OPERATIONS RESEARCH ASSIGNMENT II TO BE DONE IN GROUPS OF 5 STUDENTS DUE DATE IS 27/02/2019 MR. KICHE

SECTION A.

Q1) Find the dual program of the following linear programming problem.

$$\begin{gathered} Maximize \qquad z = 5x_1 - 2x_2 \\ subject\ to \\ 3x_1 + 2x_2 \geq 16 \\ x_1 - x_2 \leq 4 \\ x_1 \geq 5 \\ x_1 \geq 0, \qquad x_2\ is\ unconstrained \end{gathered}$$

Q2) Find the dual program of the following linear programming problem.

$$\begin{gathered} Minimize \qquad Z = 30x_1 - 50x_2 + 10x_3 \\ subject\ to \\ 3x_1 + 2x_2 - x_3 \geq 44 \\ x_1 - x_2 + x_3 = 7 \\ x_1\ is\ unconstrained, \qquad x_2 \geq 0, \quad x_3 \geq 0, \end{gathered}$$

Q3) Find the dual program of the following linear programming problem.

$$\begin{gathered} Minimize \qquad P = 16x - 2y - 5z \\ subject\ to \\ x + 4y - z \geq 120 \\ x + y + 3z \leq 130 \\ x \geq 0, \qquad y \geq 0,\ \ z\ \ is\ unrestricted. \end{gathered}$$

SECTION B.

Q1.) A company has 5 salesmen and 5 customers to attend to on a particular day. The company has estimated the savings in dollars associated with assigning a particular salesman to a specific client. These estimates are as given in the table below

		Clients				
		1	2	3	4	5
	A	30	37	40	28	40
	B	40	24	27	21	36
Salesmen	C	40	32	33	30	35
	D	25	38	40	36	36
	E	29	62	41	34	39

Determine who should be assigned which client and the maximum profit the company can achieve from the allocations.

Q2) A company has fixed funds to undertake three projects through contractors. Five contractors have already applied to do the job and each has submitted a quotation for each project from which the company has estimated the saving associated with allocating a given project to a specific contractor. The figures are given in the table below and the company's policy is to give one project per contractor.

	Project		
Contractor	1	2	3
A	1020	1080	1050
B	1500	1410	1050
C	1110	750	1050
D	1080	1020	1080
E	1470	1290	1590

Determine who should be assigned which project and the maximum saving the company can make.

Q3) Five applicants are competing for four jobs . The scores from aptitude tests related to the four vacancies are given below. It is believed the tests measure an applicant possible performance in the job.

	JOBS			
APPLICANTS	1	2	3	4
A	18	15	12	25
B	9	11	10	15
C	12	10	14	16
D	9	10	10	21
E	14	18	26	26

Determine who should be assigned which job in order to maximize overall output.

SECTION C.

Q1.) Solve the following linear programming problem using the simplex method:

$$Maximize \qquad z = 14x + 15y$$
$$subject\ to$$
$$13x + 15y \leq 80$$
$$-12x - 17y \geq -120$$
$$x \geq 0, \quad y \geq 0$$

Q2) Use the simplex method to obtain the optimal solution of the dual of following linear programming model

$$Minimize \quad P = 70x_1 + 50x_2$$
$$subject\ to$$
$$40x_1 + 30x_2 \leq 2400$$
$$-20x_1 - 10x_2 \geq 1000$$
$$x_1 \geq 0, x_2 \geq 0$$

Q3) Use the simplex method to obtain the optimal solution of the following linear programming model

$$Maximize \quad Z = 35x_1 + 50x_2$$

$$subject\ to$$

$$3x_1 + x_2 \leq 30$$

$$x_1 + 2x_2 \leq 15$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1, x_2 \geq 0$$

Q4) Solve the dual of the following linear programming problem using the simplex method.

$$Maximize \qquad P = 20x + 30y + 45z$$
$$subject\ to$$
$$20x + 40y + 30z \leq 800$$
$$30x + 20y + 40z \leq 800$$
$$20x + 10y + 30z \leq 1000$$

$$x \geq 0, \qquad y \geq 0, \quad z \geq 0$$

Q5) Solve the following linear programming problem using the simplex method.

$$Minimize \qquad P = 2100y_1 + 2400y_2 + 10y_3 - 70y_4$$
$$subject\ to$$
$$25y_1 + 15y_2 + y_3 \geq 250$$
$$20y_1 + 30y_2 - y_3 - y_4 \geq 300$$
$$y_1 \geq 0, \qquad y_2 \geq 0, \qquad y_3 \geq 0\ , \qquad y_4 \geq 0$$

SECTION D.

$(Q1)$ Explain ways in which the CPM type of networks differ from PERT networks

$Q2)$ A small project is composed of 8 activities whose time estimates are listed below

	Activity		Time in weeks		
	i	j	Optimistic(a)	Most likely(M)	Pessimistic(b)
A	1	2	2	5	8
B	2	3	4	7	10

C	2	4	4	9	11
D	3	5	6	10	20
E	4	6	1	3	5
F	4	5	3	6	9
G	5	7	4	5	12
H	6	7	6	8	10

i) Develop a PERT network for the project .
ii) Determine the expected value and the variance for every activity.
iii) Calculate EST and LCT for every node.
iv) Find the critical path for the project .
v) Compute the probability of completing the project in 36 weeks .

Q3) A small project is composed of seven activities whose time estimates in hours are given below.

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
a	1	1	2	1	2	2	3
b	7	7	8	1	14	8	15
M	1	4	2	1	5	5	6

a=optimistic time.

b=pessimistic time

M=Most likely time.

i) Draw the project network.
ii) Find the expected duration and variance of each activity.
iii) Determine the critical path.
iv) Find the expected project completion time.

v) Calculate the probability that the project will be completed three weeks earlier than expected.

vi) If the project's due date is 18 weeks, find the probability of not meeting the due date.

Q4) A small project is composed of 7 activities whose time estimates in weeks are listed below:

Activity	Predecessors	Optimistic	Most likely	Pessimistic
A	-	1	2	4
B	-	5	6	7
C	-	2	4	5
D	A	1	3	4
E	C	4	5	7
F	A	3	4	5
G	B,D,E	1	2	3

i) Draw the network.

ii) Calculate the expected duration and variance of every task.

iii) Determine the critical path.

iv) Calculate the expected project duration and the variance of the project duration based on network analysis.

v) Calculate the probability that the project will be completed on or before a deadline of 10 weeks

Q5) A certain industrial project has the following data.

Activity (i,j)	A	B	C	D	E	F	G	H	I	J	K	L	M
Predecessor(s)	-	-	A	A	B	B	D,E	D,E	D,E	C,G	F,I	C,G	J,H,K
t_0	7	5	8	12	12	14	3	16	4	14	13	6	16
t_m	8	9	10	14	14.5	15	5	22	7	17	16	8	18
t_p	9	10	12	16	17	16	7	25	10	20	22	13	26

a) Explain the meaning of three time estimates

$$t_0, \quad t_m, \quad t_p$$

b) Construct the network diagram.

c) Find the critical path

d) For each activity , compute the expected time and variance.
e) Find the expected project duration and its variance.
f) Determine the probability of completing the project within 25 weeks.

SECTION E.

Q1) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean of 3 minutes.

i) What is the probability that a person arriving at the booth will have to wait?
ii) What is the average length of the queues that form time to time?
iii) The telephone department will install a second booth when convinced that an arrival would expect to have to wait at least three minutes for the phone. By how much must the flow of arrivals be increased in order to justify a second booth?

Q2) Trucks at a single platform weigh-bridge arrive according to Poisson probability distribution. The time required to weigh the truck follows an exponential probability distribution. The mean arrival rate is 12 trucks per day , and the mean service rate is 18 trucks per day. Determine the following:

i) The probability that no trucks are in the system.
ii) The average number of trucks waiting for service.
iii) The average time a truck waits for weighing service to begin.
iv) The probability that an arriving truck will have to wait for service.

Q3) Consider the transportation problem presented in the following table:

	Destination				
Origin	**1**	**2**	**3**	**4**	**Supply**
1	250	350	360	600	150
2	550	300	450	380	60
3	400	500	260	650	140
4	600	400	660	270	110
Demand	100	120	150	90	

Use Vogel Approximation Method to determine the minimum cost of transportation.

Q4) Consider the transportation problem presented in the following table:

		Destination		
Origin	**1**	**2**	**3**	**Supply**
1	2	7	4	50
2	3	3	1	80
3	5	4	7	70
4	1	6	2	140
Demand	70	90	180	340

Use North West Corner Rule to determine the optimal allocation that minimizes the transportation cost , perform Modified Distributed Algorithm (MODI) to determine the transportation cost.

Use Minimum Cost Rule to determine the optimal allocation that minimizes the transportation cost .

Q5) i) Describe the theory of convex sets in relation to solutions of linear programming problems.

ii) With illustrations , describe the following problems in transportation problems and give solutions.

a) degeneracy
b) prohibited routes.

iv) Distinguish between weak duality and strong duality as used in duality theorem giving examples.

v) Define unboundedness as applied in linear programming problems giving appropriate example.

vi) Define a convex set C in R^n and show that any point within a convex four sided-figure with vertices $y_1,\ y_2,\ y_3,\quad and\quad y_4$ is a convex combination of these extreme points.