

Priority Queue

Overview

- A PQ is a queue where each element has a priority ranking
- Elements with a higher priority are dequeued first
- Elements must be comparable/ordered so priority can be assigned
- A PQ uses a heap to determine which item is to be dequeued (based on priority)
- **What is a Heap?**
 - A tree-based DS that satisfies the heap invariant: If A is a parent node of B then A is ordered with respect to B for all nodes, A, B in the heap
 - Max Heap: Root is the largest value
 - Min Heap: Root is the smallest value
 - * ***PQ's ARE NOT HEAPS***
 - PQ is an Abstract Data Type meaning it can be implemented with other DS also (such as an array list)
 - * ***ALL HEAPS MUST BE TREES***
 - A binary heap is a binary tree with the heap invariant. Every node must have two children
 - A complete binary tree is a tree in which at every level, except the last, is completely filled and all the nodes are as far left as possible
- **Implementation**
 - A heap is conveniently represented using an array
 - Let **i** be the parent node index
 - Left Child Index: **$2i + 1$**
 - Right Child Index: **$2i + 2$**
 - Parent Index: **$(i - 1) / 2$**

- **Adding Elements**

- Elements are inserted at the bottom left position of the heap and are "bubbled up" to adhere to the heap property




- **Removing Elements**

- The element to be removed is swapped with the bottom right element (leaf node) and the root node is "bubbled down" to adhere to the heap property
- Polling - Remove root element (no searching required, $O(\log n)$)
 - If removeElement is not at root, element must be linearly searched to find the position of the element. Therefore, removal takes $O(n)$.
 - A HashMap can be used to map each element to an index for instant search $O(1)$

Why Use It?

- Dijkstra's Shortest Path Algo
- When you need to dynamically fetch the next best or next worst element
- Used in Huffman coding
- BFS algos such as A* use PQs to grab the next most promising node
- Used by Minimum Spanning Tree algo

Big O Analysis (PQ with binary heap)

 Operation	 Big O Notation	 Explanation
<u>Binary Heap construction</u>	$O(n)$	
<u>Adding</u>	$O(\log(n))$	Restoring the heap invariant
<u>Deletion (Polling).</u>	$O(\log(n))$	Restoring the heap invariant
<u>Deletion (RemoveAt).</u>	$O(n)$	Linear Search (no hashmap) + Restoring the heap invariant

<u>Aa</u> Operation	⋮ Big O Notation	≡ Explanation
<u>Peeking</u>	O(1)	
<u>Contains</u>	O(n)	Linear Search

Code Implementation

- <https://www.geeksforgeeks.org/priority-queue-using-binary-heap/>