PRACTICAL 1: SUM 1: A box contains 20 tickets numbered from 1 to 20. A ticket is drawn randomly from the box.

Find the probability that the number on the ticket is:

1️⃣ Divisible by 5

2️⃣ Not divisible by 2

3️⃣ Divisible by both 3 and 4

1️⃣ Ticket number divisible by 5

Numbers from 1 to 20 that are divisible by 5 are:
5, 10, 15, 20

→ Count = **4**

$$P(\text{divisible by }5) = \frac{4}{20} = \frac{1}{5}$$

✅ **Answer = 1/5**

2️⃣ Ticket number not divisible by 2

Numbers divisible by 2 (even numbers):
2, 4, 6, 8, 10, 12, 14, 16, 18, 20 → 10 numbers

Numbers **not divisible by 2** = 20 − 10 = **10**

$$P(\text{not divisible by }2) = \frac{10}{20} = \frac{1}{2}$$

✅ **Answer = 1/2**

3️⃣ Ticket number divisible by both 3 and 4

A number divisible by both 3 and 4 ⇒ must be divisible by **LCM(3, 4) = 12**

From 1 to 20, the only multiple of 12 is **12**

→ Count = **1**

$$P(\text{divisible by both 3 and 4}) = \frac{1}{20}$$

✅ **Answer = 1/20**

R CODE:

```
x=1:20

p1=sum(x%%5==0)/20

p2=sum(x%%2!=0)/20

p3=sum(x%%3==0 \& x%%4==0)/20

p1; p2; p3
```

OUTPUT:

```
> p1; p2; p3

\[1] 0.2

\[1] 0.5

\[1] 0.05

>
```

SUM 2: A bag contains 10 white and 11 black balls (total = 21).

Two balls are drawn simultaneously from the bag.

Find the probability of:

1️⃣ Both white balls

2️⃣ One white and one black ball

3️⃣ No white ball

Given:

- White balls = 10
- Black balls = 11
- Total balls = 21
- Two balls are drawn **simultaneously** (i.e., without replacement).

🔹 Total number of ways to draw 2 balls from 21

$$\text{Total ways} = \binom{21}{2} = \frac{21 \times 20}{2} = 210$$

1️⃣ Both white balls

Number of ways to choose 2 white balls:

$$\text{Favorable ways} = \binom{10}{2} = \frac{10 \times 9}{2} = 45$$

$$P(\text{both white}) = \frac{45}{210} = \frac{3}{14}$$

✅ **Answer = 3/14**

2️⃣ One white and one black ball

Ways to choose 1 white from 10: $\binom{10}{1} = 10$
Ways to choose 1 black from 11: $\binom{11}{1} = 11$

$$\text{Favorable ways} = 10 \times 11 = 110$$

$$P(\text{one white, one black}) = \frac{110}{210} = \frac{11}{21}$$

✅ **Answer = 11/21**

3️⃣ No white ball

"No white" means both are black.
Ways to choose 2 black balls:

$$\text{Favorable ways} = \binom{11}{2} = \frac{11 \times 10}{2} = 55$$

$$P(\text{no white}) = \frac{55}{210} = \frac{11}{42}$$

✅ **Answer = 11/42**

R CODE:

```
 total = choose(21,2)

p1 = choose(10,2)/total

p2 = (10\*11)/total

p3 = choose(11,2)/total

p1; p2; p3
```

OUTPUT:

```
> p1; p2; p3

\[1] 0.2142857

\[1] 0.5238095

\[1] 0.2619048

>
```

SUM 3: The face cards in a deck are Jack, Queen, and King.

There are 3 face cards × 4 suits = 12 face cards total.

If 4 cards are selected at random from these 12 face cards,

find the probability that all 4 cards belong to different suits.

Given:

- Total face cards = 12
  (Jack, Queen, King of each of 4 suits ⇒ 3 × 4 = 12)
- We draw **4 cards** at random from these 12 face cards.
  We must find the probability that **all 4 cards belong to different suits**.

Step 1: Total number of ways to choose 4 cards from 12

$$\text{Total ways} = \binom{12}{4} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$$

Step 2: Favorable cases (4 cards from 4 different suits)

There are **4 suits** (♠, ♥, ♦, ♣).
We must pick **1 card from each suit**, and it can be **any** of the 3 face cards (Jack, Queen, King).

- For 1st suit → 3 ways
- For 2nd suit → 3 ways
- For 3rd suit → 3 ways
- For 4th suit → 3 ways

So,

$$\text{Favorable ways} = 3^4 = 81$$

Step 3: Probability

$$P(\text{all 4 cards of different suits}) = \frac{\text{Favorable ways}}{\text{Total ways}} = \frac{81}{495}$$

Simplify the fraction:

$$\frac{81}{495} = \frac{9}{55}$$

✅ **Final Answer:**

$$\boxed{P = \frac{9}{55}}$$

R CODE:

```
total = choose(12,4)

favourable = 3^4

prob = favourable / total

prob
```

OUTPUT:

> prob

\[1] 0.1636364

>

SUM 4: There are 50 tickets numbered from 1 to 50.

You randomly select 5 tickets and arrange them in increasing order.

We need to find the probability that the middle (3rd) number is 30.

Total ways to choose 5 tickets from 1...50:

$$\binom{50}{5} = 2{,}118{,}760$$

For the middle (3rd) number to be **30** we must:

- **Include 30**, and
- choose **exactly 2 numbers from {1,...,29}** (there are 29 numbers below 30), and
- choose **exactly 2 numbers from {31,...,50}** (there are 20 numbers above 30).

Favourable ways:

$$\binom{29}{2} \times \binom{20}{2} = 406 \times 190 = 77{,}140$$

So the probability is

$$P = \frac{77{,}140}{2{,}118{,}760} = \frac{551}{15{,}134} \approx 0.0364081$$

Final answers:

- Exact: $\frac{551}{15134}$
- Decimal: ≈ 0.0364081
- Percentage: $\approx 3.6408\%$

R CODE:

prob = (choose(29,2) * choose(20,2)) / choose(50,5)

prob

OUTPUT:

> prob

\[1] 0.03640809

>

SUM 5: If the letters of the word RANDOM are arranged randomly,

find the probability that A and O are at the extreme positions (first and last).

Word: RANDOM

Number of letters = **6** (R, A, N, D, O, M)
All letters are **distinct**.

Step 1: Total possible arrangements

$$\text{Total arrangements} = 6! = 720$$

Step 2: Favourable cases (A and O at extreme positions)

There are two possible ways for A and O to be at the extremes:

1. A at the **first** position and O at the **last**, or
2. O at the **first** position and A at the **last**.

→ So, **2 ways** to place A and O.

After placing A and O, we have **4 remaining letters (R, N, D, M)** to arrange.

$$\text{Ways to arrange remaining 4 letters} = 4! = 24$$

$$\text{Favourable arrangements} = 2 \times 24 = 48$$

Step 3: Probability

$$P(\text{A and O at extreme positions}) = \frac{48}{720} = \frac{1}{15}$$

✅ **Final Answer:**

$$\boxed{P = \frac{1}{15}}$$

R CODE:

```
prob = (2 * factorial(4)) / factorial(6)
prob
```

OUTPUT: Probability = 0.0667 (or 1/15)

PRACTICAL 2: SUM 1: Find the probability that the sum of two dice > 9, given that the first die = 5.

Given:

Two dice are rolled.
We are told that **the first die = 5**.

We need to find the probability that

$$\text{Sum of two dice} > 9$$

given the first die is 5.

Step 1: Possible outcomes (given first die = 5)

If the first die is fixed at 5,
then the second die can be **1, 2, 3, 4, 5, 6** → 6 possible outcomes.

So total outcomes = **6**

Step 2: Find outcomes where the sum > 9

$$5 + (\text{second die}) > 9 \Rightarrow \text{second die} > 4$$

So, second die = **5 or 6**

So total outcomes = **6**

Step 2: Find outcomes where the sum > 9

$$5 + (\text{second die}) > 9 \Rightarrow \text{second die} > 4$$

So, second die = **5 or 6**

→ Favourable outcomes = 2

Step 3: Probability

$$P(\text{sum} > 9 \mid \text{first die} = 5) = \frac{2}{6} = \frac{1}{3}$$

✅ **Final Answer:**

$$\boxed{P = \frac{1}{3}}$$

R CODE:

```
prob = 2/6
prob
```

OUTPUT:

```
> prob = 2/6
> prob
[1] 0.3333333
>
```

SUM 2: A problem in mathematics is given to three students whose chances of solving it are 1/3, 1/4, and 1/5 respectively.

Find:

1️⃣ The probability that the problem is solved.

2️⃣ The probability that exactly one of them will solve it.

Also, write the R code for this sum.

Given:

Three students:

- A: $P(A) = \frac{1}{3}$
- B: $P(B) = \frac{1}{4}$
- C: $P(C) = \frac{1}{5}$

Each tries independently.

---

1 Probability that the problem is solved

This means **at least one** of them solves it.

We use the complement rule:

$$P(\text{solved}) = 1 - P(\text{none solve})$$

Find $P(\text{none solve})$:

$$\begin{aligned} P(\text{none}) &= (1 - P(A))(1 - P(B))(1 - P(C)) \\ &= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \\ &= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60} = \frac{2}{5} \end{aligned}$$

✅ **Answer (1):** $\boxed{\frac{3}{5}}$

---

2 Probability that exactly one of them solves it

We calculate three separate cases:

1. **Only A solves:**

$$P(A) \times (1 - P(B)) \times (1 - P(C)) = \frac{1}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{12}{60} = \frac{1}{5}$$

2. **Only B solves:**

$$(1 - P(A)) \times P(B) \times (1 - P(C)) = \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} = \frac{8}{60} = \frac{2}{15}$$

3. **Only C solves:**

$$(1 - P(A)) \times (1 - P(B)) \times P(C) = \frac{2}{3} \times \frac{3}{4} \times \frac{1}{5} = \frac{6}{60} = \frac{1}{10}$$

Add all three:

$$P(\text{exactly one}) = \frac{1}{5} + \frac{2}{15} + \frac{1}{10}$$

Find common denominator = 30:

$$\frac{6}{30} + \frac{4}{30} + \frac{3}{30} = \frac{13}{30}$$

✅ **Answer (2):** $\boxed{\frac{13}{30}}$

R CODE:

```
p1=1/3; p2=1/4; p3=1/5

prob_solved = 1 - (1-p1)*(1-p2)*(1-p3)

prob_exactly_one = p1*(1-p2)*(1-p3) + (1-p1)*p2*(1-p3) + (1-p1)*(1-p2)*p3

prob_solved; prob_exactly_one
```

OUTPUT:

```
[1] 0.6

[1] 0.4333333

>
```

SUM 3: A bag contains red and blue marbles.

Two marbles are drawn without replacement.

The probability of selecting a red marble first and then a blue marble is 0.28.

The probability of selecting a red marble on the first draw is 0.5.

Find the probability of selecting a blue marble on the second draw, given that the first marble drawn was red.

Given:

- $P(\text{Red first}) = 0.5$
- $P(\text{Red first and Blue second}) = 0.28$
- We need to find:

$$P(\text{Blue second} \mid \text{Red first})$$

Step 1: Use conditional probability formula

$$P(\text{Blue second} \mid \text{Red first}) = \frac{P(\text{Red first and Blue second})}{P(\text{Red first})}$$

Step 2: Substitute given values

$$P(\text{Blue second} \mid \text{Red first}) = \frac{0.28}{0.5}$$

Step 3: Simplify

$$P(\text{Blue second} \mid \text{Red first}) = 0.56$$

✅ **Final Answer:**

$$\boxed{P(\text{Blue on 2nd} \mid \text{Red on 1st}) = 0.56}$$

R CODE:

```
p_red_blue = 0.28
p_red_first = 0.5
p_blue_given_red = p_red_blue / p_red_first
p_blue_given_red
```

OUTPUT:

```
[1] 0.56
>
```

SUM 4: Suppose there are two bags:

Bag I contains 3 white and 2 black balls.

Bag II contains 2 white and 4 black balls.

One ball is transferred from Bag I to Bag II, and then a ball is drawn from Bag II, and it is found to be white.

Find the probability that the transferred ball was white.

🧮 **Given:**

- **Bag I:** 3 white, 2 black → total = 5
- **Bag II:** 2 white, 4 black → total = 6
- One ball is transferred from **Bag I** → **Bag II**, then one ball is drawn from **Bag II**, which turned out to be **white**.

We need:

$$P(\text{Transferred ball was white} \mid \text{Drawn ball is white})$$

Step 1: Define events

Let

- W_1: transferred ball is **white**
- B_1: transferred ball is **black**
- W_2: drawn ball from Bag II is **white**

We need $P(W_1|W_2)$.

Step 2: Use Bayes' theorem

$$P(W_1|W_2) = \frac{P(W_1)\,P(W_2|W_1)}{P(W_1)\,P(W_2|W_1) + P(B_1)\,P(W_2|B_1)}$$

Step 3: Find each term

(a) $P(W_1)$

Probability of transferring a white from Bag I:

$$P(W_1) = \frac{3}{5}$$

(b) $P(B_1)$

$$P(B_1) = \frac{2}{5}$$

Step 4: Find $P(W_2|W_1)$

If a **white** ball is transferred, Bag II will have:

- $2 + 1 = 3$ white
- 4 black

  → total = 7 balls

$$P(W_2|W_1) = \frac{3}{7}$$

Step 5: Find $P(W_2|B_1)$

If a **black** ball is transferred, Bag II will have:

- 2 white
- $4+1=5$ black
  → total = 7 balls

$$P(W_2|B_1) = \frac{2}{7}$$

Step 6: Substitute in Bayes' formula

$$P(W_1|W_2) = \frac{\frac{3}{5}\times\frac{3}{7}}{\frac{3}{5}\times\frac{3}{7}+\frac{2}{5}\times\frac{2}{7}}$$

Simplify numerator and denominator:

$$= \frac{\frac{9}{35}}{\frac{9}{35}+\frac{4}{35}} = \frac{9}{13}$$

✅ **Final Answer:**

$$\boxed{P(\text{Transferred ball was white} \mid \text{Drawn ball is white}) = \frac{9}{13}}$$

R CODE:

```
pE1=3/5; pE2=2/5

pA_E1=3/7; pA_E2=2/7

pA = pE1*pA_E1 + pE2*pA_E2

pE1_A = (pE1*pA_E1) / pA

pE1_A
```

OUTPUT:

Probability = 0.6923

SUM 5: Three companies A, B, and C supply 25%, 35%, and 40% of the chairs respectively to a college.

It is known that 5%, 4%, and 2% of the chairs produced by A, B, and C respectively are defective.

If a chair is found to be defective, find the probability that it was supplied by firm A.

Also, write the R code for this problem.

Given

- $P(A) = 0.25,\ P(B) = 0.35,\ P(C) = 0.40$
- Defective rates: $P(D|A) = 0.05,\ P(D|B) = 0.04,\ P(D|C) = 0.02$

We want $P(A \mid D)$: probability the defective chair came from A.

Bayes formula

$$P(A \mid D) = \frac{P(A)\,P(D \mid A)}{P(A)P(D \mid A) + P(B)P(D \mid B) + P(C)P(D \mid C)}$$

Substitute values:

$$P(A \mid D) = \frac{0.25 \times 0.05}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02} = \frac{0.0125}{0.0125 + 0.014 + 0.008} = \frac{0.0125}{0.0345}$$

Simplify:

$$\frac{0.0125}{0.0345} = \frac{25}{69} \approx 0.3623$$

✅ **Final answer**

$$\boxed{P(A \mid D) = \frac{25}{69} \approx 0.3623 \text{ (about } 36.23\%)}$$

R CODE:

```
pA=0.25; pB=0.35; pC=0.40
pD_A=0.05; pD_B=0.04; pD_C=0.02
pD = pA*pD_A + pB*pD_B + pC*pD_C
pA_D = (pA*pD_A) / pD
pA_D
```

OUTPUT: [1] 0.3623188

PRACTICAL 3 : SUM 1: A fair coin is tossed 6 times.

Find the probability of getting at least 0 heads.

(That means, finding the probability that the number of heads is 0 or more.)

Given:

A **fair coin** is tossed **6 times**.

We are asked:

> Probability of getting **at least 0 heads**, i.e., number of heads ≥ 0.

Step 1: Possible outcomes

When a coin is tossed 6 times, total outcomes:

$$2^6 = 64$$

Step 2: "At least 0 heads"

The number of heads can be **0, 1, 2, 3, 4, 5, or 6** — i.e., **any possible case**.

So, **every possible outcome** satisfies this condition.

Step 3: Probability

$$P(\text{at least 0 heads}) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{64}{64} = 1$$

R CODE:

```
dbinom(0, 6, 0.5)
```

OUTPUT:

```
> dbinom(0, 6, 0.5)
[1] 0.015625
>
```

SUM 2: A student is given 12 True/False questions in an experiment.

The student guesses on each question.

Find the probability that the student answers at least 6 questions correctly.

🎯 **Given:**

- Number of questions, $n = 12$
- Each question has **2 outcomes** → True or False
- Probability of correct guess $p = \frac{1}{2}$
- Probability of wrong answer $q = 1 - p = \frac{1}{2}$

We need:

$$P(X \geq 6)$$

where X = number of correct answers.

Step 1: Use the Binomial Probability Formula

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

So,

$$P(X \geq 6) = \sum_{k=6}^{12} \binom{12}{k} \left(\frac{1}{2}\right)^{12}$$

Step 2: Simplify

Since $p = q = \frac{1}{2}$,

$$P(X \geq 6) = \frac{1}{2^{12}} \sum_{k=6}^{12} \binom{12}{k}$$

Step 3: Use binomial symmetry

For a fair coin (p = 0.5),

$$P(X \geq 6) = P(X \leq 6)$$

and because the distribution is symmetric around 6 (mean = 6),

$$P(X \geq 6) = 0.5 + \frac{1}{2^{12}} \binom{12}{6} \frac{1}{2}$$

But an easier way is to note:

$$\sum_{k=0}^{12} \binom{12}{k} = 2^{12} = 4096$$

So,

$$\sum_{k=6}^{12}\binom{12}{k}=\frac{1}{2}\times 2^{12}+\frac{1}{2}\binom{12}{6}$$

(because for even n, the middle term is counted equally on both sides)

$$=2048+\frac{1}{2}(924)=2048+462=2510$$

Step 4: Probability

$$P(X\geq 6)=\frac{2510}{4096}\approx 0.6123$$

✅ **Final Answer:**

$$\boxed{P(\text{at least 6 correct})=\frac{2510}{4096}\approx 0.6123}$$

R CODE:

```
1 - pbinom(5, 10, 0.5)
```

OUTPUT:

```
> 1 - pbinom(5, 10, 0.5)
[1] 0.3769531
>
```

SUM 3: Compute the mean, standard deviation, and variance for a binomial distribution where the probability of success (p) is 0.1 and the number of trials (n) is .

R CODE:

```
n = 60

p = 0.1

mean = n * p

variance = n * p * (1 - p)

sd = sqrt(variance)

mean; variance; sd
```

OUTPUT:

[1] 6

[1] 5.4

[1] 2.32379

SUM 4: The mean and variance of a binomial distribution are given as 16 and 18 respectively.

Find:P(X=0),P(X>0)

Given:

- Mean $\mu = 16$
- Variance $\sigma^2 = 18$

We know for a **Binomial distribution**:

$$\mu = np \quad \text{and} \quad \sigma^2 = npq$$

where $q = 1 - p$

Step 1: Use the formulas

$$np = 16 \quad \text{and} \quad npq = 18$$

Divide the second by the first:

$$\frac{npq}{np} = \frac{18}{16} \Rightarrow q = \frac{18}{16} = 1.125$$

But q **cannot be greater than 1**, so something seems off — check again:

We must have $q = \frac{\sigma^2}{\mu} = \frac{18}{16} = 1.125$, which is impossible.
Hence, there's a **contradiction**, meaning **no valid binomial distribution** can have mean = 16 and variance = 18, because:

For a binomial,

$$\sigma^2 = npq = np(1 - p)$$

and since $q = 1 - p$, we must have $\sigma^2 \leq \mu$ (because $1 - p \leq 1$).

But here, $\sigma^2 = 18 > 16 = \mu$, which violates the rule.

✅ **Conclusion:**

A binomial distribution **cannot exist** with mean = 16 and variance = 18.
Hence, $P(X = 0)$ and $P(X > 0)$ **cannot be determined**, because such parameters are **not possible** for any binomial distribution.

R CODE:

```
dbinom(0, 32, 0.5)
1 - dbinom(0, 32, 0.5)
```

OUTPUT:

```
> dbinom(0, 32, 0.5)
[1] 2.328306e-10
> 1 - dbinom(0, 32, 0.5)
[1] 1
>
```

SUM 5: In 10 tosses of an unbiased coin, find:

The probability of getting exactly 1 head.

The probability of getting at least 1 head.

Use R-code to compute these probabilities.

Given:

- Number of tosses $n = 10$
- Probability of head $p = \frac{1}{2}$
- Probability of tail $q = 1 - p = \frac{1}{2}$

(a) Probability of getting exactly 1 head

Using the **binomial probability formula**:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

Substitute $n = 10, k = 1, p = 0.5, q = 0.5$:

$$P(X = 1) = \binom{10}{1} (0.5)^1 (0.5)^9$$

$$P(X = 1) = 10 \times (0.5)^{10} = \frac{10}{1024} = 0.009765625$$

✅ So, $P(X = 1) = 0.0098$ (approx)

(b) Probability of getting at least 1 head

"At least 1 head" means **1 or more heads**, i.e. $P(X \geq 1)$.

$$P(X \geq 1) = 1 - P(X = 0)$$

Compute $P(X = 0)$:

$$P(X = 0) = \binom{10}{0}(0.5)^0(0.5)^{10} = 1 \times (0.5)^{10} = \frac{1}{1024}$$

$$P(X \geq 1) = 1 - \frac{1}{1024} = \frac{1023}{1024} = 0.9990234375$$

✅ So, $P(X \geq 1) = 0.9990$ (approx)

✅ **Final Answers:**

- $P(\text{exactly 1 head}) = 0.0098$
- $P(\text{at least 1 head}) = 0.9990$

R CODE:

```
dbinom(1, 10, 0.5)

1 - dbinom(0, 10, 0.5)
```

OUTPUT:

```
> dbinom(1, 10, 0.5)

[1] 0.009765625

> 1 - dbinom(0, 10, 0.5)

[1] 0.9990234

>
```

SUM 6 : Find the Moment Generating Function (MGF) of a Binomial Distribution where n = 7, p = 0.5,

Also, find the 1st, to 4th moments about the origin using R-code.

🎯 **Given:**

- $n = 7$
- $p = 0.5$

📘 **Formula for MGF of a Binomial Distribution:**

$$M_X(t) = (q + pe^t)^n$$

where
$q = 1 - p$

Step 1: Substitute values

$$p = 0.5, \quad q = 1 - 0.5 = 0.5, \quad n = 7$$
$$M_X(t) = (0.5 + 0.5e^t)^7$$

✅ **Final Answer:**

$$\boxed{M_X(t) = (0.5 + 0.5e^t)^7}$$

(Optional) You can simplify it as:

$$M_X(t) = \left(\frac{1 + e^t}{2}\right)^7$$

✳️ **Note:**

From this MGF, you can find:

- **Mean** = $np = 7 \times 0.5 = 3.5$
- **Variance** = $npq = 7 \times 0.5 \times 0.5 = 1.75$

R CODE:

```
n <- 4
p <- 3/4
q <- 1 - p
dbinom(0, n, p)
1 - pbinom(1, n, p)
```

OUTPUT:

```
> dbinom(0, n, p)
[1] 0.00390625
```

```
> 1 - pbinom(1, n, p)
[1] 0.9492188
>
```

SUM 7:

A perfect cube (die) is thrown a large number of times in sets of 8. The occurrence of 2 or 4 is called a success.

Find the proportion of sets in which you expect exactly 3 successes.

🎯 **Given:**

- A **die** is thrown in sets of **8 trials**.
- "Success" = getting **2 or 4**.
- We want: **Probability (exactly 3 successes)**.

Step 1: Find probability of success (p)

Since there are 6 possible outcomes on a die,
and 2 outcomes (2 or 4) are considered "success":

$$p = \frac{2}{6} = \frac{1}{3}$$

Thus,

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Step 2: Define the distribution

The number of successes in 8 trials follows a **Binomial distribution**:

$$X \sim B(n = 8, p = \tfrac{1}{3})$$

Step 3: Formula for Binomial probability

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

Here,
$n = 8, r = 3, p = \frac{1}{3}, q = \frac{2}{3}$

Step 4: Substitute values

$$P(X=3)=\binom{8}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^5$$

$$\binom{8}{3}=\frac{8!}{3!(8-3)!}=\frac{8\times7\times6}{3\times2\times1}=56$$

So,

$$P(X=3)=56\times\left(\frac{1}{3}\right)^3\times\left(\frac{2}{3}\right)^5$$

$$P(X=3)=56\times\frac{1}{27}\times\frac{32}{243}$$

$$P(X=3)=56\times\frac{32}{6561}$$

$$P(X=3)=\frac{1792}{6561}$$

✅ **Final Answer:**

$$\boxed{P(X=3)=\frac{1792}{6561}\approx 0.273}$$

R CODE:

```
n <- 8

p <- 2/6

dbinom(3, n, p)
```

OUTPUT:

```
 > dbinom(3, n, p)

[1] 0.2731291

>
```

SUM 8: A man tosses a fair coin 10 times. Find the probability of getting:

1️⃣ Heads on the first 5 tosses and tails on the next 5 tosses

2️⃣ Heads on tosses 1, 3, 5, 7, 9 and tails on 2, 4, 6, 8, 10 tosses

3️⃣ 5 heads and 5 tails

4️⃣ At least 5 heads

5️⃣ Not more than 5 heads

Setup: 10 fair coin tosses → total outcomes = $2^{10} = 1024$. Let X = number of heads.

---

1 Heads on first 5 tosses and tails on next 5 tosses

This is one specific sequence. Probability = $(1/2)^{10} = \frac{1}{1024}$.

---

2 Heads on tosses 1,3,5,7,9 and tails on 2,4,6,8,10

Also one specific sequence (order fixed). Probability = $(1/2)^{10} = \frac{1}{1024}$.

---

3 Exactly 5 heads (and 5 tails)

$$P(X = 5) = \frac{\binom{10}{5}}{2^{10}} = \frac{252}{1024} = \frac{63}{256} \approx 0.24609$$

4 At least 5 heads (i.e. $X \geq 5$)

$$P(X \geq 5) = \sum_{k=5}^{10} \frac{\binom{10}{k}}{1024} = \frac{638}{1024} = \frac{319}{512} \approx 0.62305$$

(Computed using $P(X \geq 5) = 1 - P(X \leq 4)$ and $\sum_{k=0}^{4} \binom{10}{k} = 386$.)

---

5 Not more than 5 heads (i.e. $X \leq 5$)

$$P(X \leq 5) = \sum_{k=0}^{5} \frac{\binom{10}{k}}{1024} = \frac{638}{1024} = \frac{319}{512} \approx 0.62305$$

(Note: because the binomial with $p = 0.5$ is symmetric, $P(X \geq 5) = P(X \leq 5)$ when n is even and the middle term $k = 5$ is included.)

R CODE:

```
p <- 0.5

n <- 10


# 1. Heads first 5, tails next 5

(p^5) * (p^5)


# 2. Heads on 1,3,5,7,9; tails on 2,4,6,8,10

(p^5) * (p^5)


# 3. Exactly 5 heads

dbinom(5, n, p)
```

```
# 4. At least 5 heads
1 - pbinom(4, n, p)

# 5. Not more than 5 heads
pbinom(5, n, p)
```

OUTPUT:

```
 >
> # 1. Heads first 5, tails next 5
> (p^5) * (p^5)
[1] 0.0009765625
>
> # 2. Heads on 1,3,5,7,9; tails on 2,4,6,8,10
> (p^5) * (p^5)
[1] 0.0009765625
>
> # 3. Exactly 5 heads
> dbinom(5, n, p)
[1] 0.2460938
>
> # 4. At least 5 heads
> 1 - pbinom(4, n, p)
[1] 0.6230469
>
> # 5. Not more than 5 heads
> pbinom(5, n, p)
[1] 0.6230469
>
```

PRACTICAL 4: SUM 1: A manufacturer of screws knows that 5% of his products are defective.

He sells his products in boxes of 10 items and guarantees that not more than one item in each box will be defective.

Find the probability that a box will fail to meet the guaranteed quality (i.e., more than one defective item).

- Probability of a defective screw, $p = 0.05$
- Probability of a non-defective screw, $q = 1 - p = 0.95$
- Number of screws per box, $n = 10$
- The box **fails** if **more than 1** screw is defective

  ⇒ event = $X > 1$

Here $X \sim \text{Binomial}(n = 10, p = 0.05)$

We need:

$$P(X > 1) = 1 - [P(X = 0) + P(X = 1)]$$

Step 1: Compute $P(X = 0)$

$$P(X = 0) = \binom{10}{0}(0.05)^0(0.95)^{10} = 0.95^{10} = 0.5987$$

Step 2: Compute $P(X = 1)$

$$P(X = 1) = \binom{10}{1}(0.05)^1(0.95)^9 = 10(0.05)(0.95)^9$$

$$(0.95)^9 = 0.6302$$

$$P(X = 1) = 10 \times 0.05 \times 0.6302 = 0.3151$$

Step 3: Compute $P(X > 1)$

$$P(X > 1) = 1 - (0.5987 + 0.3151) = 1 - 0.9138 = 0.0862$$

✅ **Final Answer:**

$$\boxed{P(\text{box fails}) = 0.0862}$$

R CODE:

```
n <- 10

p <- 0.05

1 - pbinom(1, n, p)
```

OUTPUT:

```
> 1 - pbinom(1, n, p)
```

[1] 0.08613836

>

SUM 2: A milk plant manufactures ghee pouches. There is a small chance of 1 in 500 for any pouch to be defective.

Each packet contains 10 pouches.

Find the approximate number of packets containing

👉 0 defective,

👉 1 defective,

👉 2 defective, and

👉 3 defective pouches

in a consignment of 5000 packets.

🎯 **Given:**

- Probability of a defective pouch = $p = \frac{1}{500} = 0.002$
- Number of pouches per packet = $n = 10$
- Number of packets = 5000
- We need the number of packets with $X = 0, 1, 2, 3$ defective pouches.

Step 1: Identify the distribution

Each packet follows a **Binomial Distribution**

$$X \sim B(n = 10, p = 0.002)$$

Since n is small and p is very small, we can use the **Poisson approximation**.

$$\text{Mean } (\lambda) = n \times p = 10 \times 0.002 = 0.02$$

Thus, $X \sim \text{Poisson}(\lambda = 0.02)$

Step 2: Poisson probability formula

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

with $e^{-0.02} = 0.9802$

Step 3: Compute each probability

(a) $P(X=0)$

$$P(X=0) = e^{-0.02} = 0.9802$$

(b) $P(X=1)$

$$P(X=1) = e^{-0.02}\frac{(0.02)^1}{1!} = 0.9802 \times 0.02 = 0.0196$$

(c) $P(X=2)$

$$P(X=2) = e^{-0.02}\frac{(0.02)^2}{2!} = 0.9802 \times 0.0002 = 0.000098$$

(d) $P(X=3)$

$$P(X=3) = e^{-0.02}\frac{(0.02)^3}{3!} = 0.9802 \times 0.000001333 = 0.00000131$$

R CODE:

```
n <- 10

p <- 1/500

packets <- 5000


x <- 0:3

prob <- dbinom(x, n, p)

packets * prob
```

OUTPUT:

```
> packets * prob

[1] 4.900895e+03 9.821433e+01 8.857004e-01 4.733202e-03

>
```

SUM 3: The mean of a Poisson distribution is 2.25.

Find the other constants of the distribution — namely the variance, standard deviation, and the probability of X = 0.

Also, write the R-code for the same.

🎯 **Given:**

Mean of Poisson distribution = $\lambda = 2.25$

For a **Poisson distribution**,

$$\text{Mean} = \text{Variance} = \lambda$$

Step 1: Variance

$$\text{Variance} = \lambda = 2.25$$

Step 2: Standard Deviation

$$\text{SD} = \sqrt{\text{Variance}} = \sqrt{2.25} = 1.5$$

Step 3: Probability that $X = 0$

Formula:

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

$$P(X = 0) = e^{-2.25} \approx 0.1054$$

R CODE:

```
lambda <- 2.25

variance <- lambda

sd <- sqrt(lambda)

p0 <- dpois(0, lambda)

variance; sd; p0
```

OUTPUT:

```
> variance; sd; p0

[1] 2.25

[1] 1.5

[1] 0.1053992

>
```

SUM 4: The number of arrivals at the Regional Computer Center express service counter between 12 noon and 3 p.m. follows a Poisson distribution with a mean of 1.2 arrivals per minute.

Find:

1️⃣ The probability of no arrivals during a given 1-minute interval.

2️⃣ The probability of no arrivals during a given 2-minute interval.

3️⃣ The probability of no arrivals during a given 5-minute interval.

Given: arrivals follow Poisson with rate $\lambda = 1.2$ per minute.
For a time interval of length t minutes, the mean for that interval is λt.
Probability of **no arrivals** in interval length t is

$$P(X = 0) = e^{-\lambda t}.$$

1️⃣ **For a 1-minute interval ($t = 1$):**

$$\lambda t = 1.2 \quad \Rightarrow \quad P(X = 0) = e^{-1.2} \approx 0.301194.$$

2️⃣ **For a 2-minute interval ($t = 2$):**

$$\lambda t = 2.4 \quad \Rightarrow \quad P(X = 0) = e^{-2.4} \approx 0.090718.$$

3️⃣ **For a 5-minute interval ($t = 5$):**

$$\lambda t = 6.0 \quad \Rightarrow \quad P(X = 0) = e^{-6.0} \approx 0.002479.$$

✅ **Final answers (approx):**

- 1-minute: **0.301194**
- 2-minute: **0.090718**
- 5-minute: **0.002479**

R CODE:

```
lambda1 <- 1.2
lambda2 <- 2 * lambda1
lambda5 <- 5 * lambda1

p1 <- dpois(0, lambda1)
p2 <- dpois(0, lambda2)
p5 <- dpois(0, lambda5)

p1; p2; p5
```

OUTPUT:

```
>

> p1; p2; p5

[1] 0.3011942

[1] 0.09071795

[1] 0.002478752

>
```

SUM 5: If λ is a Poisson variate such that p(x=1) = p(x=2)

1) find p(x=0)

2️⃣ The mean of the distribution (λ)

3️⃣ The variance of the distribution

🎯 **Given:**

For a **Poisson distribution**,

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

and we are told that

$$P(X = 1) = P(X = 2)$$

Step 1️⃣: Write the equality

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

Cancel $e^{-\lambda}$ from both sides:

$$\frac{\lambda}{1} = \frac{\lambda^2}{2}$$

Step 2️⃣: Solve for λ

$$2\lambda = \lambda^2$$

$$\lambda(\lambda - 2) = 0$$

Since $\lambda \neq 0$ (it's the mean of the distribution),

$$\boxed{\lambda = 2}$$

Step 3️⃣: Find $P(X = 0)$

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda}$$

Substitute $\lambda = 2$:

$$P(X = 0) = e^{-2} = 0.1353$$

Step 4️⃣: Mean and Variance

For a Poisson distribution:

$$\text{Mean} = \lambda = 2$$

$$\text{Variance} = \lambda = 2$$

R CODE:

```
 lambda <- 2
p0 <- dpois(0, lambda)
mean <- lambda
variance <- lambda
p0; mean; variance
```

OUTPUT:

```
 [1] 0.1353353
[1] 2
[1] 2
>
```

PRACTICAL 5: SUM 1: If X follows a Binomial Distribution with, n= 8, p=1/2,

find: p(x=2), p(x smaller then equal to 2), p(x > 4), mean, variance?

🎯 Given:

Binomial Distribution:

$$X \sim B(n = 8, p = \tfrac{1}{2})$$

Thus,

$$q = 1 - p = \tfrac{1}{2}$$

- **Binomial Probability Formula:**

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

Here,

$$P(X = x) = \binom{8}{x} \left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{8-x} = \binom{8}{x} \left(\tfrac{1}{2}\right)^8$$

1️⃣ $P(X = 2)$

$$P(X = 2) = \binom{8}{2} \left(\tfrac{1}{2}\right)^8 = 28 \times \tfrac{1}{256} = \tfrac{28}{256} = 0.1094$$

2️⃣ $P(X \leq 2)$

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$P(X = 0) = \binom{8}{0} \tfrac{1}{256} = \tfrac{1}{256}$$

$$P(X = 1) = \binom{8}{1} \tfrac{1}{256} = \tfrac{8}{256}$$

$$P(X = 2) = \tfrac{28}{256}$$

$$P(X \leq 2) = \tfrac{1+8+28}{256} = \tfrac{37}{256} = 0.1445$$

3️⃣ $P(X > 4)$

$$P(X > 4) = 1 - P(X \leq 4)$$

We already have $P(X \leq 2)$, now find $P(X = 3)$ and $P(X = 4)$:

$$P(X = 3) = \binom{8}{3} \frac{1}{256} = \frac{56}{256}$$

$$P(X = 4) = \binom{8}{4} \frac{1}{256} = \frac{70}{256}$$

$$P(X \leq 4) = \frac{1+8+28+56+70}{256} = \frac{163}{256}$$

$$P(X > 4) = 1 - \frac{163}{256} = \frac{93}{256} = 0.3633$$

4️⃣ **Mean and Variance**

$$\text{Mean} = np = 8 \times \frac{1}{2} = 4$$

$$\text{Variance} = npq = 8 \times \frac{1}{2} \times \frac{1}{2} = 2$$

R CODE: n <- 8

```
p <- 1/2

p_x2 <- dbinom(2, n, p)
p_le2 <- pbinom(2, n, p)
p_gt4 <- 1 - pbinom(4, n, p)

mean <- n * p
variance <- n * p * (1 - p)

p_x2; p_le2; p_gt4; mean; variance
```

OUTPUT:

```
>
> p_x2; p_le2; p_gt4; mean; variance
[1] 0.109375
[1] 0.1445313
[1] 0.3632813
```

```
[1] 4

[1] 2

>
```

SUM 2: A coin is weighted so that the probability of landing on heads is P = 0.9.

What is the probability that the first head appears on the 3rd flip (i.e., after 2 tails and then a head)?

Also, find the probability that the first head appears after the 3rd flip.

Give the R code for this problem.

🎯 **Given:**

- Probability of **Head (H)** = $p = 0.9$
- Probability of **Tail (T)** = $q = 1 - p = 0.1$

We're using a **Geometric distribution**, where

$$P(X = k) = q^{k-1}p$$

represents the probability that the **first head** appears on the **kth flip**.

1️⃣ Probability that the first head appears on the 3rd flip

That means: T, T, H

$$P(X = 3) = q^{3-1}p = q^2p = (0.1)^2 \times 0.9 = 0.009$$

✅ So,

$$\boxed{P(X = 3) = 0.009}$$

2️⃣ Probability that the first head appears after the 3rd flip

That means: no head in the first 3 flips (all tails).

$$P(X > 3) = q^3 \,, (0.1)^3 = 0.001$$

R CODE:

```
p <- 0.9   # probability of head

q <- 1 - p # probability of tail


# Probability that first head occurs on 3rd flip

p_3rd <- dgeom(3 - 1, p)


# Probability that first head occurs after 3rd flip

p_after3 <- pgeom(3, p, lower.tail = FALSE)
```

p_3rd; p_after3

OUTPUT:

```
>

> p_3rd; p_after3

[1] 0.009

[1] 1e-04

>
```

SUM 3: People prefer to sleep on long flights rather than watch movies, read, etc., with a probability of 0.7.

Consider randomly selecting 25 passengers from a particular BNG flight.

Let the random variable X denote the number of passengers who prefer sleeping on a long flight.

Find the following:

The probability that exactly 12 of those selected passengers prefer sleeping.

The probability that all passengers selected prefer sleeping.

The probability that at least 20 passengers prefer sleeping.

The average (mean) number of passengers that prefer sleeping on a long flight.

The standard deviation and interpret it.

Also, give the R code for the above.

1) Probability exactly 12 prefer sleeping

$$P(X = 12) = \binom{25}{12}(0.7)^{12}(0.3)^{13} \approx \mathbf{0.011476}$$

2) Probability all 25 prefer sleeping

$$P(X = 25) = (0.7)^{25} \approx \mathbf{1.3411 \times 10^{-4}} \text{ (about 0.0001341)}$$

3) Probability at least 20 prefer sleeping

$$P(X \geq 20) = \sum_{k=20}^{25} \binom{25}{k}(0.7)^k(0.3)^{25-k} \approx \mathbf{0.193488}$$

(About 19.35% of the samples will have 20 or more sleepers.)

4) Mean (expected number)

$$E[X] = np = 25 \times 0.7 = \mathbf{17.5}$$

5) Variance and standard deviation (and interpretation)

$$\mathrm{Var}(X) = np(1-p) = 25 \times 0.7 \times 0.3 = 5.25$$

$$\mathrm{SD}(X) = \sqrt{5.25} \approx \mathbf{2.2913}$$

R CODE:

```
n <- 25
p <- 0.7
q <- 1 - p

# 1. P(X = 12)
p_12 <- dbinom(12, n, p)

# 2. P(X = 25)
p_all <- dbinom(25, n, p)

# 3. P(X >= 20)
p_atleast20 <- 1 - pbinom(19, n, p)

# 4. Mean and Standard Deviation
mean_x <- n * p
sd_x <- sqrt(n * p * q)
```

```
p_12; p_all; p_atleast20; mean_x; sd_x
```

OUTPUT:

```
>
> p_12; p_all; p_atleast20; mean_x; sd_x
[1] 0.01147575
[1] 0.0001341069
[1] 0.1934884
[1] 17.5
[1] 2.291288
>
```

SUM 4: Sophia is a dog who loves to play catch. Unfortunately, she is not very good — the probability that she catches the ball on any given toss is 10% (p = 0.1).

Let X be the number of tosses until Sophia catches the first ball.

Then X follows a Geometric Distribution with probability of success

p = 0.1

Find:

1) The probability that Sophia will catch the ball on her second try.

2) The probability that it will take more than 3 tosses for Sophia to catch her first ball.

3) The expected number of tosses before Sophia catches her first ball.

Also, give the R code for this sum.

Let's do this step-by-step for $X \sim \text{Geometric}(p = 0.1)$ where

$$P(X = k) = (1 - p)^{k-1} p.$$

---

1) Probability Sophia catches on her 2nd try

$$P(X = 2) = (1 - 0.1)^1 \cdot 0.1 = 0.9 \times 0.1 = 0.09$$

2) Probability it takes more than 3 tosses

$$P(X > 3) = (1 - p)^3 = 0.9^3 = 0.729$$

(That's the probability the first three tosses are all failures.)

3) Expected number of tosses until first catch

For a geometric distribution,

$$E[X] = \frac{1}{p} = \frac{1}{0.1} = 10$$

✅ **Final answers:**

1. $P(X = 2) = 0.09$
2. $P(X > 3) = 0.729$
3. $E[X] = 10$

R CODE:

```
p <- 0.1


# 1. P(X = 2)

p_second_try <- dgeom(2 - 1, p)


# 2. P(X > 3)

p_more_than3 <- (1 - p)^3


# 3. Expected number of tosses

expected_tosses <- 1 / p


p_second_try; p_more_than3; expected_tosses
```

OUTPUT:

```
>

> p_second_try; p_more_than3; expected_tosses
```

```
[1] 0.09

[1] 0.729

[1] 10

>
```

SUM 5: You are to take a multiple-choice exam consisting of 100 questions, each having 5 possible responses (only one correct).

Suppose you did not study and decide to guess randomly on each question.

Let X be the number of correct answers.

Then X follows a Binomial Distribution with parameters: n= 100, p= 0.2

(probability of guessing a question correctly)

Find:

The expected score (mean) on the exam.

The probability of getting exactly 50 correct answers.

Also, give the R-code for this question.

Given:

- Number of questions: $n = 100$
- Probability of a correct guess: $p = 0.2$
- Random variable: $X \sim \text{Binomial}(n = 100, p = 0.2)$

1️⃣ Expected Score (Mean)

Formula:

$$E[X] = n \times p = 100 \times 0.2 = 20$$

✅ **Expected score = 20**

2️⃣ Probability of getting exactly 50 correct answers

Formula:

$$P(X = 50) = \binom{100}{50}(0.2)^{50}(0.8)^{50}$$

This is an extremely small probability because 50 is far above the mean (20).

R CODE:

```
n <- 100

p <- 0.2


# 1. Expected score

expected_score <- n * p


# 2. Probability of getting exactly 50 correct answers

prob_50 <- dbinom(50, n, p)


expected_score

prob_50
```

OUTPUT:

```
 >

> expected_score

[1] 20

> prob_50

[1] 1.621261e-11

>
```

SUM 6: A person is throwing a fair die repeatedly and will stop once he gets a 5.

Since there are 6 possible outcomes, the probability of success (getting a 5) is, P = 1/6 = 0.17

Now, find:

The probability that the person gets a 5 for the first time on the 3rd trial.

The probability that the person gets a 5 for the second time on the 4th trial.

Also, provide the R-code for this sum.

Given:

- A fair die → 6 faces
- Probability of success (getting a 5):

$$p = \frac{1}{6} \approx 0.17$$

- Probability of failure:

$$q = 1 - p = \frac{5}{6}$$

1️⃣ Probability that the person gets a 5 for the *first time* on the 3rd trial

For a **Geometric Distribution**:

$$P(X = k) = (1 - p)^{k-1} p$$

Substitute $p = \frac{1}{6}, k = 3$:

$$P(X = 3) = \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$$

$$P(X = 3) = \frac{25}{216} \approx 0.1157$$

✅ **Probability = 0.1157**

2️⃣ Probability that the person gets a 5 for the *second time* on the 4th trial

Here we use the **Negative Binomial Distribution** formula:

$$P(X = x) = \binom{x-1}{r-1} p^r (1 - p)^{x-r}$$

where:

- $r = 2$ (second success),
- $x = 4$ (on the 4th trial).

$$P(X = 4) = \binom{3}{1} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$$

$$P(X = 4) = 3 \times \frac{1}{36} \times \frac{25}{36} = \frac{75}{1296} \approx 0.0579$$

✅ **Probability = 0.0579**

R CODE:

```
p <- 1/6


# 1. Probability of first success on 3rd trial

prob_first_3 <- dgeom(3 - 1, p)


# 2. Probability of second success on 4th trial
```

```
prob_second_4 <- dnbinom(4 - 2, 2, p)


prob_first_3

prob_second_4
```

OUTPUT:

```
>

> prob_first_3

[1] 0.1157407

> prob_second_4

[1] 0.05787037

>
```

SUM 7: Suppose 15% of cereal boxes contain a prize.

You are determined to keep buying cereal boxes until you win a prize.

Let the probability of finding a prize in one box be p=0.15.

Find:

1. The probability that you will have to buy at most two boxes.
2. The probability that you will have to buy exactly four boxes.
3. The probability that you will have to buy more than four boxes.
4. The average number of boxes you will need to buy before you get a prize.

Also, give the R-code for this sum.

Given $p = 0.15$ and $q = 1 - p = 0.85$, the geometric pmf is

$$P(X = k) = q^{k-1}p.$$

Now compute each item.

1️⃣ **At most two boxes**:

$$P(X \le 2) = 1 - P(X > 2) = 1 - q^2 = 1 - 0.85^2 = 1 - 0.7225 = 0.2775.$$

2️⃣ **Exactly four boxes**:

$$P(X = 4) = q^3 p = 0.85^3 \times 0.15 \approx 0.09211875 \approx 0.09212.$$

3️⃣ **More than four boxes**:

$$P(X > 4) = q^4 = 0.85^4 \approx 0.52200625 \approx 0.52201.$$

4️⃣ **Average number of boxes (expected value)**:

$$E[X] = \frac{1}{p} = \frac{1}{0.15} = \frac{20}{3} \approx 6.6667.$$

R CODE: p <- 0.15

```
# 1. Probability of at most 2 boxes
p_atmost2 <- pgeom(2 - 1, p)

# 2. Probability of exactly 4 boxes
p_exact4 <- dgeom(4 - 1, p)

# 3. Probability of more than 4 boxes
p_more4 <- 1 - pgeom(4 - 1, p)

# 4. Average number of boxes before getting a prize
mean_boxes <- 1 / p

p_atmost2
p_exact4
p_more4
mean_boxes
```

OUTPUT:

```
>

> p_atmost2

[1] 0.2775

> p_exact4

[1] 0.09211875

> p_more4

[1] 0.5220063

> mean_boxes

[1] 6.666667

>
```

SUM 8: Bob is a high school basketball player who has a 70% free throw success rate (i.e., probability of making a free throw

p=0.7.

Assume all free throws are independent of one another.

Find the following probabilities:

1️⃣ What is the probability that he takes more than three shots to make his first free throw?

2️⃣ What is the probability that his first made free throw is on the third shot?

3️⃣ What is the probability that his third made free throw is on the fifth shot?

4️⃣ What is the probability that his 100th made free throw is on the 103rd shot?

Also, write the R code for these calculations.

R CODE:

```
 p <- 0.7


# 1. Probability of taking more than 3 shots for 1st success

p_more3 <- (1 - p)^3


# 2. Probability 1st success on 3rd shot
```

```
p_first3 <- dgeom(3 - 1, p)

# 3. Probability 3rd success on 5th shot
p_third5 <- dnbinom(5 - 3, 3, p)

# 4. Probability 100th success on 103rd shot
p_100th103 <- dnbinom(103 - 100, 100, p)

p_more3
p_first3
p_third5
p_100th103
```

OUTPUT:

```
 >
> p_more3
[1] 0.027
> p_first3
[1] 0.063
> p_third5
[1] 0.18522
> p_100th103
[1] 1.499471e-12
>
```

PRACTICAL 6: SUM 1:

If $X \sim N(100, 10^2)$, find:
(i) $P(X > 101)$
(ii) $P(|X - 100| < 1)$

(i) $P(X > 101)$

Convert to standard normal Z:

$$Z = \frac{X - \mu}{\sigma} = \frac{101 - 100}{10} = 0.1$$

So,

$$P(X > 101) = P(Z > 0.1)$$

From standard normal tables:

$$P(Z < 0.1) = 0.5398$$

Hence,

$$P(Z > 0.1) = 1 - 0.5398 = 0.4602$$

✅ **Answer:** $P(X > 101) = 0.4602$

(ii) $P(|X - 100| < 1)$

That means:

$$P(99 < X < 101)$$

Convert both to Z:

$$Z_1 = \frac{99 - 100}{10} = -0.1, \quad Z_2 = \frac{101 - 100}{10} = 0.1$$

So,

$$P(99 < X < 101) = P(-0.1 < Z < 0.1)$$
$$= P(Z < 0.1) - P(Z < -0.1)$$

Using symmetry of normal distribution:

$$P(Z < -0.1) = 1 - P(Z < 0.1) = 1 - 0.5398 = 0.4602$$

Therefore,

$$P(-0.1 < Z < 0.1) = 0.5398 - 0.4602 = 0.0796$$

✅ **Answer:** $P(|X - 100| < 1) = 0.0796$

R CODE:

```
# Q1
mu <- 100
sigma <- 10

# (i)
p1 <- 1 - pnorm(101, mu, sigma)

# (ii)
p2 <- pnorm(101, mu, sigma) - pnorm(99, mu, sigma)

p1
p2
```

OUTPUT:

```
>
> p1
[1] 0.4601722
> p2
[1] 0.07965567

>
```

SUM 2:

If $X \sim N(10, \sigma^2)$ and $P(X > 12) = 0.1587$,
find σ and also $P(9 \leq X \leq 11)$.

Solution:

Given $P(X > 12) = 0.1587 \Rightarrow P(X \leq 12) = 0.8413$.
From Z-table, $z = 1$.
So, $\frac{12-10}{\sigma} = 1 \Rightarrow \sigma = 2$

Now, $P(9 \leq X \leq 11) = P(X \leq 11) - P(X \leq 9)$

Step 1: Find σ

We know

$$P(X > 12) = 0.1587 \implies P(X \le 12) = 1 - 0.1587 = 0.8413$$

From the **Z-table**,

$$P(Z \le 1) = 0.8413$$

So,

$$Z = \frac{12 - 10}{\sigma} = 1$$
$$\Rightarrow \frac{2}{\sigma} = 1 \Rightarrow \sigma = 2$$

✅ **Standard deviation, $\sigma = 2$**

Step 2: Find $P(9 \le X \le 11)$

We standardize:

$$Z_1 = \frac{9 - 10}{2} = -0.5, \quad Z_2 = \frac{11 - 10}{2} = 0.5$$
$$P(9 \le X \le 11) = P(-0.5 \le Z \le 0.5)$$

From the Z-table:

$$P(Z \le 0.5) = 0.6915, \quad P(Z \le -0.5) = 0.3085$$

So,

$$P(-0.5 \le Z \le 0.5) = 0.6915 - 0.3085 = 0.3830$$

✅ $P(9 \le X \le 11) = 0.3830$

R CODE:

```
# Q2
mu <- 10
sigma <- 2

p_9_11 <- pnorm(11, mu, sigma) - pnorm(9, mu, sigma)
p_9_11
```

OUTPUT:

```
>
> p_9_11 <- pnorm(11, mu, sigma) - pnorm(9, mu, sigma)
> p_9_11
[1] 0.3829249
```

>

SUM 3:

If $X \sim N(\mu, \sigma^2)$, and $P(X < 89) = 0.90$, $P(X < 94) = 0.93$, find μ and σ.

Step 1️⃣: Convert both to standard normal Z

$$Z = \frac{X - \mu}{\sigma}$$

So:

$$P(X < 89) = 0.90 \Rightarrow Z_1 = 1.2816$$

$$P(X < 94) = 0.93 \Rightarrow Z_2 = 1.4758$$

Step 2️⃣: Write the two equations

From $Z = \frac{X-\mu}{\sigma}$:

1️⃣ For $X = 89$:

$$\frac{89 - \mu}{\sigma} = 1.2816$$

2️⃣ For $X = 94$:

$$\frac{94 - \mu}{\sigma} = 1.4758$$

Step 3️⃣: Subtract (2) − (1)

$$\frac{94 - 89}{\sigma} = 1.4758 - 1.2816$$

$$\frac{5}{\sigma} = 0.1942$$

$$\sigma = \frac{5}{0.1942} = 25.74$$

✅ **Standard deviation,** $\sigma = 25.74$

Step 4️⃣: Substitute to find μ

Use equation (1):

$$\frac{89 - \mu}{25.74} = 1.2816$$

$$89 - \mu = 33.01$$

$$\mu = 89 - 33.01 = 55.99$$

✅ **Mean,** $\mu = 55.99 \approx 56.0$

R CODE:

```
# Q3
z1 <- 1.28
z2 <- 1.48
x1 <- 89
x2 <- 94

sigma <- (x2 - x1) / (z2 - z1)
mu <- x1 - z1 * sigma

mu
sigma
```

OUTPUT:

```
>
> mu
[1] 57
> sigma
[1] 25

>
```

SUM 4:

A normal population of 1000 employees has a mean income of ₹800 per day and variance 400.
Find the number of employees whose income is:

(i) between ₹750 and ₹820

(ii) more than ₹700

(iii) less than ₹760

Solution:

Mean = 800, SD = √400 = 20, n = 1000.

R CODE:

```r
# Q4
mu <- 800
sigma <- 20
n <- 1000

# (i)
p1 <- pnorm(820, mu, sigma) - pnorm(750, mu, sigma)
num1 <- n * p1

# (ii)
p2 <- 1 - pnorm(700, mu, sigma)
num2 <- n * p2

# (iii)
p3 <- pnorm(760, mu, sigma)
num3 <- n * p3

num1
num2
num3
```

OUTPUT:

```
>
> num1
[1] 835.1351
> num2
[1] 999.9997
> num3
[1] 22.75013

>
```

SUM 5:

The lifetime of torch batteries follows $N(50, 9)$.
Find the probability that lifetime will be:
(i) more than 53
(ii) less than 45
(iii) between 53 and 55.

R CODE:

```
# Q5
mu <- 50
sigma <- 3

# (i)
p1 <- 1 - pnorm(53, mu, sigma)

# (ii)
p2 <- pnorm(45, mu, sigma)

# (iii)
p3 <- pnorm(55, mu, sigma) - pnorm(53, mu, sigma)

p1
p2
p3
```

OUTPUT:

```
>
> p1
[1] 0.1586553
> p2
[1] 0.04779035
> p3
[1] 0.1108649
```

```
>
```

SUM 6:

If $X \sim N(45, 25)$, find:

(i) $P(X > 48)$

(ii) $P(X < 40)$

(iii) $P(42 < X < 50)$

(iv) $P(47 < X < 49)$

R CODE:

```
# Q6
mu <- 45
sigma <- 5

p1 <- 1 - pnorm(48, mu, sigma)
p2 <- pnorm(40, mu, sigma)
p3 <- pnorm(50, mu, sigma) - pnorm(42, mu, sigma)
p4 <- pnorm(49, mu, sigma) - pnorm(47, mu, sigma)

p1
p2
p3
p4
```

OUTPUT:

```
>
> p1
[1] 0.2742531
> p2
[1] 0.1586553
> p3
[1] 0.5670916
> p4
[1] 0.1327229
```

>

SUM 7:

The municipal corporation of a city installed 8000 bulbs with mean lifespan 1100 hours and SD 180 hours. Find the number of bulbs with:

(i) $X > 1150$

(ii) $X < 1100$

(iii) $1090 < X < 1150$

R CODE:

```
# Q7

mu <- 1100

sigma <- 180

n <- 8000


# (i)

p1 <- 1 - pnorm(1150, mu, sigma)

num1 <- n * p1


# (ii)

p2 <- pnorm(1100, mu, sigma)

num2 <- n * p2


# (iii)

p3 <- pnorm(1150, mu, sigma) - pnorm(1090, mu, sigma)

num3 <- n * p3


num1

num2

num3
```

OUTPUT:

```
>
> num1
[1] 3124.732
> num2
[1] 4000
> num3
[1] 1052.485

>
```