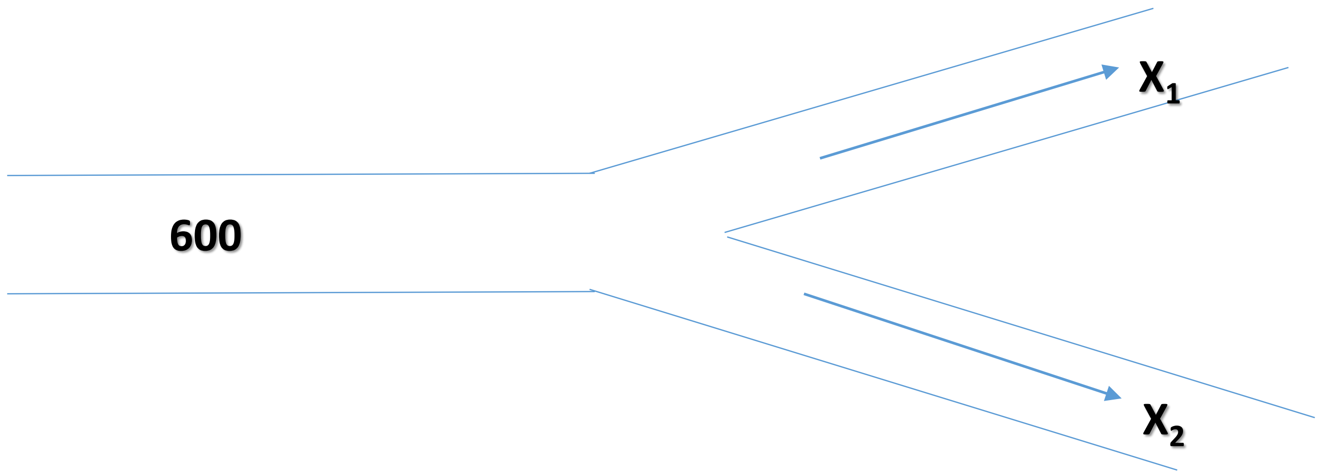
LINEAR ALGEBRA APPLICATIONS

Areas

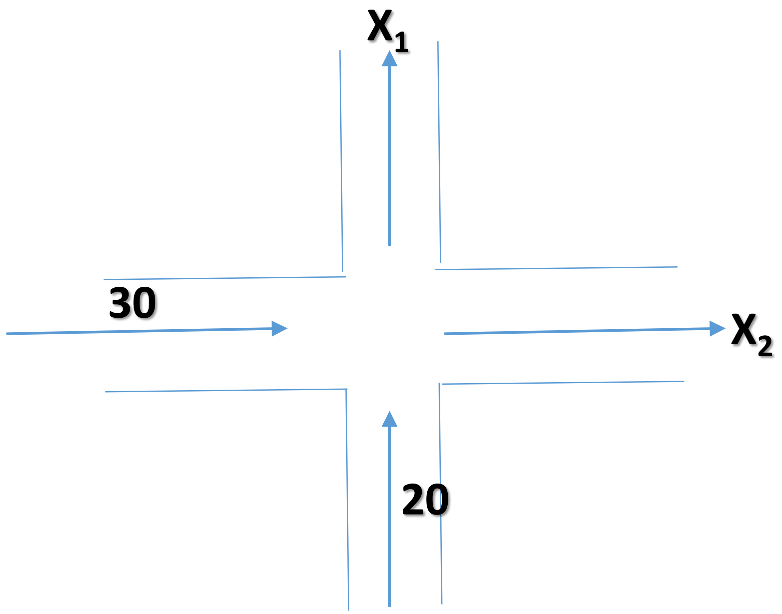
1. Signal Analysis – A signal is a sequence of numbers. Signals can be in form of audio, video or image. Signal Analysis use Fourier analysis
2. Facial Recognition – PCA Principal Component Analysis is a linear algebraic technique used to analyze facial features
3. Predictions in Linear Models
4. Ranking in search engines
5. Traffic Control models

TRAFFIC CONTROL

- Rule: Traffic inflow at a junction = outflow

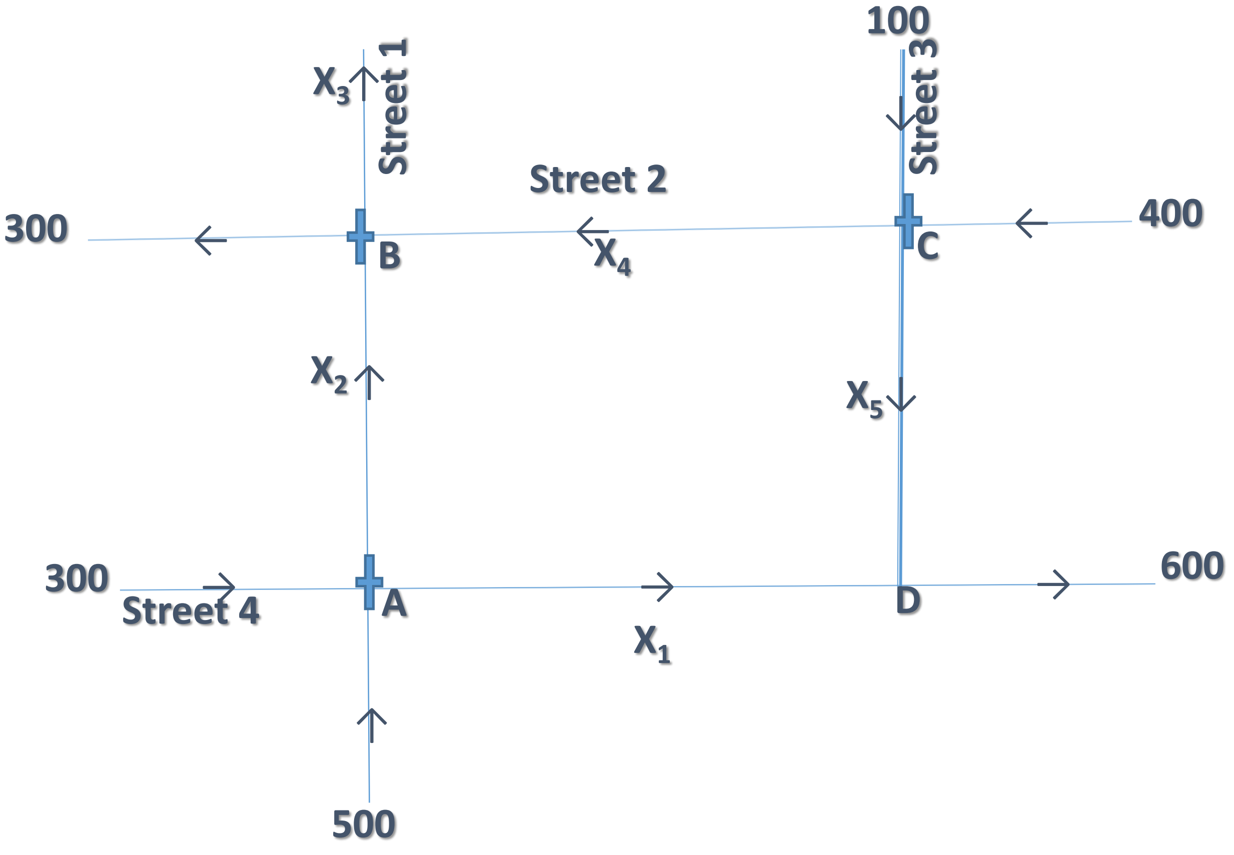


$600 = X_1 + X_2$



$30 + 20 = X_2 + X_1$

Several Junctions Scenario



JUNCTION	TRAFFIC IN FLOW	TRAFFIC OUT FLOW
A	300 + 500	$X_1 + X_2$
B	$X_2 + X_4$	$X_3 + 300$
C	100 + 400	$X_4 + X_5$
D	$X_1 + X_5$	600

System of Linear Equations

$X_1 + X_2 = 800$

$X_2 - X_3 + X_4 = 300$

$X_4 + X_5 = 500$

$X_1 + X_5 = 600$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 800 \\ 0 & 1 & -1 & 1 & 0 & 300 \\ 0 & 0 & 0 & 1 & 1 & 500 \\ 1 & 0 & 0 & 0 & 1 & 600 \end{bmatrix}$$

Row Echelon Form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 600 \\ 0 & 1 & 0 & 0 & -1 & 200 \\ 0 & 0 & 1 & 0 & 0 & 400 \\ 0 & 0 & 0 & 1 & 1 & 500 \end{bmatrix}$$

$X_1 + X_5 = 600$

$X_2 - X_5 = 200$

$X_3 = 400$

$X_4 + X_5 = 500$

Write Solution in terms of X_5

$X_1 = 600 - X_5$

$X_2 = 200 + X_5$

$X_3 = 400$

$X_4 = 500 - X_5$

X_5 must be <=500 e.g.

suppose $X_5 = 400$, $X_4=100$, $X_3=400$, $X_2=600$, X1=200

Reduce column 6 by dividing by 100

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & -1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 & 5 \\ 1 & 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

Row echelon

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \cancel{x_6} \end{array} \leftarrow \text{unknowns} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & -1 & 1 & 0 & 3 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{bmatrix} \end{array}$$

$x_1 + x_2 = 8$

$x_2 - x_3 + x_4 = 3$

$-x_3 + x_4 + x_5 = 1$

$x_4 + x_5 = 5$

$x_1 = 8 - x_2$

$x_2 = 3 + x_3 - x_4$

$x_3 = -1 + x_4 + x_5$

$x_4 = 5 - x_5$

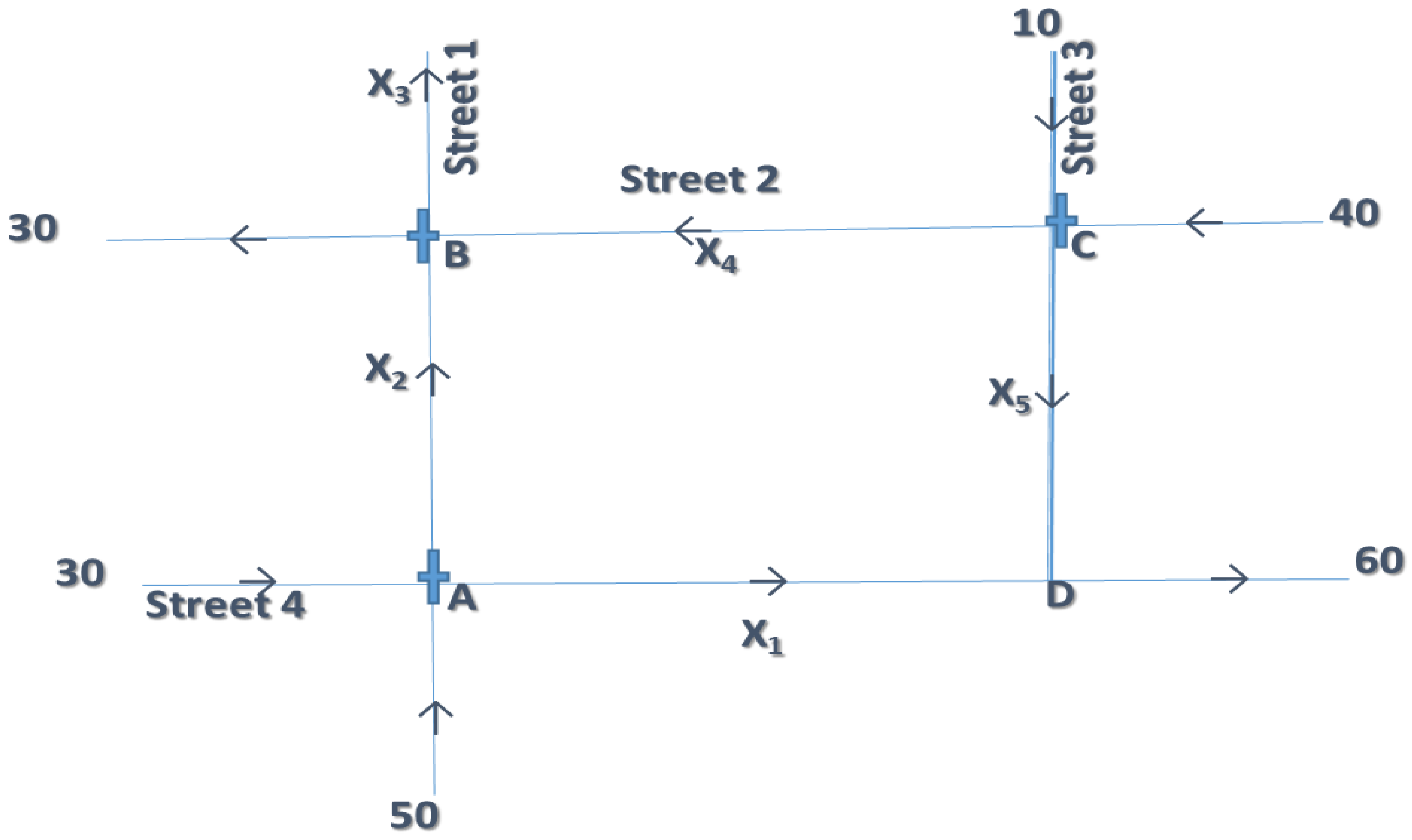
$x_5 \le 5$ e.g if $x_5 = 4$

$x_4 = 1$

$x_1 = 8 - 6 = 2$

$x_2 = 3 + 4 - 1 = 6$ $\quad x_3 = -1 + 1 + 4 = 4$

Linear Algebra applications range from signal analysis to machine learning problems. Below is a diagram depicting several road junctions and the number of vehicles counted on various streets in a span of 12 hours. Using row echelon method to perform row reduction, compute the values of X_1, X_2, X_3, X_4 and X_5; which in the end can guide the traffic control department in effective traffic management at these junctions.



SOLUTION

Subtract row 1 from row 4: $R_4 = R_4 - R_1$.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 80 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 1 & 50 \\ 0 & -1 & 0 & 0 & 1 & -20 \end{bmatrix}$$

Subtract row 2 from row 1: $R_1 = R_1 - R_2$.

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 50 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 1 & 50 \\ 0 & -1 & 0 & 0 & 1 & -20 \end{bmatrix}$$

Add row 2 to row 4: $R_4 = R_4 + R_2$.

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 50 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 1 & 50 \\ 0 & 0 & -1 & 1 & 1 & 10 \end{bmatrix}$$

Since the element at row 3 and column 3 (pivot element) equals 0, we need to swap the rows.

Find the first nonzero element in column 3 under the pivot entry.

The first nonzero element is at row 4.

Swap the rows 3 and 4:

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 50 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & -1 & 1 & 1 & 10 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$$

Multiply row 3 by -1: $R_3 = -R_3$.

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 50 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & 1 & -1 & -1 & -10 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$$

Subtract row 3 from row 1: $R_1 = R_1 - R_3$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 60 \\ 0 & 1 & -1 & 1 & 0 & 30 \\ 0 & 0 & 1 & -1 & -1 & -10 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$$

Add row 3 to row 2: $R_2 = R_2 + R_3$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 60 \\ 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -1 & -10 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$$

Add row 4 to row 3: $R_3 = R_3 + R_4$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 60 \\ 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 1 & 0 & 0 & 40 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$$

The reduced row echelon form is $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 60 \\ 0 & 1 & 0 & 0 & -1 & 20 \\ 0 & 0 & 1 & 0 & 0 & 40 \\ 0 & 0 & 0 & 1 & 1 & 50 \end{bmatrix}$ A.

Questions?
Comments?