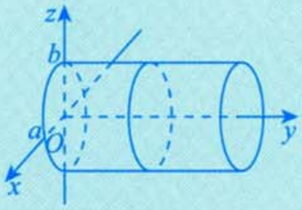
$x^2+z^2=2az$



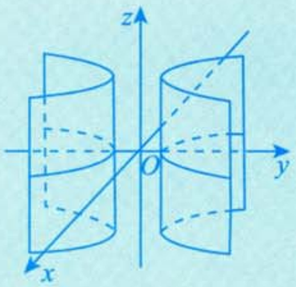
$$\left(x-\frac{a}{2}\right)^2+y^2=\left(\frac{a}{2}\right)^2$$



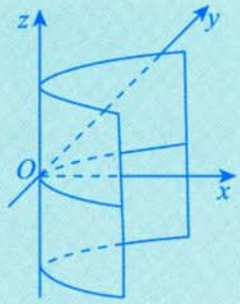
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



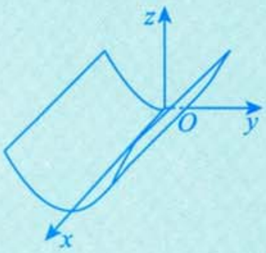
$$\frac{x^2}{a^2}+\frac{z^2}{b^2}=1$$



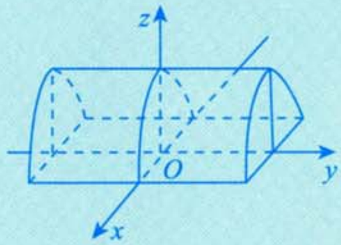
$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



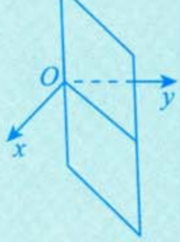
$$y^2=x$$



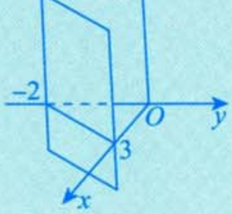
$$z=y^2$$



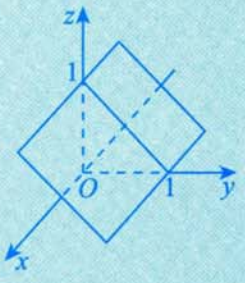
$$z=1-x^2$$



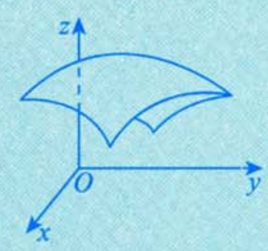
$$x-y=0$$



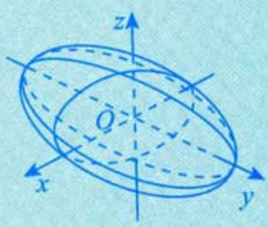
$$2x-3y-6=0$$



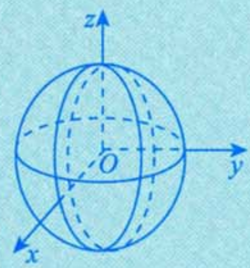
$y+z=1$



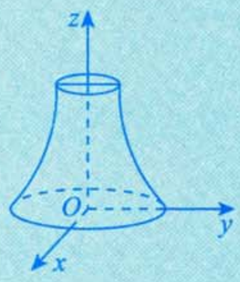
$F(x,y,z)=0$



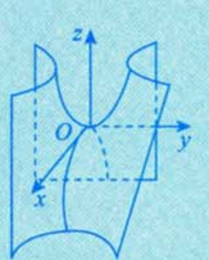
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



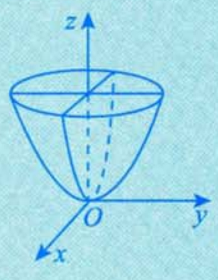
$x^2+y^2+z^2=R^2$



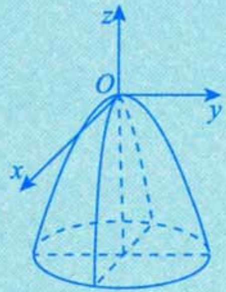
$f(\pm\sqrt{x^2+y^2},\ z)=0$



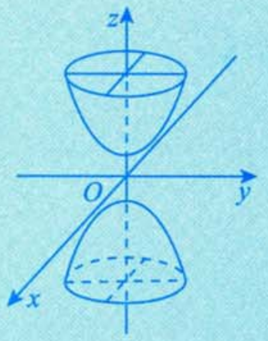
$$-\frac{x^2}{2p}+\frac{y^2}{2q}=z,\quad p,q>0$$



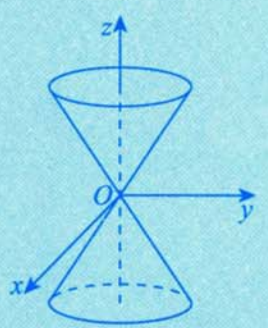
$$\frac{x^2}{2p}+\frac{y^2}{2q}=z,\quad p,q>0$$



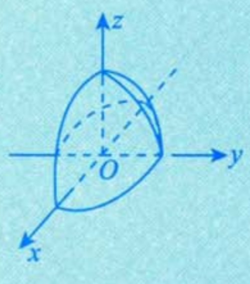
$$\frac{x^2}{2p}+\frac{y^2}{2q}=z,\quad p,q<0$$



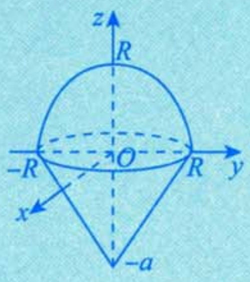
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$



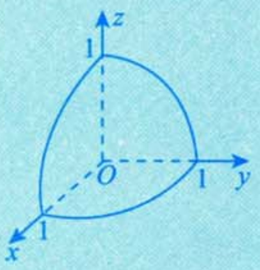
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$



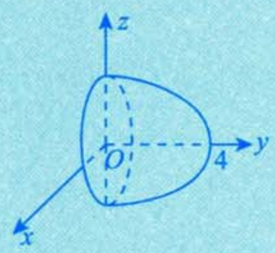
$$z=1-x^2-2y^2,\ z\geqslant 0$$



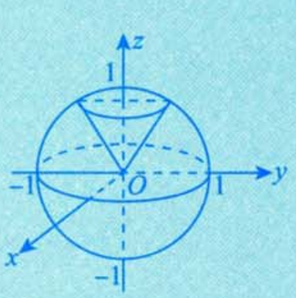
$$\begin{cases} z=\sqrt{R^2-x^2-y^2}, \\ z=\dfrac{a}{R}\sqrt{x^2+y^2}-a \end{cases}(R>0,\ a>0)$$



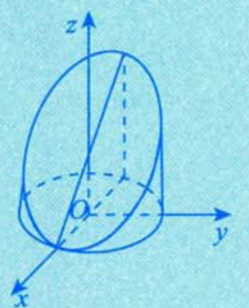
$$z=\sqrt{1-x^2-y^2},\ x\geqslant 0,\ y\geqslant 0$$



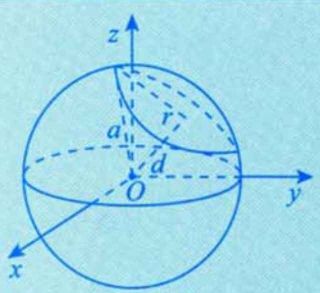
$$4-y=x^2+z^2,\quad y\geqslant 0$$



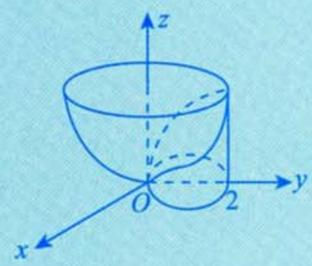
$$z=\sqrt{x^2+y^2},\quad x^2+y^2+z^2\leqslant 1$$



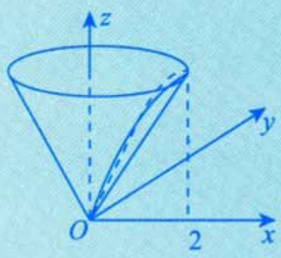
$$\begin{cases} x^2+y^2=4, \\ x+z=2, \\ z=0 \end{cases}$$



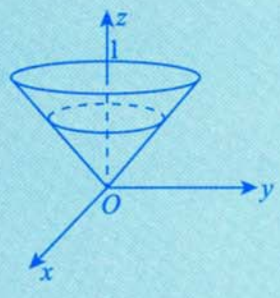
$$\begin{cases} x^2+y^2+z^2=a^2, \\ x+y+z=\dfrac{3}{2}a \end{cases} (a>0)$$



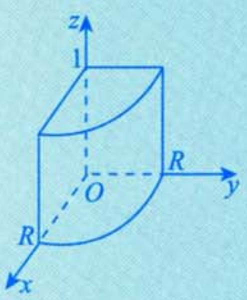
$$\begin{cases} x^2+(y-1)^2=1, \\ z=x^2+y^2 \end{cases}$$



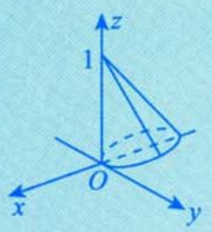
$$\begin{cases} z=\sqrt{x^2+y^2}, \\ z^2=2x \end{cases}$$



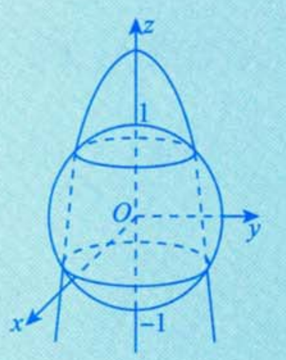
$$\begin{cases} z=\sqrt{x^2+y^2}, \\ z=1 \end{cases}$$



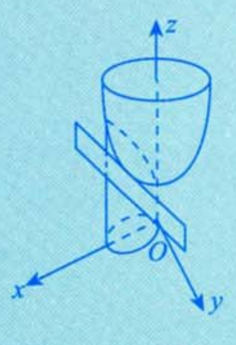
$x^2+y^2=R^2$， $x,y>0$， $0<z<1$



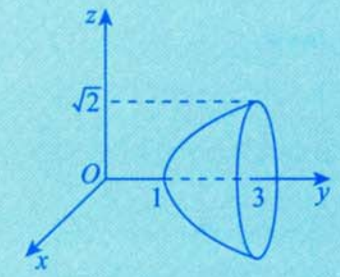
$x^2+2x(1-z)+y^2=0$， $0\leqslant z\leqslant 1$



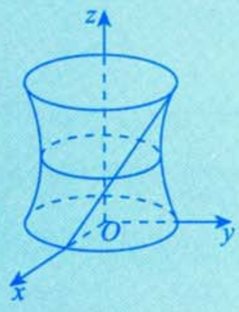
$$\begin{cases} x^2+y^2+z^2=1, \\ 6(x^2+y^2)=-z+4 \end{cases}$$



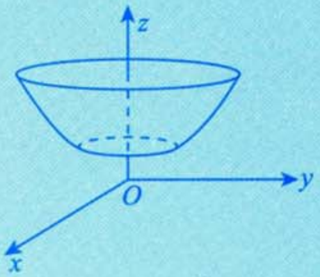
$$\begin{cases} z=x^2+y^2+1, \\ z=2x, \\ (x-1)^2+y^2=1 \end{cases}$$



$\begin{cases} z=\sqrt{y-1}, \\ x=0 \end{cases}(1\leqslant y\leqslant 3)$（绕 y 轴旋转）

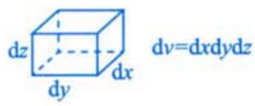


$\begin{cases} x=1-z, \\ y=z \end{cases}$（绕 z 轴旋转）



$\begin{cases} z=\mathrm{e}^{y}, \\ x=0 \end{cases}(1\leqslant y\leqslant 2)$（绕 z 轴旋转）

4 计算

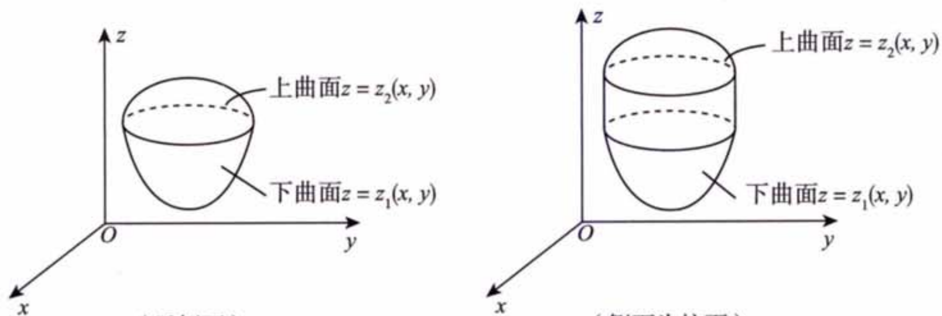


(1) 直角坐标系 .

①先一后二法（先 z 后 xy 法，也叫**投影穿线法**）.

a. 适用场合 .

Ω有下曲面 $z=z_1(x,y)$、上曲面 $z=z_2(x,y)$，无侧面或侧面为柱面，如图 18-2 所示 .



（无侧面）
(a)

（侧面为柱面）
(b)

图 18-2

b. 计算方法 .

如图 18-3 所示，有 $\iiint\limits_{\Omega} f(x,y,z)\mathrm{d}v=\iint\limits_{D_{xy}}\mathrm{d}\sigma\int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\mathrm{d}z$.